Longitudinal Oscillations in a Nonuniform Plasma*

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Longitudinal electron oscillations in a bounded plasma slab immersed in a magnetic field are analyzed using hydrodynamic equations. Resonance conditions are given for an arbitrary density distribution. In the zero magnetic field limit, the present results may account for some main features of Tonks' and Dattner's experiments. The behavior of these resonances and the possibility of damping due to thermal effects is discussed. It is found that the magnetic field tends to "push" the perturbations further into regions near the walls and keep the bulk of the plasma free from oscillations.

I. INTRODUCTION

In earlier experiments, Tonks[1] and Dattner[2] have found that a plasma column shows several resonances upon the incidence of a microwave. The theory of Herlofson,[3] assuming a uniform density and zero temperature, predicts one resonance. Gould[4] and others[5] have found that finite electron temperature introduces additional resonances. The spacing between these resonances are, however, about 100 times too small. By assuming a particular density distribution, Weissglass[6] has recently shown that the spacing is substantially increased by the presence of a density gradient.

When a longitudinal magnetic field is applied, Tonks,[1] Messiaen and Vandenplas,[7] and Crawford, Kino, and Cannara[8] have observed that the main resonance peak is splitted into two peaks. This phenomenon has been approximatively explained by Åström[9] and others.[7,8] Here again, the behavior of the additional resonances is not understood theoretically and is little known experimentally.[2,7]

The purpose of this paper is to bring out the nature and the main properties of these additional resonances. We shall consider longitudinal electrostatic oscillations in a plasma slab, having an arbitrary density distribution, immersed in a homogeneous magnetic field. (The plane geometry can well represent the cylindrical geometry with a longitudinal magnetic field for the present purposes.)

In the zero magnetic-field limit, our result is able to account for some main features of Dattner's experiment. The effect of a longitudinal magnetic field on these resonances is derived. The detailed properties of these resonances and eventual damping is discussed. It should be mentioned that a number of authors[10] have independently presented calculations accounting for the additional resonances.[2] While these authors are mainly concerned with a good agreement between theory and experiment, the present paper emphasizes the physical understanding of these resonances.

Before going into details, it is useful to have a simple physical picture of the additional resonances introduced by the finite electron temperature for a uniform density distribution. The resonance frequency ω obeys[4,5]

$$\omega^2=\omega_{pm}^2+(n+\tfrac{1}{2})^2\pi^2(KT/m_eL^2)\,, \tag{1.1}$$

$$\omega_{pm}^2=e^2N_{0m}/m_e\epsilon_0\,,\quad n=0,1,2,\ldots, \tag{1.2}$$

where K is the Boltzmann constant, T the electron temperature, e the electronic charge, m_e the electron mass, $2L$ the thickness of the plasma slab, ϵ_0 the dielectric constant in vacuum, and N_{0m} is the maximum density in the plasma. Let us now perturb the uniform plasma (Fig. 1). The displaced electrons are restored by Coulomb attraction, corresponding to the ω_{pm}^2 term in (1.1), *and* by the associated pressure gradient, which corresponds to the last term in (1.1) with $n=0$. The latter force is responsible for the additional resonances, although it is too small to explain the experimental data. In a nonuniform plasma, the perturbations may be confined to a small region, e.g., near the walls.[6] In that region, the pressure-gradient force is no longer small as compared to the Coulomb force; the spacing may be

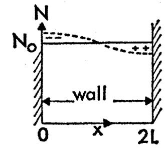

FIG. 1. Uniform density distribution superimposed by a ground-state mode electron-density perturbation.

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[1] L. Tonks, Phys. Rev. **37**, 1458 (1931).
[2] A. Dattner, Ericsson Tech. **13**, 309 (1957); Phys. Rev. Letters **10**, 205 (1963).
[3] N. Herlofson, Arkiv Fysik **3**, 15 (1951).
[4] R. W. Gould, *Linde Conference on Plasma Oscillations* (Spencer, Indiana, 1959), p. 167.
[5] There exists a great number of papers on this subject. A list of these is given in Ref. 8.
[6] P. Weissglass, Phys. Rev. Letters **10**, 206 (1963).
[7] A. M. Messiaen and P. E. Vandenplas, Physica **28**, 537 (1962); Phys. Letters **2**, 193 (1962).
[8] F. W. Crawford, G. S. Kino, and A. B. Cannara, Microwave Laboratory Report No. 1014, Stanford University, 1963 (unpublished).
[9] E. Åström, Arkiv Fysik **19**, 163 (1961).
[10] See papers by P. Weissglass, P. E. Vandenplas, and A. M. Messiaen, R. B. Hall, and F. Crawford and G. Kino, in Proceedings of the Seventh Conference on Ionization Phenomena in Gases, Paris, 1963 (unpublished); see also Microwave Laboratory Report No. 1045, Stanford University, 1963 (unpublished). The most complete numerical calculation has recently been given by J. C. Nickel, J. V. Parker, and R. W. Gould, Phys. Rev. Letters **11**, 183 (1963).

increased considerably due to the presence of a nonuniform density.

II. RESONANCE CONDITIONS FOR A NONUNIFORM PLASMA

Our plasma slab has a thickness of $2L$, a density which only varies along x, and is immersed in a magnetic field $\mathbf{B}$, directed along z. We shall, as in the earlier works,[4–6] use the hydrodynamical equations for an electron gas:

$$m_e(d\mathbf{v}/dt)=-e\mathfrak{E}-e\mathbf{v}\times\mathbf{B}-KTN^{-1}\nabla N \tag{2.1}$$

$$\partial N/\partial t+\operatorname{div}N\mathbf{v}=0 \tag{2.2}$$

$$\operatorname{div}\mathfrak{E}=-(e/\epsilon_0)(N-N_0)\,. \tag{2.3}$$

Here, $\mathbf{v}$ is the macroscopic electron velocity, N the electron density, N_0 the ion density, and $\mathfrak{E}$ the electric field. For simplicity, we have assumed $T=\text{const}$. In a uniform plasma temperature variations are known[11] to increase the last term in (1.1) by a factor of 3. We now replace the variables F in (2.1)–(2.3) by $F_0+F_1\exp(i\omega t)$, where the index 1 denotes perturbation, and linearize. Equations (2.1), (2.2), and (2.3) now becomes

$$-\omega^2\mathbf{j}_1=i\omega\frac{\epsilon_0}{e}\omega_p^2(x)\nabla V_1-i\omega\mathbf{j}_1\times\boldsymbol{\omega}_c+W^2\nabla\operatorname{div}\mathbf{j}_1-W^2\frac{\nabla N_0}{N_0}\operatorname{div}\mathbf{j}_1\,, \tag{2.4}$$

$$\nabla^2V_1=(ie/\epsilon_0\omega)\operatorname{div}\mathbf{j}_1\,, \tag{2.5}$$

where

$$\omega_p^2(x)=e^2N_0(x)/m_e\epsilon_0\,,\quad \boldsymbol{\omega}_c=e\mathbf{B}/m_e\,,\quad W^2=KT/m_e\,,\quad \mathbf{j}_1=N_0\mathbf{v}_1\,,\quad \mathbf{v}_0=0\,. \tag{2.6}$$

Here, we have put $\mathfrak{E}_1=-\nabla V_1$, where V is the electrostatic potential. In this way, we only consider longitudinal plasma oscillations and neglect their coupling to the external electromagnetic fields.

In the following we shall assume that $\partial/\partial y=\partial/\partial z=0$ in (2.4) and (2.5). In order to justify the validity of this assumption let us replace the variables F_1 in (2.4) and (2.5) by its Fourier component $F_1(x)\exp(ik_yy+ik_zz)$. Inserting this into (2.4) and (2.5) it can be found that our assumption is good if the inequalities

$$k_y^2W^2\ll\omega^2;\quad k_y^2,\quad k_y^2\frac{\omega_p^2(x)}{\omega^2}\ll k_x^2;\quad k_y\frac{\omega_c}{\omega}\ll k_x\,, \tag{2.7}$$

together with a similar set of inequalities obtained by changing k_y in (2.7) to k_z, are satisfied. Here, $k_x\equiv\partial/\partial x$ is much larger than $1/L$, as will be found later. Further, we are interested in the dipole or the quadrupole resonances in a cylindrical plasma. In the slab approximation these resonances correspond to $k_y\approx 1/L$, $2/L$. Therefore, (2.7) is in general very well satisfied. Equations (2.4) and (2.5) now yield

$$j_{1x}''+\frac{1}{W^2}[\omega^2-\omega_p^2(x)-\omega_c^2]j_{1x}=-\frac{N_0'}{N_0}j_{1x}'+i\frac{e\omega}{m_e}N_0(x)\mathfrak{E}_{1x}(0)\,, \tag{2.8}$$

where the prime denotes differentiation with respect to x and the subscript x denotes the x component.

In order to simplify the problem further, we shall assume that the wavelength of the perturbation k_x^{-1} is much smaller than the scale of the plasma inhomogeneity N_0/N_0'. The assumptions and simplifications made hitherto are consistent with the WKB approximation which will be employed. The homogeneous part of (2.8), which determines the eigenmodes, now becomes

$$j_{1x}''+W^{-2}[\omega^2-\omega_p^2(x)-\omega_c^2]j_{1x}=0\,. \tag{2.9}$$

Since this equation is in the Strum-Liouville form, its eigenvalue ω^2 is always real.

It should be pointed out that (2.9) is also valid for a cylindrical plasma column immersed in a longitudinal magnetic field, if the same approximations as those given in the preceding paragraph are used. In that case, x in (2.9) will represent the radius. However, the experiments[2,7] also contain cases for which B is perpendicular to the plasma column. Since those cases cannot be approximated by the plane geometry, they will be left out.

The surface $x=x_t$, at which the plasma dielectric constant vanishes, $\omega^2-\omega_p^2(x_t)=\omega_c^2$, is of vital physical importance. Recognizing this, (2.9) can be transformed to

$$u''+k^2u=0\,,\quad k=\lambda_D^{-1}[E-V(x)]^{1/2}\,,\quad k>0\,, \tag{2.10}$$

$$u''-\kappa^2u=0\,,\quad \kappa=\lambda_D^{-1}[V(x)-E]^{1/2}\,,\quad \kappa>0\,, \tag{2.11}$$

where

$$j_{1x}=u\,,\quad E=\frac{\omega^2-\omega_c^2}{\omega_{pm}^2}\,, \tag{2.12}$$

$$V(x)=\frac{N_0(x)}{N_{0m}}\,,\quad \lambda_D=\frac{W}{\omega_{pm}}\,. \tag{2.13}$$

If we interpret u as a time-independent Schrödinger wave, (2.10) and (2.11) then describe the discrete energy levels E of a quantum-mechanical particle trapped in a potential well given by the walls and $V(x)$, the density profile. In plasma terms, standing plasma waves are confined by $V(x)$ and the walls. Outside $V(x)$ the wave decays within a plasma wave length (de Broglie wave length in the quantum case). Thus, the resonances[1,2] may be viewed as the ground and excited states of a "plasma-wave ensemble." The situation is illustrated in Fig. 2.

In the following we shall assume that $k(x)$ varies slowly over a local plasma wavelength $2\pi/k$. Hence, (2.10) and (2.11) can be solved by a WKB approxima-

[11] D. Bohm and E. P. Gross, Phys. Rev. **75**, 1851 (1949).

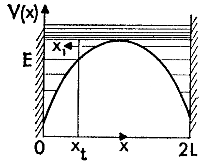

FIG. 2. "Energy levels" of a particle trapped in a potential well given by the walls and the density profile $V(x)$.

tion.[12] Here, we shall only write down the essential part of the derivation which will be useful in this paper and leave the details to the textbook. Following Schiff, we shall first express the solutions of (2.10) and (2.11) in the $x_1=(x_t-x)$ coordinate, in which the "classical turning point" $V(x_t)=E$ is the origin (Fig. 2):

$$u_1(x_1)=\xi_1^{1/2}k^{-1/2}[A_+J_{1/3}(\xi_1)+A_-J_{-1/3}(\xi_1)]\,,\quad \xi_1=\int_0^{x_1}kdx_1\,, \tag{2.14}$$

$$u_2(x_2)=\xi_2^{1/2}k^{-1/2}[B_+I_{1/3}(\xi_2)+B_-I_{-1/3}(\xi_2)]\,,\quad \xi_2=\int_{x_1}^{0}\kappa dx_1\,, \tag{2.15}$$

where J is the Bessel function and I is the modified Bessel function. We have assumed that

$$k^2=c^2x_1(1+ax_1+\cdots)\,,\quad c=(\lambda_D^2g)^{-1/2}, \tag{2.16}$$

i.e., the turning point is linear. Further, (2.14) is valid for $x_1>0$ and (2.15) for $x_1<0$ in Fig. 2. A smooth connection of u_1 and u_2 at x_t yields $B_+=-A_+$ and $B_-=A_-$. Further, at a number of Debye length away from x_t, the asymptotic formulas of the Bessel functions may be used. We require that the exponentially increasing part of u_2 vanishes in region II and obtain $B_+=-B_-$. We now get $A_+=A_-$ and hence $u_1(x_1)$ takes the asymptotic form of $k^{-1/2}\cos(\xi_1-\pi/4)$ for $x_1<0$. Returning to the x coordinate and using the boundary condition that the plasma current vanishes at the wall, $u_1(x=0)=0$, one gets the general resonance condition

$$\xi_1-\tfrac{1}{4}\pi=(n+\tfrac{1}{2})\pi\,,\quad n=0,1,2,\ldots. \tag{2.17}$$

Upon substituting ξ_1, this relation becomes

$$\int_0^{x_t}[E-V(x)]^{1/2}dx=(n+3/4)\pi\lambda_D\,, \tag{2.18}$$

$$E-V(x_t)=0. \tag{2.19}$$

III. RESONANCE CHARACTERISTICS FOR A TRAPEZOIDAL DENSITY PROFILE

For arbitrary density distributions, (2.18) needs a numerical treatment. In the following we shall confine ourselves to a trapezoidal density distribution. The density is constant throughout the centerpart of the plasma slab. At a distance f from the walls, it decreases linearly with a slope g^{-1} (see Fig. 3). Such a density distribution should give us the general features of the resonances characteristics in an actual experiment. It should be pointed out that for this particular distribution, (2.14) and (2.15) are *exact* rather than a WKB approximation.

The trapezoidal density profile is symmetric with respect to $x=L$. For $x<L$, it is

$$V(x)=1\,,\quad f<x<L\,,\qquad V(x)=V(0)+x/g\,,\quad x<f. \tag{3.1}$$

Using (3.1), (2.18) and (2.19) yield

$$E=(\omega^2-\omega_c^2)/\omega_{pm}^2=V(0)+[\tfrac{3}{2}\pi(n+\tfrac{3}{4})\lambda_D/g]^{2/3},\quad E<1\,, \tag{3.2}$$

$$x_t=[\tfrac{3}{2}\pi(n+\tfrac{3}{4})]^{2/3}(\lambda_D^2g)^{1/3}\,, \tag{3.3}$$

These results are very accurate even for small n at which the WKB method formally fails. Actually, the factor $\frac{3}{4}$ will be altered only a few percent for $n=0$ if the exact (2.14) is used instead of its asymptotic form.

In the following we shall discuss the zero magnetic-field case, $\omega_c=0$, in more detail. For $\omega>\omega_{pm}$, (2.10) and (2.14) are valid for $x<f$. The solution in this region is to be connected to the solution u_B of (2.10) in the region B $(f<x<2L-f)$ at $x=f$ (Fig. 3). Because $V=1$ in region B, (2.10) yields

$$u_B=C\sin qx+D\cos qx\,,\qquad q=\lambda_D(E-1)^{1/2}, \tag{3.4}$$

where C, D are constants. In order to connect u_B with u_1, we take the leading terms in the expansion[12] of u_1 near $x=f=x_t$:

$$u_1=A_+\alpha_+(f-x)+A_-\alpha_-\,, \tag{3.5}$$

$$\alpha_+=(\tfrac{2}{3})^{1/2}(\tfrac{1}{3}c)^{1/3}\Gamma^{-1}(\tfrac{4}{3})\,,\quad \alpha_-=(\tfrac{2}{3})^{1/2}(\tfrac{1}{3}c)^{-1/3}\Gamma^{-1}(\tfrac{2}{3})\,, \tag{3.6}$$

where Γ is the gamma function. The following conditions determine the constants: (1) $u_B'(x=L)=0$. We require that the perturbed density N_1 to be antisymmetric with respect to the center of the slab, $x=L$, because the corresponding modes in a cylindrical case are the ones which radiate (positive on one side and negative on the other side of the plasma column). This condition is fulfilled if $N_1(x=L)=0$, which reduces to condition (1) by using the linearized form of (2.2) and the relation $N_0v_{1x}=j_x=u_B$. (2) The boundary condition $u_1(x=0)=0$. (3) The continuity condition at

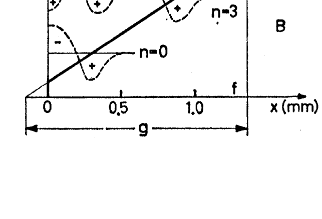

FIG. 3. The full curve shows density distribution. The dashed curve shows the form of the perturbed density distribution, $N_1(x)$, as estimated from (2.2) and (2.14).

[12] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), pp. 185–190.

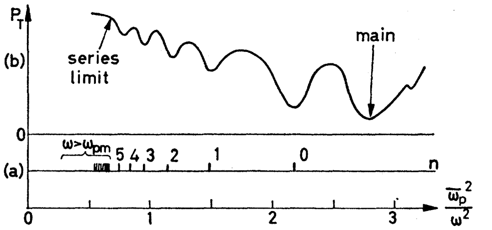

FIG. 4. (a) Position of resonances assuming a density distribution of Fig. 2 and $\omega_{pm}^2 = 1.5\langle\omega_p\rangle_{av}^2$. (b) A schematic curve obtained from Dattner's experiment; microwave frequency 1.5 GHz, mercury temp. 20°C, tube diameter 32.4 mm. P_T denotes the transmitted power.

f, $u_1(f)=u_B(f)$, and finally (4) $u_1'(f)=u_B'(f)$. These four conditions now yield the dispersion relation for $\omega>\omega_{pm}$:

$$\frac{J_{1/3}(\xi_{10})}{J_{-1/3}(\xi_{10})}\frac{\alpha_-}{\alpha_+} = -\frac{1}{q}\frac{\cos q(L-f)}{\sin q(L-f)}, \quad \xi_{10}=\frac{2c}{3}f^{3/2}. \quad (3.7)$$

It is difficult to obtain the resonance frequencies ω explicitly. However, in the limit of $q\to 0$, i.e., ω barely exceeds ω_{pm}, (3.7) takes the simple form

$$\cos q(L-f)=0, \quad (3.8)$$

which gives

$$\omega^2=\omega_{pm}^2+(n+\tfrac{1}{2})^2\pi^2[kT/m(L-f)^2], \quad \omega\gtrsim\omega_{pm}. \quad (3.9)$$

For $\omega\gg\omega_{pm}$, we may approximate V in (2.10) and (2.13) by its mean value. The solution of (2.10) is again sinusoidal of the type of (3.4) and the resonance condition is

$$\omega^2=\langle\omega^2_p\rangle_{av}+(n+\tfrac{1}{2})^2\pi^2(kT/mL^2), \quad \omega\gg\langle\omega_p\rangle_{av}, \quad (3.10)$$

where the $\langle\ \rangle_{av}$ denotes the mean value throughout the slab.

In Dattner's experiments we find typically $\omega_{pm}^2 = 3\times10^{20}s^{-1}$, $T=3\times10^4$ °K, and $\lambda_D=0.04$ mm. Assuming $V(0)=0.1$ and $g=1.5$ mm, (3.2), (3.9) and (3.10) then yield the resonances positions shown in Fig. 4(a). Compared with a typical experimental curve, [Fig. 4(b)] we find a qualitative agreement in spacing. The corresponding standing density waves $N_1(x)$ are illustrated by the dashed lines in Fig. 3. We see here that our $n=0, 1, \ldots$ correspond to the second, third, etc. resonances in the experiment. The main resonance falls outside our series and is attributed to the dipole mode.[3]

When a magnetic field is present, ω^2 is to be replaced by $\omega^2-\omega_c^2$ in (3.9) and (3.10). For a given ω, the net effect of the magnetic field is to "lift up" the density profile $V(x)$ by an amount of ω_c^2/ω^2_{pm}. Applying this operation to Fig. 3, we see that the magnetic field tends to "push" the perturbations further into the wall region and keep the bulk of plasma unperturbed.

IV. PHYSICAL INTERPRETATION AND DISCUSSION

From (3.2) we estimate the spacing between the resonances $\Delta(\omega^2-\omega_c^2)/\omega_{pm}^2$ to be of the order of $(\lambda_D/g)^{2/3}$. Further, it is important to note that electron oscillations in an inhomogeneous plasma has a "natural" wavelength scale given by the particular combination $(\lambda_D^2 g)^{1/3}$, as is estimated by x_t/n given in (3.3). The presence of an inhomogeneity of the scale of g puts a constraint on the wavelength of plasma oscillations, which constraint is absent in a homogeneous plasma.

The physics involved in the mentioned effect and the variation of wavelength with x [See Fig. 3 or (2.14)] can be understood somewhat better using the force balance model discussed in Sec. I. The perturbation near x_t (Fig. 3), is mainly restored by the Coulomb force, since $\omega\gtrsim\omega_p(x)$. As the Coulomb force [$\propto N_0(x)$] *decreases* away from x_t, the pressure gradient force must step in and *help up* to balance the inertia force ($\propto\omega$), which is independent of x. The pressure-gradient force ($\propto KT$) is increased by decreasing the wavelength and, moreover, the local wavelength and its change with x is evidently determined by the local Coulomb force (or density) and its change with x. The parameters determing the wavelength are therefore KT, N_0, and g and our findings in the preceeding paragraph is therefore not surprising. The situation is illustrated in Fig. 3 with $n=3$, where the wavelength decreases towards the lower density region near the wall.

It is interesting to note that ω is entirely determined by the density near the walls as long as $\omega<\omega_{pm}$ (3.2). As soon as $\omega>\omega_{pm}$, however, (3.9) and (3.10) shows that the spacing in ω is determined by the bulk of the plasma; the plasma near the wall *suddenly* looses its importance when ω passes ω_{pm}. This may provide a "series limit" effect[2] if we assume that the radiations having $\omega>\omega_{pm}$ (see Fig. 4) are not observed. The reason seems to be that the width of these resonances, as is judged from the experimental data (Fig. 4), is much larger than the spacing between them; these resonances are smeared out.

Further, for $\omega<\omega_{pm}$, the energy of the microwave radiation is used to excite a small portion of plasma near the walls. The damping (Landau and collisional) of plasma oscillations (~absorbed power) is therefore small, and the amplitude of the oscillations (~reflected power) is large. As ω is increased and eventually $\omega>\omega_{pm}$, energy dissipation occurs over a larger volume of plasma, the absorbed power is increased and the reflected power is decreased. The latter power (amplitude) may eventually become so small as to contribute to the "series-limit" effect. This physical picture agrees qualitatively with the amplitude effects observed.[2]

It is desirable to treat these oscillations from the Vlasov equations since an eventual damping effect may show up. The problem is very difficult because the unperturbed orbit of an electron moving in a zero-order electric field ($\propto N_0'/N_0$) between two walls is highly complicated. Nevertheless, it is useful to estimate the damping of our resonant oscillations using a simple physical model which is well known[13] to describe the Landau damping effect in a homogeneous plasma. Consider a density perturbation which for the moment is stationary in space (Fig. 3). After a time $(k_xW)^{-1}$, the perturbations having a wavelength of $2\pi/k_x$ will be "dissipated" by the thermal motion W. We now let the perturbations oscillate with a period of ω^{-1}. Evidently, the damping effect due to temperature motion (Landau damping) is not important if the dissipation time $(k_xW)^{-1}$ is long compared to the oscillation period ω^{-1}:

$$\omega \geqslant k_xW. \tag{4.1}$$

[13] M. N. Rosenbluth, Danish Atomic Energy Commission, Risø Report No. 18, 1960, p. 197 (unpublished).

Let $k_x=2\pi/\lambda_x$ with $\lambda_x=2\partial x_t/\partial n$, and make use of (3.2) and (3.3) with $\omega_c=0$. (4.1) can be reduced to

$$\{1+\omega_p{}^2(0)/[\omega^2-\omega^2{}_p(0)]\}^{1/2}\geqslant 1\,, \tag{4.2}$$

where $\omega^2{}_p(0)$ is the plasma density at the wall. This inequality is certainly not strong and in fact the equality sign is valid for zero wall density. According to this estimate, therefore, thermal damping effects cannot be excluded and a further investigation is physically significant.

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Fermi Surface of Ferromagnetic Nickel*

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A survey of nonmagnetic band structures near the top of the 3*d* band is made. The aim is to combine these band structures with a wide variety of experimental data to determine the exchange splittings of the *d* bands ΔE_{dd} and the *s-p* conduction band ΔE_{ss}. Saturation magnetization, *g* factors, and high-field Hall data are analyzed and compared with the effect of *s-d* hybridization on the number of *s* electrons. One concludes that if the neck observed in magnetoresistance studies is associated with the same band edge as the Cu neck, $\Delta E_{dd}\lesssim 0.8$ eV. It appears that $\Delta E_{ss}\lesssim \Delta E_{dd}/2$. Recent optical rotation data of Krinchik are interpreted as giving a direct measurement of ΔE_{dd}. The value obtained is (0.6 ± 0.1) eV, in good agreement with the values obtained from other data.

1. INTRODUCTION

ENORMOUS progress has been made recently in extending our knowledge of the electronic structure of metals through a variety of experiments which determine certain properties of the Fermi surface.[1] Fawcett and Reed have recently studied the transverse magnetoresistance and Hall coefficient of Ni.[2,3] By combining their results with the results of saturation magnetization,[4] gyromagnetic resonance,[4] and Faraday rotation measurements,[5] it may be possible to obtain a rather precise picture of certain portions of the Fermi surface of ferromagnetic Ni.

* Supported in part by the National Science Foundation.
† Guggenheim Fellow with a grant-in-aid from the Sloan Foundation.

[1] *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, Inc. New York, 1960).
[2] E. Fawcett and W. A. Reed, Phys. Rev. Letters **9**, 336 (1962).
[3] E. Fawcett and W. A. Reed, Phys. Rev. **131**, 2463 (1963).
[4] C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, Inc., New York, 1953), pp. 166–171.
[5] G. S. Krinchik and R. D. Naralieva, Zh. Eksperimic Teor. Fiz. **36**, 1022 (1959) [translation: Soviet Phys.—JEPT **36**, 724 (1959)]. G. S. Krinchik and A. A. Gorbacher, Fiz. Metal. i Metalloved. **11**, 203 (1961).

Before undertaking an analysis of the experimental data we must make certain assumptions about the band structure of Ni. All Fermi surface measurements tend to be almost too microscopic. Because the measurements are confined to the neighborhood of $E=E_F$, one views the band structure through a slit that is energetically very narrow. Many different band models of $\mathbf{E}_n(\mathbf{k})$, where n labels bands, often fit the same data with apparently equal success. It is therefore necessary at the outset to attempt to define certain rules for physically plausible band structures. If the rules are correct, reasonable models which fit experiment naturally will emerge from the analysis.

For nontransition metals this prescription has been carried through with great success by Harrison.[6] His rule is to apply the nearly free-electron model. Ashcroft[7] has extended the pseudopotential treatment to characterize allowed Fermi surface topologies. We know that narrow d bands cannot be treated in this fashion, and

[6] W. Harrison, Ref. 1, p. 28.
[7] N. W. Ashcroft, Phys. Letters 4, 292 (1963).