# Polarization Effects in the Production of Intermediate Bosons*

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For the coherent production of intermediate bosons by neutrinos or muons in the coulomb field of nuclei, we evaluate total cross sections, angular distributions, and spectra of the leptons appearing in the production reaction, and in particular polarization effects, e.g., the components of the polarization vector of the boson as a function of production angle and energy, and the angular distributions of charged leptons from decay in flight, which are characterized by the strong circular polarization of the boson. The angular distributions of $2\pi$ decay are obtained also. In the calculation, we use the Weizsäcker-Williams method in covariant form, another demonstration that this convenient calculatory shortcut lends itself perfectly to the evaluation of polarization effects.

## I. INTRODUCTION

EXPERIMENTAL evidence for the existence of the intermediate vector boson $W$, as postulated by Lee and Yang,[1] has not been unfavorable in the results of the well-known neutrino experiment of Danby *et al.*[2] If actually there is such a particle transmitting the weak interactions, one should be able to produce it relatively copiously with leptons, the cross section being proportional to the first power of the weak-interaction coupling constant. Neutrinos are preferable as a producing agent by their absence of strong or electromagnetic interactions, but muons may be suitable also since their electromagnetic cross sections are small. Total cross sections for $W$ production by neutrinos have been obtained by various workers[3–5]; the latter authors also stated the circular polarization of the boson.[6] In the present work, we obtain angular distributions, spectra and total cross sections of the production reactions

$$\nu+Z\rightarrow Z+\mu^-+W^+ \tag{1}$$

and

$$\mu^++Z\rightarrow Z+\bar{\nu}+W^+, \tag{2}$$

where the nucleus acts coherently, and also of the "charge conjugate" reactions of $\bar{\nu}$ (1*c*) and $\mu^-$ (2*c*) on $Z$. Further, we calculate angular distributions of the subsequent decays in flight,

$$W^+\rightarrow l^++\nu, \tag{3}$$

where $l$ is an electron or muon, as well as

$$W^+\rightarrow \pi^++\pi^0, \tag{4}$$

which may have a rate[7] comparable to reaction (3), and of the charge conjugate decays (3*c*), (4*c*). In addition, the polarization vector of $W$ is obtained as a function of production angle and energy, and it is seen that the $W^+$ in (1), (2) has a high degree of circular polarization, left-handed for process (1), but in case (2) with a sign depending on the anomalous magnetic moment of the $W$.

The method of calculation used is the Weizsäcker-Williams method in covariant form,[8] which provides an effective economy of calculation without appreciable loss of accuracy. The applicability of this method for the calculation of polarization effects, already pointed out earlier,[9,10] is again demonstrated here.

In Sec. II, the $l-\nu$ and the $2\pi$ decay of the intermediate boson are discussed, in particular if the latter is polarized. In Sec. III, we obtain cross sections for $W$ production by a neutrino or a muon on a free photon, which are needed in the Weizsäcker-Williams calculation. The covariant spectrum of equivalent photons is presented in Sec. IV. In Sec. V, three- and four-dimensional forms of the density matrix of the vector boson are discussed, and are subsequently utilized to obtain the results presented in Sec. VI, i.e., the angular distributions of charged particles in $W$ decay; there, the total cross sections and the spectra and angular distributions of leptons produced together with the boson are also given.

## II. DECAY OF THE $W$

The decay (3) is described by an interaction Lagrangian

$$\mathfrak{L}_{\rm int}=ig\psi_l^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_\nu\varphi_\lambda^\star+\text{H.c.}, \tag{5}$$

where $g$ is related to the vector coupling constant of the universal Fermi interaction, $G_V=10^{-5}m_p^{-2}$ ($m_p$= proton mass), by $g=2^{-\frac{1}{4}}G_V^{\frac{1}{2}}M$, with the mass $M$ of the intermediate boson. The symbol $\star$ on the boson field $\varphi_\lambda$ means Hermitean conjugate for $\lambda=1, 2, 3$, and minus

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957); Phys. Rev. Letters **4**, 307 (1960).
[2] G. Danby, J.-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962).
[3] T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Letters **7**, 429 (1961).
[4] V. V. Solov'ev and I. S. Tsukerman, Zh. Eksperim. i Teor. Fiz. **42**, 1252 (1962) [translation: Soviet Phys.—JETP **15**, 868 (1962)].
[5] J. S. Bell and M. Veltman, Phys. Letters **5**, 94 (1963).
[6] J. S. Bell and M. Veltman, Phys. Letters **5**, 151 (1963).
[7] J. Bernstein and G. Feinberg, Phys. Rev. **125**, 1741 (1962).
[8] A. M. Badalyan and Ya. A. Smorodinskii, Zh. Eksperim. i Teor. Fiz. **40**, 1231 (1961) [translation: Soviet Phys.—JETP **13**, 865 (1961)].
[9] S. Sarkar, Nuovo Cimento **21**, 410 (1961).
[10] A. M. Badalyan, Zh. Eksperim. i Teor. Fiz. **43**, 608 (1962) [translation: Soviet Phys.—JETP **16**, 436 (1963)].

Hermitean conjugate[11] for $\lambda=4$. We shall assume that the lepton in Eq. (5) is massless, since one may show that this leads to errors of order $(m/M)^2$ only, with $m$ the lepton mass, even for decay in flight. It follows then that the $l^{\pm}$ in Eq. (3), (3c) are created with 100% right- and lefthandedness, respectively, from the chirality conserving character of Eq. (5). Calling $q$ the four-momentum of the $W$, we find for the energy of the charged decay lepton

$$l=\tfrac{1}{2}M^2(q_0-\mathbf{q}\cdot\hat{l})^{-1}, \tag{6}$$

where $\mathbf{l}$ is its momentum, and $\hat{l}=\mathbf{l}/l$. The free boson satisfies a Proca equation, equivalent to a Klein-Gordon equation for its components and a subsidiary condition

$$\partial\varphi_\lambda/\partial x_\lambda=0. \tag{7}$$

One may therefore choose three independent polarization four-vectors (two transverse states of linear polarization, one longitudinal state),

$$\begin{aligned}\epsilon^{(1)}&=(\mathbf{a}\times\mathbf{q}/|\mathbf{a}\times\mathbf{q}|,0),\\ \epsilon^{(2)}&=[\mathbf{q}\times(\mathbf{a}\times\mathbf{q})/|\mathbf{q}\times(\mathbf{a}\times\mathbf{q})|,0],\\ \epsilon^{(3)}&=M^{-1}(q_0\mathbf{q}/|\mathbf{q}|,i|\mathbf{q}|),\end{aligned} \tag{8}$$

normalized to $\epsilon_\lambda{}^\star\epsilon_\lambda=1$, and satisfying

$$\sum_i \epsilon_\lambda{}^{(i)\star}\epsilon_\mu{}^{(i)}=\delta_{\lambda\mu}+M^{-2}q_\lambda q_\mu. \tag{9}$$

Circular basis states may be chosen as

$$\epsilon^{(\pm)}=2^{-1/2}(\epsilon^{(1)}\pm i\epsilon^{(2)}). \tag{10}$$

All of these satisfy $\epsilon^{(i)}\cdot q=0$. Equation (5) leads to a decay rate

$$\frac{dw}{d\Omega_l}=\frac{g^2l^2}{2\pi^2M^2q_0}(\epsilon^\star\cdot l\epsilon\cdot\nu+\epsilon^\star\cdot\nu\epsilon\cdot l-\epsilon^\star\cdot\epsilon l\cdot\nu+\epsilon_\alpha{}^\star\epsilon_\lambda l_\rho\nu_\sigma\epsilon_{\rho\alpha\sigma\lambda}) \tag{11}$$

with $\nu=q-l$ the neutrino four-momentum, for both reaction (3) and its charge conjugate. In the rest system of the $W$, we have thus an angular distribution

$$\frac{dw}{d\Omega_l}=\frac{G_VM^3}{2^4\pi^2\sqrt{2}}[1-(\boldsymbol{\varepsilon}\cdot\hat{l})^2] \tag{12}$$

for Eqs. (3) and (3c), if the boson is linearly polarized; i.e., the leptons come out predominantly normal to the boson polarization. For a circular polarization of the $W$ in the $\pm\boldsymbol{\varepsilon}^{(3)}$ direction, we find

$$\frac{dw}{d\Omega_l}=\frac{G_VM^3}{2^5\pi^2\sqrt{2}}[1\pm\boldsymbol{\varepsilon}^{(3)}\cdot\hat{l}]^2 \tag{13}$$

for $W^+$ decay, but with the $\pm$ sign replaced by $\mp$ for $W^-$ decay. Thus in Eq. (13), the charged lepton appears predominantly along the circular $W^+$ polarization direction (or opposite to it in $W^-$ decay), which agrees with the simple helicity arguments. The total decay rate is found from the above as[1]

$$w=G_VM^3/6\pi\sqrt{2}. \tag{14}$$

The matrix element for $2\pi$ decay of the boson may be written down[7] on the basis of the conserved vector current hypothesis[12]:

$$\mathfrak{M}=\frac{\sqrt{2}gF_\pi(M^2)}{(8q_0E_cE_0)^{1/2}}\epsilon_\lambda{}^\star Q_\lambda, \tag{15}$$

with $F_\pi(M^2)$ the pion form factor at the value of the $W$ mass, and $Q_\lambda=(p_c-p_0)_\lambda$, where $p_{c,0}$ are the four vectors of the charged and neutral pion in reactions (4), (4c). It leads to an angular distribution of $\pi^\pm$:

$$\frac{dw}{d\Omega_c}=\frac{g^2F_\pi{}^2(M^2)}{8\pi^2q_0}\frac{p_c{}^3}{M^2E_c-2m_\pi{}^2q_0}|\epsilon_\lambda{}^\star Q_\lambda|^2, \tag{16}$$

or again, with neglect of the pion mass $m_\pi$,

$$\frac{dw}{d\Omega_c}\cong\frac{g^2F_\pi{}^2(M^2)p_c{}^2}{8\pi^2M^2q_0}|\epsilon_\lambda{}^\star Q_\lambda|^2, \tag{17}$$

with

$$p_c=\tfrac{1}{2}M^2(q_0-\mathbf{q}\cdot\hat{p}_c)^{-1}. \tag{18}$$

In the $W$ rest system, and with a linearly polarized $W$, the angular distribution of $\pi^\pm$ is, from Eq. (16),

$$\frac{dw}{d\Omega_c}=\frac{g^2}{32\pi^2}MF_\pi{}^2\left(1-\frac{4m_\pi{}^2}{M^2}\right)^{3/2}(\boldsymbol{\varepsilon}\cdot\hat{p}_c)^2, \tag{19}$$

thus, the $\pi$'s being emitted along and against the polarization vector. Circularly polarized bosons give an angular distribution

$$\frac{dw}{d\Omega_c}=\frac{g^2}{64\pi^2}MF_\pi{}^2\left(1-\frac{4m_\pi{}^2}{M^2}\right)^{3/2}[1-(\boldsymbol{\varepsilon}^{(3)}\cdot\hat{p}_c)^2], \tag{20}$$

the $\pi$'s coming out in a plane normal to the direction of circular polarization. Equations (19), (20) apply to both $W^+$ and $W^-$ decay. The total decay rate follows as

$$w=\frac{G_VM^3F_\pi{}^2(M^2)}{24\pi\sqrt{2}}\left(1-\frac{4m_\pi{}^2}{M^2}\right)^{3/2}, \tag{21}$$

as shown earlier.[7]

### III. *W* PRODUCTION ON A FREE PHOTON

A recent investigation of the field theory of vector bosons is due to Lee and Yang.[13] A boson-photon three-vertex (boson $p,\alpha\rightarrow p',\beta$ with emission or absorption

[11] G. Wentzel, *Einführung in die Quantenthorie der Wellenfelder* (Franz Deuticke, Vienna, 1943), p. 72.

[12] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1957).

[13] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

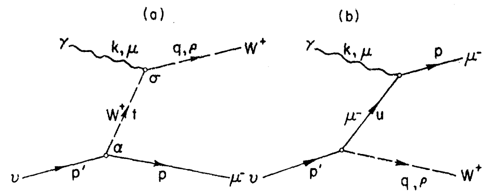


FIG. 1. Feynman diagrams for reaction (1).

of a photon $k$, $\mu$, polarization four-vector $e_\mu$) is described by

$$\mathcal{L}_{\rm int}=e\epsilon_\alpha\epsilon_\beta{}^\star e_\mu[\delta_{\alpha\beta}(p+p')_\mu-\delta_{\alpha\mu}(p+\kappa(p-p'))_\beta-\delta_{\beta\mu}(p'+\kappa(p'-p))_\alpha], \quad (22)$$

$\kappa$ being the anomalous magnetic moment of the $W$, and the boson propagator may be taken as

$$S_{\mu\nu}=-i(\delta_{\mu\nu}+M^{-2}p_\mu p_\nu)/(p^2+M^2). \quad (23)$$

In lowest order, the two diagrams of Fig. 1 contribute to the process [needed in the Weizsäcker-Williams treatment of reaction (1)]

$$\nu+\gamma\rightarrow\mu^-+W^+ \quad (24)$$

and give rise to a matrix element[14]

$$\mathfrak{M}=-ege_\mu\epsilon_\rho{}^\star u_l{}^\dagger(p)\gamma_4T_{\mu\rho}(1+\gamma_5)u_\nu(p'), \quad (25)$$

$$T_{\mu\rho}=(\Gamma_\mu)_{\rho\sigma}\frac{\delta_{\alpha\sigma}+M^{-2}t_\alpha t_\sigma}{t^2+M^2}\gamma_\alpha+\gamma_\mu\frac{1}{u_\lambda\gamma_\lambda-im}\gamma_\rho, \quad (26)$$

$$(\Gamma_\mu)_{\rho\sigma}=-(q+t)_\mu\delta_{\rho\sigma}+[t+\kappa(t-q)]_\rho\delta_{\mu\sigma}+[q+\kappa(q-t)]_\sigma\delta_{\mu\rho}, \quad (27)$$

$$t=q-k=p'-p,\quad u=p-k=p'-q.$$

The assignment of the momenta may be read off from the diagrams in Fig. 1; in particular, $p'$ is the initial neutrino. This matrix element may be shown to be gauge invariant.

Cross sections for reaction (2) have also been obtained before.[4,15] To calculate it using the Weizsäcker-Williams method, we need to consider the reaction

$$\mu^++\gamma\rightarrow\bar\nu+W^+, \quad (28)$$

whose matrix element is given by the two diagrams of Fig. 2 as

$$\mathfrak{M}'=-ege_\mu\epsilon_\rho{}^\star v_l{}^\dagger(p_+)\gamma_4T_{\mu\rho}'(1+\gamma_5)v_\nu(\bar p), \quad (29)$$

with $v$ the negative-energy spinors,

$$s'=p_++k=\bar p+q,$$

and

$$T_{\mu\rho}'=(\Gamma_\mu)_{\rho\sigma}\frac{\delta_{\alpha\sigma}+M^{-2}t_\alpha t_\sigma}{t^2+M^2}\gamma_\alpha+\gamma_\mu\frac{-1}{s_\lambda'\gamma_\lambda+im}\gamma_\rho. \quad (30)$$

[14] We use Pauli's notation for four-products and Dirac matrices; e.g., $a\cdot b=\mathbf{a}\cdot\mathbf{b}+a_4b_4$, $a_4=ia_0$.
[15] M. E. Ebel and W. D. Walker, Phys. Rev. 122, 1639 (1961).

With the notation $P=(p,p_+)$, $P'=(p',\bar p)$, $X=(u,s')$ appropriate for reactions (24) and (28), respectively, neglecting terms of order $(m/M)^2$ and using the dirac equations and the relations $e\cdot k=0$, $\epsilon\cdot q=0$, Eqs. (26) and (30) may be simplified and written in unified form as

$$T_{\mu\rho}=\frac{-2q_\mu\gamma_\rho+(1+\kappa)(\delta_{\mu\rho}k_\alpha\gamma_\alpha-k_\rho\gamma_\mu)}{t^2+M^2}+\frac{\pm2P_\mu-\gamma_\mu k_\lambda\gamma_\lambda}{X^2+m^2}\gamma_\rho; \quad (31)$$

the upper (lower) sign will always refer to the $\nu(\mu)$ initiated reaction. The cross sections for reactions (24), (28), after summation (or averaging) over photon and lepton polarizations, are obtained in invariant form as

$$d\sigma_\gamma=\frac{\alpha G_V M^2}{4\sqrt2}\frac{dt^2}{s^4}\frac{d\varphi}{2\pi}\epsilon_\rho\epsilon_\sigma{}^\star Q_{\sigma\rho}, \quad (32)$$

$$d\sigma_\gamma'=\frac{\alpha G_V M^2}{8\sqrt2}\frac{dt^2}{s'^4}\frac{d\varphi}{2\pi}\epsilon_\rho\epsilon_\sigma{}^\star Q_{\sigma\rho}, \quad (33)$$

with $e^2=4\pi\alpha=4\pi/137$, and where the $W$ polarization is still unspecified. The role of the azimuth $\varphi$ will be discussed later. The additional factor $\frac12$ in Eq. (33) comes from the initial spin average for unpolarized muons. We have

$$Q_{\sigma\rho}=\mathrm{Tr}(1+\gamma_5)\Lambda_\nu(P')\bar T_{\mu\rho}\Lambda_l(P)T_{\mu\sigma}, \quad (34)$$

where $\bar T_{\mu\rho}$ differs from $T_{\mu\rho}$ by the replacement $\gamma_\mu\gamma_\lambda\gamma_\rho\rightarrow\gamma_\rho\gamma_\mu\gamma_\lambda$, and with lepton projection operators

$$\Lambda_\nu(P')=-iP_\alpha'\gamma_\alpha,$$
$$\Lambda_l(P)=-i(P_\alpha\gamma_\alpha\pm im)\cong-iP_\alpha\gamma_\alpha, \quad (35)$$

the latter approximation leading again only to the neglect of terms $\sim(m/M)^2$.

If one evaluates the matrix elements for the reactions (24c) and (28c) with charge conjugate leptons and bosons, one obtains exactly the same results as above, the only difference being in the sign of terms proportional with $m$, which were neglected by us. This result agrees with Theorem 1 of Lee, Markstein, and Yang.[3]

The expression (34) consists of an even and odd part,

$$Q_{\sigma\rho}=Q_{\sigma\rho}{}^{(e)}+Q_{\sigma\rho}{}^{(o)}, \quad (36)$$

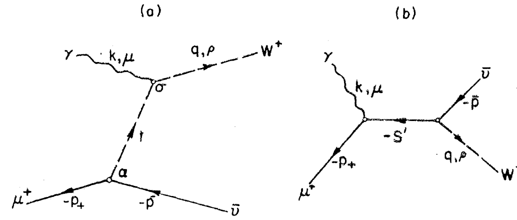


FIG. 2. Feynman diagrams for reaction (2).

which are given by

$$\tfrac{1}{16}Q_{\sigma\rho}{}^{(i)}=\frac{A_{1\sigma\rho}{}^{(i)}+(1+\kappa)A_{2\sigma\rho}{}^{(i)}+(1+\kappa)^2A_{3\sigma\rho}{}^{(i)}}{(l^2+M^2)^2}+\frac{B_{1\sigma\rho}{}^{(i)}\pm\bar{B}_{1\sigma\rho}{}^{(i)}+(1+\kappa)(B_{2\sigma\rho}{}^{(i)}\pm\bar{B}_{2\sigma\rho}{}^{(i)})}{(l^2+M^2)(X^2+m^2)}+\frac{C_{\sigma\rho}{}^{(i)}}{(X^2+m^2)^2}. \quad (37)$$

The terms $A_{k\sigma\rho}{}^{(i)}$, etc., are listed in the Appendix.

## IV. EQUIVALENT PHOTON SPECTRUM

The conventional Weizsäcker-Williams method is inapplicable to our case since there exists no rest system for the incident neutrino. The exact covariant formulation of photon exchange reactions given by Gribov *et al.*[16] can also not be used, since its derivation assumes a summing over the polarization of the final particles and does not seem immediately generalizable to the polarized case. The covariant Weizsäcker-Williams method of Badalyan and Smorodinskii,[8] however, is appropriate for polarization calculations[10] and shall be used here. We shall also show that its results agree for high energies ($p'\gg M$, calling $p'=|\mathbf{p}'|$) with Gribov's results. Since it was derived for initial particles with mass, we shall trivially reformulate it here for incident massless particles.

The idea is to obtain a cross section of type of Eq. (1) by multiplying the cross section of Eq. (24) on a free photon by an equivalent photon spectrum representing the nucleus $Z$ and integrating over it; the exchanged photon is thus treated as being on the energy shell. The Fourier components of the field contributing the exchanged photon,

$$A(k)=2\pi Zek^{-2}P\delta(k\cdot P)F(k^2), \quad (38)$$

with $P=P_1+P_2$, $P_{1,2}$ being the momenta of the nucleus before and after the photon exchange (and $k=P_1-P_2$), and $F$ being the nuclear form factor, may be gauge-transformed into

$$A(k)=2\pi Zek^{-2}\delta(k\cdot P)\mathfrak{F}e'F(k^2), \quad (39)$$

with

$$\mathfrak{F}^2=P^2+k^2(s\cdot P)^2(s\cdot k)^{-2}, \quad (40)$$

and the space-like polarization four-vector of the pseudophoton (which is thus linearly polarized) is

$$e'=\mathfrak{F}^{-1}(P-ks\cdot P(s\cdot k)^{-1}); \quad (41)$$

the four-vector $s$ is given by

$$s=p'+k, \quad (42)$$

$p'$ being the incident particle in the photon-exchange reaction. We find the properties $e'\cdot e'=1$ and

$$e'\cdot s=0, \quad (43)$$

i.e., the photon has no scalar component in the center-of-momentum (CM) system of $p'$ and $k$, where $\mathbf{s}=0$. The Lorentz condition is however no longer satisfied in our new gauge, i.e., $e'\cdot k\neq 0$. The energy of the pseudophotons $k_0$ in the CM system may be written invariantly as

$$\omega=-s\cdot k(-s^2)^{-1/2}, \quad (44)$$

and the number of equivalent photons in the invariant interval $d^4k$ is therefore

$$dN(s^2,k^2)=I(\omega,k^2)d^4k=\frac{-1}{\omega s^2}\int \tau_{\mu\nu}s_\mu s_\nu dV, \quad (45)$$

integrated over a space-like 3-surface orthogonal to $s$, with $\tau_{\mu\nu}$ the energy-momentum tensor of the electromagnetic field, since Eq. (45) reduces to $\int\tau_{00}dV/k_0$ in the CM system. If the expression for $\tau_{\mu\nu}$ is inserted, we find

$$dN(s^2,k^2)=\frac{Z^2\alpha}{2\pi^2}\frac{F^2(k^2)}{k^4}\mathfrak{F}^2\frac{s\cdot k}{s\cdot P}\left(1-\frac{1}{2}\frac{s^2k_1{}^2}{(s\cdot k)^2}\right)\times\delta(k\cdot P)d^4k, \quad (46)$$

where

$$k_1=k-e'(e'\cdot k), \quad (47)$$

and the last two factors may be rewritten as

$$d^4k\delta(k\cdot P)=\delta(P_2{}^2+M_N{}^2)d^4P_2=\frac{-ds^2dk^2d\phi}{8[(p'\cdot P_1)^2-p'^2P_1{}^2]^{1/2}}, \quad (48)$$

with $\phi$ the azimuth of $\mathbf{P}_2$; integration over it will result in an averaging over the equivalent photon polarization [as was already done in obtaining Eqs. (32), (33)], as shown in the following. If we write

$$dN=dN'(d\phi/2\pi), \quad (49)$$

then the Weizsäcker-Williams cross section is given by

$$d\sigma_{WW}=2dN'\int\frac{d\phi}{2\pi}(d\sigma_\gamma)_{\text{pol}}, \quad (50)$$

where $(d\sigma_\gamma)_{\text{pol}}$ is the cross section of $\gamma+p'\to$ final state, with a linearly polarized $\gamma$ on the energy shell ($k^2=0$); the factor 2 follows from considerations of incident flux.

It may be shown now that, as far as calculation of $d\sigma_\gamma$ is concerned (where $k^2\cong 0$), the equivalent quanta travel along $\mathbf{P}_1$, have their linear polarization vector $\mathbf{e}'$ normal to $\mathbf{P}_1$, and that $\mathbf{e}'$ lies in the $\mathbf{P}_1$, $\mathbf{k}$ plane ($\mathbf{k}$ being here considered the nuclear recoil),[10] which in the laboratory system is also the plane of $\mathbf{p}'$ and the nuclear

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[16] V. N. Gribov, V. A. Kolkunow, L. B. Okun', and V. M. Shekhter, Zh. Eksperim. i Teor. Fiz. **41**, 1839 (1961) [translation: Soviet Phys.—JETP **14**, 1308 (1962)].

recoil, or the plane of $\mathbf{p}'$ and $\mathbf{P}_2$, since the aximuth $\phi$ is invariant under the Lorentz transformation from the CM to the laboratory system. For the proof, write $P_2=P_1-k$ and obtain

$$k^2=2P_1\cdot k\,, \tag{51}$$

from which follows[17]

$$k_0=\mathbf{v}\cdot\mathbf{k}-(k^2/2E_1)\,, \tag{52}$$

where $\mathbf{v}=\mathbf{P}_1/E_1$. Since in $d\sigma_\gamma$, $k^2\cong 0$, we get

$$k_0=\mathbf{v}\cdot\mathbf{k}\,. \tag{53}$$

Decomposing $\mathbf{k}$ into components parallel and normal to $\mathbf{v}$,

$$\mathbf{k}=\mathbf{k}_{\parallel}+\mathbf{k}_\perp\,,\quad \mathbf{k}_{\parallel}=\mathbf{v}k_0v^{-2}\,, \tag{54}$$

we may write

$$k^2\cong 0=k_\perp^2+k_0^2(v^{-2}-1)\,; \tag{55}$$

in a system with $v\cong 1$, $k_0$ need not be $\cong 0$, but we must have

$$k_\perp\cong 0\,, \tag{56}$$

or $\mathbf{k}\|\mathbf{P}_1$. Naively, $\mathbf{k}$ could be thought of being the propagation direction of the pseudophotons, and the first statement were proved. However, since now in the CM system $\mathbf{e}'\cdot\mathbf{k}\neq 0$, the photon polarization were not transverse. We must thus, following Smorodinskii,[8] consider $\mathbf{k}_1$ (with $\mathbf{k}_1\cdot\mathbf{e}'=0$) as the propagation direction. But since we may rewrite

$$k_1=\mathfrak{F}^{-2}[kP^2+k^2(s\cdot P)(s\cdot k)^{-1}(2P_1-k)]\,, \tag{57}$$

and using (56) for $d\sigma_\gamma$, $\mathbf{k}_1$ is $\|\mathbf{P}_1$, proving the first statement. Further, $\mathbf{e}'$ lies in the $\mathbf{P}_1$, $\mathbf{k}$ plane from its definition (41), and since $k_1\cdot e'=0$ and $d\sigma_\gamma$ "sees" $\mathbf{k}_1\|\mathbf{P}_1$, $\mathbf{e}'$ is $\perp\mathbf{P}_1$. Taking, then, $\phi$ as the azimuth between the $\mathbf{p}'$, $\mathbf{e}'$ plane and the $\mathbf{p}'$, $\mathbf{P}_2$ plane, and carrying out the integration over $d\phi$, we obtain from (50):

$$d\sigma_{WW}=2dN'(d\sigma_\gamma)_{\text{unpol}}\,. \tag{58}$$

The expression for $dN'$ may be shown to agree with the term containing $a$ of Gribov *et al.*[16] in the high-energy limit, $p'\gg M$ [the $b$ term of Gribov has to be disregarded anyway since $b$ is unknown; in our case, it may be considered negligible if the leptons in reactions (1), (2) are muons rather than electrons,[4] and for this reason, we shall always think of muons produced with the $W$ only]: From the energy denominators in $d\sigma_\gamma$, it follows that in reactions (1), (2), the orders of magnitude of $k$ are in the laboratory system (with $M_N$ the mass of the nucleus):

$$k_{\parallel}\sim M^2/p'\,,\quad k_\perp\sim M\,,\quad k_0\sim M^2/M_N\,; \tag{59}$$

(thus, if we take a heavy nucleus, $A^{-1}\ll 1$, we have $k_0\ll|\mathbf{k}|$, and moreover $p'\cong|\mathbf{p}|+q_0$); we then obtain

$$-s\cdot P_1\gg -s\cdot k\,,\quad (s\cdot P_1)^2\gg s^2P_1^2\,, \tag{60}$$

and considering also the magnitude of the limits in the Weizsäcker-Williams integration over $dk^2$, which, as discussed later, are determined by the nuclear form factor and are of order $(M^2/p')^2$ (lower) and $m_\mu^2$ (upper limit), we finally find

$$d\sigma_{WW}\cong\frac{Z^2\alpha}{2\pi}\frac{F^2(k^2)}{k^2}(d\sigma_\gamma)\left[1+\frac{P_1^2(p'\cdot k)^2}{k^2(P_1\cdot p')^2}\right]\frac{ds^2}{p'\cdot k}dk^2\,, \tag{61}$$

in agreement with Ref. 16 if $p'\gg M$ [so that the additional term $(p'\cdot k)/(p'\cdot P_1)$ in the square bracket of Ref. 16 may also be dropped], which we shall always assume in the following. The way we derived the pseudophoton spectrum, however, does not place any restrictions on the polarizations of the outgoing particles, in contrast to Ref. 16.

Limits on the $s^2$ and $k^2$ integrations are given kinematically $(s=p'+k)$ and from the properties of the nuclear form factor $F(k^2)$. If we assume $F=0$ for $k^2\geqslant K^2$, we find, using the approximations after Eq. (59):

$$(M+m)^2\leq -s^2\leq 2p'K\,, \tag{62}$$

$$(s^2/2p')^2\leq k^2\leq K^2\,. \tag{63}$$

We shall now insert $d\sigma_\gamma$ from Eq. (32) (for the case of the $\mu$-initiated reaction, $d\sigma_\gamma'$, the treatment is almost identical and shall not be described here; the main difference is the additional factor $\frac{1}{2}$.) The calculation will be done for the specific target material copper, $Z=29$, $A=63.5$; the corresponding form factor (of Fermi type) is given graphically in Ref. 5 and may be approximated by the polynomial

$$F(k^2)=1-3(k/K)^2+2(k/K)^3\,, \tag{64}$$

with $K=0.2132m_p$. We find, carrying out the $k^2$ integration:

$$d\sigma_{WW}=\frac{Z^2\alpha^2G_VM^2}{2\pi\sqrt{2}}\frac{ds^2}{s^6}\frac{d\varphi}{2\pi}Bdt^2\epsilon_\rho\epsilon_\sigma{}^\star Q_{\sigma\rho}\,, \tag{65}$$

with

$$\begin{aligned}B=&(1+6Y^2)\ln Y+\tfrac{1}{2}(1-Y^2)+4Y^2(1-Y)\\&+\tfrac{3}{2}(2+3Y^2)(1-Y^2)-\tfrac{4}{3}(1+3Y^2)(1-Y^3)\\&-\tfrac{1}{4}(9-4Y^2)(1-Y^4)+(12/5)(1-Y^5)\\&-\tfrac{2}{3}(1-Y^6)\,,\end{aligned} \tag{66}$$

$$Y=-s^2/M^2\xi\,, \tag{67}$$

and with the initial energy parameter

$$\xi=2p'K/M^2\,. \tag{68}$$

Since Eq. (65) is in invariant form, we may evaluate it in the laboratory system $(\mathbf{P}_1=0)$. The laboratory quantities $|\mathbf{p}|$, $|\mathbf{k}|$, and $\vartheta$ needed in $d\sigma_\gamma$ are found from $k^2=0$ and the statements after Eq. (50) by solving

$$\begin{aligned}|\mathbf{p}'|+|\mathbf{k}|&=p_0+q_0\,,\\|\mathbf{p}'|-|\mathbf{k}|&=|\mathbf{p}|\cos\vartheta+|\mathbf{q}|\cos\theta\,,\\|\mathbf{p}|\sin\vartheta&=|\mathbf{q}|\sin\theta\,,\end{aligned} \tag{69}$$

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[17] I. Ya. Pomeranchuk and I. M. Shmushkevich, Nucl. Phys. **23**, 452 (1961).

with $\theta=\measuredangle(\mathbf{p}',\mathbf{q})$, $\vartheta=\measuredangle(\mathbf{p}',\mathbf{p})$, and considering $|\mathbf{p}'|$, $|\mathbf{q}|$, and $\theta$ as given. Note that the azimuth $\varphi$ remained invariant under the Lorentz transformation along $\mathbf{p}'$ that brought the nucleus $\mathbf{P}_1$ to rest. We now have to go over from variable $ds^2dt^2$ to $dxd\gamma$, where

$$x=|\mathbf{q}|/|\mathbf{p}'|\,,\quad \gamma=\cos\theta\,. \tag{70}$$

The Jacobian of the transformation is found to be

$$ds^2dt^2=[4s^2\mathbf{p}'^2x^2/x_0(2-x_0-x\gamma)]dxd\gamma \tag{71}$$

with $x_0=q_0/p'$, and one obtains the limits

$$\begin{aligned}\xi^{-1}\leqslant x\leqslant 1\,,\\ 1-(M^2/\mathbf{p}'^2)[(1-x)(x\xi-1)/2x^2]\leqslant\gamma\leqslant 1\,.\end{aligned} \tag{72}$$

The differential cross section of reaction (1) [and similarly (2)] is found immediately from Eq. (65) using Eq. (9). We may still remark that for $p'\gg M$, the angular distributions are mostly forward, since from the resonance denominators in $Q_{\sigma\rho}$ follows the order-of-magnitude estimate

$$\theta\lesssim\frac{M}{|\mathbf{q}|}\left(\frac{|\mathbf{p}|}{|\mathbf{p}'|}\right)^{1/2},\quad \vartheta\lesssim\frac{M}{(|\mathbf{p}|\,|\mathbf{p}'|)^{1/2}}\,. \tag{73}$$

If one considers polarization effects, introduction of the density matrix is in order.

### V. DENSITY MATRIX OF $W$

The intermediate boson has spin 1, it is however represented in covariant formulation by a four-vector $\varphi_\lambda$; thus, although a $3\times3$ density matrix $\rho'$ should be sufficient to describe the $W$, we have in our calculation formally to deal with a $4\times4$ Hermitean density matrix of the $W$ produced in reaction (1):

$$\rho=\tau\rho_{\text{in}}\tau^\dagger\,, \tag{74}$$

where $\tau$ is the transition matrix, and the density matrix of the initial state $\rho_{\text{in}}=1$ in our case. For a pure state $i$, the density matrix is

$$\rho_i=\epsilon^{(i)}\epsilon^{(i)\star}\,, \tag{75}$$

of trace 1; a general density matrix is a superposition of such,

$$\rho=\sum_n c_n\rho_n \tag{76}$$

with $\sum c_n=1$. The density matrix of the $W$ produced in reaction (1) is, from Eq. (65),

$$\rho_{\alpha\beta}=\left[B(\mathrm{Tr}Q+M^{-2}q_\rho q_\sigma Q_{\rho\sigma})\frac{x^2}{2-x_0-x\gamma}\frac{dxd\gamma}{s^4x_0}\frac{d\varphi}{2\pi}\right]^{-1}\times BQ_{\alpha\beta}\frac{x^2}{2-x_0-x\gamma}\frac{dxd\gamma}{s^4x_0}\frac{d\varphi}{2\pi}, \tag{77}$$

with all those factors retained in numerator and denominator that contain variables to be integrated over. The probability to find a $W$ in a specific pure state is given by

$$P_i=\mathrm{Tr}\rho_i\rho\,. \tag{78}$$

To find the probability for decay in flight of the $W$ produced in reaction (1), one gets from Eqs. (11) or (17)

$$dw/d\Omega_{l,c}=\mathrm{Tr}\rho R\,, \tag{79}$$

where for $l-\nu$ decay

$$R_{\alpha\beta}=(g^2l^2/2\pi^2M^2q_0)(l_\alpha\nu_\beta+\nu_\alpha l_\beta-\delta_{\alpha\beta}l\cdot\nu+l_\rho\nu_\sigma\epsilon_{\rho\alpha\sigma\beta})\,, \tag{80}$$

for $2\pi$ decay,

$$R_{\alpha\beta}=(g^2F_\pi{}^2(M^2)p_c{}^2/8\pi^2M^2q_0)Q_\alpha Q_\beta\,. \tag{81}$$

From these equations, the decay angular distributions have been calculated. It is gratifying to note that $q_\alpha R_{\alpha\beta}=0$, since then the terms in $\rho$ of Eq. (77) that contain a $q_\alpha$ give no contribution; a general $R_{\alpha\beta}$ is a superposition of expressions $\epsilon_\alpha\epsilon_\beta{}^\star$ and thus has to satisfy $q_\alpha R_{\alpha\beta}=0$, since $\epsilon\cdot q=0$. Strictly speaking, terms in $\rho$ that do not vanish when contracted with $q$ would have to be removed from Eq. (77) by a projection operator.

For the question of the polarization state of the produced $W$, one preferably deals with the $3\times3$ density matrix $\rho'$. In entirely differential form, $\rho'$ reads

$$\rho_{ij}'=\epsilon_\alpha{}^{(i)\star}\epsilon_\beta{}^{(j)}Q_{\alpha\beta}(\mathrm{Tr}Q+M^{-2}q_\rho q_\sigma Q_{\rho\sigma})^{-1}\,, \tag{82}$$

of unit trace, from which the probability of a pure state $m$ of density matrix

$$(\rho'_m)_{ij}=\epsilon^{(i)\star}\cdot\epsilon^{(m)}\epsilon^{(m)\star}\cdot\epsilon^{(j)} \tag{83}$$

may again be found by

$$P_m=\mathrm{Tr}\rho_m'\rho'\,. \tag{84}$$

In the calculation, we take as basis vectors the right-handed set

$$\begin{aligned}\epsilon^{(1)}&=(\mathbf{p}'\times\mathbf{q}/|\mathbf{p}'\times\mathbf{q}|,0)\,,\\ \epsilon^{(2)}&=[\mathbf{q}\times(\mathbf{p}'\times\mathbf{q})/|\mathbf{q}\times(\mathbf{p}'\times\mathbf{q})|,0]\,,\\ \epsilon^{(3)}&=(q_0\mathbf{q}/M|\mathbf{q}|,i|\mathbf{q}|/M)\,.\end{aligned} \tag{85}$$

The expression (82) must be compared with the general 8-parameter form of the $3\times3$ density matrix[18]

$$\rho'=\tfrac{1}{3}[1+\boldsymbol{\alpha}\cdot\mathbf{t}+\beta_{ij}(t_it_j+t_jt_i)] \tag{86}$$

with $\mathbf{t}$ the operators of angular momentum of unity, $\boldsymbol{\alpha}$, the polarization pseudovector, and $\beta_{ij}$ the 5-parameter symmetric traceless quadrupolarization tensor. One finds, e.g., for a pure state of circular polarization in the $\pm3$ direction:

$$\begin{aligned}\alpha_1=\alpha_2=0\,,\quad \alpha_3=\pm\tfrac{3}{2}\,;\\ \beta_{11}=\beta_{22}=-\tfrac{1}{4}\,,\quad \beta_{33}=\tfrac{1}{2}\,;\quad \beta_{ij}=0(i\neq j)\,.\end{aligned} \tag{87}$$

[18] R. H. Dalitz, Proc. Phys. Soc. (London) A65, 172 (1952).

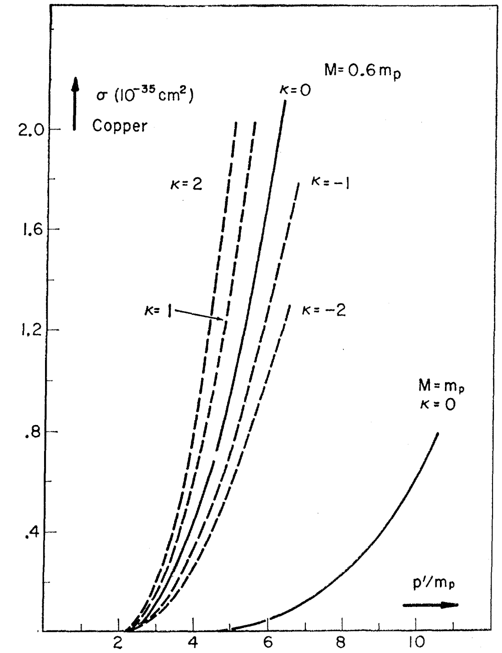


FIG. 3. Total cross section of reaction (1), $\nu+Z\to Z+\mu^-+W^+$ in a Cu target.

We have also calculated this polarization vector, whose components may be written as

$$\alpha_i=-3\,\mathrm{Im}\rho_{jk}'\ (ijk\ \text{cyclic}) \tag{88}$$

for the produced $W$, using the basis of Eq. (85).

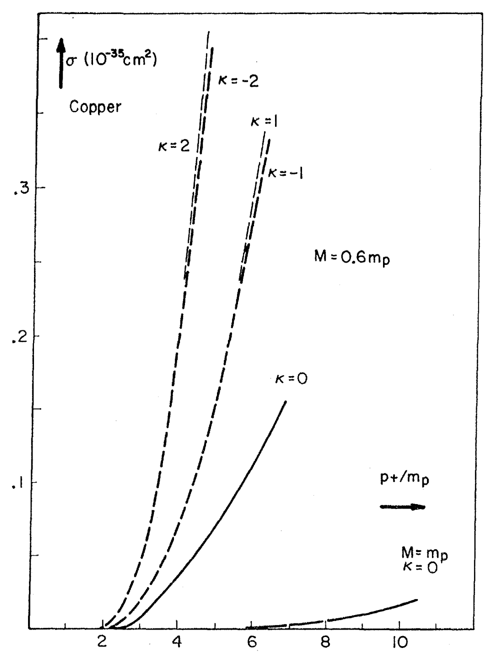


FIG. 4. Total cross section of reaction (2), $\mu^++Z\to Z+\bar{\nu}+W^+$, in a Cu target.

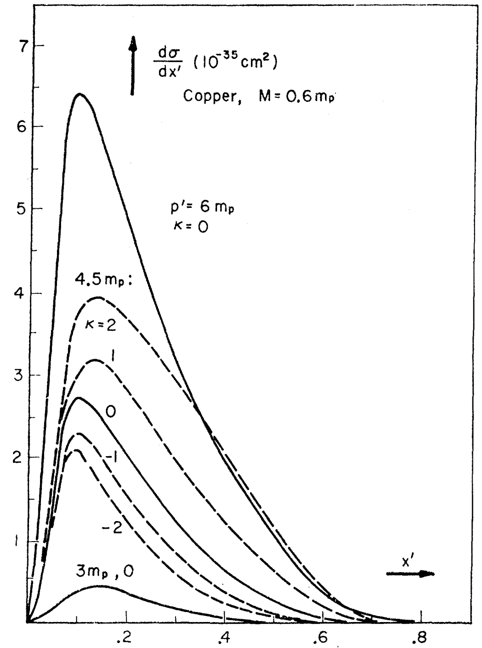


FIG. 5. Spectra of $\mu$ produced in reaction (1), for $M=0.6m_p$.

## VI. RESULTS AND DISCUSSION

Formula (65), with $\epsilon_\rho\epsilon_\sigma{}^\star$ replaced by Eq. (9), gives the angular distributions, spectra, and after integration, the total cross section of the $W$ production reactions. Figure 3 presents the total cross section of reaction (1) in a Cu target, for two values $M=0.6m_p$ and $1m_p$ and plotted versus $p'/m_p$. The numerical values are in good

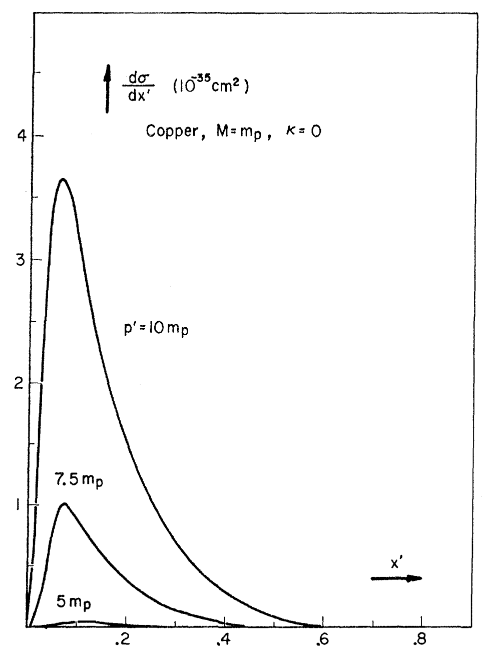


FIG. 6. Spectra of $\mu$ produced in reaction (1), for $M=m_p$.

agreement with those of Ref. 5 (for $\kappa=0$),[19] which demonstrates the accuracy of the Weizsäcker-Williams method. One notices that there is a strong dependence on the anomalous magnetic moment $\kappa$ of the $W$. Figure 4 presents the total cross section of reaction (2), which turns out to be much smaller than that of reaction (1). This is however only a threshold effect [caused by the difference of the diagrams of Figs. (1b) and (2b): diagram (1b) gives a much larger contribution than diagram (2b) near threshold], whereas the asymptotic expressions for $\sigma_{WW}$ of reactions (1) and (2) for $\xi\rightarrow\infty$ (in the second case, $\xi=2p_+K/M^2$, calling $p_+=|\mathbf{p}_+|$) agree with each other except for the additional factor $\frac{1}{2}$ for reaction (2): We have[3]

$$\sigma_{WW}\rightarrow(Z^2\alpha^2G_V/6\pi\sqrt{2})(\kappa-1)^2\ln^3\xi. \tag{89}$$

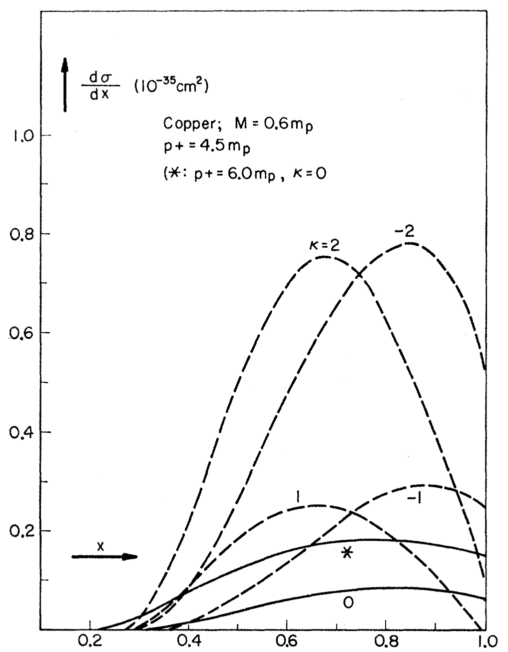


FIG. 7. Spectra of $W$ produced in reaction (2).

One should note, however, that this asymptotic form is of rather academic interest only, being valid at those extremely large energies where $\ln^3\xi\gg\ln^2\xi$.

In this work, no incoherent processes have been considered, i.e., the reactions (1), (2) occurring in the field of a single proton which is knocked out of the nucleus in the course of the process. Only near threshold would this reaction be of importance: from Ref. 5, ratios of incoherent/coherent total cross sections for reactions

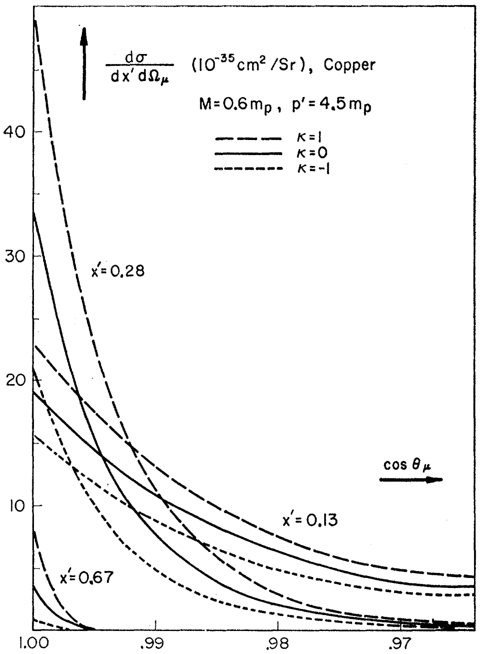


FIG. 8. Angular distributions of muon in reaction (1).

(1) are $\sim$0.6 for $M=m_p$, $p'=10m_p$ and $\sim$0.40 and 0.25, for $M=0.6m_p$ and $p'=4.5m_p$ and $6m_p$, respectively. The emitted proton may possibly be observed.

Figures 5 and 6 show laboratory spectra of the muon produced in reaction (1) plotted versus $x'=|\mathbf{p}|/p'$; one sees that the muon is generally of low energy, and the $W$ thus of high energy; the $W$ spectra are approximately the same if $x'$ is replaced by $1-x$, $x=|\mathbf{q}|/p'$. The polari-

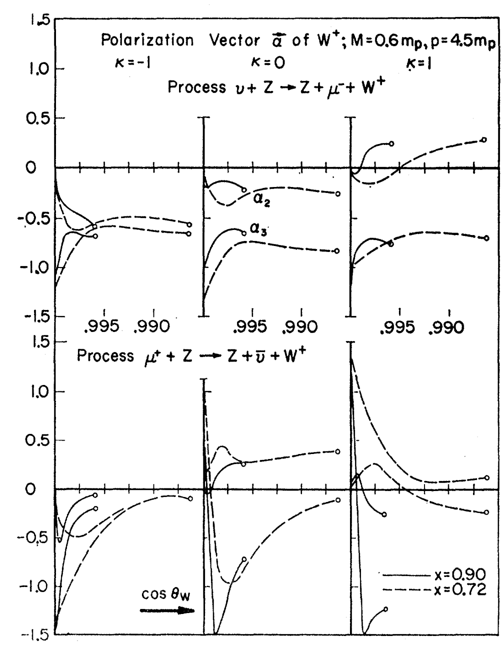


FIG. 9. Polarization vector of boson in reactions (1) and (2).

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[19] *Note added in proof.* Our results are below those of Ref. 5 by less than 10%, between $p'/m_p=3$ and 10. Disagreements reported between exact calculations of $W$ pair production by photons [W. Williamson and G. Salzman, Phys. Rev. Letters **11**, 224 (1963)] and earlier Weizsäcker-Williams results are due to the fact that the latter represent asymptotic expressions, valid at $\gtrsim10^3$ BeV only.

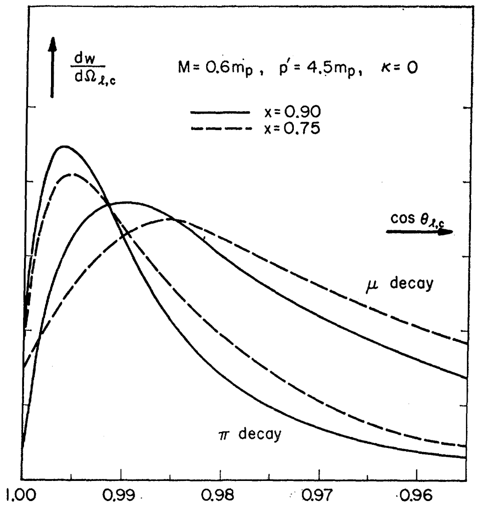


FIG. 10. Angular distribution of charged decay lepton and pion from boson produced in reaction (1).

zation of the produced $\mu^-$ is almost 100% left-handed [or 100% right-handed for $\mu^+$ in reaction (1c)], from the structure of the matrix element in Eq. (25).

In Fig. 7, we plotted the $W$ spectra of reaction (2); they are much broader than for reaction (1), and we see that the near equality of total cross sections for $\kappa=\pm|\kappa|$ is probably accidental, since the spectra are rather different.

Finally, Fig. 8 presents angular distributions of the produced $\mu$ in reaction (1), which are strongly forward, ve estimated above.

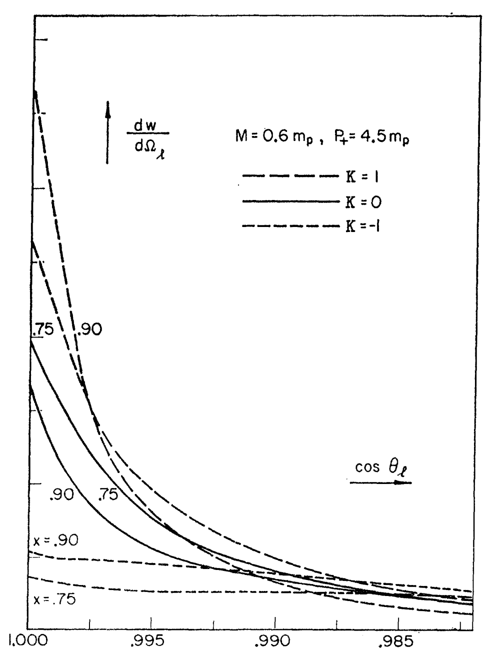


FIG. 11. Angular distribution of charged decay lepton from boson produced in reaction (2).

The polarization vector $\boldsymbol{\alpha}$ of the boson has been evaluated from Eq. (88) in the differential case; we find the first component $\alpha_1\equiv 0$ [with respect to the basis of Eq. (85)], which is to be expected from symmetry. The remaining two components are shown in Fig. 9; we see that in reaction (1), the boson $W^+$ has a large degree of left-handed circular polarization [right-handed for the $W^-$ in reaction (1c) where $\boldsymbol{\alpha}$ has the opposite sign], which is especially large in the forward direction. In reaction (2), $W^+$ is still strongly circularly polarized, but the sign of the polarization depends on the anomalous boson moment. Again, the $W^-$ circular polarization is opposite to that of the $W^+$. Here and in other figures, we present the results at $p'$ or $p_+=4.5m_p$ and for $M=0.6m_p$ only, but they do not change qualitatively at other values of these parameters.

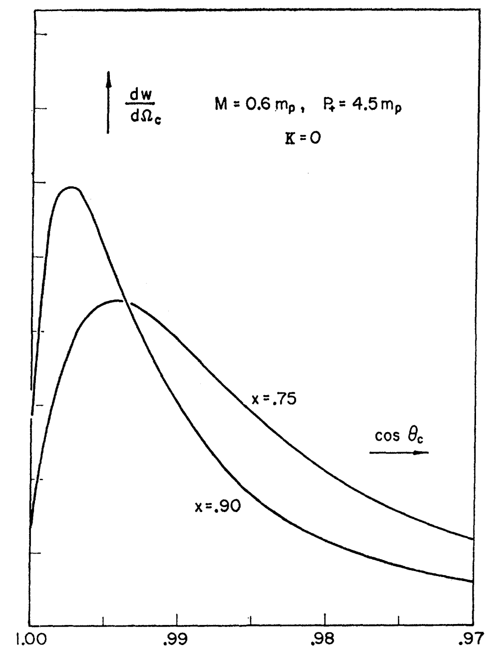


FIG. 12. Angular distribution of charged decay pion from boson produced in reaction (2).

The angular distributions of the decay reactions (3) and (4) have been calculated for decay in flight of the $W$ produced in reactions (1) and (2), which are of course characterized by the strong circular boson polarization and are clearly different from those where the boson had been unpolarized. Equation (79) has been used for the calculation, and we have plotted against the cosine of the lepton (or pion) angle measured from the direction $\mathbf{p}'$ or $\mathbf{p}_+$, with the intermediate angles $\theta$, $\varphi$ of the $W$ integrated out in numerator and denominator of Eq. (77) separately. The $W$ energy $x$ has also been kept differential and fixed; in the actual experiment, the produced $\mu$ in reaction (1) is observed, and its energy determines the $W$ energy from $p'\cong p+q_0$.

For process (1), the decay angular distributions of the charged lepton and pion are plotted in Fig. 10, on an arbitrary scale. They reflect the circular $W$ polarization: the $\mu^+$, which in the $W^+$ rest system tends to come out along the circular polarization which is left-handed, has in the laboratory its peak kinematically shifted away from the forward direction. The $\pi^+$, which in the $W^+$ rest system tends to come out normally to the $W^+$ circular polarization, has its peak nearer to the forward direction than the $\mu^+$. The same angular distribution applies to the "charge conjugate" cases, since although the sign of the $W$ polarization reverses, the decay-$\mu$ angular distribution with respect to this circular polarization reverses also. Only the case $\kappa=0$ is presented, since we found a weak dependence on $\kappa$.

Finally, in Figs. 11 and 12, we present the angular distributions of the decay lepton and pion, respectively, from the boson produced in reaction (2). We see that Fig. 11 reflects the reversal of circular $W$ polarization as $\kappa$ goes from 1 to $-1$ (as seen in Fig. 9) from right- to left-handed, by a shift from an angular distribution that was strongly forward peaked, to a much broader one, i.e., backward in the boson rest system.

The numerical calculations have been performed on the University of Michigan IBM 7090 electronic computer, with a computing time of approximately one hour.


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## APPENDIX

The tensors needed in Eq. (37) are given by

$$A_{1\sigma\rho}{}^{(e)}M^2(P_\rho'P_\sigma+P_\sigma'P_\rho-P'\cdot P\delta_{\rho\sigma})$$

$$A_{2\sigma\rho}{}^{(e)}=-\tfrac{1}{2}[P\cdot q(P_\rho'k_\sigma+P_\sigma'k_\rho)+P'\cdot q(P_\rho k_\sigma+P_\sigma k_\rho) -P'\cdot k(P_\rho q_\sigma+P_\sigma q_\rho)-P\cdot k(P_\rho'q_\sigma+P_\sigma'q_\rho)]$$

$$A_{3\sigma\rho}{}^{(e)}=-\tfrac{1}{4}[2\delta_{\rho\sigma}P'\cdot kP\cdot k-P'\cdot k(P_\sigma k_\rho+P_\rho k_\sigma) -P\cdot k(P_\sigma'k_\rho+P_\rho'k_\sigma)]$$

$$B_{1\sigma\rho}{}^{(e)}=-[P_\rho'(P\cdot qk_\sigma+q\cdot kP_\sigma-P\cdot kq_\sigma) +P_\sigma'(P\cdot qk_\rho+q\cdot kP_\rho-P\cdot kq_\rho) +\delta_{\rho\sigma}(P'\cdot qP\cdot k-P'\cdot kP\cdot q-P'\cdot Pq\cdot k)]$$

$$\bar{B}_{1\sigma\rho}{}^{(e)}=2P\cdot q(P_\rho'P_\sigma+P_\sigma'P_\rho-P'\cdot P\delta_{\rho\sigma})$$

$$B_{2\sigma\rho}{}^{(e)}=P\cdot k(P_\rho'k_\sigma+P_\sigma'k_\rho-P'\cdot k\delta_{\rho\sigma})$$

$$\bar{B}_{2\sigma\rho}{}^{(e)}=-\tfrac{1}{2}[P_\rho(P'\cdot kP_\sigma+P\cdot kP_\sigma'-P'\cdot Pk_\sigma) +P_\sigma(P'\cdot kP_\rho+P\cdot kP_\rho'-P'\cdot Pk_\rho)]$$

$$C_{\sigma\rho}{}^{(e)}=P\cdot k(P_\rho'k_\sigma+P_\sigma'k_\rho-P'\cdot k\delta_{\rho\sigma})$$

$$A_{1\sigma\rho}{}^{(0)}=M^2P_\alpha'P_\beta\epsilon_{\alpha\rho\beta\sigma}$$

$$A_{2\sigma\rho}{}^{(0)}=-\tfrac{1}{2}P_\alpha'P_\beta[(q_\nu k_\sigma-k_\nu q_\sigma)\epsilon_{\alpha\rho\beta\nu}+(k_\nu q_\rho-k_\rho q_\nu)\epsilon_{\alpha\sigma\beta\nu}]$$

$$A_{3\sigma\rho}{}^{(0)}=\tfrac{1}{4}P_\alpha'P_\beta(k_\rho k_\nu\epsilon_{\alpha\sigma\beta\nu}-k_\sigma k_\nu\epsilon_{\alpha\rho\beta\nu})$$

$$B_{1\sigma\rho}{}^{(0)}=-\tfrac{1}{2}[P_\rho'P_\beta q_\mu k_\nu\epsilon_{\beta\mu\nu\sigma}-P'\cdot Pq_\mu k_\nu\epsilon_{\rho\mu\nu\sigma} +P_\rho P_\alpha'q_\mu k_\nu\epsilon_{\alpha\mu\nu\sigma}+q\cdot kP_\alpha'P_\beta\epsilon_{\alpha\rho\beta\sigma} -q_\sigma P_\alpha'P_\beta k_\nu\epsilon_{\alpha\rho\beta\nu}+k_\sigma P_\alpha'P_\beta q_\mu\epsilon_{\alpha\rho\beta\mu}-(\rho\leftrightarrow\sigma)]$$

$$\bar{B}_{1\sigma\rho}{}^{(0)}=2P\cdot qP_\alpha'P_\beta\epsilon_{\alpha\rho\beta\sigma}$$

$$B_{2\sigma\rho}{}^{(0)}=\tfrac{1}{2}(P'\cdot kP_\beta k_\nu\epsilon_{\beta\rho\nu\sigma}+P\cdot kP_\alpha'k_\nu\epsilon_{\alpha\rho\nu\sigma}) +\tfrac{1}{2}[P_\alpha'P_\beta k_\rho k_\nu\epsilon_{\alpha\beta\nu\sigma}-(\rho\leftrightarrow\sigma)]$$

$$\bar{B}_{2\sigma\rho}{}^{(0)}=-\tfrac{1}{2}[P_\alpha'P_\beta P_\rho k_\lambda\epsilon_{\alpha\lambda\beta\sigma}-P_\alpha'P_\beta k_\rho P_\mu\epsilon_{\alpha\mu\beta\sigma}-(\rho\leftrightarrow\sigma)]$$

$$C_{\sigma\rho}{}^{(0)}=P\cdot kP_\alpha'k_\lambda\epsilon_{\alpha\rho\lambda\sigma}.$$