example where $E(e^-)=3.3$ eV, energy transferred from the ion will be $T_m=13.2$ eV, and the energy ratio will be the same as before. However, H_2^+ holds an outer electron somewhat less firmly than does H^+, and the two act more independently, hence, affecting more electrons. A similar argument holds for H_3^+. The dissociation of the neutrals[18] will also contribute to the higher δ_0/δ_1 ratios.

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Galvanomagnetic Effects in Heavily Doped *p*-Type Germanium*

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The Hall effect and magnetoresistance in heavily doped *p*-type germanium have been measured. The samples had gallium impurity concentrations between 10^{18} and 10^{19} cm^{-3}. The measurements were made at 77, 20.4, 4.2, and 1.4°K, with magnetic fields ranging up to 30 kG. The results are compared with the predictions of simple transport theory applied to carriers in the unperturbed valence band of germanium. The data at 77°K can be adequately described on the basis of this model. At very low temperatures, however, the data are in disagreement with the predictions of the model. The magnetoresistance effects, in particular, are anomalous; the transverse component shows a linear dependence on magnetic field, and a longitudinal magnetoresistance is observed.

1. INTRODUCTION

THIS paper describes an investigation of galvanomagnetic effects in heavily doped *p*-type germanium. The samples had gallium impurity concentrations between 10^{18} and 10^{19} cm^{-3}. The measurements were made at 77, 20.4, 4.2, and 1.4°K, with magnetic fields ranging up to 30 kG.

To a rough approximation the ground state of a hole in a localized level around a gallium impurity is described by an effective Bohr radius of 70 Å.[1] When there are more than 10^{18} gallium atoms per cm^3, the neighboring impurities are so close together that the concept of states localized around an impurity loses its meaning.

The band structure in such disordered materials has been the subject of several theoretical investigations,[2] but as yet there is no agreement about the nature and the density of the carrier states. Many of the experimental studies of such materials, however, yield results which are consistent with the following model. The carriers are assumed to occupy states in the unperturbed band; as the temperature is lowered there is no freeze out so that the number of carriers remains constant, the only effect is to change the degree of statistical degeneracy of the carriers. For example, the magnetic susceptibility[3] and infrared reflectivity[4] of *n*-type germanium have been measured. The density of states effective mass deduced from these experimental results agrees with the value found by cyclotron resonance experiments on pure samples. Keesom and Seidel[5] measured the specific heat of *n*- and *p*-type samples in the temperature range between 0.5 and 4.2°K. Their experiments determine essentially the density of states at the Fermi level, and the results are in agreement with the values calculated on the basis of a degenerate Fermi gas in the unperturbed band.

The purpose of the present investigation was to measure the Hall effect and magnetoresistance in *p*-type germanium, and compare the results with the predictions of simple transport theory applied to such a model. The results at 77°K are in reasonable agreement with the theory. However at liquid-helium temperatures the galvanomagnetic effects show an anomalous behavior. The most striking features are a linear dependence of the transverse magnetoresistance on the magnetic field and the presence of a longitudinal magnetoresistance.

2. TRANSPORT THEORY FOR A TWO-BAND MODEL

The structure of the valence band in pure germanium is known from cyclotron resonance experiments.[6,7] The surfaces of constant energy are two sets of warped spheres and degenerate at $k=0$. These give rise to two kinds of holes, the heavy and the light ones; the ratio of the number of heavy holes to the number of light

* Work supported by U. S. Army Research Office.

[1] S. Golin, Bull. Am. Phys. Soc. **8**, 224 (1963).

[2] *Proceedings of the International Conference on the Physics of Semiconductors, Exeter, 1962* (The Institute of Physics and the Physical Society, London); see papers by V. L. Bonch-Bruevich, p. 216; E. M. Conwell and B. W. Levinger, p. 227; P. A. Wolff, p. 220.

[3] R. Bowers, Phys. Rev. **108**, 683 (1957).

[4] W. G. Spitzer, F. A. Trumbore, and R. A. Logan, J. Appl. Phys. **32**, 1822 (1961).

[5] P. H. Keesom and G. Seidel, Phys. Rev. **113**, 33 (1959).

[6] G. Dresselhaus, A. F. Kip, and C. Kittel, Phys. Rev. **98**, 368 (1955).

[7] D. M. S. Bagguley, R. A. Stradling, and J. S. Whiting, Proc. Roy. Soc. (London) **A262**, 340 (1961).

holes is about twenty-four. In addition there is a split-off band at $k=0$ lying about 0.3 eV below the band edge. For the samples discussed in this paper the carrier concentrations are low enough that the presence of this band can be disregarded.

In applying transport theory to conduction by the holes the following assumptions are made: (1) both the heavy and the light hole bands are spherical; (2) the conductivity in a band can be described by a relaxation time τ which is proportional to ϵ^s, where ϵ is the energy of the carrier and s is a parameter which depends on the scattering mechanism; (3) the scattering mechanism is the same for both types of carriers, and the effects of interband scattering can be neglected. The galvanomagnetic coefficients in this case are well known and we quote only the results here, using a notation similar to that used by Putley.[8]

We denote the heavy and the light hole bands by the subscripts 1 and 2, respectively, and write

$$\left.\begin{array}{lll}\sigma_{10}=p_1e\mu_1 & R_{10}=r/p_1e & \beta=p_1/p_2\\ \sigma_{20}=p_2e\mu_2 & R_{20}=r/p_2e & \alpha=\mu_2/\mu_1\end{array}\right\}, \qquad (1)$$

the zeros in the subscripts standing for the values of the quantities at zero magnetic field. The zero field conductivity of a sample is denoted by σ_0 and the value of the Hall coefficient extrapolated to zero field by R_0. In all the expressions it is assumed that σ is measured in Ω^{-1} cm^{-1}, R in cm^3 C^{-1}, and H in gauss. The analysis is made for $\beta=24$ and $s=\frac{3}{2}$; some of the expressions are given in terms of s to show how the value chosen for this parameter would affect the calculations.

R_0 is given by

$$R_0=[(\beta+1)(\beta+\alpha^2)(\beta+\alpha)^{-2}](r/pe), \qquad (2)$$

where $p=p_1+p_2$, and r is given by

$$r=\tfrac{3}{2}[(2s+\tfrac{3}{2})/(s+\tfrac{3}{2})^2] \times\{F_{1/2}(\eta^*)F_{2s+1/2}(\eta^*)/[F_{s+1/2}(\eta^*)]^2\}, \qquad (3)$$

η^* is the reduced Fermi energy, i.e., $\eta^*=\zeta/kT$, and F is the Fermi function. Tables of the required Fermi functions can be found in Tauc.[9] The values of r as a function of η^* for the case $s=\frac{3}{2}$ are given in Fig. 1.

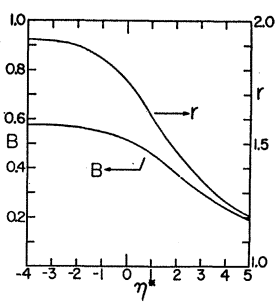

FIG. 1. The values of r [Eq. (3)], and of B [Eq. (6)], as a function of the reduced Fermi energy η^* for the scattering index s equal to $\frac{3}{2}$.

[8] E. H. Putley, *The Hall Effect and Related Phenomena* (Butterworths Scientific Publications, Ltd., London, 1960), Chap. 3.

[9] J. Tauc, *Photo and Thermoelectric Effects in Semiconductors* (Pergamon Press, Inc., New York, 1962), p. 225.

The field dependence of the Hall effect with two bands and arbitrary statistical degeneracy is somewhat complicated, and in view of the uncertainties in the experimental determinations, a detailed analysis is not warranted. If we omit the variations of R_1 and R_2 with magnetic field and consider only the variation of R due to two band effects, then for $R_0\sigma_0H\ll10^8$,

$$\begin{gathered}[(R_0-R)/R_0](R_0\sigma_0H)^{-2}\times10^{16}=f_1;\\ f_1=\beta\alpha^2(\beta+1)(\alpha-1)^2(\beta+\alpha^2)^{-3}.\end{gathered} \qquad (4)$$

The values of f_1 are given in Fig. 2.

The phenomenological description of the magnetoresistance effect in a cubic crystal can be made in terms of three constants b, c, and d.[10] If with respect to the crystal axes the direction cosines of the current vector are ι_1, ι_2, ι_3, and the direction cosines of the

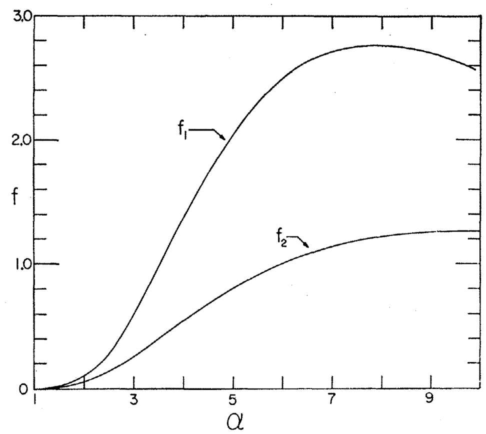

FIG. 2. The values of f_1 [Eq. (4)], and of f_2 [Eq. (6)] versus the ratio of the conductivity mobilities α.

magnetic field vector are η_1, η_2, η_3 then

$$(\Delta\rho/\rho H^2)=b+c(\sum\iota_j\eta_j)^2+d\sum\iota_j^2\eta_j^2.$$

If the bands are spherical

$$b+c=0,\quad d=0 \qquad (5)$$

and the magnitude of b is given by

$$b\times10^{16}=[B(f_2+1)+f_2]R_0^2\sigma_0^2,$$
$$\begin{gathered}B=[(s+\tfrac{3}{2})(3s+\tfrac{3}{2})/(2s+\tfrac{3}{2})^2] \times\{F_{s+1/2}(\eta^*)F_{3s+1/2}(\eta^*)/[F_{2s+1/2}(\eta^*)]^2\}-1,\\ f_2=\beta\alpha(\alpha-1)^2(\beta+\alpha^2)^{-2}.\end{gathered} \qquad (6)$$

The values of B as a function of η^* for the case $s=\frac{3}{2}$ are given in Fig. 1; the magnitude of f_2 for different values of α are shown in Fig. 2.

The equations in this section summarize the expected relations between the various measured quantities; α has been treated as a parameter to be determined from the data.

[10] F. Seitz, Phys. Rev. **79**, 372 (1950).

3. SAMPLES

The samples used in these experiments were cut from a single crystal grown in the $\langle 01\bar{1}\rangle$ direction by the normal Czochralski pulling technique. The length of the ingot was about 7 cm; the concentration of gallium varied by about a factor of ten from one end of the ingot to the other. Four slices were cut perpendicular to the growth axis of the ingot. From each slice two samples were taken, sample (a) with the length along $\langle 100\rangle$, and sample (b) with the length along $\langle 011\rangle$. These samples were ground down to rectangular plates with dimensions approximately $7\times1.4\times0.8$ mm, and etched in CP_4. Soldered contacts were used for making resistivity and Hall effect measurements. Two pairs of probes were used on each sample, and they gave results which agreed very closely with each other. The temperature and field dependence of the Hall effect were, within experimental error, the same for the samples (a) and (b) from a given slice. Thus we may conclude that there were no marked inhomogeneities in any of the specimens. In order to check the reproducibility of the results measurements were made on a bridge-shaped sample cut from a different ingot. The results were completely consistent with the measurements discussed here.

TABLE I. The values of R_0 and ρ_0 at 77°K, the direction of the current, and the plane in which H was rotated in determining the orientation dependence of the magnetoresistance.

Sample	R_0 cm^3 C^{-1}	ρ_0 Ω cm	$\langle i\rangle$	H $\perp$ to
Ia	8.46	0.0139	100	100
b			011	100
IIa	5.87	0.0106	100	011
b			011	100
IIIa	4.13	0.0081	100	100
b			011	100
IVa	0.95	0.0026	100	011
b			011	100

Conventional dc potentiometric methods were used in the experiments. The error in the measurement of the resistance is estimated to be less than 0.02%, and the error in the determination of the magnetic field to be less than 0.5%.

4. MEASUREMENTS AT 77°K

The values of R_0 and ρ_0 for the samples are given in Table I. The direction of the current and the plane in which H was rotated in determining the orientation dependence of the magnetoresistance are also indicated in the table.

The field dependence of the Hall coefficient for the different samples is shown in Fig. 3, where R is plotted against H^2. There is some scatter in the experimental points, but all the samples behave in the same way and show an approximately linear decrease of R with H^2.

TABLE II. The carrier concentration, the Fermi energy at absolute zero, the reduced Fermi energy at 77°K, and the transverse magnetoresistance coefficient at 77°K.

Sample	p cm^{-3}	ζ_0/k °K	η^* at 77°K	$(\Delta\rho/\rho H^2)(10^8/R_0\sigma_0)^2$ expntl.	calc. ($\alpha=3.5$)
I	1.2×10^{18}	130	1.0	1.1	1.04
II	1.6×10^{18}	160	1.5	1.0	0.99
III	2.1×10^{18}	190	2.0	0.9	0.93
IV	7.2×10^{18}	430	5.3	1.1	0.7

The values of $[(R_0-R)/R_0](R_0\sigma_0H)^{-2}\times10^{16}$ lie around 1.3. Equation (4) has been derived taking into account only the variation of R due to two band effects, hence it gives the maximum expected value of α, and so comparison with Fig. 2 suggests that α is less than four.

The values of p, η^*, and ζ_0 are listed in Table II. In calculating these the following procedure has been used. An approximate value of η^* is obtained by setting $\alpha=3.5$ and $r=1.5$ in Eq. (2), and using the standard relations

$$\zeta_0=(h^2/8m^*)(3p/\pi)^{2/3},\quad \tfrac{2}{3}(\zeta_0/kT)^{3/2}=F_{1/2}(\eta^*),$$

where m^*, the average density of states mass for both types of holes together, is 0.37 m_0.[11] This value of η^* is then used to get a new value of r from which the final values of p, η^*, and ζ_0 are obtained.

The final values of r used are consistent with the

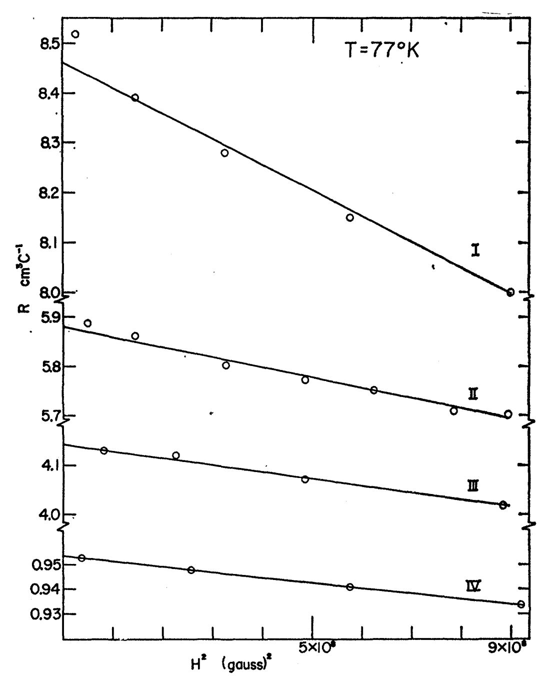

FIG. 3. Field dependence of the Hall coefficient at 77°K. The values given are the mean of the values for the samples (a) and (b) from a given slice.

[11] B. Lax and J. G. Mavroides, Phys. Rev. 100, 1650 (1955).

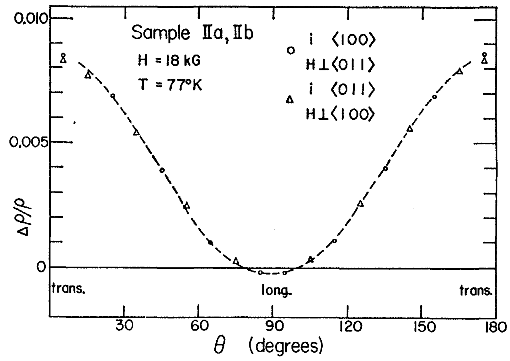

FIG. 4. Orientation dependence of the magnetoresistance at 77°K for the samples IIa and IIb.

results of Bernard, Roth, and Straub.[12] They determined the gallium concentration in heavily doped samples by different techniques, and found r varying from 1.6 to 1.0 as the gallium concentration changed from 10^{18} to 10^{19} cm^{-3}.

To within the accuracy of the measurements the longitudinal magnetoresistance is zero for all the samples. For each slice the transverse magnetoresistance of the sample (a) is the same as that of the sample (b). The variation of $\Delta\rho/\rho$ as the magnetic field is rotated from a transverse to a longitudinal direction is illustrated in Fig. 4. The orientation dependence of the magnetoresistance shows that the symmetry relations of Eq. (5) are obeyed, so that the spherical band approximation is a satisfactory one.

The field dependence of the transverse magnetoresistance for the different samples is shown in Fig. 5, where $\Delta\rho/\rho$ is plotted against H^2. It is seen that $\Delta\rho/\rho$ is approximately proportional to H^2; saturation effects are evident in the purer samples. The values of b have been found by extrapolating the slope of the curve to zero magnetic field. The experimental values of $b/(R_0\sigma_0)^2$ are given in Table II, together with the

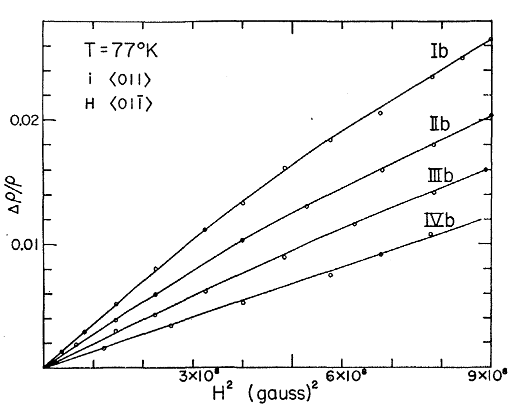

FIG. 5. Field dependence of the transverse magnetoresistance at 77°K for the samples I, II, III, and IVb.

[12] W. Bernard, H. Roth, and W. D. Straub, Bull. Am. Phys. Soc. **8**, 224 (1963); and private communication.

values calculated from Eq. (6) with α set equal to 3.5. There is good agreement between the two except for sample IV. For samples I, II, and III the experimental error in the value of $b/(R_0\sigma_0)^2$ is estimated to be 10%, arising chiefly from the uncertainty in the extrapolation procedure that has to be used for determining b. Sample IV has a very low resistivity and magnetoresistance and the experimental results can have a large error. It is also possible that for this sample the carrier concentration is so high that the influence of the nonparabolic nature of the bands becomes significant.

Other values of the parameters s and α have been tried, but they do not give as close an agreement with the observed Hall and magnetoresistance effects. The value of $\frac{3}{2}$ for the parameter s supports the idea that the dominant scattering mechanism is that due to the screened Coulomb potential of the ionized impurities.[13]

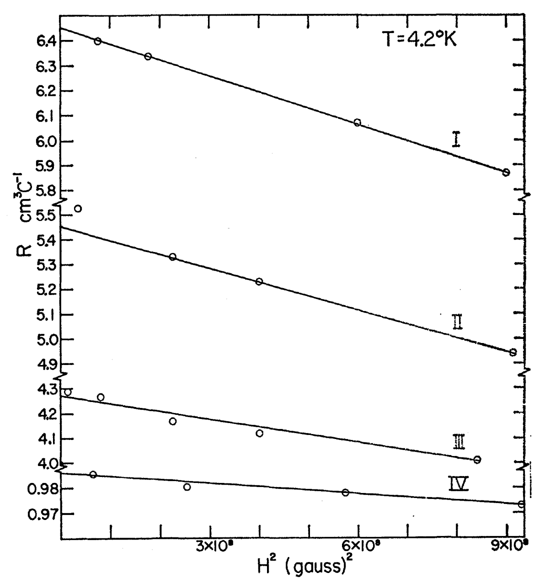

FIG. 6. Field dependence of the Hall coefficient at 4.2°K. The values given are the mean of the values for the samples (a) and (b) from a given slice.

5. MEASUREMENTS AT 4.2 AND 1.4°K

The results given in Sec. 4 indicate that the simple two carrier model is quite adequate for describing the general features of the galvanomagnetic effects at 77°K. The measurements at lower temperatures, however, are at variance with the predictions of the model. The resistivity and the Hall effect show deviations from the expected values, and very anomalous magnetoresistance effects are observed.

At liquid-helium temperatures, the carriers should behave like a degenerate Fermi gas. In this case, if the scattering were due to ionized impurities, the resistivity should have increased.[13] But the zero field resistivity of the samples at 4.2 and 1.4°K is the same as at 77°K.

[13] R. Mansfield, Proc. Phys. Soc. (London) **B69**, 76 (1956).

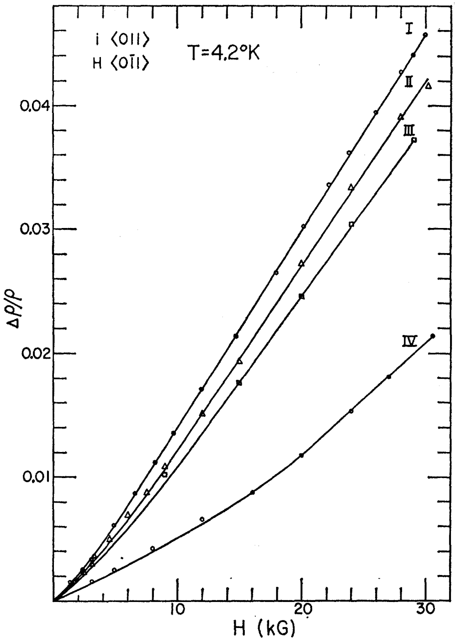

FIG. 7. Field dependence of the transverse magnetoresistance at 4.2°K for the samples I, II, III, and IVb.

Furthermore the Hall coefficient for a single band becomes independent of magnetic field so that the variation of R with H should be given by Eq. (4). The behavior of R at 4.2°K is shown in Fig. 6. (The 77°K data are shown in Fig. 3.) The values of $[(R_0-R)/R_0](R_0\sigma_0H)^{-2}\times10^{16}$, which should be given by f_1, are roughly 4.5, 4.2, 2, and 1 for the samples I, II, III, and IV, respectively. An examination of Fig. 2 shows that for samples I and II the variation of R with field is too large to be accounted for by a two band effect.

The magnetoresistance data can be summarized as follows:

(1) To within the experimental accuracy the results for sample (a) are the same as those for the sample (b) from the same slice.

(2) The transverse magnetoresistance at 4.2°K is shown in Fig. 7. Except at very low fields $\Delta\rho/\rho$ increases linearly with H.

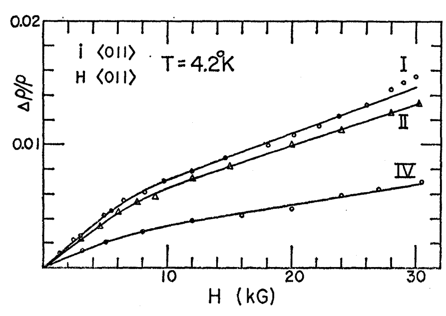

FIG. 8. Field dependence of the longitudinal magnetoresistance at 4.2°K for samples I, II, and IVb.

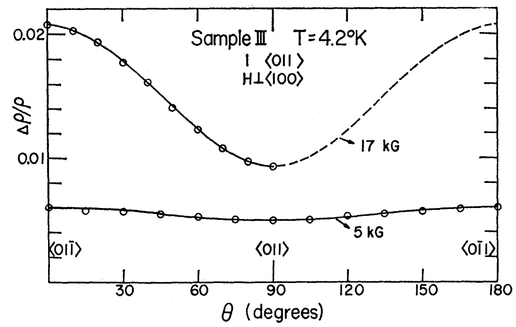

FIG. 9. Orientation dependence of the magnetoresistance at 4.2°K for the sample IIIb.

(3) A longitudinal magnetoresistance is observed which is comparable to the transverse component for fields below 6 kG, but then shows a less rapid increase with field, as illustrated in Fig. 8.

(4) The orientation dependence of the magnetoresistance as the field is rotated from a transverse to a longitudinal direction follows the relation $\Delta\rho/\rho=a_1+a_2\sin^2\theta$, where θ is the angle between $\mathbf{i}$ and $\mathbf{H}$. The results for sample IIIb are shown in Fig. 9.

(5) When the temperature is lowered from 4.2 to 1.4°K the magnetoresistance increases. The transverse component becomes linear with field down to the lowest fields. This is shown in Fig. 10, in which the data for sample IIb are plotted. Samples I and III show similar behavior; measurements at 1.4°K were not made on sample IV.

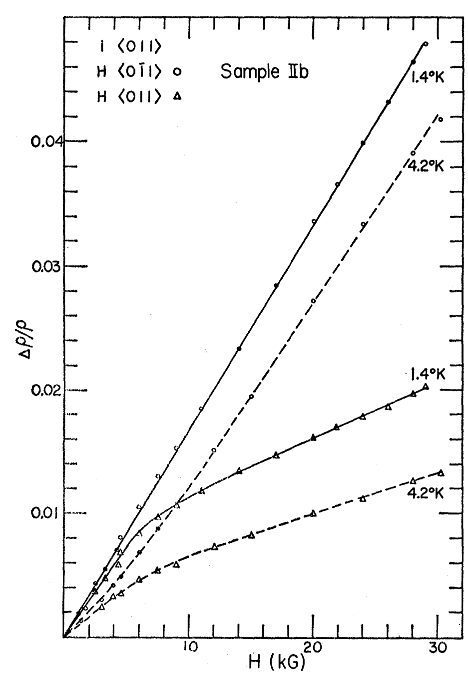

FIG. 10. Field dependence of the transverse and of the longitudinal magnetoresistance for sample IIb at 1.4°K and at 4.2°K.

The linear dependence of the transverse magnetoresistance on H, the presence of a longitudinal component, and the increase of $\Delta\rho/\rho$ on going from 4.2 to 1.4°K are phenomena which are completely different from the expected behavior. The discussion in Sec. 2 has been based on simple transport theory applied to carriers in a normal band. Within the framework of this model there are some effects which can be investigated to see if they could account for the observed anomalies. These are the warping of the bands, quantum effects in the magnetoresistance,[14] and the interaction of the spins of the heavy holes with the magnetic field. We discuss these effects briefly.

The results for the samples with the current along ⟨100⟩ are the same as those for the samples with the current along ⟨011⟩. Thus $\Delta\rho/\rho$ depends on the angle between **i** and **H** but does not depend on the orientation of the current with respect to the crystal axes. So it does not seem likely that the warping of the bands is responsible for the observed behavior.

The effects of the Landau quantization on the resistivity become important if $\omega\tau\gg 1$, where ω is the cyclotron frequency. For the heavy holes $\omega\tau\ll 1$. For the light holes, even if we assume the mobility to be six times larger than that of the heavy holes, $\omega\tau$ is smaller than unity at the highest fields that have been used. Hence quantum effects in the magnetoresistance should not play a significant role in the present experiments.

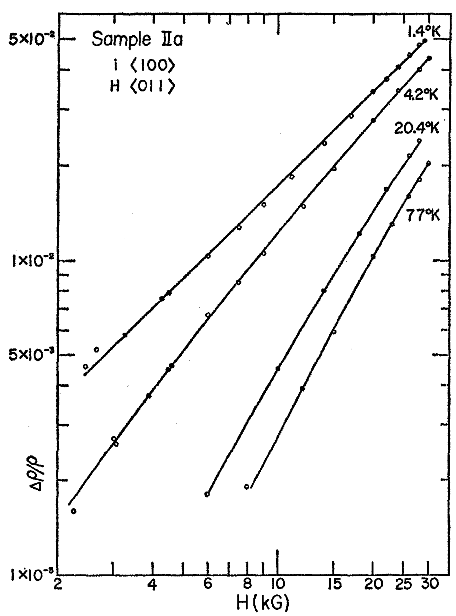

FIG. 11. Log-log plot of $\Delta\rho/\rho$ versus H at four different temperatures for sample IIa.

Theoretical calculations show that the g factor for the heavy holes is very large and anisotropic; in the spherical band approximation we have

$$g\approx 30|\cos\phi|-6.4,$$

where ϕ is the angle between the momentum of the hole and the magnetic field.[15,16] We attempted a calculation of the effect of this interaction on the conductivity along the following lines. The components of the conductivity tensor are given by

$$\sigma_{ij}=\frac{e^2}{4\pi^3}\int\frac{\tau v_i v_j}{|\mathrm{grad}_k\epsilon|}dS, \tag{7}$$

where the symbols have their usual meaning, and the integration is over the Fermi surface.[17] In the absence of a magnetic field the Fermi surface is a sphere defined by the relation $\hbar^2k^2=2m^*\zeta_0$. When a magnetic field is applied the Fermi surface splits up into two sheets, corresponding to the two directions of spin for the holes, and is defined by

$$\hbar^2k^2=2m^*(\zeta_0\pm\tfrac{1}{2}g\beta H).$$

If it is assumed that the only effect of the magnetic field is to change the surface of integration in this manner then a calculation based on Eq. (7) does not yield a linear dependence of $\Delta\rho/\rho$ on H. However, the picture is very oversimplified, and a theory of the effects of the paramagnetic interaction on the transport phenomena might prove useful in interpreting the data.

On the other hand, it is entirely possible that the description of the carriers by Bloch functions, and the treatment of the transport process by the assumption of a relaxation time, are both totally inappropriate for heavily doped materials. Several theoretical approaches

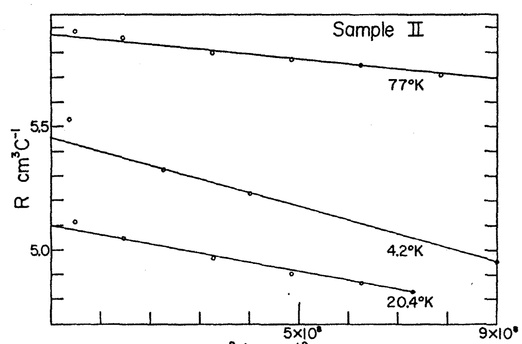

FIG. 12. Field dependence of the Hall coefficient at 77, 20.4, and 4.2°K for the samples II.

[14] E. N. Adams and R. W. Keyes, in *Progress in Semiconductors*, edited by A. F. Gibson (John Wiley & Sons, Inc., New York, 1962), Vol. 6, p. 87.

[15] Y. Yafet, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1963), Vol. 14, p. 32.

[16] R. Bowers and Y. Yafet, Phys. Rev. **120**, 62 (1960).

[17] A. H. Wilson, *The Theory of Metals* (Cambridge University Press, Cambridge, England, 1958), p. 196.

have been tried,[2,18–20] but they are as yet not in a form where their predictions of galvanomagnetic effects can be compared with experimental results.

6. MEASUREMENTS AT 20.4°K

Some measurements were made on samples IIa and IIb at 20.4°K. The zero field resistivity at this temperature is 0.0109 Ω cm. The transverse magnetoresistance is shown in Fig. 11, in which $\Delta\rho/\rho$ is plotted against H on a log-log scale. The behavior of R as a function of H^2 is shown in Fig. 12. The results at other temperatures are also indicated in these figures to facilitate a comparison between the data at different temperatures for the same sample.

[18] T. Kasuya, J. Phys. Soc. Japan **13**, 1096 (1958).
[19] T. Matsubara and Y. Toyozawa, Progr. Theoret. Phys. (Kyoto) **26**, 739 (1961).
[20] Y. Toyozawa, J. Phys. Soc. Japan **17**, 986 (1962).

ACKNOWLEDGMENTS

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High-Frequency Resistivity of Degenerate Semiconductors

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(Received 24 September 1963)

The high-frequency and long-wavelength resistivity due to the electron-proton (optical) interaction is calculated for degenerate semiconductors. This result is compared with the resistivity arising from electron-ion collisions calculated previously.

I. INTRODUCTION

In a recent paper, the high-frequency and long-wavelength resistivity of degenerate semiconductors was obtained[1] from a model of an electron gas moving in a medium of randomly distributed ions. Hence, the electron-ion collisions were responsible for the absorptive part of the conductivity and therefore the resistivity.

However degenerate semiconductors such as InSb, InP, GeP, etc., are ionic to a small degree and their conduction electrons interact with the polarized vibrations of the lattice (optical phonons). Therefore, the electron-phonon interaction is an additional mechanism which gives rise to an absorptive part of the conductivity. In the following pages we shall calculate the high-frequency and long-wavelength resistivity which arises from the electron-phonon (optical) interaction.

Our calculations are based on an expression for the resistivity[1] previously obtained by the author.[2] However, in order to perform the machine calculation, we compute the resistivity for a model semiconductor in which the frequency spectrum of the phonons is constant for all wave numbers, and the temperature of the system is taken to be zero.

Our calculations below show that for realistic semiconductors the resistivity due to the electron-phonon interaction is about 20% of the resistivity arising from electron-ion collisions.

[1] A. Ron and N. Tzoar, Phys. Rev. (to be published).
[2] N. Tzoar, Phys. Rev. **133**, 1213 (1964).

II. CALCULATIONS OF THE RESISTIVITY

We use here the notation and the definitions of Ref. 2. Our starting point will be Eq. (36) of Ref. 2 which gives the expression for the high-frequency and long-wavelength resistivity. After some algebra we obtain

$$R(\omega)=\frac{16}{3\omega_p{}^4m^2\omega}\int_0^q dqq^6|C_q|^2\frac{P}{4\pi}\int_{-\infty}^{+\infty}dx \times\left[\coth\left(\frac{\beta x}{2}\right)-\coth\frac{\beta}{2}(x+\omega)\right] \times\mathrm{Im}\frac{1}{\mathcal{E}(q,\,x+\omega)}\,\mathrm{Im}D_q{}^+(x)\,,\quad(1)$$

where $D_q{}^\dagger(x)=2\omega_q[(x+i\eta)^2-\omega_q{}^2]^{-1}$, $\eta=0_+$; $\omega_p=(4\pi e^2n/m)^{1/2}$ is the classical plasma frequency, and $\mathcal{E}(q,\omega)$ is the dielectric function of the electron gas.

We calculate now $R(\omega)$ for a simplified model in which $\omega_q=\omega_l=$constant for any wave number q, and the temperature of the system is taken to be zero (i.e.,