Λ-Nucleon Interactions at Low and Intermediate Energies*

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The low-energy Λ-nucleon interaction has been represented by several effective central potentials which have a hard core of radius 0.4 F and four-parameter attractive wells with an interaction for large separations suggested by the two-pion-exchange mechanism. These potentials, which are consistent with the binding-energy data for the light hypernuclei, were taken to represent the Λ-nucleon interaction in even-parity states for energies greater than those which play a role in the binding of the light hypernuclei. These potentials and others of reduced strength were taken to represent the interaction in odd-parity states. The binding energy D of a Λ particle in nuclear matter and Λ-nucleon scattering cross sections were calculated with these potentials. The calculated cross sections and empirical cross sections for energies less than 76 MeV in the zero-momentum frame are in agreement only if the interaction in odd-parity states is taken to be (approximately) as strong as that in even-parity states. These effective central potentials of equal strength in even-parity and odd-parity states lead to values of D close to 40 MeV.

I. INTRODUCTION

INFORMATION about the strength of the Λ-nucleon interaction has been obtained primarily from analyses of the binding-energy data for the light hypernuclei.[1–6] Although the Λ-nucleon interaction may well contain noncentral components,[7–9] existing knowledge of the structure of the light hypernuclei is not sufficiently detailed to allow the determination of more than the parameters of effective central potentials which represent both central and noncentral components of the Λ-nucleon interaction.[1] Moreover, it has not been possible to establish the presence of possible three-body Λ-nucleon interactions in these analyses.[6] Even for assumed two-body central potentials, however, analyses of the binding-energy data have not been sufficiently precise to allow a determination of parameters characterizing both the range and depth of Λ-nucleon interaction potentials.[1,2] When the Λ-nucleon interactions are represented by effective two-body central potentials, the binding energies of the light hypernuclei are determined primarily by the S-wave interactions.[10] Analyses of the binding-energy data for the light hypernuclei have, therefore, led to specification of the depths of effective central two-body potentials of assumed ranges, which represent the Λ-nucleon interaction in S states.[1–5]

The most reliable analyses of the binding energies of the light hypernuclei are those for ${}_\Lambda H^3$ and ${}_\Lambda He^5$. These analyses have led to specification of depth parameters characterizing the spin-averaged Λ-nucleon interaction in S states for each system, for assumed shapes and ranges.[1–5] For effective central two-body potentials, the average interactions in these systems are

$$\bar{V}_3=(3V_s+V_t)/4 \quad \text{for} \quad {}_\Lambda H^3, \tag{1a}$$

and

$$\bar{V}_5=(3V_t+V_s)/4 \quad \text{for} \quad {}_\Lambda He^5, \tag{1b}$$

in terms of the singlet and triplet Λ-nucleon potentials V_s and V_t. The combination of potentials (1a) depends upon the currently accepted assumption that the singlet interaction is more attractive than the triplet.[11] The combination (1b), on the other hand, is independent of assumptions about the relative strength of V_s and V_t, the spin of the (alpha-particle) nucleon core being zero.

Parameters characterizing the average potentials $\bar{V}_3$ and $\bar{V}_5$ have been specified for potentials with and without hard cores.[1–5] The well-established existence of a hard core in the nucleon-nucleon interaction suggests that a hard core is also a characteristic of the Λ-nucleon interaction. It is, therefore, reasonable to represent Λ-nucleon interactions by potentials with hard cores in situations where potentials are meaningful. Recent detailed analyses of the hypertriton ${}_\Lambda H^3$ in terms of hard-core potentials have indicated that the zero-energy scattering length

$$\bar{a}_3 \approx -2.0 \text{ F} \tag{2a}$$

provides an approximate characterization of $\bar{V}_3$ for

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[1] R. H. Dalitz and B. W. Downs, Phys. Rev. **111**, 967 (1958).

[2] B. W. Downs and R. H. Dalitz, Phys. Rev. **114**, 593 (1959).

[3] R. H. Dalitz, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company Ltd., London, 1961), p. 103; and other references cited there.

[4] B. W. Downs, D. R. Smith, and T. N. Truong, Phys. Rev. **129**, 2730 (1963).

[5] K. Dietrich, R. Folk, and H. J. Mang, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961* (Heywood and Company Ltd., London, 1961), p. 165.

[6] A. R. Bodmer and S. Sampanthar, Nucl. Phys. **31**, 251 (1962).

[7] D. B. Lichtenberg and M. Ross, Phys. Rev. **107**, 1714 (1957); F. Ferrari and L. Fonda, Nuovo Cimento **9**, 842 (1958).

[8] J. J. de Swart and C. Dullemond, Ann. Phys. (N. Y.) **16**, 263 (1961).

[9] J. J. de Swart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962).

[10] See Ref. 1, especially Appendix C.

[11] For discussions of the evidence supporting this assignment, see Ref. 3 and R. H. Dalitz, in a paper presented at the (CERN) International Conference on Hyperfragments, St. Cergue, Switzerland, March 1963, and subsequently issued as Report No. EFINS-63-29 by the Enrico Fermi Institute for Nuclear Studies, University of Chicago (unpublished).

hard-core potentials with ranges suggested by consideration of the two-pion-exchange (TPE) mechanism.[12] Although the five-body hypernuclear system has not been studied in as great detail as has the three-body system, studies which have been made indicate that the zero-energy scattering length

$$\bar{a}_5 \approx -0.7_7 \text{ F} \tag{2b}$$

provides an approximate characterization of the average potential $\bar{V}_5$ for ranges suggested by the TPE mechanism.[1,13]

The Λ-nucleon interaction in states with relative orbital angular momentum $l>0$ (primarily P states) can play a significant role in determining the binding energy B_Λ of a Λ particle in a heavy hypernucleus.[6,13,14] Since the state of a single Λ particle bound in a hypernucleus is not restricted by the Pauli principle, the binding energy B_Λ will approach a limiting value D as the mass number A of the hypernucleus approaches infinity, on account of the saturation of the central density in heavy nuclei. The experimental determination of D is currently the object of considerable study. Although estimates of D ranging from 21 to 40 MeV have been made,[15] recent measurements of the binding energies of hypernuclei with mass numbers in the range $60 \lesssim A \lesssim 100$ have led to the estimate that a value of D in the neighborhood of 30 MeV is most likely on the basis of current data.[16] The asymptotic binding energy D has been calculated in terms of phenomenological two-body interaction potentials $\bar{V}_5$ with ranges suggested by consideration of the TPE mechanism and having zero-energy scattering lengths approximately equal to that given in (2b).[1,6,13,14] When these S-wave potentials have been assumed to be appropriate also to higher angular-momentum states, the calculated values of D have been considerably in excess of 30 MeV for potentials without hard cores[1,6] and for hard-core potentials with two-parameter attractive wells having relatively long ranges suggested by consideration of the TPE mechanism[13,14]; for hard-core potentials having two-parameter attractive wells of shorter range, also suggested by the TPE mechanism, values $D \approx 30$ MeV have been obtained.[13] Comparison of the results of these calculations for the longer ranged attractive wells with the smaller empirical estimates ($D \lesssim 30$ MeV) have led to the suggestion that the Λ-nucleon interaction in higher angular momentum states (at least in P states) may be less attractive than it is in S states.[6,14] The shorter ranges, which have been used in these calculations,[13] were suggested by effective range theory, in which the introduction of a hard core leads to a significant compression of a two-parameter attractive well. The longer ranges have been fixed with partial or complete neglect of this compression.[13,14] The use of compressed attractive wells leads to relatively small contributions to D from states with $l>0$. Therefore, the use of the shorter ranged attractive wells without suppression of the interaction in higher angular momentum states is, to some extent, equivalent to the use of the longer ranged wells with some suppression.

The Λ-nucleon interaction in states with relative orbital angular momentum $l>0$ will play a significant role in Λ-nucleon scattering for incident Λ-particle energies of about 20 MeV or more in the laboratory. It is to be expected that extensive scattering data (when these become available for the Λ-nucleon system) will ultimately lead to an understanding of the Λ-nucleon interaction in these higher angular momentum states, just as they have in the case of the nucleon-nucleon interaction. The existing scattering data can be used at the present time to make a rough check of assumptions about the Λ-nucleon interaction in states with $l>0$.[8,9,17] Alexander *et al.*[18] have reported fourteen Λ-proton elastic-scattering events, which led to an average cross section of (22.3 ± 5.9) mb for energies in the range 30–168 MeV in the zero-momentum frame. Sixteen Λ-proton elastic-scattering events have been reported by Arbuzov *et al.*[19] for energies in the range 32–320 MeV in the zero-momentum frame; these data led to an average cross section of (42 ± 16) mb. The most recently reported elastic-scattering data are those of Groves,[20] who obtained an average cross section of (20 ± 5) mb for zero-momentum frame energies in the range 5–320 MeV on the basis of 26 events. Alexander *et al.* and Groves also reported average cross sections for the parts of their energy ranges above and below 76 and 68 MeV, respectively. Although the average cross sections reported in these three papers are approximately consistent within the quoted errors, the angular distributions are not. Arbuzov *et al.* reported a pronounced forward-backward asymmetry, 13 of the 16 observed Λ particles having been scattered into the backward hemisphere. Alexander *et al.* and Groves found no such evidence of a pronounced forward-backward asymmetry.

It is the purpose of this paper to report the results of a study of the effects of partial suppression of the Λ-nucleon interaction in odd-parity states on D and on Λ-nucleon scattering cross sections. Central potentials with hard cores and four-parameter attractive wells

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[12] Values $\bar{a}_3 = -1.8$ F and -2.7 F have been obtained by D. R. Smith and B. W. Downs [Phys. Rev. **133**, B461 (1964)] for potentials with a hard-core radius of 0.4 F and two-parameter exponential wells with range parameters $R=0.395$ F and 0.847 F, respectively. Differences in $\bar{a}_3$ of this magnitude would not have a very large effect in the calculations reported in this paper; see Fig. 1.

[13] B. W. Downs and W. E. Ware, Phys. Rev. **133**, B133 (1964).

[14] J. D. Walecka, Nuovo Cimento **16**, 342 (1960).

[15] A summary of empirical estimates of D is given in Ref. 13.

[16] D. H. Davis, R. Levi Setti, M. Raymund, O. Skjeggestad, G. Tomasina, J. Lemonne, P. Renard, and J. Sacton, Phys. Rev. Letters **9**, 464 (1962).

[17] J. S. Kovacs and D. B. Lichtenberg, Nuovo Cimento **13**, 371 (1959).

[18] G. Alexander, J. A. Anderson, F. S. Crawford, Jr., W. Laskar, and L. J. Lloyd, Phys. Rev. Letters **7**, 348 (1961).

[19] B. A. Arbuzov, E. N. Kladnitskaya, V. N. Penev, and R. N. Raustov, Zh. Eksperim. i Teor. Fiz. **42**, 979 (1962) [translation: Soviet Phys.—JETP **15**, 676 (1962)].

[20] T. H. Groves, Phys. Rev. **129**, 1372 (1963).

were used in these calculations in order to provide for various distributions of attraction without over-all compression of the attractive well. Two-body S-wave potentials of this type, which are consistent with the hypernuclear binding energy data, are described in Sec. II. The values of D to which these potentials lead when they are applied to all angular momentum states are given in Sec. III, and the amount of odd-parity suppression which may be indicated is discussed. For the calculation of Λ-nucleon scattering cross sections, these S-wave potentials were taken to represent the interaction in all even-parity states, and a variety of odd-parity interactions was considered. The resulting cross sections are given and discussed in Sec. IV. The validity of the representation of the Λ-nucleon interaction by effective central potentials is briefly discussed in Sec. V.

II. THE Λ-NUCLEON POTENTIALS FOR S-WAVE INTERACTIONS

Central two-body potentials, incorporating the effects of possible noncentral components, are assumed to provide a reliable representation of Λ-nucleon interactions in the light hypernuclei. Such potentials, whose parameters are deduced from analyses of the binding energies of the light hypernuclei, represent Λ-nucleon interactions in S states.[10] We represented these interactions by five-parameter potentials of the form

$$\begin{aligned} V(r) &= \infty && r<c \\ &= -U && c<r<c+b \\ &= -W \exp[-2(r-c)/R] && r>c+b\,. \end{aligned} \tag{3}$$

Three of the parameters (c, R, and W) in (3) were fixed once and for all; three values were taken for b; and the fifth parameter U was determined by the requirement that the potential combinations (1) have the scattering lengths (2) for each choice of b.

The hard-core radius

$$c=0.4\ \mathrm{F} \tag{4a}$$

was chosen as being representative of the short-range repulsions characteristic of strong interactions between baryons. The range parameter R, which determines the asymptotic form of the potential (3), was chosen to represent the TPE mechanism, which is expected to dominate the Λ-nucleon interaction in the region of large separations. The value,

$$R=0.847\ \mathrm{F}\,, \tag{4b}$$

which was used, corresponds to an intrinsic range of 1.5 F for an exponential potential; the same intrinsic range for a Yukawa potential corresponds to a Yukawa range parameter equal to one-half the pion Compton wavelength, appropriate to the TPE mechanism.

The outstanding feature of the Λ-nucleon interaction in S states is the strong spin dependence indicated by Eqs. (1) and (2). After having fixed the parameters c and R, we still had three parameters in the potential form (3) with which account of this spin dependence could be taken. We assumed that the parameters W and b are spin-independent and, consequently, that the depth U of the inner square well provides the entire spin dependence of the S-wave Λ-nucleon interaction represented by the potential (3). The decision to choose the depth W of the asymptotic exponential well to be spin-independent was suggested by the result of a calculation of the TPE contribution to the Λ-nucleon scattering matrix. This calculation by Schrils and Downs[21] led to a spin-independent TPE contribution to the zero-energy Λ-nucleon scattering length. This result suggests that the spin dependence of the S-wave Λ-nucleon interaction may be primarily a characteristic of the region of small separations where exchange mechanisms other than TPE make significant contributions to the interaction. This inner region is represented in (3) by a spin-dependent square well.

The value

$$W=150\ \mathrm{MeV}\,, \tag{4c}$$

which was taken for the depth of the asymptotic exponential well, was suggested by the potential obtained by Gupta[22,23] to represent the TPE contribution to the low-energy nucleon-nucleon interaction. Under the assumption of a universal pion-baryon interaction and with the neglect of the mass differences among the baryons, the low-energy TPE Λ-nucleon potential can be obtained from Gupta's nucleon-nucleon potential by setting $[\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)]=0$ and by replacing the pseudoscalar-pseudoscalar coupling-constant product $(g_{NN\pi}{}^2)^2$ by $(g_{NN\pi}{}^2)(g_{\Lambda\Sigma\pi}{}^2)$.[7–9,21] With the parameters (4b) and (4c), the asymptotic exponential in (3) provides a good representation of this TPE Λ-nucleon potential (which is spin-independent!) in the region of very large separations ($r>2$ F) if $(g_{NN\pi}{}^2/4\pi\hbar c)(g_{\Lambda\Sigma\pi}{}^2/4\pi\hbar c)\approx(13)^2$.[24,25] Although there is some evidence to indicate that the appropriate use of the Gupta potential is justified in the description of nucleon-nucleon scattering,[26] the adaptation of the Gupta potential to the Λ-nucleon interaction has not been justified. In particular, it has been pointed out by Schrils and Downs that the neglect of the mass differences among the baryons may not be justified in the calculation of the TPE contribution to the Λ-nucleon potential.[27]

[21] R. Schrils and B. W. Downs, Phys. Rev. **131**, 390 (1963).

[22] S. N. Gupta, Phys. Rev. **117**, 1146 (1960); and Nuovo Cimento **18**, 823 (1960).

[23] For a discussion of limitations on the use of Gupta's TPE potential, see G. Breit, Ann. Phys. (N. Y.) **16**, 346 (1961).

[24] The use of the value $W=150$ MeV, to which this value of the coupling-constant product corresponds, was taken to be representative; in particular, its use is not intended to imply the equality of the coupling constants $g_{NN\pi}$ and $g_{\Lambda\Sigma\pi}$. Values $W=100$ MeV and 200 MeV were also considered in Ref. 25.

[25] Budh Ram, thesis, University of Colorado, 1963 (unpublished).

[26] G. Breit, K. E. Lassila, H. M. Ruppel, and M. H. Hull, Jr., Phys. Rev. Letters **6**, 138 (1961); and G. Breit, Rev. Mod. Phys. **34**, 766 (1962).

[27] See Ref. 21, especially footnote 35.

TABLE I. Potential parameters and effective ranges for average potentials $\bar{V}_3$ and $\bar{V}_5$.

b (F)	$\bar{U}_3$ (MeV)	$r_{0,3}$ (F)	$\bar{U}_5$ (MeV)	$\bar{r}_{0,5}$ (F)	$W\exp(-2b/R)$ (MeV)
0.7	$96._0$	3.5_4	$51._3$	7.5_5	29.7
1.1	$43._3$	3.8_1	$27._6$	7.8_2	11.2
1.5	$24._1$	4.2_7	$16._1$	8.5_3	4.3

Three values

$$b=\begin{Bmatrix}0.7\\1.1\\1.5\end{Bmatrix}\text{ F} \tag{4d}$$

of the range of the inner square well were used to allow for different extents of the spin-dependent inner region of the potential (3). For each value of b, the depths $\bar{U}_3$ and $\bar{U}_5$ of the inner square wells of the potential combinations (1) were determined by the requirement that the corresponding potentials $\bar{V}_3$ and $\bar{V}_5$ have the scattering lengths (2). The depths U_s and U_t of the inner wells of the singlet and triplet potentials are related to $\bar{U}_3$ and $\bar{U}_5$ by equations of the form (1) since all other parameters in the potential (3) have been taken to be spin-independent.

Scattering lengths for the potential (3) with the parameters given in Eqs. (4) have been calculated as a function of the depth U of the inner square well. The range of calculated scattering lengths was large enough to include the scattering lengths appropriate to the singlet and triplet potentials V_s and V_t. The results of these calculations are given in Fig. 1.

The values of $\bar{U}_3$ and $\bar{U}_5$ which correspond to the scattering lengths $\bar{a}_3=-2.00$ F and $\bar{a}_5=-0.77$ F are given in Table I, along with the corresponding effective ranges $\bar{r}_{0,3}$ and $\bar{r}_{0,5}$ of the full average potentials $\bar{V}_3$ and $\bar{V}_5$. The depths $W\exp(-2b/R)$ of the outer exponential wells at the interface between inner and outer wells are also given in Table I for comparison. The depths U_s and U_t of the inner square wells of the singlet and triplet potentials, which correspond to the average potentials of Table I, are given in Table II, along with the low-energy scattering parameters of the full singlet and triplet potentials V_s and V_t.

III. THE BINDING ENERGY OF A Λ-PARTICLE IN NUCLEAR MATTER

The independent-pair approximation has been used to estimate the asymptotic binding energy D of a Λ-particle in nuclear matter by Walecka[14] and, subsequently, by Downs and Ware.[13] In this approximation, the binding energy is given by

$$D=-D_C+D_A$$
$$=-[4/(2\pi)^3](M_N^*/\mu^*)^3\int_0^{\mu^*k_F/M_N^*}d\mathbf{k}\times\int e^{-i\mathbf{k}\cdot\mathbf{r}}\bar{V}_5(r)\psi_{BG}(\mathbf{k},\mathbf{r})d\mathbf{r} \tag{5}$$

for nuclear matter which is taken to be a spin-saturated collection of an equal number of neutrons and protons, having a Fermi momentum k_F. The effective mass of a nucleon in nuclear matter is M_N^*, and the reduced effective mass of a Λ-nucleon pair is μ^*. The wave function $\psi_{BG}(\mathbf{k},\mathbf{r})$ is the solution of the self-consistent Bethe-Goldstone equation for the relative motion of a Λ-nucleon pair imbedded in nuclear matter and having relative momentum $\mathbf{k}$. The two parts D_C and D_A of the binding energy D arise from the hard core and the attractive well in the average Λ-nucleon potential $\bar{V}_5$, respectively.

For the calculations reported in this section we used (5) to estimate the values of the asymptotic binding energy D which result when the central S-wave potentials, whose parameters are given in Table I, are assumed to be appropriate to all angular-momentum states. The method of calculation was that described by Downs and Ware[13]; and the constants which they used were also used here: The density of nuclear matter was taken to be

$$\rho=0.172\text{ nucleons/F}^3, \tag{6a}$$

TABLE II. Potential parameters and low-energy scattering parameters for singlet and triplet potentials.

b (F)	U_s (MeV)	a_s (F)	$r_{0,s}$ (F)	U_t (MeV)	a_t (F)	$r_{0,t}$ (F)
0.7	$118._4$	-3.8_0	2.6_9	$29._0$	-0.4_9	12.9_6
1.1	$51._2$	-3.4_4	3.0_4	$19._8$	-0.4_5	14.4_5
1.5	$28._1$	-3.2_8	3.4_8	$12._1$	-0.4_3	16.7_0

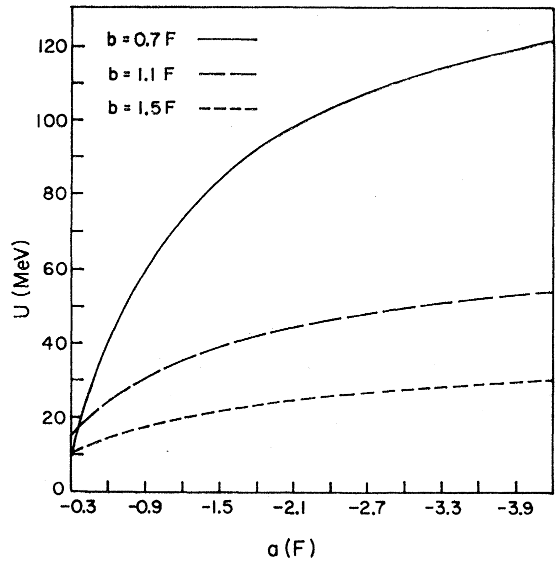

FIG. 1. Depths U of inner square wells as functions of scattering length a of entire potential.

appropriate to the central density of heavy nuclei[28] and corresponding to the Fermi momentum

$$k_F = 1.36_6 \mathrm{F}^{-1}; \quad (6b)$$

and the baryon effective masses were taken to be

$$M_N^* = 0.735 M_N \quad (7a)$$

and

$$M_\Lambda^* = M_\Lambda. \quad (7b)$$

It has been found that the S-wave solution of the Bethe-Goldstone equation for the interaction of two nucleons in nuclear matter is well approximated by the solution corresponding to the hard-core interaction alone (that is, when the attractive well is neglected in the Bethe-Goldstone equation).[29,30] We have therefore assumed that the S-wave component of $\psi_{BG}(\mathbf{k},\mathbf{r})$ can be replaced in (5) by the S-wave solution for a hard-core interaction alone. The contributions to D which arise from higher partial waves have been estimated with the help of several other approximations.

The first three partial-wave contributions $D_C{}^l$ to the hard-core part of (5) have been calculated by Downs and Ware[13] for the hard-core radius (4a) and the Fermi momentum (6b):

$$D_C{}^l = \begin{Bmatrix} 54._7 \\ 2._6 \\ <0._1 \end{Bmatrix} \mathrm{MeV} \quad \text{for} \quad l = \begin{Bmatrix} 0 \\ 1 \\ 2 \end{Bmatrix}. \quad (8)$$

The value of $D_C{}^0$ was obtained by numerical integration of (5) with the S-wave solution of the Bethe-Goldstone equation for the hard-core interaction alone; and the values of $D_C{}^1$ and $D_C{}^2$ were obtained from approximate expressions suggested by Gomes[31] and Walecka.[14,30]

In order to estimate the partial-wave contributions $D_A{}^l$ to the attractive-well part of (5), we have approximated $\psi_{BG}(\mathbf{k},\mathbf{r})$ by the function

$$\psi(\mathbf{k},\mathbf{r}) = 1.05\{1 - \exp[-2k_F(r-c)]\} \exp(i\mathbf{k}\cdot\mathbf{r}), \quad (9)$$

suggested by Downs and Ware.[13] The S-wave component of this function gives a good representation of the S-wave solution of the Bethe-Goldstone equation (for the hard-core interaction alone) for all values of $\mathbf{k}$ which contribute to (5) and for all values of r for which the Λ-nucleon interaction is significant; and its approximation of the partial-wave solutions of the Bethe-Goldstone equation for $l>0$ is expected to be at least as good as the Born approximation.[13]

With the function (9), the total attractive contribution D_A to (5) can be expressed in closed form for simple direct (nonexchange) potentials such as (3); and the partial-wave contributions $D_A{}^l$ can easily be evaluated by numerical integration. The results of such calculations for the potentials of Table I are given in Table III. The average potentials $\bar{V}_5$ are identified in the first two columns of Table III. The next four columns contain values of the binding energy D and the first three partial-wave contributions D^l; these values include both attractive and core contributions. The values of $D^{l>2}$, which are between 0.5 and 0.7 MeV, indicate that interactions in states with $l>2$ make negligibly small contributions to D.

TABLE III. The binding energy D and the partial-wave contributions to D.

b (F)	$\bar{U}_5$ (MeV)	D (MeV)	D^0 (MeV)	D^1 (MeV)	D^2 (MeV)
0.7	$51._3$	$41._3$	$15._7$	$21._3$	$3._7$
1.1	$27._6$	$40._3$	$13._1$	$22._8$	$3._9$
1.5	$16._1$	$40._1$	$9._0$	$26._0$	$4._4$

The main features of Table III are that all the potentials lead to values of D close to 40 MeV and that the P-wave contribution to D is dominant in each case. Larger values of b correspond to greater proportions of attraction in the region of large separations. This leads to increases in $D^{l>0}$ with increasing b and to corresponding decreases in D_0, the net effect being a small decrease in D with an increase in b. The same effects appear if different values of W are used with a given value of b: The larger the value of W, the larger are D and the partial-wave contributions $D^{l>0}$ and the smaller is D^0 (at least for values of W in the range 100–200 MeV and the values of b considered here).[32]

The values of D given in Table III are consistent with the largest empirical estimates,[15] but are considerably in excess of the currently preferred value of about 30 MeV.[16] Reductions of about 40% in the attractive interactions in odd-parity states would lead to values $D \approx 30$ MeV for all the potentials considered here. If a value of D close to 30 MeV is finally established empirically, such a reduction (at least in P states) would be indicated by these calculations, in accordance with the suggestion of Walecka.[14] In this connection, it should be noted that even greater reductions might be rerequired if three-body Λ-nucleon interactions make an appreciable positive contribution to D,[33] while having a smaller effect in determining the average Λ-nucleon interaction in ${}_\Lambda He^5$.

[28] R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

[29] L. C. Gomes, J. D. Walecka, and V. F. Weisskopf, Ann. Phys. (N. Y.) 3, 241 (1958).

[30] J. D. Walecka, thesis, Massachusetts Institute of Technology, 1958.

[31] L. C. Gomes, thesis, Massachusetts Institute of Technology, 1958.

[32] The smallest value of D obtained in similar calculations in Ref. 25 was $38._5$ MeV for $W=100$ MeV and $b=0.7$ F, the smallest values considered for each parameter. The value $D=42._7$ MeV was obtained in Ref. 13 for an average potential having a hard-core radius 0.4 F and a two-parameter exponential well having the range parameter (4b).

[33] See J. D. Chalk, III, and B. W. Downs, Phys. Rev. 132, 2728 (1963), and other references cited there.

IV. Λ-NUCLEON SCATTERING

In this section we report the results of calculations of Λ-nucleon scattering cross sections for four energies in the range for which experimental data have been reported.[18–20] The S-wave potentials, whose parameters are given in Table II, were assumed to be appropriate to the Λ-nucleon interaction in all even-parity states. For the interaction in odd-parity states, three alternatives were considered:

$$(U_{\text{odd }l}, W_{\text{odd }l}) = (U_{\text{even }l}, W_{\text{even }l}) \tag{10a}$$

$$= (0.6U_{\text{even }l}, 0.6W_{\text{even }l}) \tag{10b}$$

$$= (0,0)\text{—hard core only.} \tag{10c}$$

The variety of choices (10) was made in order to investigate the effects on the cross sections of possible reductions in the strength of the interaction suggested by calculations of the asymptotic binding energy D, such as those reported in Sec. III.

The very-low-energy (S-wave) Λ-nucleon scattering cross sections corresponding to the potentials of Table II can be obtained from the scattering parameters in that table. In order to calculate the cross sections for higher energies, we employed a partial-wave analysis in which the scattering amplitudes are calculated directly and in which the phase shifts do not appear explicitly.[34] The amplitude for scattering from a central potential can be expressed as the sum of partial scattering amplitudes

$$f(\theta) = \sum_l S_l P_l(\cos\theta) \tag{11a}$$

in terms of the complex partial amplitude coefficients

$$S_l = \mathcal{R}_l + i\mathcal{I}_l. \tag{11b}$$

The differential cross section is the absolute square of (11a), and the total cross section is

$$\sigma = (4\pi/k) \sum_l \mathcal{I}_l. \tag{12}$$

The phase shifts can be obtained from the relation

$$\delta_l = \tan^{-1}(\mathcal{I}_l/\mathcal{R}_l). \tag{13}$$

For a potential which vanishes beyond some radius $r=d$, the partial amplitude coefficients are

$$S_l = \frac{-(2l+1)d}{[kdh_l^{(1)}(kd)]^2 Z_l} + \frac{i(2l+1)dj_l(kd)}{kdh_l^{(1)}(kd)}, \tag{14a}$$

where

$$Z_l = \frac{d}{R_l(d)}\left(\frac{dR_l(r)}{dr}\right)_{r=d} = \begin{cases} -1+ikd, & \text{for } l=0, \\ -(l-1)+kdh_{l-1}^{(1)}(kd)/h_l^{(1)}(kd), & \text{for } l>0. \end{cases} \tag{14b}$$

[34] See, for example, B. W. Downs, Am. J. Phys. **30**, 248 (1962).

TABLE IV. Partial amplitude coefficients in Fermis for even-parity potentials of Table II with $b=1.1$ F and odd-parity potentials given in (10a).

S	l	$E=20$ MeV $\mathcal{R}_l$	$E=20$ MeV $\mathcal{I}_l$	$E=40$ MeV $\mathcal{R}_l$	$E=40$ MeV $\mathcal{I}_l$	$E=75$ MeV $\mathcal{R}_l$	$E=75$ MeV $\mathcal{I}_l$	$E=150$ MeV $\mathcal{R}_l$	$E=150$ MeV $\mathcal{I}_l$
0	0	0.585	0.324	0.282	0.089	0.027	0.001	−0.168	0.064
	1	0.669	0.111	0.871	0.287	0.797	0.356	0.478	0.170
	2	0.077	0.001	0.203	0.008	0.406	0.047	0.626	0.166
	3	0.009	···	0.038	···	0.105	0.002	0.262	0.020
	4	0.001	···	0.009	···	0.033	···	0.097	0.002
1	0	0.050	0.002	−0.095	0.009	−0.199	0.061	−0.236	0.162
	1	0.334	0.027	0.399	0.055	0.332	0.053	0.117	0.009
	2	0.062	···	0.148	0.004	0.251	0.018	0.310	0.039
	3	0.008	···	0.035	···	0.091	0.002	0.184	0.010
	4	0.001	···	0.009	···	0.032	···	0.087	0.002

In (14), $R_l(r)$ is the solution of the radial Schrödinger equation corresponding to the angular momentum eigenvalue l; and, for the calculations of this section, $\hbar k$ is the relative momentum in the zero-momentum frame.

The first five partial amplitude coefficients (14a) were calculated for the singlet and triplet potentials described in connection with (10) for the energies 20, 40, 75, and 150 MeV in the zero-momentum frame.[35,36] The real and imaginary parts of these coefficients are given in Tables IV and V for the potentials with $b=1.1$ F; $S=0$ and 1 identifies the singlet and triplet spin states. In these table dashes indicate that the corresponding coefficients are zero to three decimal places. For the odd-parity interaction (10c), the partial amplitude coefficient $S_{\text{odd }l}$ is just the second term on the right-hand side of (14a) evaluated at $d=c$, the hard-core radius. The differential cross sections for the statistical mixture

$$(d\sigma/d\Omega)^{(i)} = \tfrac{1}{4}(d\sigma/d\Omega)_s + \tfrac{3}{4}(d\sigma/d\Omega)_t, \tag{15}$$

TABLE V. Partial amplitude coefficients in fermis for odd-parity potentials (10b) with $b=1.1$ F.

S	l	$E=20$ MeV $\mathcal{R}_l$	$E=20$ MeV $\mathcal{I}_l$	$E=40$ MeV $\mathcal{R}_l$	$E=40$ MeV $\mathcal{I}_l$	$E=75$ MeV $\mathcal{R}_l$	$E=75$ MeV $\mathcal{I}_l$	$E=150$ MeV $\mathcal{R}_l$	$E=150$ MeV $\mathcal{I}_l$
0	1	0.323	0.025	0.435	0.066	0.427	0.089	0.231	0.036
	3	0.006	···	0.024	···	0.064	0.001	0.153	0.007
1	1	0.175	0.007	0.202	0.014	0.152	0.011	−0.002	···
	3	0.006	···	0.023	···	0.056	0.001	0.111	0.004

[35] The partial amplitude coefficients with $l=5$–8 were also calculated in Ref. 25 for some of these potentials at 75 and 150 MeV. The contributions of these higher partial waves are negligibly small.

[36] The partial amplitude coefficients (14a) were evaluated at $d=5$ F, the coefficients being essentially independent of d for larger values. Starting with the exact solution at the outer edge of the square well, the radial Schrödinger equation was integrated out to $d=5$ F with a fourth-order Runge-Kutta program on the University of Colorado IBM 709 computer. With the integration interval 0.003 F which was used, the coefficients in Table IV should be correct to two decimal places.

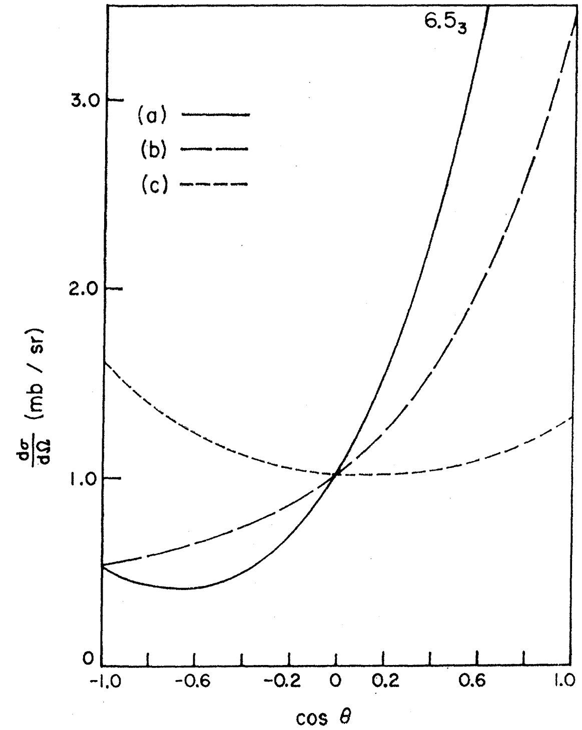

FIG. 2. Average differential cross sections at 20 MeV for even-parity potentials having $b=1.1$ F and odd-parity potentials (10). Curve (a) corresponds to odd-parity potential (10a), etc. Forward scattering cross section for curve (a) is indicated.

appropriate to the scattering of unpolarized particles, are given in Figs. 2–5 for the potentials with $b=1.1$ F. The superscript $(i=a,\ b,\ c)$ identifies the odd-parity potential given in (10a,b,c). The corresponding potentials with $b=0.7$ F lead to relatively smaller contributions to the cross sections from the higher partial waves; and the contributions of the higher partial waves are relatively greater for the potentials with $b=1.5$ F. Similar changes have been found for changes in W for a given value of b, the larger values of W leading to relatively greater contributions from the higher partial waves.[25] These reflect the different distributions of attraction in the region of large separations mentioned in connection with Table III. The main effect of an increase in b (or W) is to increase the magnitude of the oscillations in the angular distributions.[25]

The total cross sections $\sigma^{(i)}$ corresponding to the statistical mixture (15) are given in Table VI for the potentials discussed in connection with (10). The energy dependence of the effect of odd-parity suppression can easily be understood in terms of the partial amplitude coefficients given in Table IV. For 40 and 75 MeV, the contributions of the P-wave amplitudes to the total cross sections $\sigma^{(a)}$ are the dominant ones; consequently, the appreciable odd-parity suppression considered here leads to quite small cross sections for these energies. Even parity contributions to $\sigma^{(a)}$ are larger than the

TABLE VI. Total cross section in millibarns.

b	$E=20$ MeV			$E=40$ MeV			$E=75$ MeV			$E=150$ MeV		
(F)	$\sigma^{(a)}$	$\sigma^{(b)}$	$\sigma^{(c)}$	$\sigma^{(a)}$	$\sigma^{(b)}$	$\sigma^{(c)}$	$\sigma^{(a)}$	$\sigma^{(b)}$	$\sigma^{(c)}$	$\sigma^{(a)}$	$\sigma^{(b)}$	$\sigma^{(c)}$
0.7	$26._1$	$20._8$	$19._0$	$18._2$	$8._9$	$6._2$	$17._4$	$8._3$	$6._2$	$16._4$	$11._9$	$12.^2$
1.1	$22._7$	$16._4$	$14._4$	$18._2$	$7._6$	$4._4$	$18._0$	$9._2$	$6._8$	$17._3$	$14._2$	$14.^8$
1.5	$21._3$	$13._0$	$10._2$	$19._8$	$8._0$	$3._9$	$19._6$	$12._1$	$9._8$	$19._0$	$17._1$	$17._8$

P-wave contributions for 20 MeV (S wave) and 150 MeV (S and D wave). The dependence of the total cross sections on b can be understood from the fact that an increase in the value of b leads to relatively greater contributions from the higher partial waves.

If the values of D and scattering cross sections to which effective central potentials lead are to be compared to investigate the consistency of the descriptions, the scattering energies should be in (or at least close to) the range of energies which contribute to D. The maximum energy in the two-body zero-momentum frame which contributes to D in Eq. (5) is 32.5 MeV for the parameters (6) and (7) used here. This energy is about the minimum energy for which scattering data were reported by Alexander *et al.*[18] and by Arbuzov *et al.*[19]; it is also near the middle of the energy range for a six-event sample for which Groves[20] reported an average cross section.

Groves[20] reported an average cross section (22 ± 10)

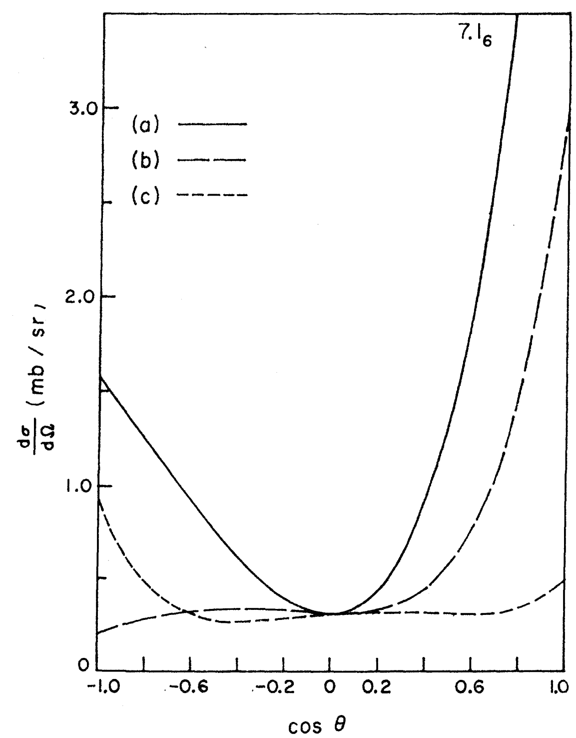

FIG. 3. Average differential cross sections at 40 MeV for even-parity potentials having $b=1.1$ F and odd-parity potentials (10). Curve (a) corresponds to odd-parity potential (10a), etc. Forward scattering cross section for curve (a) is indicated.

mb for six events in the energy range 5–68 MeV; and Alexander *et al.*[18] obtained the average cross section (24±9.3) mb for seven events in the range 32–76 MeV. These measured cross sections are consistent with the cross sections $\sigma^{(a)}$ given in Table VI, but not with the cross sections $\sigma^{(b)}$ and $\sigma^{(c)}$ for 40 and 75 MeV. If our use of effective central potentials is justified for scattering energies as high as these,[37] an established cross section of about 20 mb for energies near 40 MeV, which the current data indicate, could rule out odd-parity suppressions of the magnitude we have considered in (10b,c). Taken with the results of Sec. III, this would imply an asymptotic binding energy D close to 40 MeV because the odd-parity suppression (10b) leads to a value of D close to 30 MeV and (10c), to an even smaller value.

The cross sections of Table VI can also be compared with empirical cross sections in higher ranges. For twenty events in the energy range 68–320 MeV, Groves[20] reported an average cross section (19±5) mb; and Alexander *et al.*[18] obtained an average cross section (20.4±7.7) mb for seven events in the range 76–168 MeV. The cross sections $\sigma^{(a)}$ of Table VI for 75 and

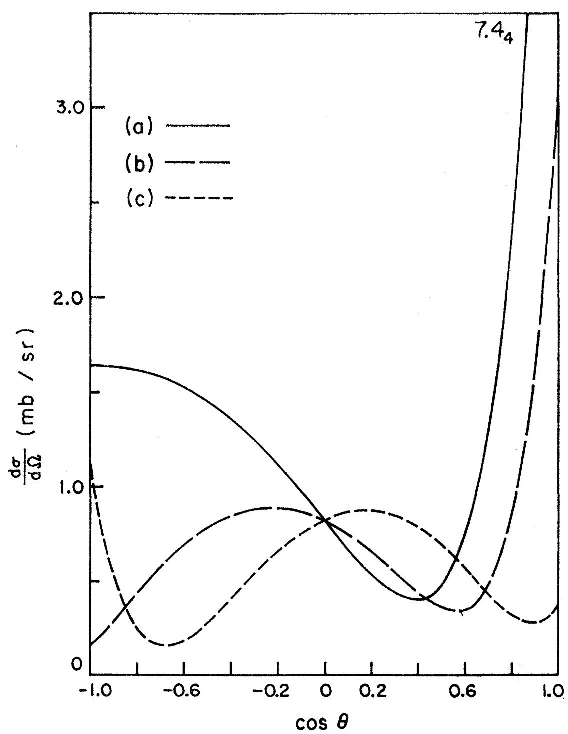

FIG. 4. Average differential cross sections at 75 MeV for even-parity potentials having $b=1.1$ F and odd-parity potentials (10). Curve (a) corresponds to odd-parity potential (10a), etc. Forward scattering cross section for curve (a) is indicated.

[37] In Ref. 17, Kovacs and Lichtenberg demonstrated that a spin-orbit potential could make a significant contribution to the scattering cross sections at these energies. There is, however, no experimental data which requires for its explanation the presence of a potential such as they used.

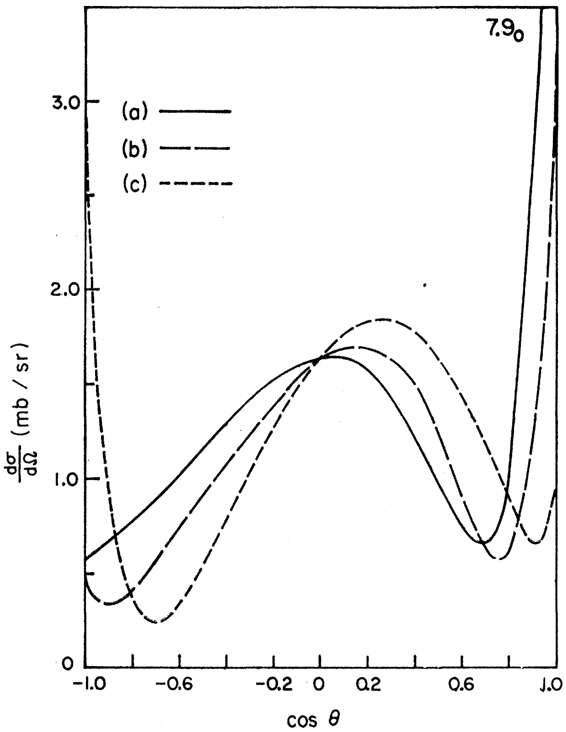

FIG. 5. Average differential cross sections at 150 MeV for even-parity potentials having $b=1.1$ F and odd-parity potentials (10). Curve (a) corresponds to odd-parity potential (10a), etc. Forward scattering cross section for curve (a) is indicated.

150 MeV and most of the suppressed cross section $\sigma^{(b)}$ and $\sigma^{(c)}$ for 150 MeV are consistent with these results, the effect of odd-parity suppression being relatively small at the higher energy. The cross sections $\sigma^{(a)}$ decrease slightly with increasing energy, as do the most probable values of the average cross sections of Groves and of Alexander *et al.* On the other hand, the cross sections $\sigma^{(b)}$ and $\sigma^{(c)}$ increase with increasing energy above about 40 MeV. The average cross section (42±16) mb reported by Arbuzov *et al.*[19] for the energy range 32–320 MeV is not consistent with any of the cross sections of Table VI in this energy range; and it is difficult to reconcile the angular distributions of Figs. 2–5 with the predominance of backward-scattered Λ particles reported by them.

V. CONCLUDING REMARKS

The potentials which we have used were suggested by hypernuclear binding energy data, which is determined by the Λ-nucleon interaction for energies up to about 30 MeV. The comparison of experimental cross sections with those calculated with these potentials can be expected to be more meaningful for the energies 20 and 40 MeV, which we have considered, than they can for the higher energies 75 and 150 MeV. Such comparisons at the lower energies indicated that agreement can be attained only if the interaction in odd-parity states is approximately as strong as it is in even-parity states; and this suggests that a value of D close to 40 MeV may be expected.

The validity of the representation of the Λ-nucleon interaction by effective central potentials is doubtful at the higher energies we have considered. At these energies, the effects of possible noncentral components may be important[8,9,17,37] and quite different from their effects in hypernuclei. Moreover, there may be an appreciable effect from the presence of the Σ-production channel.[8,9] The cross sections reported here for the higher energies are, therefore, to be considered only as the contributions of those components of the Λ-nucleon interaction which can reasonably be represented by effective central potentials at low energies.

That the presence of the Σ-production channel can have a pronounced effect in Λ-nucleon scattering has been emphasized by de Swart *et al.*[8,9] In particular, de Swart and Dullemond[8] have calculated Λ-nucleon scattering cross sections with hyperon-nucleon potentials deduced from phenomenological nucleon-nucleon potentials under the assumption of a universal pion-baryon interaction. Their cross sections have a prominent peak in the neighborhood of the Σ-production threshold (about 76 MeV in the zero-momentum frame), and have values above that threshold which are appreciably larger than the average empirical cross sections of Groves[20] and of Alexander *et al.*[18] Although their cross sections are consistent with the average empirical cross section of Arbuzov *et al.*[19], the measured and calculated angular distributions appear to be inconsistent. The cross sections $\sigma^{(a)}$ to which our effective central potentials lead for energies above the Σ-production threshold are closer to the empirical cross sections of Groves and of Alexander *et al.* than are those of de Swart and Dullemond. Considering the preliminary nature of the scattering data, however, it is probably too early to draw a conclusion from this comparison.

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Spin and Parity Analysis at All Production Angles*†

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Bohr's symmetry method is applied to an unstable spin-j state X, which is produced in a reaction $A+B\rightarrow C+X$ and then decays according to $X\rightarrow D+E$. Particles A, B, C, D are assumed to be spinless, and E is either a spinless particle or a gamma ray. Parity is conserved in production, but not necessarily in decay. The angular distribution of E, in the rest system of X, is $I(\theta)=\frac{1}{2}\Sigma a_L P_L(\cos\theta)$, where $L\leqslant 2j$ and the polar angle θ is measured from the normal to the production plane. The coefficients a_L depend upon the production angle δ and upon the dynamics of the production. It is proved here that the sign of the maximum-complexity coefficient a_{2j} depends only upon the parity of X, and that the magnitude of a_{2j} is not zero but lies between bounds which depend upon j and the parity alone. The implied test for j and the parity has the following advantages: (1) The spin j is equal to half the largest L in $I(\theta)$. Addition of a small amount of a higher P_L, which always improves the fit, is forbidden by the lower bound of a_{2j}. (2) The bounds of a_{2j} are independent of δ. Any (perhaps biased) average over δ may be performed before expanding $I(\theta)$ in the P_L. (3) All the data are condensed into a single test quantity a_{2j}, whose statistical error is reliably known.

1. INTRODUCTION

SUPPOSE an unstable particle or state X is produced in the reaction

$$A+B\rightarrow C+X \tag{1.1}$$

and then decays according to

$$X\rightarrow D+E, \tag{1.2}$$

where A, B, C, and D have spin zero, and E is either a spinless particle or a gamma ray. It was first pointed out by Bohr[1] that conservation of parity in the production reaction implies a symmetry condition for the spin state of X, and consequently also for its decay products. Bohr found that if the spin j of state X is equal to unity, then the angular distribution of its decay is given (for spinless E) by

$$I(\theta)=(3/4\pi)\cos^2\theta, \tag{1.3}$$

if the intrinsic parity is unchanged in the production process, and by

$$I(\theta)=(3/8\pi)\sin^2\theta, \tag{1.4}$$

* Work performed under the auspices of the U. S. Atomic Energy Commission.
† This work was reported briefly at the Chicago Meeting of the American Physical Society [M. Peshkin, Bull. Am. Phys. Soc. 8, 514 (1963)].

[1] A. Bohr, Nucl. Phys. 10, 486 (1959).