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Possible Effects of Strong Interactions in the Feinberg-Pais Theory of Weak Interactions*

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We show that, in the simplest model where strong interaction effects are included in the baryon vertices and propagators only, the peratization of the nuclear vector β-decay coupling strength is no longer equal to the corresponding peratization of the μ-decay coupling strength. This might suggest a vastly different role of strong interactions in the weak interaction (peratization) theory of Feinberg and Pais, in order to conform with the observed equality of nuclear vector β-decay and μ-decay strengths.

RECENTLY, a new field theory of weak interactions, called the peratization theory, was proposed by Feinberg and Pais.[1] It is a theory which seeks to include higher-order weak interaction effects, within the framework of intermediate vector meson theory. In the realm of pure weak interaction physics, peratization has led to some remarkable new conclusions, which are different from the conclusions drawn from the lowest-order graph. Thus, in the most famous example of the μ-meson decay, the effective decay strength peratizes to $\frac{3}{4}g^2/m^2$,[2] which is to be compared with the lowest-order result, g^2/m^2. By their argument, the peratization of the nuclear β-decay strength should, in the absence of strong interactions, result in the same reduction. This would conform with the observed equality of the two β-decay strengths. It remains to be seen, however, if strong interactions could affect this equality of peratization.

In this brief note we wish to point out some interesting possibilities in this connection. If we assume the vector current to be strictly conserved, and the axial vector current conserved only in the high-energy limit, then, by adhering to the peratization procedure of Feinberg and Pais, we arrive very naturally at a break of equality of the peratization of nuclear and μ-meson β-decay strengths. More precisely, we have found this to be true for the simplest model where strong interactions are included in the modified baryon vertices and propagators. This result makes more difficult an understanding of the role of strong interactions in weak peratization theory, as presently formulated by Feinberg and Pais.

We begin with a restatement of the peratization result in the case of μ decay. Peratization, in the main, seeks to include the higher-order weak-interaction effects through the solution of a Bethe-Salpeter equation. The Lagrangian is, specifically, chosen to be

$$igW_\rho\{\bar{\mu}\gamma_\rho(1+\gamma_5)\nu_\mu+\bar{e}\gamma_\rho(1+\gamma_5)\nu\}+\text{h.c.} \qquad (1)$$

In words, one sums over an infinite set of uncrossed ladder graphs, taking, for the first iteration, only the most divergent singularity of each nth-order graph into account. This is equivalent to using the $q_\mu q_\nu$ part of the vector meson propagator

$$\Delta_{\mu\nu}=\frac{1}{q^2+m^2}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m^2}\right)$$

in each graph. [Actually, in their paper, Feinberg and Pais used only $p_{2\mu}''p_{2\nu}''$ part of $q_\mu q_\nu=(p_2''-p_2')_\mu(p_2''-p_2')_\nu$ for their first iteration. As they have shown, however, the difference in solution is of higher order in g^2. The convenience of keeping the full $q_\mu q_\nu$ in our discussion will be apparent later.]

The statement, then, is that the sum of all the uncrossed ladder graphs, with the consistent replacement of $\Delta_{\mu\nu}(q)$ everywhere by $(q_\mu q_\nu/m^2)/(q^2+m^2)$, even for $n=1$, the lowest-order graph, leads to a μ-decay matrix element (at $q\to 0$)

$$\bar{e}\gamma_\rho(1+\gamma_5)\nu_e\bar{\nu}_\mu\gamma_\lambda(1+\nu_5)\mu\left[-\frac{1}{4}\frac{g^2}{m^2}\delta_{\lambda\rho}\right]. \qquad (2)$$

Upon adding the $\delta_{\mu\nu}$ part of $\Delta_{\mu\nu}$ for the lowest-order graph, one recovers the famous $\frac{3}{4}g^2/m^2$ result of peratization theory for μ decay.

We turn now to a consideration of nuclear β decay. We assume the Lagrangian to be given by

$$igW_\rho\bar{e}\gamma_\rho(1+\gamma_5)\nu_e+igW_\rho(J_\rho{}^V+\eta J_\rho{}^A), \qquad (3)$$

where the strangeness conserving currents are

$$Z^{-1}J_\rho{}^V=\bar{n}'\gamma_\rho p'+\cdots,$$
$$Z_A{}^{-1}J_\rho{}^A=\bar{n}'\gamma_\rho\gamma_5 p'+\cdots, \qquad (4)$$

with the vector current strictly conserved and the axial vector current conserved in the high-energy limit. The currents are defined in terms of renormalized operators, $n'=Z^{-1/2}n$, $p'=Z^{-1/2}p$, while the renormalized coupling constants are, respectively, $g_1{}^V=g$ for the vector case and $g_1{}^A=gZZ_A{}^{-1}\equiv\eta g$ for the axial vector case. Z_A is the usual axial vector vertex function strong interaction renormalization constant.

Now consider the renormalized vector vertex function, $V_\rho{}^c(p,q)$. It must satisfy the Ward-Takahashi identity[3] (to lowest order in g, but to all orders in strong

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[1] G. Feinberg and A. Pais, Phys. Rev. **131**, 2724 (1963).
[2] This is the result for sum over uncrossed ladder graphs (see below). It changes when one includes the so-called crossed ladder graphs.
[3] J. C. Ward, Phys. Rev. **78**, 182 (1950); Y. Takahashi, Nuovo Cimento **6**, 370 (1957).

interactions) on account of the conserved vector current hypothesis:

$$q_\rho V_\rho{}^c(p,q)=iS_F{}^{c^{-1}}(p-q)-iS_F{}^{c^{-1}}(p)\,. \tag{5}$$

A direct consequence of the identity is that the vertex function itself as a whole cannot vanish as $q_\rho \to \infty$. That is to say, the form factors contained in the vertex function cannot all vanish at the same time sufficiently rapidly for the whole vertex function to vanish. On the other hand, the vertex function could be either finite (and nonzero) or infinite at the high virtual energy limit. This result is true for the case where $Z\neq 0$. [As $q_\rho \to \infty$, we have $S_F{}^{c^{-1}}(q)\to Z(i\gamma\cdot q)$.]

The corresponding result for the spin-zero vertex function would be that it must diverge at least linearly in q_ρ as $q_\rho\to\infty$, if $Z\neq 0$.

In this connection, we digress momentarily and point out a common error in principle in the introduction and interpretation of a cutoff in, say, the radiative correction calculations. Consider the case $Z\neq 0$. Then it is certainly erroneous to replace, say, the charge vertex function for spin-zero meson, $(2p-q)_\mu$, by

$$(2p-q)_\mu[\Lambda^2/(q^2+\Lambda^2)]$$

and relate Λ to the physical pion charge form factor structure. This is a criticizm of principle, because, in practice, it is a moot point in quantum electrodynamics. For in that case one is at liberty to use a Landau gauge for the photon propagator, $D_{\mu\nu}=D_F(\delta_{\mu\nu}-q_\mu q_\nu/q^2)$, and since Γ_μ occurs everywhere with $D_{\mu\nu}$ multiplying it, the product $\Gamma_\mu D_{\mu\nu}$ could very easily damp out without violating the Ward-Takahashi identity. For the $Z\neq 0$ case, the criticizm shifts to the propagator since then the propagator should be changed to have the correct asymptotic behavior.

Let us return now to the weak interaction vector meson theory. The high virtual energy limit of the $V_\rho{}^c$ can, as pointed out above, be either finite (and nonzero) or infinite, for $Z\neq 0$; and zero, finite, or infinite for $Z=0$. The bare $V_\rho=\gamma_\rho$ is of course finite for all q_ρ. We shall for the moment rule out the possibility that $V_\rho{}^c$ blows up violently on account of the strong interactions. Otherwise, the Feinberg-Pais iteration procedure where only $q_\lambda q_\rho V_\rho{}^c$ is kept in the kernel would be wrong since $\delta_{\lambda\rho}V_\rho{}^c$ would then dominate the integral rather than the $q_\rho q_\lambda V_\rho{}^c$ term. Thus, in order to adhere to the usual peratization program we assume the infinite behavior of $V_\rho{}^c$ to be less than q^2, for $Z\neq 0$.

Similar asymptotic limit holds for the axial vector vertex function if one assumes that the axial vector current is conserved in the high-energy limit. Thus the corresponding identity

$$q_\rho A_\rho{}^c(p,q)=I(p,q)+iS_F{}^{c^{-1}}(p-q)\gamma_5+i\gamma_5 S_F{}^{c^{-1}}(p) \tag{6}$$

in the $q_\rho\to\infty$ limit reads, by our assumption,

$$\eta q_\rho A_\rho{}^c \to Z\gamma\cdot q\gamma_5+O(1)\,. \tag{7}$$

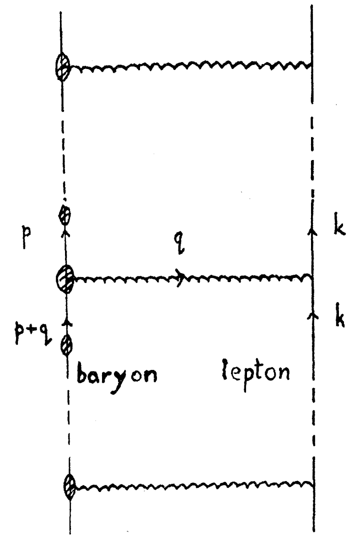

FIG. 1. A typical uncrossed ladder graph with strong interaction effects modifying the baryon vertices and propagators. The shaded parts in the diagram represent such effects.

We also assume that A_ρ does not blow up at infinity as fast as q^2 on account of the strong interactions.

We proceed now to the peratization of nuclear β decay. For clarity, we consider the sum over the ladder graphs rather than the integral equation. (The same conclusion holds, of course, for the integral equation solution.) For each rung of ladder we have (Fig. 1)

$$g^2 S_F{}^c(p+q)\{V_\rho{}^c+\eta A_\rho{}^c\}\Delta_{\rho\lambda}(q)\cdot\frac{1}{k-q}\gamma_\lambda(1-\gamma_5)\,. \tag{8}$$

By our assumption that $V_\rho{}^c$, $A_\rho{}^c$ do not blow up violently at infinity, peratization would require consideration of only the $q_\rho q_\lambda$ part of $\Delta_{\rho\lambda}$. But the $q_\rho q_\lambda$ part is known from the Ward-Takahashi identity. Thus the dominant behavior of the factors at $q_\rho\to\infty$ is, by our previous results,

$$\frac{g^2}{q^2+m^2}\gamma\cdot q(1+\gamma_5)\gamma\cdot q(1+\gamma_5)\,,$$

which, is the same behavior as in μ decay. The peratization of such a leading divergence is straightforward in the formalism. It leads to a nuclear β-decay matrix element at $(n-p)_\mu\to 0$, zero momentum transfer,

$$\bar u(p)(Z\gamma_\rho+Z\gamma_\rho\gamma_5)u(n)\bar u(e)\gamma_\rho(1+\gamma_5)V(\nu_e)\times\left[-\frac{1}{4}\frac{q^2}{m^2}\right]+\bar u(p)(\gamma_\rho+\eta\gamma_\rho\gamma_5)u(n)\bar u(e)\gamma_\rho\times(1+\gamma_5)V(\nu_e)\left[\frac{g^2}{m^2}\right]. \tag{9}$$

The extra Z factor comes from renormalization of the external lines. Rewritten, we find the following interesting results for the effective coupling strengths:

$$G^V=(g^2/m^2)(1-Z/4)\,,$$
$$G^A=\eta(g^2/m^2)(1-Z/4\eta)\,. \tag{10}$$

Clearly since $Z\neq 1$, the peratization of nuclear vector β decay will not be equal to the corresponding μ-decay result.

The presence of a strangeness changing current does not affect the argument above essentially, provided we assume again a conservation of the current in the high-energy limit to make a definite statement about the leading divergences. The coupled set of integral equations can again be diagonalised and solved, and leads to a peratization result of the form

$$G^V=(1-\tfrac{1}{4}Z)(g^2/m^2)+O(g^2g_s^2,g^4) \qquad (11)$$

and so long as g_s (=renormalized coupling constant for strangeness changing current) is not $\gg g$, the leading term would still be $(1-\frac{1}{4}Z)g^2/m^2$.

The arguments given here, within the framework of the usual procedure of peratization where one starts with the $q_\rho q_\lambda$ part of $\Delta_{\rho\lambda}$, are quite general and, in a sense, natural. Thus, all the assumptions made were necessary for following the usual procedure of peratization and conservation of the current took care of the rest. Hence, if we insist on the conservation of current, and wish to understand the equality of μ-decay and nuclear β-decay strengths we might have to resort to the following possibilities: (*i*) a different procedure of peratization is called for the presence of strong interactions; where, for instance, the $\delta_{\lambda\rho}$ part of $\Delta_{\rho\lambda}$ is the dominant term in the integral. V_ρ^c should blow up quite drastically at infinity in that case. The correspondence with the pure weak-decay peratization would of course be lost. (ii) Strong interaction so overwhelms weak (and also electromagnetic) interaction that no meaningful separation of its effects on vertices and propagators can nor should be made. This is a possibility which defeats and defines analysis. (iii) Weak interaction overwhelms strong and electromagnetic interaction at virtual high energies in some very special way so that peratization can be carried out with total neglect of the strong interactions.

On the other hand, if we give up the strict conservation of the vector current, but continue to assume it to be conserved in the high-energy limit, we could restore equality of the two β-decay strengths. The result for G^V would read

$$G^V=\eta'\frac{g^2}{m^2}\left(1-\frac{Z}{4\eta'}\right), \qquad (12)$$

which could mysteriously lead to $\frac{3}{4}g^2/m^2$. In other words, this would mean that, from the point of view of peratization theory, the experimentally observed equality of μ-meson and nuclear β-decay strengths is an indication of the nonconservation of vector current.

In a forthcoming paper, done in collaboration with H. S. Mani, the result (10) will be generalized to include, based upon a power-counting argument, all crossed and uncrossed ladder graphs. Thus, if $G_\mu=\kappa g^2/m^2$ when all graphs are included, then $G^V=(g^2/m^2)\times(1-[1-\kappa]Z)$.

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