have been tried,[2,18–20] but they are as yet not in a form where their predictions of galvanomagnetic effects can be compared with experimental results.

6. MEASUREMENTS AT 20.4°K

Some measurements were made on samples IIa and IIb at 20.4°K. The zero field resistivity at this temperature is 0.0109 Ω cm. The transverse magnetoresistance is shown in Fig. 11, in which $\Delta\rho/\rho$ is plotted against H on a log-log scale. The behavior of R as a function of H^2 is shown in Fig. 12. The results at other temperatures are also indicated in these figures to facilitate a comparison between the data at different temperatures for the same sample.

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ACKNOWLEDGMENTS

The author is profoundly grateful to Dr. P. H. Keesom for his help and guidance throughout the course of this work. He is indebted to Dr. W. M. Becker for valuable discussions, and for the use of his apparatus; and he thanks Miss Louise Roth for growing the crystals used in these experiments.

PHYSICAL REVIEW VOLUME 133, NUMBER 4A 17 FEBRUARY 1964

High-Frequency Resistivity of Degenerate Semiconductors

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(Received 24 September 1963)

The high-frequency and long-wavelength resistivity due to the electron-proton (optical) interaction is calculated for degenerate semiconductors. This result is compared with the resistivity arising from electron-ion collisions calculated previously.

I. INTRODUCTION

In a recent paper, the high-frequency and long-wavelength resistivity of degenerate semiconductors was obtained[1] from a model of an electron gas moving in a medium of randomly distributed ions. Hence, the electron-ion collisions were responsible for the absorptive part of the conductivity and therefore the resistivity.

However degenerate semiconductors such as InSb, InP, GeP, etc., are ionic to a small degree and their conduction electrons interact with the polarized vibrations of the lattice (optical phonons). Therefore, the electron-phonon interaction is an additional mechanism which gives rise to an absorptive part of the conductivity. In the following pages we shall calculate the high-frequency and long-wavelength resistivity which arises from the electron-phonon (optical) interaction.

Our calculations are based on an expression for the resistivity[1] previously obtained by the author.[2] However, in order to perform the machine calculation, we compute the resistivity for a model semiconductor in which the frequency spectrum of the phonons is constant for all wave numbers, and the temperature of the system is taken to be zero.

Our calculations below show that for realistic semiconductors the resistivity due to the electron-phonon interaction is about 20% of the resistivity arising from electron-ion collisions.

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II. CALCULATIONS OF THE RESISTIVITY

We use here the notation and the definitions of Ref. 2. Our starting point will be Eq. (36) of Ref. 2 which gives the expression for the high-frequency and long-wavelength resistivity. After some algebra we obtain

$$R(\omega)=\frac{16}{3\omega_p{}^4m^2\omega}\int_0^q dqq^6|C_q|^2\frac{P}{4\pi}\int_{-\infty}^{+\infty}dx \times\left[\coth\left(\frac{\beta x}{2}\right)-\coth\frac{\beta}{2}(x+\omega)\right] \times\mathrm{Im}\frac{1}{\mathcal{E}(q,\,x+\omega)}\,\mathrm{Im}D_q{}^+(x)\,, \quad (1)$$

where $D_q{}^\dagger(x)=2\omega_q[(x+i\eta)^2-\omega_q{}^2]^{-1}$, $\eta=0_+$; $\omega_p=(4\pi e^2n/m)^{1/2}$ is the classical plasma frequency, and $\mathcal{E}(q,\omega)$ is the dielectric function of the electron gas.

We calculate now $R(\omega)$ for a simplified model in which $\omega_q=\omega_l=$constant for any wave number q, and the temperature of the system is taken to be zero (i.e.,

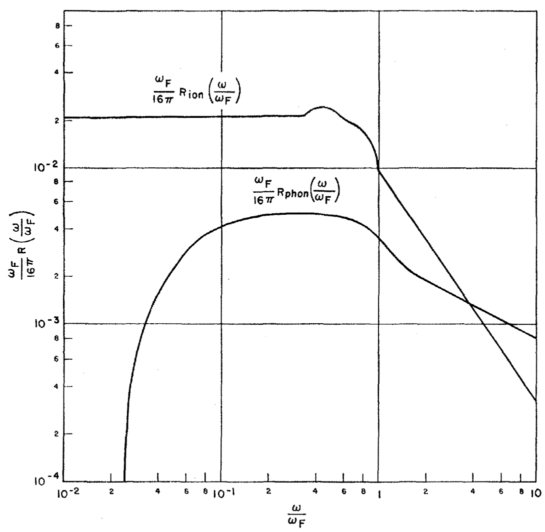

FIG. 1. Plot of the functions $(\omega_F/16\pi)R_{\rm ion}(\omega/\omega_F)$ and $(\omega_F/16\pi)R_{\rm phonon}(\omega/\omega_F)$, respectively.

$\beta\rightarrow\infty$). We obtain

$$R(\Omega)=\frac{16\pi}{\omega_F}12\pi^3(\alpha r_s)\left(\frac{\hbar\omega_F}{e^2/a_0}\right)\frac{1}{\mathcal{E}_0}\left(\frac{\omega_l^2-\omega_t^2}{\omega_t^2}\right)\Omega_l \cdot\frac{1}{\Omega}\int_0^\infty dk k^4\frac{\mathcal{E}_i(k,\Omega-\Omega_l)}{|\mathcal{E}(k,\Omega-\Omega_l)|^2}\theta(\Omega-\Omega_l)\,, \quad (2)$$

where $\mathcal{E}_i(k,\omega)$ is the imaginary part of $\mathcal{E}(k,\omega)$. Here ω_F is the Fermi frequency, $\alpha=(4/9\pi)^{1/3}$, $r_s=r_0/a_0$; where r_0 is the mean radius per electron, and $a_0=\hbar^2/me^2$ is the Bohr radius, ϵ_0 is the static dielectric constant and ω_l, ω_t are, respectively, the longitudinal and transverse frequencies of the optical vibrations. In Eq. (2), $k=q/2q_F$ where q_F is the Fermi wave number and Ω, Ω_l are, respectively, ω/ω_F and ω_l/ω_F, and $\theta(X)=1$ for $X>0$, $\theta(X)=0$ for $X<0$.

The asymptotic value of $R(\Omega)$ for frequencies $\Omega\gg1$, $\Omega_l<1$, is given by[3]

$$R^{(\infty)}(\Omega)=\frac{16\pi}{\omega_F}\frac{\pi^3}{2}(\alpha r_s)^2\left(\frac{\hbar\omega_F}{e^2/a_0}\right)\frac{1}{\mathcal{E}_0}\left(\frac{\omega_l^2-\omega_t^2}{\omega_t^2}\right)\Omega_l\Omega^{-1/2}\,, \quad (3)$$

where we used to expression for $\mathcal{E}(q,\omega)$ given by Glick and Ferrell.[4]

The computation of Eq. 2 has been carried out on an IBM-7094 for a degenerate semiconductor and is displayed in Fig. 1 together with the resistivity given in Ref. 1. We have chosen the effective electron mass and charge to be $m/100$ and $e/(10)^{1/2}$, respectively,[5] and the electron density to be 10^{18} electrons/cc. As for the photon part we have chosen[6] $\omega_l=10^{13}$ rad/sec and $\omega_l^2/\omega_t^2=\epsilon_0/\epsilon_\infty=3/1$, where ϵ_∞ is the crystal dielectric constant in the limit $\omega\rightarrow\infty$.

In conclusion, the resistivity $R(\omega)$ has a threshold at $\omega=\omega_l$, a wide peak around $\omega\sim0.4\,\omega_F$ and for $\omega>\omega_p$, $R(\omega)$ behaves as a constant, times $\omega^{-1/2}$. The resistivity due to the electron-phonon interaction is of the order of 20% of the resistivity arising from electron-ion collisions and in principle could be detected experimentally.

ACKNOWLEDGMENT

We would like to thank Miss V. Rogers of the Whippany Laboratories for the numerical computation.

[3] For more details, see Appendix of Ref. 1.

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