Evaluation of the Ginzburg-Landau-Abrikosov-Gor'kov Parameters from a Nonlocal, Nonlinear Theoretical Model

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Expressions for the Ginzburg-Landau-Abrikosov-Gor'kov (GLAG) parameters such as the weak-field penetration depth δ_0, the dimensionless parameter κ, and the range of order parameter δ_0/κ have been obtained from a simple, nonlocal, nonlinear theoretical model previously used to calculate the critical fields of superconducting films. We have shown that in the local limit, these expressions yield results which are in quite good agreement with those obtained by other techniques. We have also obtained simple expressions for these parameters which are valid in the thin film limit. From these limiting formulas, it is apparent that κ can get large in thin films, and it is estimated that κ exceeds the critical value of $1/\sqrt{2}$ in indium films somewhat thinner than 1000 Å. We also conclude that in the thin limit, one would expect the order parameter to be nearly constant across the film thickness.

IN previous papers,[1,2] hereafter referred to as I and II, respectively, a simple nonlocal, nonlinear theoretical model using the Pippard kernel was shown to be in good agreement with the experimentally observed variation of film critical field with thickness and composition. More recent results on the temperature dependence of critical field[3] indicate that the agreement between the theoretical model and experiment is improved by using the BCS kernel instead of the Pippard kernel. In I it was pointed out that, although explicit relations were obtained only for susceptibility and critical field, one might also obtain nonlocal relations for the Ginzburg-Landau weak field parameter δ_0. Once δ_0 has been calculated, one can easily obtain nonlocal expressions for the other Ginzburg-Landau-Abrikosov-Gor'kov parameters, such as κ and δ_0/κ, the range of order.

In this communication, we should like to point out that the values of κ and δ_0/κ calculated from this model in the impure or local limit are in good agreement with those calculated in quite a different manner by Gor'kov[4] and Caroli, de Gennes, and Matricon.[5] Further, we shall show how these quantities can be calculated at any temperature for films having any values of thickness and intrinsic mean free path. In particular, we shall calculate simple expressions valid in the thin film limit. We shall show, using parameters previously determined from critical field measurements on thin indium films, that the quantity κ exceeds the critical value $1/\sqrt{2}$ for pure indium films thinner than 750–1000 Å.

First, let us calculate δ_0. In I it is pointed out that the Ginzburg-Landau theory gives an explicit relationship between the weak field penetration depth δ_0 and the weak field susceptibility of a film χ, which is

$$\chi/\chi_0 = 1-(\delta_0/a)\tanh(a/\delta_0)\,, \qquad (1)$$

where χ_0 is the bulk weak field susceptibility and a is the half-thickness of the film. Using the methods developed by Schrieffer,[6] which are discussed in I and II, χ/χ_0 can be calculated from the kernel of any theory of superconductivity. For specular scattering, one can express the susceptibility in terms of an infinite series.

$$(\chi/\chi_0)_{\rm spec} = 1-(2/a^2)\sum_{n=0}^{\infty}[k_n^2+K(k_n)]^{-1}\,, \qquad (2)$$

where $k_n=(2n+1)\pi/2a$ and $K(k)$ is the kernel obtained from the relationship between the current density and vector potential. From (1) and (2) we can obtain a direct relationship between δ_0 and $K(k)$ which is

$$(\delta_0/a)\tanh(a/\delta_0) = (2/a^2)\sum_{n=0}^{\infty}[k_n^2+K(k_n)]^{-1}. \qquad (3)$$

If for $K(k)$, we use the Pippard kernel, δ_0 will be a function of only two nonlocal variables, $\xi_0\lambda_L^2/a^3$ and ξ/a:

$$\delta_0 = F(\xi_0\lambda_L^2/a^3,\ \xi/a)\,, \qquad (4)$$

where ξ_0 is the coherence length in pure material, λ_L is the London penetration depth, and ξ is the effective coherence distance which is related to the intrinsic mean free path l by the relation

$$1/\xi = 1/\xi_0 + 1/l. \qquad (5)$$

It is clear that from relation (4) we can calculate how δ_0 varies with thickness, mean free path, and temperature. In general, F must be numerically evaluated. However, in certain limits, one can obtain simple analytic expressions for δ_0. In the thin film limit, $\xi/a \gg 1$, we obtain from (2), using the Pippard kernel, the following expression[2]:

$$(\chi/\chi_0)_{\rm spec} \simeq 0.495(a^3/\xi_0\lambda_L^2)^{1/2}\,, \qquad (6)$$

which is independent of ξ/a. In this limit δ_0/a will be

[1] A. M. Toxen, Phys. Rev. **127**, 382 (1962).
[2] A. M. Toxen and M. J. Burns, Phys. Rev. **130**, 1808 (1963).
[3] A. M. Toxen, Intern. Conf. Science of Superconductivity, Colgate University, August 1963 (unpublished).
[4] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **37**, 1407 (1959) [English transl.: Soviet Phys.—JETP **10**, 998 (1960)].
[5] C. Caroli, P. G. de Gennes, and J. Matricon (to be published).
[6] J. R. Schrieffer, Phys. Rev. **106**, 47 (1957).

large and Eq. (1) yields

$$\chi/\chi_0 \simeq (1/3)(a/\delta_0)^2. \quad (7)$$

Combining (6) and (7) we obtain for $\xi/a \gg 1$,

$$\delta_0 \simeq 0.821(\xi_0 \lambda_L^2/a)^{1/2}. \quad (8)$$

Thus, we see that δ_0 should vary inversely as the square root of the thickness. For diffuse scattering, one obtains

$$\delta_0 \simeq 0.943(\xi_0 \lambda_L^2/a)^{1/2}. \quad (9)$$

Had we used the BCS[7] kernel instead of Pippard, relations very similar to (8) and (9) would have been obtained, differing only by a multiplicative factor which is a slowly varying function of temperature and is about one in magnitude.

For the local limit, we obtained in II the expression

$$\delta_0 = (\xi_0 \lambda_L^2/\xi)^{1/2}, \quad (10)$$

for specular boundary conditions and the Pippard kernel. In a bulk sample, the local limit is obtained for $\xi/\lambda \ll 1$ (where λ is the bulk penetration depth), which for a pure sample ($l/\xi_0 \gg 1$), will obtain only near T_c. Therefore, for a pure, bulk sample near T_c,

$$\delta_0 \simeq \lambda_L. \quad (11)$$

One can also reach the local limit by decreasing the electronic mean free path. Thus, in a thin film the local limit obtains for $l/a \ll 1$; in a bulk sample for $l/\lambda \ll 1$. And so for these cases,

$$\delta_0 \simeq \lambda_L(\xi_0/l)^{1/2}. \quad (12)$$

This is just the expression obtained by Pippard and Caroli *et al.* for the weak field penetration depth of a dirty superconductor.

Next, let us consider the quantity δ_0/κ. This characteristic length, which is temperature dependent as we shall see later, defines the breadth of the transition region between normal and superconducting phases.[8] The parameter δ_0/κ has been called by some the "range of order," since it characterizes the distance over which the order parameter varies, and it corresponds to ξ_T, the "coherence length" discussed by Caroli, de Gennes, and Matricon.[5] To avoid confusion with the coherence lengths ξ and ξ_0 appearing in (5), we shall refer to δ_0/κ as the "range of order parameter" and use for it the symbol L. Using for κ the Ginzburg-Landau[8] expression, as modified by Gor'kov,[4]

$$\kappa = (2\sqrt{2}e/\hbar c)H_c\delta_0^2, \quad (13)$$

where H_c is the bulk critical field, we get the following expression for L:

$$L = \delta_0/\kappa = \hbar c/2\sqrt{2}eH_c\delta_0. \quad (14)$$

[7] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[8] V. L. Ginzburg and L. D. Landau, Zh. Eksperim. i Teor. Fiz. **20**, 1064 (1950).

First, let us consider the local limit. We obtain, by substituting (10) into (14) the result

$$L = (\hbar c/2\sqrt{2}eH_c)[\xi/\xi_0\lambda_L^2]^{1/2}. \quad (15)$$

If we now extract all of the temperature-dependent terms, we obtain

$$L = \frac{\hbar c}{2\sqrt{2}e}\left\{\frac{H_0}{H_c(t)}\frac{\lambda_L(0)}{\lambda_L(t)}\right\}\left[\frac{\xi}{H_0^2\xi_0\lambda_L^2(0)}\right]^{1/2}, \quad (16)$$

where H_0 is the bulk critical field at $T=0°$K and t is the reduced temperature. As we showed in a previous communication,[9]

$$H_0^2\xi_0^2\lambda_L^2(0) = 3c^2\hbar^2/2\pi^2e^2. \quad (17)$$

Therefore, in the local limit, we obtain the desired expression:

$$L = \frac{\pi}{2\sqrt{3}}\left[\frac{H_0}{H_c(t)}\frac{\lambda_L(0)}{\lambda_L(t)}\right](\xi_0\xi)^{1/2}, \quad (18)$$

which gives us the complete temperature dependence. At $T=0°$K,

$$L = (\pi/2\sqrt{3})(\xi_0\xi)^{1/2} = 0.91(\xi_0\xi)^{1/2}. \quad (19)$$

Near T_c, we can obtain from the BCS theory an approximate expression for the term in brackets in relation (18). From Eq. (C40) of BCS,

$$\lambda_L(0)/\lambda_L(t) \simeq [2(1-t)]^{1/2} \quad (20)$$

for $t \simeq 1$. We can also obtain an expression for H_0/H_c near T_c, for

$$H_c \simeq -(dH_c/dt)_{t=1}(1-t). \quad (21)$$

But from Eq. (3.53) of BCS,

$$(dH_c/dt)_{t=1} = -1.82H_0. \quad (22)$$

Therefore, at $t \simeq 1$,

$$H_c(t)/H_0 \simeq 1.82(1-t). \quad (23)$$

If we now substitute (20) and (23) into (18), we obtain the limiting value of L near T_c:

$$L \simeq 0.70(\xi_0\xi)^{1/2}[T_c/(T_c-T)]^{1/2}. \quad (24)$$

In a dirty superconductor, $\xi \simeq l$, and we see that (24) is quite similar to (IV.3) of Caroli *et al.*

In a pure superconductor, $\xi \simeq \xi_0$, one would ordinarily reach the local limit only in a bulk sample very near T_c so that $\xi_0/\lambda \ll 1$. Under these conditions, we obtain from (24)

$$L \simeq 0.70\xi_0[T_c/(T_c-T)]^{1/2}, \quad (25)$$

which is of interest since it gives a direct relation between the two fundamental lengths L and ξ_0.

In like fashion, we can derive expressions for κ in the local limit which are virtually identical to those derived by Gor'kov and Caroli, de Gennes, and Matricon. By

[9] A. M. Toxen and P. M. Marcus (to be published).

substituting (10) into (13), we obtain for κ the following expression valid in the local limit:

$$\kappa=(2\sqrt{2}e/\hbar c)H_c(\xi_0\lambda_L^2/\xi). \quad (26)$$

By separating the temperature dependent terms, we obtain from (26)

$$\kappa=\frac{2\sqrt{2}e}{\hbar c}\left[\frac{H_c}{H_0}\frac{\lambda_L^2(t)}{\lambda_L^2(0)}\right]\frac{H_0\xi_0\lambda_L^2(0)}{\xi}. \quad (27)$$

By substituting (17) into (27) we then obtain the desired result.

$$\kappa=\frac{2\sqrt{3}}{\pi}\left[\frac{H_c(t)}{H_0}\frac{\lambda_L^2(t)}{\lambda_L^2(0)}\right]\frac{\lambda_L(0)}{\xi}. \quad (28)$$

Using (20) and (23) the quantity in brackets is easily evaluated near T_c where its value is 0.91. At 0°K it is obviously 1. Thus we see that κ is very nearly temperature independent on this model. In a bulk pure superconductor where $\xi=\xi_0$, the local limit will obtain only near T_c under ordinary circumstances. Under these conditions we obtain from (28) the result

$$\kappa=1.00[\lambda_L(0)/\xi_0], \quad (29)$$

which is in excellent agreement with the result obtained by Gor'kov.[4] In a dirty superconductor, $\xi=l$, and we obtain

$$\kappa=1.00[\lambda_L(0)/l],\quad T=T_c$$

and

$$\kappa=1.10[\lambda_L(0)/l],\quad T=0°\mathrm{K}, \quad (30)$$

in agreement with the results of Caroli *et al.* By using the definition of $\lambda_L(0)$ in terms of the normal state electronic properties, one can easily obtain the well-known result of Gor'kov. This is done as follows:

$$\lambda_L(0)=[3c^2/8\pi e^2N(0)v_F^2]^{1/2}, \quad (31)$$

and

$$l=3\sigma/2v_FN(0)e^2. \quad (32)$$

From (31) and (32), we obtain

$$\kappa=\lambda_L(0)/l=ec\gamma^{1/2}/2\pi^{3/2}k\sigma, \quad (33)$$

where γ is the coefficient of the electronic specific heat. $N(0)$ is the density of states at the Fermi surface, which is related to γ by the relation $N(0)=3\gamma/2\pi^2k^2$, σ is the normal state conductivity, v_F is the Fermi velocity, and k is Boltzmann's constant.

Let us consider at this point what we have derived thus far. Using a nonlocal, nonlinear model for δ_0, we have shown that it is possible to calculate δ_0, L, and κ as a function of the relevant parameters such as temperature, mean free path, thickness, and the electronic properties of the materials (through ξ_0 and λ_L). We have derived explicit expressions for δ_0, L, and κ in the local limit, and shown that these expressions are virtually identical to the results of others in this limit; i.e., we have demonstrated that the results of this model are consistent with the results derived by Gor'kov, Caroli, *et al.*, etc. What we shall do next is produce explicit relations for these parameters in the thin film limit, where the electrodynamics is nonlocal.

From (8) or (9), the expression for δ_0 can be written as

$$\delta_0=A(\xi_0\lambda_L^2/a)^{1/2}, \quad (34)$$

where A is 0.82 for specular scattering and 0.94 for diffuse. Had we used the BCS kernel, we would have obtained a similar result but with A a slowly varying function of temperature, although still about one in value. If (34) is substituted into (14) we obtain after a little algebra

$$L=(0.91/A)[H_0/H_c(t)][\lambda_L(0)/\lambda_L(t)](\xi_0a)^{1/2}. \quad (35)$$

From (35) we get

$$L=(0.91/A)(\xi_0a)^{1/2}\quad\text{for}\quad T=0°\mathrm{K}.$$
$$=(0.70/A)(\xi_0a)^{1/2}[T_c/(T_c-T)]^{1/2}\quad\text{for}\quad T\simeq T_c. \quad (36)$$

Thus, we see that in a thin film one would expect the range of order to be proportional to the square root of the thickness and the square root of the coherence length. One can also obtain expressions for κ in the thin limit by substituting (34) into (13).

$$\kappa=(2\sqrt{2}e/\hbar c)A^2H_c(\xi_0\lambda_L^2/a). \quad (37)$$

From (37) we obtain, as previously,

$$\kappa=1.10A^2\left[\frac{H_c(t)}{H_0}\frac{\lambda_L^2(t)}{\lambda_L^2(0)}\right]\frac{\lambda_L(0)}{a}. \quad (38)$$

From (38) we obtain, the limiting results

$$\kappa=A^2[\lambda_L(0)/a],\qquad T=T_c$$
$$=1.10A^2[\lambda_L(0)/a],\quad T=0°\mathrm{K}. \quad (39)$$

Thus, we see from (38) or (39) that in the thin limit κ is proportional to the London penetration depth and inversely proportional to the thickness. Hence one would expect κ to become quite large in a thin film. At the recent superconductivity conference,[10] Douglass suggested that a thin film would be a type II super conductor, even in a tangential magnetic field. From (39) one can see that the condition of $\kappa>1/\sqrt{2}$ can easily be satisfied. In fact, if we use the value of $\lambda_L(0)$ previously determined for indium from critical field measurements, 360 Å, using the Pippard kernel and specular boundary conditions, we obtain for the critical half thickness, below which type II behavior could occur:

$$a_{\mathrm{critical}}\simeq 375\ \text{Å}\quad\text{at}\quad T=0°\mathrm{K}. \quad (40)$$

Assuming diffuse scattering, one obtains a slightly greater critical thickness,

$$a_{\mathrm{critical}}\simeq 490\ \text{Å}\quad\text{at}\quad T=0°\mathrm{K}. \quad (41)$$

[10] D. H. Douglass, Jr., Intern. Conf. Science of Superconductivity, Colgate University, August 1963 (unpublished).

Both values, however, give a critical half thickness below 500 Å. Thus, we see that a film somewhat thinner than 1000 Å *could* break up into a mixed state. Whether it does, however, depends on whether it is energetically favorable to do so. There is, though, one further piece of information. Even though κ is greater than 0.707 in a thin film, the order will probably be constant across the film thickness d, for we can show that $L>d$ for $d<d_{\text{critical}}$. From (35)

$$a/L=\frac{A}{0.91}\left[\frac{H_c(t)}{H_0}\frac{\lambda_L(t)}{\lambda_L(0)}\right](a/\xi_0)^{1/2}. \tag{42}$$

For diffuse scattering, which gives the slightly larger estimate, we obtain from (42) the result

$$a/L=0.45\,,\quad \text{at}\quad T=0°\text{K}\,,\quad a=a_{\text{critical}}. \tag{43}$$

For thinner films, higher temperatures, or specular scattering, a/L is even smaller. Thus we see that the order probably does not vary across the film thickness in a thin film.

To summarize our results, we have shown that a simple nonlocal model previously used with some success to calculate the critical fields of superconducting films can also be used to calculate the Ginzburg-Landau-Abrikosov-Gor'kov parameters such as the weak field penetration depth δ_0, the dimensionless parameter κ, and the range of order parameter $L=\delta_0/\kappa$. We have calculated these quantities in the local limit and have shown that the results are in good agreement with those obtained by other techniques. We have also obtained simple expressions for these parameters which are valid in the thin limit. From these limiting formulas, we show that κ can get large in a thin film and estimate that κ exceeds the critical value of $1/\sqrt{2}$ in indium films somewhat thinner than 1000 Å. We also conclude that in the thin limit, one would expect the order parameter to be constant across the film thickness.

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Test of Special Relativity or of the Isotropy of Space by Use of Infrared Masers*

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The highly monochromatic frequencies of optical or infrared masers allow very sensitive detection of any change in the round-trip optical distance between two reflecting surfaces. Hence, comparison of the frequencies of two masers with axes perpendicular to each other allows an improved experiment of the Michelson-Morley type, or a very precise examination of the isotropy of space with respect to light propagation. Two He-Ne masers were mounted with axes perpendicular on a rotating table carefully isolated from acoustical vibrations. Their frequency difference was found to be constant to within 30 cps over times as short as about one second, or to one part in 10^{13} of the maser frequency, which is near 3×10^{14} cps. Rotation of the table through 90° produced repeatable variations in the frequency difference of about 275 kc/sec, presumably because of magnetostriction in the Invar spacers due to the earth's magnetic field. Examination of this variation over six consecutive hours shows that there was no relative variation in the maser frequencies associated with orientation of the earth in space greater than about 3 kc/sec. Hence there is no anisotropy or effect of either drift larger than 1/1000 of the small fractional term $(v/c)^2$ associated with the earth's orbital velocity. This preliminary version of the experiment is more precise by a factor of about 3 than previous Michelson-Morley experiments. There is reason to hope that improved versions will allow as much as 2 more orders of magnitude in precision, and that similar techniques will also yield considerably improved precision in an experiment of the Kennedy-Thorndike type.

OPTICAL and infrared masers make possible and attractive a number of new experiments, and refinements of old ones, where great precision in measurement of length is needed. On type is the examination of the isotropy of space for light propagation, or more specifically the examination of what effects the earth's velocity or various other fields may have on the velocity of light. We have completed the first stages of an experiment with He-Ne masers which can be regarded as equivalent to a Michelson-Morley experiment of improved precision. These preliminary tests show that the effect of "ether drift" is less than 1/1000 of that which might be produced by the earth's orbital velocity.

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I. INTRODUCTION

A synoptic treatment of the connections between measurements in coordinate systems in relative motion