with helicities $\lambda=\pm\frac{1}{2}$. Expressions (A3) and (A4) are correct up to a phase factor which is not important. With the help of (A3) and (A4), it is now easy to obtain all the equations of (4). We shall show below the calculation of $R_{n_1\bar{n}_1}(\theta)$ only.

Let us define A, B, C, D to be the four helicity amplitudes with well-defined incoming $p\bar{p}$ helicities and with $++$, $+-$, $-+$, $--$ helicities for the outgoing $\Lambda\bar{\Lambda}$, where A, B, C, D can be any column of the matrix in Table I. Hence, the helicity amplitudes for polarized $\Lambda\bar{\Lambda}$ will be as follows

Λ along n_1, $\bar{\Lambda}$ along n_1: $\theta_1=\frac{1}{2}[(A+C)+(B+D)]$,

Λ along n_1, $\bar{\Lambda}$ along $-n_1$: $\theta_2=\frac{1}{2}[(A+C)-(B+D)]$,

Λ along $-n_1$, $\bar{\Lambda}$ along n_1: $\theta_3=\frac{1}{2}[(A-C)+(B-D)]$,

Λ along $-n_1$, $\bar{\Lambda}$ along $-n_1$: $\theta_4=\frac{1}{2}[(A-C)-(B-D)]$.

The probability of finding Λ polarized along n_1 and simultaneously $\bar{\Lambda}$ polarized along $-n_1$ is now given by

$$\frac{|\theta_2|^2+|\theta_3|^2-|\theta_1|^2-|\theta_4|^2}{|\theta_1|^2+|\theta_2|^2+|\theta_3|^2+|\theta_4|^2},$$

which is equal to

$$\frac{-2\,\mathrm{Re}[AD^*+BC^*]}{|A|^2+|B|^2+|C|^2+|D|^2}.$$

If we assume that the beam and the target are unpolarized, we can sum all these 4 columns of the matrix in Table I and to obtain

$$R_{n_1\bar{n}_1}(\theta)=\frac{2\,\mathrm{Re}[\phi_6\phi_6^*-\phi_3\phi_4^*-\phi_1\phi_2^*-\phi_5\phi_5^*]}{|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+2|\phi_5|^2+2|\phi_6|^2},$$

i.e., the one given in (4). Similarly we can derive the remaining expressions in Eqs. (4).

PHYSICAL REVIEW VOLUME 133, NUMBER 2B 27 JANUARY 1964

Inelastic $\pi^+\pi^-$, $K^0\bar{K}^0$, and K^+K^- Collisions in Unitary Symmetry*

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The G conjugation invariance in the isospin representation and its analogs in the u-spin and v-spin representations of unitary symmetry are applied to the inelastic channels of reactions initiated in the $\pi^+\pi^-$, $K^0\bar{K}^0$, and K^+K^- systems. Relations among differential cross sections for various particle-antiparticle pairs are obtained with and without R invariance.

INTRODUCTION

THE object of this paper is to obtain relations among the cross sections for various particle-antiparticle pairs in the inelastic channels of reactions initiated in collisions of π^+ and π^-, K^0 and $\bar{K}^0$, and K^+ and K^- within the scheme of unitary symmetry.[1,2] For reactions initiated in the $\pi^+\pi^-$ system the eigenstates of pairs of pseudoscalar mesons, vector mesons, baryon and antibaryon, and baryon isobar and antibaryon isobar are constructed under the conservation of isospin and G conjugation invariance.[3] Relations among the cross sections for the possible pairs are then immediately obtained.

For reactions initiated in the $K^0\bar{K}^0$ system, the isospin representation cannot be used to carry out the same program since it is not an eigenstate of G conjugation. In order to do so it is found necessary to go to the u-spin representation[4] of unitary symmetry and invoke the analog of G conjugation (henceforth called U conjugation) of which the $K^0\bar{K}^0$ system is an eigenstate.

For reactions initiated in the K^+K^- system, the previous representations cannot be used to carry out the program since it is not an eigenstate of the G nor of the U conjugation. It is then found necessary to go to the v-spin representation[4] and invoke the analog of the G and U conjugations (henceforth called V conjugation) of which the K^+K^- system is an eigenstate. These results can then be correlated to each other since the isospin, the u-spin, and the v-spin representations are canonically related to one another under unitary symmetry.

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I. REACTIONS INITIATED IN THE CENTER OF MOMENTUM OF THE $\pi^+\pi^-$ SYSTEM

A. $\pi^+\pi^-$ in Odd Orbital States

The initial state[5]:

$$\frac{1}{\sqrt{2}}[\pi^+\pi^- - \pi^-\pi^+].$$

The following are the particle-antiparticle eigenstates with the same set of quantum numbers as the initial state[6–8]:

$$I_3 = Q(charge) = 0, \quad I = 1, \quad G = +1$$

$$\frac{1}{\sqrt{2}}[\pi^+\pi^- - \pi^-\pi^+], \qquad \tfrac{1}{2}[-K^+K^- + K^-K^+ + K^0\bar{K}^0 - \bar{K}^0K^0], \qquad \frac{1}{\sqrt{2}}[-\rho^+\rho^- + \rho^-\rho^+],$$

$$\tfrac{1}{2}[-K^{*+}K^{*-} + K^{*-}K^{*+} + K^{*0}\bar{K}^{*0} - \bar{K}^{*0}K^{*0}], \qquad \tfrac{1}{2}[-p\bar{p} + \bar{p}p + n\bar{n} - \bar{n}n], \qquad \tfrac{1}{2}[-\Xi^0\bar{\Xi}^0 + \bar{\Xi}^0\Xi^0 + \Xi^-\bar{\Xi}^+ - \bar{\Xi}^+\Xi^-],$$

$$\tfrac{1}{2}[\Sigma^+\bar{\Sigma}^- - \bar{\Sigma}^-\Sigma^+ - \Sigma^-\bar{\Sigma}^+ + \bar{\Sigma}^+\Sigma^-], \qquad \tfrac{1}{2}[\Sigma^0\bar{\Lambda} + \bar{\Lambda}\Sigma^0 - \Lambda\bar{\Sigma}^0 - \bar{\Sigma}^0\Lambda], \qquad \tfrac{1}{2}[-\Xi^{*0}\bar{\Xi}^{*0} + \bar{\Xi}^{*0}\Xi^{*0} + \Xi^{*-}\bar{\Xi}^{*+} - \bar{\Xi}^{*+}\Xi^{*-}],$$

$$\tfrac{1}{2}[Y_1^{*+}\bar{Y}_1^{*-} - \bar{Y}_1^{*-}Y_1^{*+} - Y_1^{*-}\bar{Y}_1^{*+} + \bar{Y}_1^{*+}Y_1^{*-}], \qquad \frac{1}{2\sqrt{10}}[-3(N^{*++}\bar{N}^{*--} - \bar{N}^{*--}N^{*++} - N^{*-}\bar{N}^{*+} + \bar{N}^{*+}N^{*-})$$
$$-(N^{*+}\bar{N}^{*-} - \bar{N}^{*-}N^{*+} - N^{*0}\bar{N}^{*0} + \bar{N}^{*0}N^{*0})].$$

Thus, there exist the following relations among the differential cross sections for possible pairs[9,10]:

$$(K^+K^-) = (K^0\bar{K}^0), \quad (K^{*+}K^{*-}) = (K^{*0}\bar{K}^{*0}), \quad (p\bar{p}) = (n\bar{n}), \quad (\Xi^0\bar{\Xi}^0) = (\Xi^-\bar{\Xi}^+), \quad (\Sigma^+\bar{\Sigma}^-) = (\Sigma^-\bar{\Sigma}^+),$$
$$(\Sigma^0\bar{\Lambda}) = (\Lambda\bar{\Sigma}^0), \quad (\Xi^{*0}\bar{\Xi}^{*0}) = (\Xi^{*-}\bar{\Xi}^{*+}), \quad (Y_1^{*+}\bar{Y}_1^{*-}) = (Y_1^{*-}\bar{Y}_1^{*+}),$$
$$(N^{*++}\bar{N}^{*--}) = (N^{*-}\bar{N}^{*+}) = 9(N^{*+}\bar{N}^{*-}) = 9(N^{*0}\bar{N}^{*0}). \tag{1.1}$$

In addition, the following pairs may be present:

$$(\pi^+\pi^-), \quad (\rho^+\rho^-). \tag{1.2}$$

The R conjugation[1] eigenvalue of the initial state is -1. Thus, when R invariance of the octets is invoked, in place of the fifth and sixth eigenfunctions of the above, we have

$$8^{-1/2}[-p\bar{p} + \bar{p}p + n\bar{n} - \bar{n}n - \Xi^0\bar{\Xi}^0 + \bar{\Xi}^0\Xi^0 + \Xi^-\bar{\Xi}^+ - \bar{\Xi}^+\Xi^-],$$

and the eighth eigenfunction does not occur. Thus, in place of the third and fourth relations in (1.1), we obtain

$$(p\bar{p}) = (n\bar{n}) = (\Xi^0\bar{\Xi}^0) = (\Xi^-\bar{\Xi}^+), \tag{1.3}$$

and the pairs $(\Sigma^0\bar{\Lambda})$ and $(\Lambda\bar{\Sigma}^0)$ do not occur.

B. $\pi^+\pi^-$ in Even Orbital States

The initial state:

$$\frac{1}{\sqrt{2}}[\pi^+\pi^- + \pi^-\pi^+].$$

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[5] The dependence upon the momenta $\mathbf{k}$ and $-\mathbf{k}$ in the center-of-momentum system is:

$$(1/\sqrt{2})[\pi^+(\mathbf{k})\pi^-(-\mathbf{k}) - \pi^-(\mathbf{k})\pi^+(-\mathbf{k})].$$

[6] The orbital (l) and spin (s) quantum numbers of each pair are determined by $G = (-1)^{l+s+I}$.

[7] The operations of G upon isospin doublets and isospin triplets which are needed in constructing the eigenfunctions are well known (see Refs. 3 and 8). For the isospin quartet baryon isobars the following holds:

$$G\begin{pmatrix} N^{*++} \\ N^{*+} \\ N^{*0} \\ N^{*-} \end{pmatrix} = \begin{pmatrix} \bar{N}^{*+} \\ -\bar{N}^{*0} \\ \bar{N}^{*-} \\ -\bar{N}^{*--} \end{pmatrix}.$$

[8] The complete set of eigenfunctions of the $K\bar{K}$ system is found in M. Goldhaber, T. D. Lee, and C. N. Yang, Phys. Rev. **112**, 1796 (1958).

[9] (K^+K^-) stands for the differential cross section $\sigma(K^+(\mathbf{k})\ K^-(-\mathbf{k}))$. The equality of (K^+K^-) and the differential cross section $\sigma(K^-(\mathbf{k})K^+(-\mathbf{k}))$ for the charge-conjugated pair is understood.

[10] $(K^0\bar{K}^0)$ in this particular case arises from $[K^0\bar{K}^0 - \bar{K}^0K^0] = -i[K_1^0K_2^0 - K_2^0K_1^0]$ when the CP invariance of the weak decays of K_1^0 and K_2^0 is postulated. Thus, for example, $(K^+K^-) = (K^0\bar{K}^0)$ implies $\sigma(K^+(\mathbf{k})K^-(-\mathbf{k})) = \sigma(K^-(\mathbf{k})K^+(-\mathbf{k})) = \sigma(K_1^0(\mathbf{k})K_2^0(-\mathbf{k})) = \sigma(K_2^0(\mathbf{k})K_1^0(-\mathbf{k}))$.

The following are the particle-antiparticle eigenstates with $I=0$ and $I=2$:

$$I_3=Q=0,\quad I=0,\quad G=+1$$

$$\frac{1}{\sqrt{3}}[\pi^+\pi^-+\pi^-\pi^++\pi^0\pi^0],\quad \eta\eta,\quad \tfrac{1}{2}[-K^+K^--K^-K^+-K^0\bar{K}^0-\bar{K}^0K^0],\quad \frac{1}{\sqrt{3}}[-\rho^+\rho^--\rho^-\rho^+-\rho^0\rho^0],$$

$$\phi\phi,\quad \tfrac{1}{2}[-K^{*+}K^{*-}-K^{*-}K^{*+}-K^{*0}\bar{K}^{*0}-\bar{K}^{*0}K^{*0}],\quad \tfrac{1}{2}[-p\bar{p}-\bar{p}p-n\bar{n}-\bar{n}n],$$

$$\tfrac{1}{2}[-\Xi^0\bar{\Xi}^0-\bar{\Xi}^0\Xi^0-\Xi^-\bar{\Xi}^+-\bar{\Xi}^+\Xi^-],\quad \frac{1}{\sqrt{2}}[\Lambda\bar{\Lambda}+\bar{\Lambda}\Lambda],\quad 6^{-1/2}[\Sigma^+\bar{\Sigma}^-+\bar{\Sigma}^-\Sigma^++\Sigma^-\bar{\Sigma}^++\bar{\Sigma}^+\Sigma^-+\Sigma^0\bar{\Sigma}^0+\bar{\Sigma}^0\Sigma^0],$$

$$\tfrac{1}{2}[-\Xi^{*0}\bar{\Xi}^{*0}-\bar{\Xi}^{*0}\Xi^{*0}-\Xi^{*-}\bar{\Xi}^{*+}-\bar{\Xi}^{*+}\Xi^{*-}],\quad \frac{1}{\sqrt{2}}[Z^-\bar{Z}^++\bar{Z}^+Z^-],$$

$$6^{-1/2}[Y_1^{*+}\bar{Y}_1^{*-}+\bar{Y}_1^{*-}Y_1^{*+}+Y_1^{*-}\bar{Y}_1^{*+}+\bar{Y}_1^{*+}Y_1^{*-}+Y_1^{*0}\bar{Y}_1^{*0}+\bar{Y}_1^{*0}Y_1^{*0}],$$

$$8^{-1/2}[-N^{*++}\bar{N}^{*--}-\bar{N}^{*--}N^{*++}-N^{*-}\bar{N}^{*+}-\bar{N}^{*+}N^{*-}-N^{*+}\bar{N}^{*-}-\bar{N}^{*-}N^{*+}-N^{*0}\bar{N}^{*0}-\bar{N}^{*0}N^{*0}],$$

$$I_3=Q=0,\quad I=2,\quad G=+1$$

$$6^{-1/2}[\pi^+\pi^-+\pi^-\pi^+-2\pi^0\pi^0],\quad 6^{-1/2}[-\rho^+\rho^--\rho^-\rho^++2\rho^0\rho^0],\quad 12^{-1/2}[\Sigma^+\bar{\Sigma}^-+\bar{\Sigma}^-\Sigma^++\Sigma^-\bar{\Sigma}^++\bar{\Sigma}^+\Sigma^--2\Sigma^0\bar{\Sigma}^0-2\bar{\Sigma}^0\Sigma^0],$$

$$12^{-1/2}[Y_1^{*+}\bar{Y}_1^{*-}+\bar{Y}_1^{*-}Y_1^{*+}+Y_1^{*-}\bar{Y}_1^{*+}+\bar{Y}_1^{*+}Y_1^{*-}-2Y_1^{*0}\bar{Y}_1^{*0}-2\bar{Y}_1^{*0}Y_1^{*0}],$$

$$8^{-1/2}[-N^{*++}\bar{N}^{*--}-\bar{N}^{*--}N^{*++}-N^{*-}\bar{N}^{*+}-\bar{N}^{*+}N^{*-}+N^{*+}\bar{N}^{*-}+\bar{N}^{*-}N^{*+}+N^{*0}\bar{N}^{*0}+\bar{N}^{*0}N^{*0}].$$

Thus, the relations among the differential cross sections for the possible pairs are[11]:

$$\begin{aligned}
&(K^+K^-)=(K^0\bar{K}^0),\qquad (K^{*+}K^{*-})=(K^{*0}\bar{K}^{*0}),\\
&(p\bar{p})=(n\bar{n}),\qquad (\Xi^0\bar{\Xi}^0)=(\Xi^-\bar{\Xi}^+),\\
&(\Sigma^+\bar{\Sigma}^-)=(\Sigma^-\bar{\Sigma}^+),\qquad (\Xi^{*0}\bar{\Xi}^{*0})=(\Xi^{*-}\bar{\Xi}^{*+}),\\
&(Y_1^{*+}\bar{Y}_1^{*-})=(Y_1^{*-}\bar{Y}_1^{*+}),\qquad (N^{*++}\bar{N}^{*--})=(N^{*-}\bar{N}^{*+}),\\
&(N^{*+}\bar{N}^{*-})=(N^{*0}\bar{N}^{*0}).
\end{aligned}\tag{1.4}$$

In addition, the following pairs may be present[12]:

$$\begin{aligned}
&(\pi^+\pi^-),\qquad (\rho^+\rho^-),\qquad (Y_1^{*0}\bar{Y}_1^{*0}),\qquad (Z^-\bar{Z}^+),\\
&(\eta\eta),\qquad (\pi^0\pi^0),\qquad (\phi\phi),\qquad (\rho^0\rho^0),\qquad (\Lambda\bar{\Lambda}),\\
&(\Sigma^0\bar{\Sigma}^0).
\end{aligned}\tag{1.5}$$

When R invariance of the octets is invoked (eigenvalue $+1$), in place of the seventh and eighth eigenfunctions of the set ($I_3=Q=0, I=0, G=+1$), we obtain

$$8^{-1/2}[-p\bar{p}-\bar{p}p-n\bar{n}-\bar{n}n \\ -\Xi^0\bar{\Xi}^0-\bar{\Xi}^0\Xi^0-\Xi^-\bar{\Xi}^+-\bar{\Xi}^+\Xi^-].$$

Hence, instead of the third and fourth relations in (1.4), we obtain the relation (1.3).

[11] $(K^0\bar{K}^0)$ in this particular case arises from $[K^0\bar{K}^0+\bar{K}^0K^0]=[K_1^0K_1^0+K_2^0K_2^0]$ under the CP invariance of the weak decays of K_1^0 and K_2^0. Thus, $(K^0\bar{K}^0)=\sigma(K_1^0(\mathbf{k})K_1^0(-\mathbf{k}))=\sigma(K_2^0(\mathbf{k})K_2^0(-\mathbf{k}))$.

[12] The unitary singlet and the $I=0$ member of the unitary octet vector mesons ω_1 and ω_8, respectively, should occur mixed in nature because of violation of unitary symmetry as follows

$$\omega_1=\omega\cos\theta-\phi\sin\theta,$$
$$\omega_8=\phi\cos\theta+\omega\sin\theta,$$

where θ is the mixing angle and ω and ϕ are the physical unitary singlet and $I=0$ member of the octet, respectively. Thus, under the mixing, ϕ that occurs in (1.5) should be replaced by ω_8. See S. L. Glashaw, Phys. Rev. Letters **11**, 48 (1963); and J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1963); and (to be published).

II. REACTIONS INITIATED IN THE CENTER OF MOMENTUM OF THE $K^0\bar{K}^0$ SYSTEM

The irreducible representations of unitary symmetry in the u-spin space are characterized by the charge Q and the u-spin quantum numbers.[4] The appropriate correspondences between members of the irreducible representations in the isospin and u-spin representations can easily be established, and the properties of the U conjugation as the direct analog of the G conjugation can be readily worked out.[13] Thus, the results obtained in the previous section are directly transferable, and we state only the results.

A. $K^0\bar{K}^0$ in Odd Orbital States

The relations among the differential cross sections for the possible pairs are:

$$\begin{aligned}
&(\pi^-\pi^+)=(K^-K^+),\qquad (\rho^-\rho^+)=(K^{*-}K^{*+}),\\
&(\Sigma^-\bar{\Sigma}^+)=(\Xi^-\bar{\Xi}^+),\qquad (p\bar{p})=(\Sigma^+\bar{\Sigma}^-),\\
&(n\bar{n})=(\Xi^0\bar{\Xi}^0),\qquad (\Sigma^0\bar{\Lambda})=(\Lambda\bar{\Sigma}^0),\\
&(N^{*+}\bar{N}^{*-})=(Y_1^{*+}\bar{Y}_1^{*-}),\qquad (N^{*0}\bar{N}^{*0})=(\Xi^{*0}\bar{\Xi}^{*0}),\\
&(N^{*-}\bar{N}^{*+})=(Z^-\bar{Z}^+)=9(Y_1^{*-}\bar{Y}_1^{*+})=9(\Xi^{*-}\bar{\Xi}^{*+}).
\end{aligned}\tag{2.1}$$

[13] All properties of the G operator can be appropriately transferred to those of the U operator. Only states of zero baryon number and electric charge can be eigenstates of $U=C\exp(i\pi u_2)$. $U=C(-1)^u$ for $u_3=0$ systems; in general $U=(-1)^{l+s+u}$. A baryon u-spin doublet (Ref. 4), e.g., $(P, -\Sigma^+)$, and its U-conjugated antibaryon doublet, $(-\bar{\Sigma}^-, -\bar{P})$, may be compounded (Fermi-Yang model) to conduct pseudoscalar mesons ($^1s_0$ orbital) and vector mesons ($^3s_1$ or $^3d_1$ orbital). From $U=(-1)^{l+s+u}$ we find that the u-spin triplet $(K^0, M_1{}^u, -\bar{K}^0)$ and singlet $(M_0{}^u)$ pseudoscalar mesons have U parities -1 and $+1$, respectively. Similarly the u-spin triplet $(K^{0*}, M_1{}^{u*}, -\bar{K}^{0*})$ and singlet vector mesons $(M_0{}^{u*})$ have U parities $+1$ and -1, respectively. The conservation of U parity may be used along the u-spin conservation in discussing strong interactions under unitary symmetry.

In addition, the following pairs may be present[10]:

$$(K^0\bar{K}^0),\qquad (K^{*0}\bar{K}^{*0}).\tag{2.2}$$

When R invariance of the octets is invoked, in place of the third and fourth relations in (2.1), we obtain

$$(p\bar{p})=(\Xi^-\bar{\Xi}^+)=(\Sigma^+\bar{\Sigma}^-)=(\Sigma^-\bar{\Sigma}^+),\tag{2.3}$$

and the pairs $(\Sigma^0\bar{\Lambda})$ and $(\Lambda\bar{\Sigma}^0)$ do not occur.

B. $K^0\bar{K}^0$ in Even Orbital States

The relations among the differential cross sections for the possible pairs are:

$$\begin{aligned}
&(\pi^+\pi^-)=(K^-K^+),\qquad (\rho^-\rho^+)=(K^{*-}K^{*+}),\\
&(\Sigma^-\bar{\Sigma}^+)=(\Xi^-\bar{\Xi}^+),\qquad (p\bar{p})=(\Sigma^+\bar{\Sigma}^-),\qquad (n\bar{n})=(\Xi^0\bar{\Xi}^0),\\
&(N^{*+}\bar{N}^{*-})=(Y_1^{*+}\bar{Y}_1^{*-}),\qquad (N^{*0}\bar{N}^{*0})=(\Xi^{*0}\bar{\Xi}^{*0}),\\
&(N^{*-}\bar{N}^{*+})=(Z^-\bar{Z}^+),\qquad (Y_1^{*-}\bar{Y}_1^{*+})=(\Xi^{*-}\bar{\Xi}^{*+}).
\end{aligned}\tag{2.4}$$

In addition, the following pairs may be present[11,14]:

$$\begin{aligned}
&(K^0\bar{K}^0),\qquad (K^{*0}\bar{K}^{*0}),\qquad (Y_1^{*0}\bar{Y}^{*0}),\\
&(N^{*++}\bar{N}^{*--}),\qquad (M_0^uM_0^u),\qquad (M_1^uM_1^u),\\
&(M_0^{u*}M_0^{u*}),\qquad (M_1^{u*}M_1^{u*}),\qquad (B_0^u\bar{B}_0^u),\\
&(B_1^u\bar{B}_1^u),
\end{aligned}\tag{2.5}$$

where

$$\begin{aligned}
(M_0^uM_0^u)&=(\pi^0\pi^0)+(\pi^0\eta)+(\eta\pi^0)+(\eta\eta),\\
(M_1^uM_1^u)&=(\eta\eta)+(\pi^0\eta)+(\eta\pi^0)+(\pi^0\pi^0),\\
(M_0^{u*}M_0^{u*})&=(\rho^0\rho^0)+(\rho^0\phi)+(\phi\rho^0)+(\phi\phi),\\
(M_1^{u*}M_1^{u*})&=(\phi\phi)+(\rho^0\phi)+(\phi\rho^0)+(\rho^0\rho^0),\\
(B_0^u\bar{B}_0^u)&=(\Sigma^0\bar{\Sigma}^0)+(\Lambda\bar{\Sigma}^0)+(\Sigma^0\bar{\Lambda})+(\Lambda\bar{\Lambda}),\\
(B_0^u\bar{B}_1^u)&=(\Lambda\bar{\Lambda})+(\Lambda\bar{\Sigma}^0)+(\Sigma^0\bar{\Lambda})+(\Sigma^0\bar{\Sigma}^0),
\end{aligned}\tag{2.6}$$

each with ratios 9:3:3:1, respectively.[15]

When R invariance of the octets is invoked, in place of the third and fourth relations in (2.4), we obtain the relation (2.3).

III. REACTIONS INITIATED IN THE CENTER OF MOMENTUM OF THE K^+K^- SYSTEM

The irreducible representations of unitary symmetry in the v-spin space are characterized by $Q-Y$ and the v-spin quantum numbers.[4] The appropriate correspondences between members of the irreducible representations in the isospin, u-spin, and v-spin representations can easily be established, and the properties of V conjugation as the direct analog of the G and U conjugations can be readily worked out.[16] Thus, the results of the previous sections are directly transferable, and we state only the results.

A. K^+K^- in Odd Orbital States

The relations among the differential cross sections for the possible pairs are[10]:

$$\begin{aligned}
&(K^0\bar{K}^0)=(\pi^+\pi^-),\qquad (K^{*0}\bar{K}^{*0})=(\rho^+\rho^-),\\
&(\Xi^0\bar{\Xi}^0)=(\Sigma^+\bar{\Sigma}^-),\qquad (\Sigma^-\bar{\Sigma}^+)=(n\bar{n}),\\
&(\Xi^-\bar{\Xi}^+)=(p\bar{p}),\qquad (\Sigma^0\bar{\Lambda})=(\Lambda\bar{\Sigma}^0),\\
&(Y_1^{*-}\bar{Y}^{*+})=(N^{*0}\bar{N}^{*0}),\qquad (\Xi^{*-}\bar{\Xi}^{*+})=(N^{*+}\bar{N}^{*-}),\\
&(Z^-\bar{Z}^+)=(N^{*++}\bar{N}^{*--})=9(\Xi^{*0}\bar{\Xi}^{*0})=9(Y_1^{*+}\bar{Y}_1^{*-}).
\end{aligned}\tag{3.1}$$

In addition, the following pairs may be present:

$$(K^-K^+),\quad (K^{*-}K^{*+}).\tag{3.2}$$

When R invariance of the octets is invoked, in place of the third and fourth relations in (3.1), we obtain

$$(\Xi^0\bar{\Xi}^0)=(\Sigma^+\bar{\Sigma}^-)=(\Sigma^-\bar{\Sigma}^+)=(n\bar{n}),\tag{3.3}$$

and the pairs $(Z^0\bar{\Lambda})$ and $(\Lambda\bar{\Sigma}^0)$ do not occur.

B. K^+K^- in Even Orbital States

The relations among the differential cross sections for the possible pairs are[11]:

$$\begin{aligned}
&(K^0\bar{K}^0)=(\pi^+\pi^-),\qquad (K^{*0}\bar{K}^{*0})=(\rho^+\rho^-),\\
&(\Xi^0\bar{\Xi}^0)=(\Sigma^+\bar{\Sigma}^-),\qquad (\Sigma^-\bar{\Sigma}^+)=(n\bar{n}),\qquad (\Xi^-\bar{\Xi}^+)=(p\bar{p}),\\
&(Y_1^{*-}\bar{Y}_1^{*+})=(N^{*0}\bar{N}^{*0}),\qquad (\Xi^{*-}\bar{\Xi}^{*+})=(N^{*+}\bar{N}^{*-}),\\
&(Z^-\bar{Z}^+)=(N^{*++}\bar{N}^{*--}),\qquad (\Xi^{*0}\bar{\Xi}^{*0})=(Y_1^{*+}\bar{Y}_1^{*-}).
\end{aligned}\tag{3.4}$$

In addition, the following pairs may be present[17]:

$$\begin{aligned}
&(K^+K^-),\qquad (K^{*+}K^{*-}),\qquad (Y_1^{*0}\bar{Y}_1^{*0}),\qquad (N^{*-}\bar{N}^{*+}),\\
&(M_0^vM_0^v),\qquad (M_1^vM_1^v),\qquad (M_0^{v*}M_0^{v*}),\\
&(M_1^{v*}M_1^{v*}),\qquad (B_0^v\bar{B}_0^v),\qquad (B_1^v\bar{B}_1^v),
\end{aligned}\tag{3.5}$$

[14] The canonical transformation from the isospin representation to the u-spin representation mixes $I=0$, $I_3=0$ and $I=1$, $I_3=0$ members of each octet. M_0^u and M_1^u, M_0^{u*} and M_1^{u*}, and B_0^u and B_1^u are the $I=0$, $I_3=0$ and $I=1$, $I_3=0$ members of the pseudoscalar meson, vector meson, and baryon octets, respectively.

[15] Under $\phi\omega$ mixing ϕ should be replaced by ω_8 (see Ref. 12).

[16] All properties of the G operator can be appropriately transferred to those of the V operator. Only states of zero baryon number and $Q-Y=0$ can be eigenstates of $V=C\exp(i\pi v_2)$. $V=C(-1)^v$ for $v_3=0$ systems; in general $V=(-1)^{l+s+v}$. A baryon v-spin doublet (Ref. 4), e.g., (n,Σ^-), and its V-conjugated antibaryon doublet, $(\bar{\Sigma}^+,-\bar{n})$, may be compounded (Fermi-Yang model) to construct pseudoscalar mesons ($^1s_0$ orbital) and vector mesons ($^3s_1$ or $^3d_1$ orbital). From $V=(-1)^{l+s+v}$ we find that the v-spin triplet $(K^-, M_1^v, -K^+)$ and singlet (M_0^v) pseudoscalar mesons have V parities -1 and $+1$, respectively. Similarly, the v-spin triplet $(K^{*-}, M_1^{v*}, -K^{*+})$ and singlet (M_0^{v*}) vector mesons have V parities $+1$ and -1, respectively. The conservation of V parity may be used along with v-spin conservation in discussing strong interactions under unitary symmetry.

[17] The canonical transformation from the isospin representation to the v-spin representation mixes $v=0$, $v_3=0$ and $v=1$, $v_3=0$ members of each octet. M_0^v and M_1^v, M_0^{v*} and M_1^{v*}, and B_0^v and B_1^v are the $v=0$, $v_3=0$ and $v=1$, $v_3=0$ members of the pseudoscalar meson, vector meson, and baryon octets, respectively.

where each of the pairs from $(M_0{}^vM_0{}^v)$ to $(B_1{}^v\bar{B}_1{}^v)$ has the same further expansion as the corresponding pair in (2.6).

When R invariance of the octets is invoked, in place of the third and fourth relations in (3.4), we have the relation (3.3).

DISCUSSION

The results obtained in the previous sections are based on unitary symmetry in its exact limit. Thus, they are presumably applicable to high-energy collisions in which mass differences among members within each irreducible representation may not be significant.

In the derivation of the equalities between differential cross sections in the previous sections it has been necessary to assume that the initial state corresponds to either even or odd orbital angular momentum, but not to a mixture of the two. However, when the initial state is a mixture of odd and even angular momenta, these same equalities still hold in the total cross sections (integrated over angles and summed over spin states), since states of different angular momenta do not interfere.

The isospin, u-spin, and v-spin representations are canonically related to each other under unitary symmetry.[4,18] Thus, provided that the initial energies and orbital states are the same, the corresponding differential cross sections (and spin distributions) of the set (1.1), (1.2), (1.3), (1.4), (1.5), the set (2.1), (2.2), (2.3), (2.4), (2.5), and the set (3.1), (3.2), (3.3), (3.4), (3.5) must be the same (for the equalities within each set the above restrictions must be noted). In particular, if resonances or final-state interactions are found in a differential cross section of one set, the same effects must exist in the corresponding differential cross sections in the other sets.

The principal results of this work may be summarized as follows.

Let us label the outer members of the octets and the corresponding members of the decuplet of baryon isobars by "*abcdef*" going around in a circle. Under the restrictions on the orbital quantum numbers mentioned earlier, for both even and odd orbital states, in the two-body reaction channels initiated by a meson pair taken across, e.g., "*ad*," we find for the differential cross sections the relations $(a\bar{a})=(d\bar{d})$, $(b\bar{b})=(c\bar{c})$, and $(e\bar{e})=(f\bar{f})$ for each of the octets and the decuplet. Further, for the same initial energies and orbital states of the meson pairs "*ad*," "*be*," and "*cf*," we have $[(a\bar{a})=(d\bar{d})]_{ad}=[(c\bar{c})=(f\bar{f})]_{cf}=[(e\bar{e})=(b\bar{b})]_{eb}$ for each of the octets and the decuplet (the subscripts ad, cf, eb signify the entrance-channel meson pairs). Extending this procedure we obtain the following three sets of equalities among the differential cross sections:

$$[(a\bar{a})=(d\bar{d})]_{ad}=[(c\bar{c})=(f\bar{f})]_{cf}=[(e\bar{e})=(b\bar{b})]_{eb},$$
$$[(b\bar{b})=(c\bar{c})]_{ad}=[(d\bar{d})=(e\bar{e})]_{cf}=[(f\bar{f})=(a\bar{a})]_{eb},$$
$$[(e\bar{e})=(f\bar{f})]_{ad}=[(a\bar{a})=(b\bar{b})]_{cf}=[(c\bar{c})=(d\bar{d})]_{eb}.$$

For example, the differential cross sections $(K^+K^-)=(K^0\bar{K}^0)$ from $\pi^+\pi^-$, $(\pi^-\pi^+)=(K^-K^+)$ from $K^0\bar{K}^0$, and $(K^0\bar{K}^0)=(\pi^+\pi^-)$ from K^+K^- are equal, provided the initial energies and the orbital quantum numbers are the same (e.g., a pure s or a pure p orbital).[19]

When R invariance of the octets is introduced, we have from (1.3), (2.3), and (3.3) the following additional equalities in the baryon-antibaryon channels:

$$[(b\bar{b})=(c\bar{c})=(e\bar{e})=(f\bar{f})]_{ad},$$
$$[(d\bar{d})=(e\bar{e})=(a\bar{a})=(b\bar{b})]_{cf},$$
$$[(f\bar{f})=(a\bar{a})=(c\bar{c})=(d\bar{d})]_{eb}.$$

Further, for the same initial energies and orbital states of the entrance-channel meson pairs, these combine with the previous equalities to yield:

$$\begin{aligned}&[(b\bar{b})=(c\bar{c})=(e\bar{e})=(f\bar{f})]_{ad}\\&=[(d\bar{d})=(e\bar{e})=(a\bar{a})=(b\bar{b})]_{cf}\\&=[(f\bar{f})=(a\bar{a})=(c\bar{c})=(d\bar{d})]_{eb}.\end{aligned}$$

These equalities among differential cross sections may be used to test unitary symmetry.

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[18] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Nuovo Cimento **23**, 236 (1962); also paper presented at the International Conference on Nucleon Structure, Stanford University, 1963 (unpublished).

[19] The experimentally observed s-wave production and the final-state interactions of $K\bar{K}$ by $\pi^+\pi^-$ should have their analogs in $\pi^-\pi^+$ and K^-K^+ production by $K^0\bar{K}^0$ and $K^0\bar{K}^0$ and $\pi^+\pi^-$ production by K^+K^-. However, at the energy of the experiment the violation of unitary symmetry manifested in mass differences may make comparisons with the predictions difficult. For the experimental observations, see A. R. Erwin, G. A. Hoyer, R. A. March, W. D. Walker, and T. P. Wangler, Phys. Rev. Letters **9**, 34 (1962); Gideon Alexander, Orin I. Dahl, Laurance Jacobs, George R. Kalbfleisch, Donald H. Miller, Alan Rittenberg, Joseph Schwartz, and Gerald A. Smith, Phys. Rev. Letters **9**, 460 (1962).