Effect of Pairing Correlations on Nuclear Gamma-Ray Transitions*

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Several examples of the effect of pairing correlations on nuclear gamma-ray transitions are presented. It is shown that even if collective coupling exists, gamma-ray transitions of electric multipole type between low-lying quasiparticle states whose free shell-model orbit energies are symmetrically related to the Fermi energy may be extremely retarded. In this work the factor $(U_iU_f-V_iV_f)$ which arises from the pairing correlation is directly estimated from empirical even-odd mass difference data. The agreement of the calculated transition probabilities with experimental results is good. A shell-model calculation taking into account the pairing correlation has also been made for transitions of $E2$ type between $d_{3/2}$ and $s_{1/2}$ orbits in Sn^{117} and Sn^{118} and the results are compared with that of the quasiparticle approximation. It is shown that the quasiparticle approximation is more improved by using renormalized wave functions which are obtained by projecting out only the terms having the correct number of particles than by taking into account only the blocking effect except for special cases.

I. INTRODUCTION

THE theory of superconductivity has also been successfully applied to investigation of the effect of the pairing correlation on various properties of nuclei. One of the most interesting effects is a hindrance of certain nuclear gamma-ray transitions of electric multipole type by the pairing correlation, since this effect is quite different from what would be predicted by the pure shell model (or the shell model with a diagonal pairing energy). If the initial and final quasiparticle states are both near the Fermi energy, the gamma-ray transitions may be seriously hindered.[1] This was suggested by Grin and independently by Kisslinger and Sorensen.[2,3] Experimental evidence for this effect has been found for $E2$ transitions in Sn^{118}, Sn^{120}, and Sb^{122}.[4–6] Such evidence is, however, still limited to the single closed shell (SCS) or SCS-plus-one nucleon nuclei. Therefore, it would be of great value to make a more systematic investigation of this effect for more general non-SCS nuclei.

Under the pairing correlation theory, intrinsic excitation spectra of nuclei are described in terms of independent quasiparticles.[3,7] Any quasiparticle state, however, may not always remain as a pure one, but may couple to other quasiparticle states or to collective excitation modes through certain kinds of residual interactions. The latter coupling plays an especially important role, since it may cause large enhancement effects for gamma-ray transitions. Moreover, this coupling effect is expected to increase as one moves away from closed-shell regions because of a decrease of the nuclear rigidity (or stability) for the collective oscillation. As was noted by KS, however, this coupling is strongly reduced by the pairing correlation for quasiparticles near the Fermi surface.[8] Thus, as far as the low excited quasiparticle states are concerned, it may be reasonable to treat the collective coupling by perturbation theory (Sec. II). Other residual interactions between quasiparticles are ignored in the present work for simplicity. This approach may be too simple for the treatment of the real problem to be quantitatively correct; however, in view of our scanty knowledge about the residual interaction, it seems to be worthwhile to compare such a simple calculation with experiment.

In order to minimize ambiguity of the calculation caused by unsuitable choice of parameters, the hindrance factor $(U_iU_f-V_iV_f)$ of the pairing correlation is determined in this work from empirical data on even-odd mass difference. The nucleon effective charge, an additional positive charge arising from the collective coupling, is also estimated from the empirical data

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[1] Preliminary reports on the effect of pairing correlation on gamma-ray transition probabilities are found in H. Ikegami and T. Udagawa, Japan Atomic Energy Research Institute Report No. JAERI-1020, 1962, p. 129 (unpublished); T. Udagawa and H. Ikegami, Japan Atomic Energy Research Institute Report No. JAERI-1020, 1962, p. 138 (unpublished).

[2] Yu T. Grin, Zh. Eksperim. i Teor. Fiz. **39**, 138 (1960) [English transl.: Soviet Phys.—JETP **12**, 100 (1961)].

[3] L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 9 (1960). This paper will be referred to here as KS.

[4] H. Ikegami and T. Udagawa, Phys. Rev. **124**, 1518 (1961).

[5] H. H. Bolotin, A. Li, and A. Schwarzschild, Phys. Rev. **124**, 213 (1961).

[6] E. der Mateosian and M. L. Sehgal, Phys. Rev. **129**, 2195 (1963).

[7] S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 9 (1960).

[8] This is a special case of a more general one [see Eqs. (18) and (22)].

(Sec. III). Then the calculated results are compared with experimental data on $E2$ and $E3$ transitions (Sec. IV). Although the approach is simple, the calculation agrees well with experimental results.

A shell-model calculation using the pairing Hamiltonian has also been made for Sn^{117} and Sn^{118}. Transition probabilities for the $E2$ transitions between $d_{3/2}$ and $s_{1/2}$ neutron orbits have been calculated and the results compared with those of the quasiparticle approximation. The effects of blocking are also discussed (Sec. V).

II. EVALUATION OF THE NUCLEAR GAMMA-RAY TRANSITION PROBABILITY

The transition probability for radiation of a photon of multipole order λ and of the frequency ω is given by[9]

$$T(\lambda)=\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar^2}\left(\frac{\omega}{c}\right)^{2\lambda+1}B(\lambda)\,, \tag{1}$$

where $B(\lambda)$ is the reduced transition probability and is written as

$$B(\lambda)=\frac{1}{2J_i+1}|\langle\Psi_f\|\mathfrak{M}(\lambda\mu)\|\Psi_i\rangle|^2. \tag{2}$$

$\mathfrak{M}(\lambda\mu)$ is the electric multipole operator of order $(\lambda\mu)$;

$$\mathfrak{M}(\lambda\mu)=\mathfrak{M}_p(\lambda\mu)+\mathfrak{M}_{\text{coll}}(\lambda\mu)\,, \tag{3}$$

$$\mathfrak{M}_p(\lambda\mu)=\sum_{i=1}^{A} e_i r_i^\lambda Y_{\lambda\mu}(\theta_i\phi_i)\,, \tag{4}$$

$$\mathfrak{M}_{\text{coll}}(\lambda\mu)=(3/4\pi)ZeR_0^\lambda\alpha_{\lambda\mu}. \tag{5}$$

Here $\mathfrak{M}_p(\lambda\mu)$ and $\mathfrak{M}_{\text{coll}}(\lambda\mu)$ are the single-particle part and the collective part of the multipole operator, respectively. e_i is the charge of the ith nucleon; i.e., $e_i=e$ for protons and $e_i=0$ for neutrons. R_0 is the nuclear radius; in making numerical estimates we have made the conventional assumption

$$R_0=1.2\text{A}^{1/3}\times10^{-13}\text{ cm}.$$

In Eq. (5), $\alpha_{\lambda\mu}$ indicates the collective deformation parameter of the multipole order (λ,μ) which may be rewritten in terms of creation and annihilation operators $b_{\lambda\mu}{}^*$ and $b_{\lambda\mu}$ for phonons as follows:

$$\alpha_{\lambda\mu}=(\hbar\omega_\lambda/2C_\lambda)^{1/2}[b_{\lambda\mu}+(-)^\mu b_{\lambda-\mu}{}^*]. \tag{6}$$

Here $\hbar\omega_\lambda$ and C_λ are the energy associated with each phonon and the rigidity of the collective oscillation of order λ, respectively.

Now, we consider a gamma-ray transition whose initial and final states are quasiparticle states coupled to the collective oscillations. We assume with KS that the coupling Hamiltonian has the following form:

$$H=\sum_\lambda H_\lambda=-\frac{3}{4\pi}\sum_\lambda R_0{}^\lambda\chi_\lambda\left(\frac{\hbar\omega_\lambda}{2C_\lambda}\right)^{1/2}\times\sum_{j_1j_2m_1m_2\mu}\langle j_1m_1|r^\lambda Y_{\lambda\mu}|j_2m_2\rangle(U_{j1}U_{j2}-V_{j1}V_{j2})\times[d_{j_1m_1}{}^\dagger d_{j_2m_2}b_{\lambda\mu}+\text{c.c.}]. \tag{7}$$

Here χ_λ is the coupling constant, while $d_{jm}{}^\dagger$ and d_{jm} are creation and annihilation operators of a quasiparticle in a state $|jm\rangle$, respectively. The quantities U_j and V_j are usual fractional occupation parameters given by

$$U_j^2=\tfrac{1}{2}[1+(\epsilon_j-\lambda/E_j)],\quad V_j^2=\tfrac{1}{2}[1-(\epsilon_j-\lambda/E_j)], \tag{8}$$

with

$$E_j=[(\epsilon_j-\lambda)^2+\Delta^2]^{1/2}. \tag{9}$$

E_j and ϵ_j are single quasiparticle and the (free) particle energies of orbit j, respectively, while λ and Δ are the Fermi energy and half the energy gap, respectively. The Hamiltonian has exactly the same form as the last term of Eq. (39) of KS except that Eq. (7) includes the collective coupling of all multipole orders. Besides, KS disregarded the excitation of the closed-shell core in their coupling Hamiltonian but rather took into account this core excitation effect by renormalizing the parameters of the residual interaction. In the present investigation, however, Eq. (7) is derived by considering the excitation of the whole nucleus. Then, the Hamiltonian [Eq. (7)] is essentially equivalent to the BM Hamiltonian for a single particle coupled to a surface oscillator and thus one can set

$$k_\lambda=(3/4\pi)AR_0{}^\lambda\chi_\lambda\langle j|r^\lambda|j\rangle_u\,, \tag{10}$$

where k_λ is the coupling constant defined by BM. The Hamiltonian (7), however, differs from the one in BM by the factor $(U_{j1}U_{j2}-V_{j1}V_{j2})\lesssim1$, as we are considering the coupling of quasiparticles (rather than that of free particles).

The basic wave functions are denoted by $|j,RP_\lambda;I\rangle$ in which the quasiparticles are coupled to give a spin of j; P_λ phonons of spin λ are coupled to give a spin R; and then j and R are coupled to give a total spin of I. The initial- and final-state wave functions are given in perturbation theory limited to perturbation of one-phonon states.

$$\Psi=|j_\nu 00;I_\nu\rangle+\sum_{j'}\frac{\langle j'\lambda1I_\nu|H_\lambda|j_\nu00I_\nu\rangle}{E_{j\nu}-(E_{j'}+\hbar\omega_\lambda)}|j'\lambda1I_\nu\rangle,\quad \nu\equiv i,f. \tag{11}$$

If we set

$$E_{j_i}-E_{j_f}=\hbar\omega\,, \tag{12}$$

[9] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 27, No. 16 (1953). This paper will be referred to here as BM.

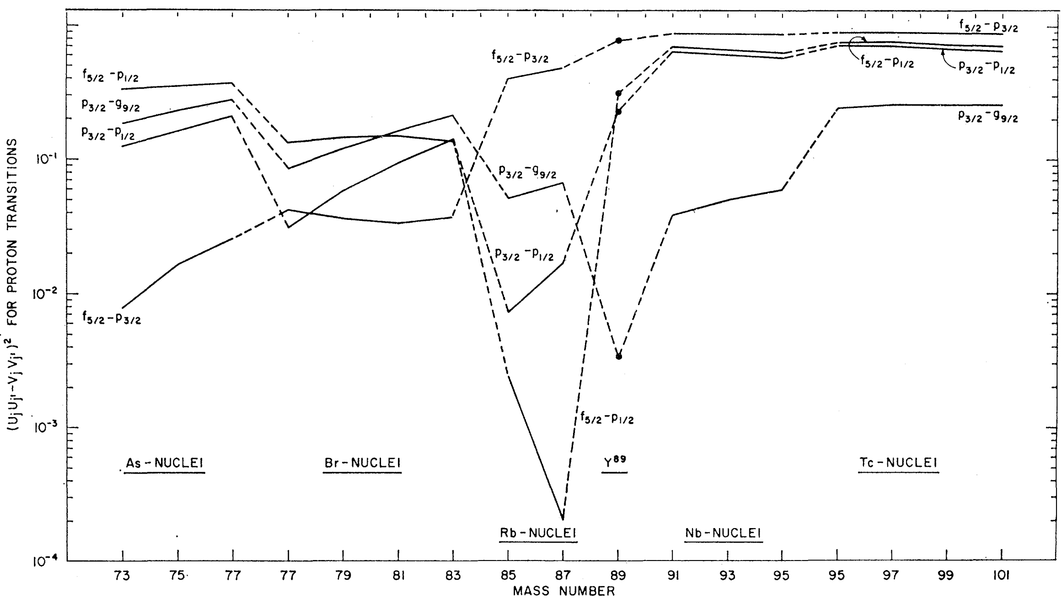

FIG. 1. $(U_iU_f-V_iV_f)^2$ factor of $E2$ proton transitions of $f_{5/2}\rightleftarrows p_{3/2}$, $f_{5/2}\rightleftarrows p_{1/2}$, $p_{3/2}\rightleftarrows p_{1/2}$, and of $p_{3/2}\rightleftarrows g_{9/2}$ in nuclei around the region $N=50$. The values of λ, Δ, and of free shell-model orbital energies are taken from Ref. 13.

where ω without subscript is the frequency of the emitted gamma ray; then the reduced matrix element for the transition between single quasiparticle states is

$$\begin{aligned}\langle\Psi_{j_f}\|\mathfrak{M}(\lambda\mu)\|\Psi_{j_i}\rangle &= e_p\langle j_f\|r^\lambda Y_{\lambda\mu}\|j_i\rangle(U_{ji}U_{jf}-V_{ji}V_{jf}) \\ &+\left(\frac{3R_0^\lambda}{4\pi}\right)^2 ZeA\frac{\chi_\lambda}{C_\lambda}\frac{\omega_\lambda^2}{\omega_\lambda^2-\omega^2}\langle j_f\|r^\lambda Y_{\lambda\mu}\|j_i\rangle(U_{ji}U_{jf}-V_{ji}V_{jf});\end{aligned} \quad (13)$$

$e_p=e$ for proton transition, $e_p=0$ for neutron transition. As can be easily seen, the result is essentially the same with those derived by BM and by Raz[10] except for an additional factor caused by the pairing correlation. The first term of the above formula represents the transition matrix element of single quasiparticle while the second term represents the collective influence due to the interaction of the quasiparticles with the collective modes of the nucleus. Then the term

$$e_{\rm eff}(\lambda)\equiv\left(\frac{3R_0^\lambda}{4\pi}\right)^2 AZe\frac{\chi_\lambda}{C_\lambda}\frac{\omega_\lambda^2}{\omega_\lambda^2-\omega^2} =\frac{3R_0^\lambda}{4\pi}\frac{Ze}{C_\lambda}\frac{k_\lambda}{\langle|r^\lambda|\rangle_u}\frac{\omega_\lambda^2}{\omega_\lambda^2-\omega^2} \quad (14)$$

fills the role of an additional positive effective charge given to each quasiparticle and causes an enhancement of the electric gamma-ray transition of multipole order λ. As is seen from Eq. (13) there may be some competition between the pairing correlation and the collective effects. The fact that the collective effect is weak for the closed-shell nuclei is just the reason why evidence on the effect of pairing correlation on gamma-ray transitions has previously been limited to SCS nuclei. It is of value to note here that the factor $(U_{ji}U_{jf}-V_{ji}V_{jf})$ may change rapidly with neutron or proton number in many cases (see Sec. III), while the effective charge varies gradually. Thus, even if there is some cancellation between effects of pairing correlation and of collective coupling, still one can isolate the effect of pairing correlation by investigating the neutron or proton number dependence of transition matrix elements.

In the same way as for Eq. (13), the transition matrix element between double-quasiparticle states is also given by

$$\begin{aligned}\langle\Psi_{J_f}\|\mathfrak{M}(\lambda,\mu)\|\Psi_{J_i}\rangle &=(2J_i+1)^{1/2}(2J_f+1)^{1/2}(U_{ji}U_{jf}-V_{ji}V_{jf}) \\ &\times W(J_0j_iJ_f\lambda;J_ij_f)(e_p+e_{\rm eff}(\lambda))\langle j_f\|r^\lambda Y_{\lambda\mu}\|j_i\rangle, \quad j_i\neq j_f.\end{aligned} \quad (15)$$

Here, two quasiparticles of spin j_0 and j_i are coupled to give a spin of J_i, while spin j_0 and j_f are coupled to give a spin of J_f. Equation (15) is also applicable for the case $j_0=j_i$ or j_f, while in the case of $j_i=j_f\equiv j$, the

[10] B. James Raz, Phys. Rev. **120**, 169 (1960).

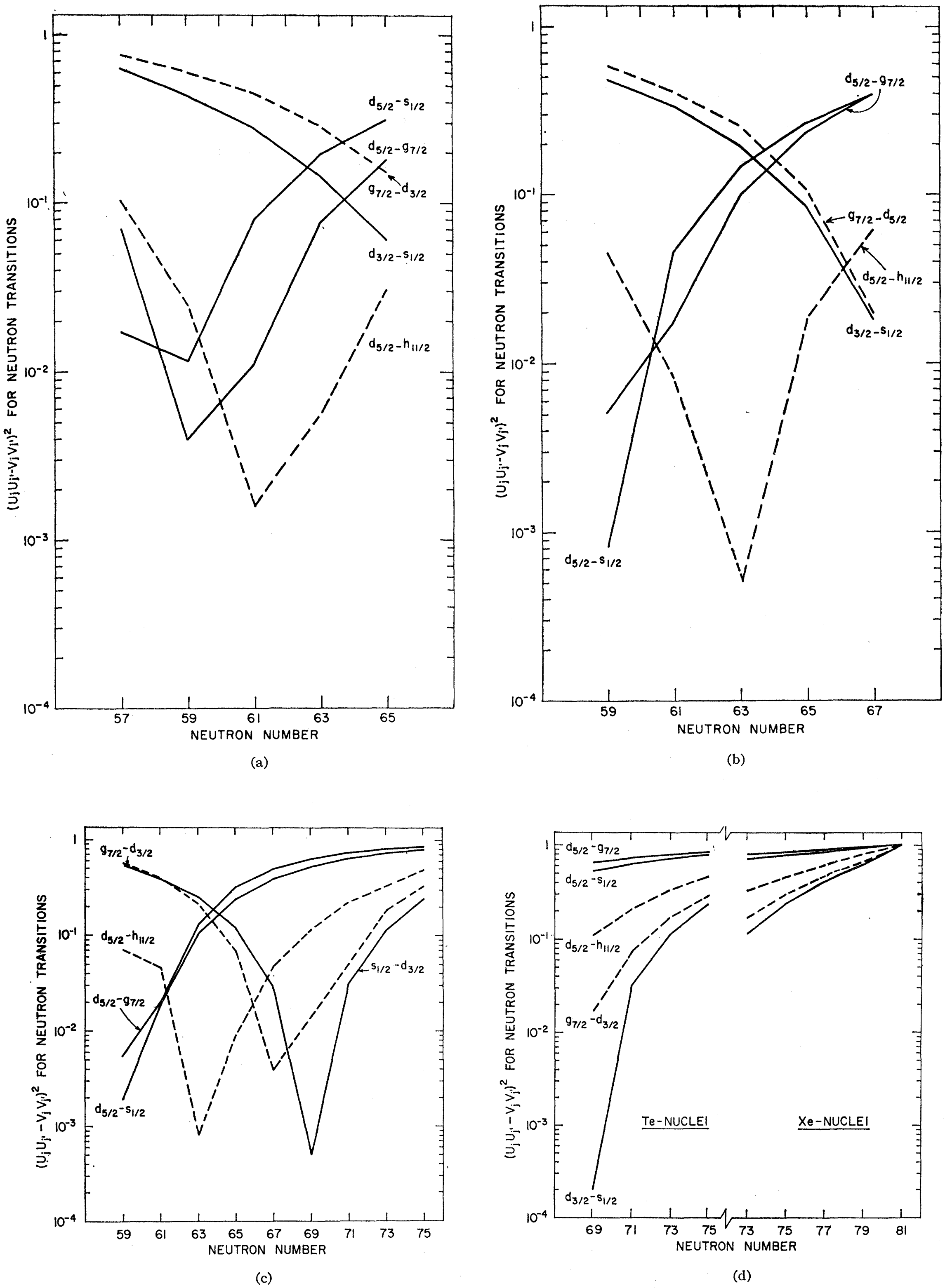

FIG. 2. $(U_iU_f-V_iV_f)^2$ factor of $E2$ neutron transitions of $d_{5/2}\rightleftarrows s_{1/2}$, $d_{5/2}\rightleftarrows h_{11/2}$, $s_{1/2}\rightleftarrows d_{3/2}$, and of $s_{1/2}\rightleftarrows h_{11/2}$ in nuclei around the region $Z=50$ [Pd (a), Cd (b), Sn (c), Te and Xe (d)]. Curves are obtained by taking parameter values of Ref. 13.

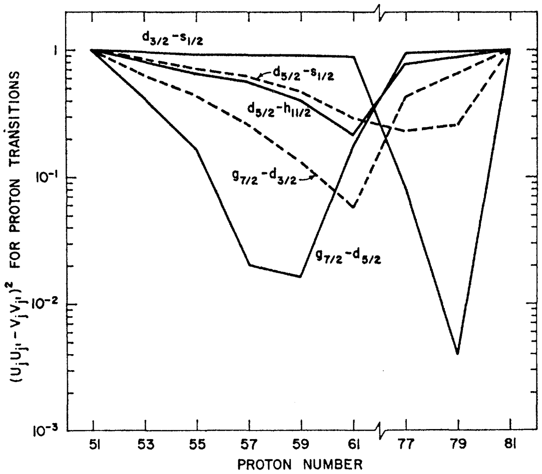

FIG. 3. $(U_iU_f-V_iV_f)^2$ factor of proton transition of $g_{7/2}\rightleftarrows d_{5/2}$, $g_{7/2}\rightleftarrows d_{3/2}$, $d_{5/2}\rightleftarrows h_{11/2}$, $s_{1/2}\rightleftarrows d_{3/2}$, and of $s_{1/2}\rightleftarrows h_{11/2}$ in nuclei around the region $N\approx 82$, 124. The values of λ, Δ, and of free shell-model orbital energies are taken from Ref. 13.

transition matrix element has the form:

$$\begin{aligned}\langle\Psi_{J_f}\|\mathfrak{M}(\lambda\mu)\|\Psi_{J_i}\rangle &= (2J_i+1)^{1/2}(2J_f+1)^{1/2}[(U_j^2-V_j^2)\\ &\times W(j_0jJ_f\lambda;J_ij)\langle j\|r^\lambda Y_{\lambda\mu}\|j\rangle+(U_{j0}^2-V_{j0}^2)\\ &\times W(jj_0J_f\lambda;J_ij_0)\langle j_0\|r^\lambda Y_{\lambda\mu}\|j_0\rangle(-)^{J_i+J_f}]\\ &\times[e_p+e_{\text{eff}}(\lambda)].\end{aligned}\tag{16}$$

Equation (16) is also applicable for the case $j_0=j$.[11]

III. EVALUATION OF $(U_iU_f-V_iV_f)^2$ FACTOR AND EFFECTIVE CHARGE

A. $(U_iU_f-V_iV_f)^2$ Factor

The factor $(U_iU_f-V_iV_f)^2$ can be written in a more explicit form as follows:

$$(U_iU_f-V_iV_f)^2=\frac{1}{2}\left[1+\frac{(\epsilon_i-\lambda)(\epsilon_f-\lambda)-\Delta^2}{E_iE_f}\right].\tag{17}$$

If

$$\epsilon_i+\epsilon_f=2\lambda,\tag{18}$$

then the factor $(U_iU_f-V_iV_f)^2$ vanishes for any value of Δ, and the nuclear gamma-ray transition of electric multipole type between these states is expected to be extremely hindered [see Eq. (13)]. Several examples of the calculated values of this factor are illustrated in Figs. 1 to 3 for the nuclei in the regions $N\approx 50$, $Z\approx 50$, $N\approx 82$, $N\approx 124$. When $\epsilon_i+\epsilon_f\sim 2\lambda$, the factor $(U_iU_f-V_iV_f)^2$ varies very rapidly with neutron (or proton) number through the variation of λ. Thus, for the corresponding nuclei, one can expect sudden changes of the reduced transition probabilities from nucleus to nucleus. Moreover, in general, slight changes of the values of the parameters ϵ, λ, and Δ causes a considerable variation of the factor. Since it is not always clear what values of these parameters to use, especially, for non-SCS nuclei,[12,13] it seems to be reasonable to estimate the factor $(U_iU_f-V_iV_f)$ as directly as possible from the empirical data. In the present investigation, we have extracted the values of λ and Δ from the empirical even-odd mass difference data and the gamma transition energy in the following way.

The even-odd mass difference P_p or P_n is given in terms of the total binding energy $E(Z,N)$ of the nucleus (Z,N) by

$$\begin{aligned}P_p(Z,N)&=E(Z,N)+E(Z-2,N)-2E(Z-1,N)\\&=S_p(Z-1,N)-S_p(Z,N),\end{aligned}\tag{19a}$$

$$\begin{aligned}P_n(Z,N)&=E(Z,N)+E(Z,N-2)-2E(Z,N-1)\\&=S_n(Z,N-1)-S_n(Z,N),\end{aligned}\tag{19b}$$

where S_p and S_n are separation energies of proton and neutron, respectively, and are tabulated in detail in the paper by Yamada and Matsumoto.[14]

On the other hand, P_p (or P_n) is related to the lowest quasiparticle energy E of the proton (or neutron) as

$$E\cong\tfrac{1}{2}P_{p(n)}(Z,N).$$

Then, if the final state of the gamma transition is the ground state of an odd-mass nucleus, one can set

$$E_f\cong\tfrac{1}{2}P_{p(n)}(Z,N)$$

and (20)

$$E_i\cong\tfrac{1}{2}P_{p(n)}(Z,N)+\hbar\omega.$$

Here the subscriptions p and n correspond to the lowest lying proton and neutron states, respectively. By using above quantities one can estimate the value of the factor $(U_iU_f-V_iV_f)^2$ as follows:

$$\begin{aligned}(U_iU_f-V_iV_f)^2&=\frac{(E_i-E_f)^2}{4E_iE_f}\left[\left(\frac{E_i+E_f}{\epsilon_i-\epsilon_f}\right)^2-1\right]\\&\equiv D(E_i,E_f).\end{aligned}\tag{21}$$

[11] Equations (15) and (16) are similar to results obtained by Ford and Levinson except for the $(U_iU_f-V_iV_f)$ factor and $e_{\text{eff}}(\lambda)$ [K. W. Ford and C. Levinson, Phys. Rev. **100**, 1 (1955)]. By putting $j_i=j_f\equiv j$ and $\omega=0$ and by taking into account of the projection factor, the Eqs. (13) and (16) can be used to calculate the electric 2^λ-pole moment of single and double quasiparticle states.

[12] The values of parameters λ and Δ of non-SCS spherical nuclei are tabulated by Tamura and Udagawa and very recently by Kisslinger and Sorensen (Ref. 13). T. Tamura and T. Udagawa, Progr. Theoret. Phys. (Kyoto) **26**, 947 (1961).

[13] L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. **35**, 853 (1963).

[14] M. Yamada and Z. Matsumoto, J. Phys. Soc. Japan **16**, 1497 (1961).

TABLE I. Single-particle energies of proton and neutron orbits (MeV).

Orbit pair	Nuclei	Present calculation	KS [a]	TU [b]	KS [c]	(d,p) (d,t) [d]
$(d_{5/2}-g_{7/2})_p$	La, Pr	0.70	1.00	0.78	0.62~0.63	···
$(g_{9/2}-p_{3/2})_p$	As, Br, Rb	2.85	2.8	2.8	2.83~2.93	···
$(h_{11/2}-d_{5/2})_n$	Pd, Cd	2.60	2.8	2.4	2.57~2.62	1.41
$(s_{1/2}-d_{5/2})_n$	Cd	1.41	1.9	1.9	1.38~1.36	0.95
$(d_{3/2}-s_{1/2})_n$	Sn, Te	0.30	0.30	0.30	1.46~1.56	0.28

[a] See Ref. 3. [b] See Ref. 12. [c] See Ref. 13. [d] See Ref. 15.

It is easily seen that the factor vanishes when

$$E_i-E_f=0, \quad \text{for} \quad \epsilon_i\neq\epsilon_f, \tag{22}$$

which is equivalent to the condition (18).

The energies ϵ_j used in the present paper are listed in Table I, together with those used by Kisslinger and Sorenson[3,13] and Tamura and Udagawa.[12] Some values ϵ_j derived from (d,p) and (d,t) experiments[15] are also listed. For most cases, these sets of energies yield similar results when used in Eq. (21), but the large energy difference, $\epsilon_{d_{3/2}}-\epsilon_{s_{1/2}}$, of Ref. 13 makes an important difference.

The equation $E_f-E_i\simeq\hbar\omega$ which comes from Eq. (20) assumes that the dominant parts of $P_p(Z,N)$ and $P_n(Z,N)$ are due to the short-range pairing correlation. P_p and P_n will be affected, of course, by other residual interactions, but it may be expected that the effect of these shifts (ΔE_i and ΔE_f) on Eq. (21) will be small. The agreement of the estimates made on this assumption ($\Delta E_i\approx\Delta E_f$) with experimental results may indicate that the assumption is rather good. The assumption must be, however, corrected in the case of double quasiparticle states in even-even nuclei. Figure 4 shows the observed $d_{3/2}-s_{1/2}$ level energy differences in Sn nuclei. The points for Sn^{118} and Sn^{120} correspond to $(d_{3/2}h_{11/2})_{7-}-(s_{1/2}h_{11/2})_{5-}$ level energy differences. Interpolated[16] effective $d_{3/2}-s_{1/2}$ energy spacings from those for odd-mass nuclei indicate an energy shift of the order of 0.18 MeV in Sn^{118} and Sn^{120}. The shift may be caused by other kinds of residual interaction, such as the neglected multipole components of the interaction of the $h_{11/2}$ quasiparticle with the $d_{3/2}$ and $s_{1/2}$ quasiparticles. For consistency, we use the corrected level energy differences rather than the experimental values for double quasiparticle states. The agreement of the calculated transition probabilities with the experimental ones, suggests the present discussion is reasonable. (See Sec. IV a.)

[15] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 1441 (1961).

[16] The interpolation is carried out so that the energy value of $E(7^--5^-)_{Sn^{118}}-E(7^--5^-)_{Sn^{120}}=40$ keV is unchanged.

B. Effective Charge

The effective charge defined by Eq. (14) plays an important role, especially in the case of $E2$ transitions, because of rather large values of χ_2/C_2 and low phonon energies [see Eq. (13)]. In the original paper by Bohr,[17] it was suggested that the coupling constant k_2 is of the order of 40 MeV. Later, Glendenning obtained a value of about 20 MeV from an analysis of level spacings of low-lying states in Te nuclei,[18] while the KS value for the strength parameter, $\chi_2=(110/A)(4\pi/5)\langle j|r^2|j\rangle_u^{-2}$, corresponds to $k_2\cong66$ MeV. As was noted in Sec. II, the KS value for χ_2 is too large for our purposes, since this value was obtained by renormalizing the closed-shell core effect into this coupling constant. In this connection, it is easily shown that the experimental values of C_2 obtained in the manner of KS are always larger than those obtained in the manner by BM by a factor of about $(A/Z)^2$. In fact, it was shown by

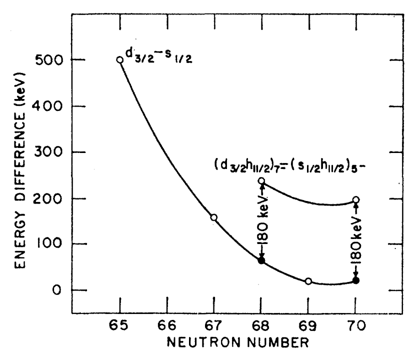

FIG. 4. Observed $d_{3/2}-s_{1/2}$ level energy differences in Sn nuclei. Open circles show the experimental values. The points for Sn^{118} and Sn^{120} correspond to $(d_{3/2}h_{11/2})_{7-}-(s_{1/2}h_{11/2})_{5-}$ level differences. Interpolated effective $d_{3/2}-s_{1/2}$ level energy differences from those for odd-mass nuclei are shown by closed circles. The factor $(U_iU_f-V_iV_f)$ for the transition between the 7− and 5− states in Sn^{118} and Sn^{120} was calculated by use of the interpolated effective $d_{3/2}-s_{1/2}$ level energy difference.

[17] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **26**, No. 14 (1952).

[18] N. K. Glendenning, Phys. Rev. **119**, 213 (1960).

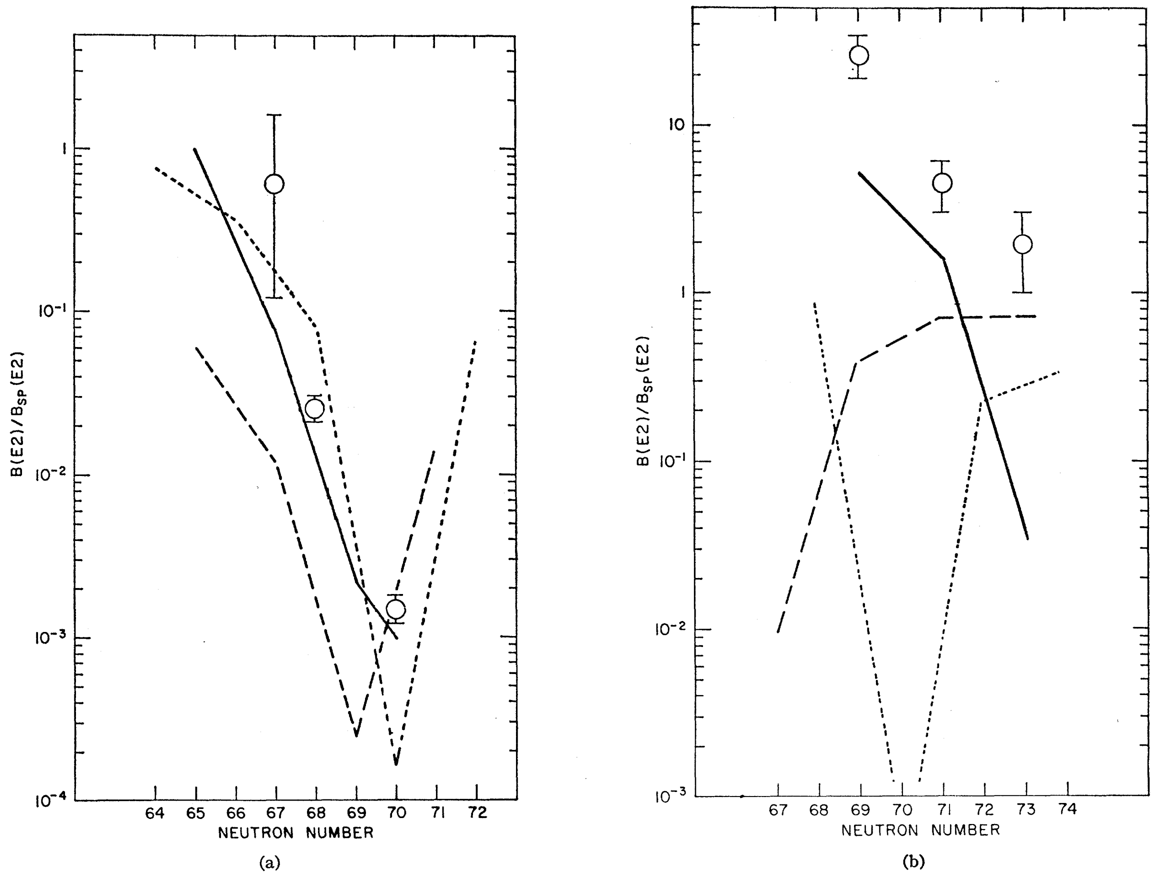

FIG. 5(a), (b). Reduced $E2$ transition probabilities in the single proton transition scale, $B(E2)/B_{sp}(E2)$, for neutron transitions $d_{3/2} \rightleftarrows s_{1/2}$ in Sn and Te nuclei. Open circles show experimental values, while solid lines are calculated ones from Eqs. (13) and (15). Dotted and broken lines are obtained by taking parameter values λ and Δ of Refs. 12 and 13, respectively.

Yoshida[19] that one must use about half the KS value of χ_2 if one takes into account directly the effect of the nuclear core as well as that of the nuclear cloud. Thus, in this paper, we take the value $k_2 \cong 30$ MeV, and use values of C_2 obtained from Coulomb excitation experiments.[20] The phonon energies of odd-mass nuclei are assumed to be approximately the same as those of neighboring even-even nuclei.

In the case of gamma-ray transitions of higher electric multipolarity, the energy factor $\omega_\lambda^2/\omega_\lambda^2-\omega^2$ associated with the effective charge can be set equal to unity, since phonon energies $\hbar\omega_\lambda$ are much larger than the transition energies considered here. Now, information on k_λ (or χ_λ) and C_λ for $\lambda > 2$ is very limited. It is known that the effective charge of one-phonon transitions of $E3$ type is about 0.5 for Ca, Ni, Sr, Sn, and Pb nuclei, while the effective charge for $E4$ transitions is of the order of 1.2 for Pb^{208}.[21] Single-particle transitions in closed-shell-plus-unpaired-nucleon (or hole) nuclei may also supply information on the effective charges of nucleons, since in such cases one can neglect the effect of the pairing correlation on the transition. Bayman *et al.*[22] found that the effective proton charges for $E2$, $E3$, and $E5$ transitions in Zr^{90} are 1.8∼2.0. Analysis of the $E4$ transition between the 2.43- and 2.16-MeV states in Mo^{93} suggests that the effective neutron charge in that case is 0.75. Because of the very gradual change of effective charge for $\lambda > 2$ with nuclear mass number, it would seem to be reasonable to assume the value $e_{\text{eff}}(\lambda) \simeq$ in Eq. (14) for all nuclei.

[19] S. Yoshida, Nucl. Phys. **28**, 380 (1962).

[20] K. Alder, A. Bohr, T. Huus, B. R. Mottelson, and A. Winter, Rev. Mod. Phys. **28**, 432 (1956); P. H. Stelson and F. K. McGowan, Phys. Rev. **110**, 489 (1958).

[21] R. Helm, Phys. Rev. **104**, 1466 (1956); H. Crannell, R. Helm, H. Kendall, J. Oeser, and M. Yearien, *ibid.* **123**, 923 (1961); T. Tamura and T. Udagawa, Nucl. Phys. **35**, 382 (1961); O. Hansen and O. Nathan, in *Proceedings of the Rutherford Jubilee International Conference*, edited by J. E. Birks (Heywood and Company, Ltd., Manchester, 1961), p. 267.

[22] B. F. Bayman, A. S. Reiner, and R. K. Sheline, Phys. Rev. **115**, 1627 (1959).

TABLE II. $B(E2)/B_{sp}(E2)$ for proton and neutron transition.

Nuclide	Energy (MeV) Initial state	Final state	Configuration Initial state	Final state	$B_{exp}(E2)/B_{sp}(E2)$	$B_{cal}(E2)/B_{sp}(E2)$
			(a) Proton transitions			
$_{57}La^{139}$	0.166	g.s.	$d_{5/2}$	$g_{7/2}$	$\lesssim 0.01^a$	0.008
$_{59}Pr^{141}$	0.145	g.s.	$g_{7/2}$	$d_{5/2}$	$\sim 0.17^a$	0.005
			(b) Neutron transitions			
$_{48}Cd^{109}$	0.058	g.s.	$s_{1/2}$	$d_{5/2}$	0.16^b	0.04
$_{48}Cd^{111}$	0.247	g.s.	$d_{5/2}$	$s_{1/2}$	0.21^b	0.28
$_{48}Cd^{113}$	0.582	g.s.	$d_{5/2}$	$s_{1/2}$	29^b	47
$_{50}Sn^{117}$	0.160	g.s.	$d_{3/2}$	$s_{1/2}$	$0.6,^c\ 0.1^d$	0.08
$_{50}Sn^{118}$	2.55	2.29	$(d_{3/2}h_{11/2})_{7-}$	$(s_{1/2}h_{11/2})_{5-}$	0.026^e	0.015
$_{50}Sn^{119}$	0.024	g.s.	$d_{3/2}$	$s_{1/2}$	$<1^f$	2.2×10^{-3}
$_{50}Sn^{120}$	2.49	2.29	$(d_{3/2}h_{11/2})_{7-}$	$(s_{1/2}h_{11/2})_{5-}$	$1.5\times10^{-3\ g}$	1.1×10^{-3}
$_{52}Te^{121}$	0.214	g.s.	$d_{3/2}$	$s_{1/2}$	26^h	4.8
$_{52}Te^{123}$	0.159	g.s.	$d_{3/2}$	$s_{1/2}$	4.5^h	1.4
$_{52}Te^{125}$	0.035	g.s.	$d_{3/2}$	$s_{1/2}$	$\lesssim 2^i$	0.04

[a] G. T. Ewan, private communication to *Nuclear Data Group*, 1954; M. A. Grace, C. E. Johnson, R. G. Scurlock, and R. T. Taylor, Phil. Mag. **7**, 1087 (1962); H. de Waard and T. R. Gerholm, Nucl. Phys. **1**, 281 (1956).
[b] Bull. Am. Phys. Soc. **1**, 389 (1956); A. Maier and K. P. Meyer, Helv. Phys. Acta **30**, 611 (1957); P. L. Simms and R. M. Steffen, Phys. Rev. **108**, 1459 (1957); F. K. McGowan and P. H. Stelson, *ibid.* **109**, 901 (1958); M. Nozawa, unpublished data presented at Osaka meeting of J. Phys. Soc. Japan, April 1962.
[c] R. K. Golden and S. Frankel, Phys. Rev. **102**, 1053 (1956); H. D. Hamilton, Z. Grabowski, and J. E. Thun, Nucl. Phys. **29**, 21 (1962). The lifetime of the 11/2 state is taken from M. Schmorak, A. C. Li, and A. Schwarzschild, [Phys. Rev. **130**, 727 (1963)].
[d] From Coulomb excitation experiment, [D. S. Andreev, V. D. Vasilev, G. M. Gusinski, K. I. Erokhina, and I. Kh. Lemberg, Izv. Akad. Nauk SSSR Ser. Fiz. **25**, 832 (1961)].
[e] References 4 and 5.
[f] J. L. Olsen, L. G. Mann, and M. Linder, Phys. Rev. **106**, 985 (1957), and from L subshell ratios, [J. W. Mihelich, *ibid.* **87**, 646 (1952)].
[g] References 4 and 5 and H. Ikegami, Phys. Rev. **120**, 2185 (1960).
[h] N. Goldberg and S. Frankel, Phys. Rev. **100**, 1350 (1955); R. L. Graham and R. E. Bell, Can J. Phys. **31**, 377 (1953); M. Schmorak, A. C. Li, and A. Schwarzschild, Phys. Rev. **130**, 727 (1963); Y. Y. Chu, O. R. Kistner, A. C. Li, S. Monaro, and M. Perlman, Phys. Rev. **133**, B1361 (1964).
[i] J. S. Geiger, R. L. Graham and I. Bergström (to be published in Nucl. Phys.).
Note added in proof: Dr. Graham reanalyzed their data and got $B_{exp}(E2)/B_{sp}(E2)\cong 4\pm 2.5$ for Te^{125}.

IV. COMPARISON WITH EXPERIMENTAL DATA

All quoted experimental data on nuclear lifetimes are corrected for competing internal-conversion electron emission and for branching. Theoretical conversion coefficients tabulated by Rose[23] are used except for the M-shell conversion coefficients, for which half the values of Rose are used. Then the experimental transition probabilities are represented in the scale of single-proton transitions, $B_{sp}(E\lambda)=(e^2/4\pi)(3/3+\lambda)^2R_0^{2\lambda}$, and compared with calculated ones.

A. $E2$ Transitions

There are several experimental data on $E2$ transitions between $d_{3/2}$ and $s_{1/2}$ neutron orbits in Sn and Te nuclei. The data illustrated in Figs. 5(a) and 5(b) show rather sudden variation of reduced transition probability with increasing neutron number. From Fig. 2, one would expect minimum transition probability for Te^{121} or Te^{122} whose neutron numbers are equal to those of Sn^{119} and Sn^{120}, respectively. However, the minimum point occurs for Te^{124} or Te^{125} rather than for Te^{121} or Te^{122}.[24] The semiempirical calculation as presented in the previous section predicts well these experimental results on $E2$ transition in Te nuclei. This suggests that the parameters λ and Δ obtained by Tamura and Udagawa[12] and by Kisslinger and Sorensen[13] must be slightly corrected in the case of non-SCS nuclei.

Another example is presented for $E2$ transitions between $d_{5/2}$ and $s_{1/2}$ states in Cd nuclei (Fig. 6). For Cd^{113}, a simple perturbation calculation now cannot be applied, since the transition energy is of the same order as the one-phonon energy. Thus, we assume tentatively that the reduced transition probability may be of the order of that of the one-phonon transition in neighboring even-even nuclei. Experimental points scatter to some extent from calculated results but are in reasonable agreement. Some other experimental data which also indicate pairing correlation effects on gamma-ray transitions of $E2$ type are given in Table II with those for Cd, Sn, and Te.

B. $E3$ Transitions

As typical examples of the pairing correlation effects on single-proton and neutron transitions of $E3$ type, experimental results on the transitions between $g_{9/2}$ and $p_{3/2}$ states in As, Br, and Rb nuclei and on the transitions between $h_{11/2}$ and $d_{5/2}$ states in Pd nuclei are presented in Table III and illustrated in Figs. 7 and 8,

[23] M. E. Rose, *Internal Conversion Coefficients* (North-Holland Publishing Company, Amsterdam, 1958).
[24] It is of value to investigate the transition probability for $E2$ transitions between $d_{3/2}$ and $s_{1/2}$ orbits in Te^{124} and Te^{125}.

TABLE III. $B(E3)/B_{sp}(E3)$ for proton and neutron transitions.

Nuclide	Energy (MeV) Initial state	Final state	Configuration Initial state	Final state	$B_{exp}(E3)/B_{sp}(E3)$	$B_{cal}(E3)/B_{sp}(E3)$
			(a) Proton transitions			
$_{33}As^{75}$	0.305	g.s.	$g_{9/2}$	$p_{3/2}$ [a]	0.15 [b]	2.1×10^{-3}
$_{33}As^{77}$	0.475	g.s.	$g_{9/2}$	$p_{3/2}$	<1.0 [c]	6.0×10^{-3}
$_{35}Br^{77}$	0.107	g.s.	$g_{9/2}$	$p_{3/2}$	1.4×10^{-2} [d]	2.6×10^{-4}
$_{35}Br^{79}$	0.208	g.s.	$g_{9/2}$	$p_{3/2}$	3.0×10^{-2} [d]	1.0×10^{-3}
$_{37}Rb^{81}$	0.085	g.s.	$g_{9/2}$	$p_{3/2}$	2.5×10^{-3} [e]	1.7×10^{-4}
			(b) Neutron transitions			
$_{46}Pd^{105}$	0.489	g.s.	$h_{11/2}$	$d_{5/2}$ [f]	$<3.5\times10^{-2}$ [g]	1.7×10^{-3}
$_{46}Pd^{107}$	0.216	g.s.	$h_{11/2}$	$d_{5/2}$	2.6×10^{-3} [h]	2.8×10^{-4}
$_{46}Pd^{109}$	0.188	g.s.	$h_{11/2}$	$d_{5/2}$	3.9×10^{-4} [h]	1.9×10^{-4}
$_{46}Pd^{111}$	0.170	g.s.	$h_{11/2}$	$d_{5/2}$	8.3×10^{-5} [i]	1.8×10^{-4}
$_{48}Cd^{111}$	0.397	0.247	$h_{11/2}$	$d_{5/2}$	1.0×10^{-4} [j]	1.9×10^{-4}

[a] The ground states of these nuclei were assigned (mainly) as $p_{3/2}$ rather than $f_{5/2}$ from experimental results on magnetic moments (Ref. 25).
[b] O. I. Leipunskii, A. M. Morozov, Yu. V. Markarov, and P. A. Yampolskii, Zh. Eksperim i Teor. Fiz. **32**, 393 (1957) [English transl.: Soviet Phys.—JETP **5**, 305 (1957)].
[c] A. W. Schardt, Phys. Rev. **108**, 398 (1957).
[d] A. Goodman and A. W. Schardt, Bull. Am. Phys. Soc. **4**, 56 (1959).
[e] D. G. Karraker and D. H. Templeton, Phys. Rev. **80**, 646 (1950); W. O. Doggett, Lawrence Radiation Laboratory Report No. UCRL-3438, 1956 (unpublished).
[f] The assignments are confirmed from experimental values of magnetic moment rejecting the possibility of $(g_{7/2}{}^{\pm3})_{5/2}$ configuration.
[g] S. H. Vegors and P. Axel, Phys. Rev. **101**, 1067 (1956); R. B. Duffield and S. H. Vegors, Phys. Rev. **112**, 1958 (1958). The upper limit of the 489-keV $E3$ transition branch in Pd^{105} is estimated from original data by T. Suter, P. Reyes-Suter, W. Scheuer, E. Aasa, and G. Bäckström, Arkiv Fysik **20**, 431 (1961).
[h] T. Stribel, Z. Naturforsch. **129**, 939 (1957); A. Flammersfeld, Z. Naturforsch. **7a**, 296 (1952); J. W. Starner, Bull. Am. Phys. Soc. **4**, 99 (1959).
[i] C. L. McGinnis, Phys. Rev. **87**, 202A (1952); B. G. Dzantiev, V. N. Levkovskii, and A. D. Malievskii, Dokl. Akad. Nauk SSSR **113**, 537 (1957) [English transl.: Soviet Phys.—Doklady **2**, 135 (1958)].
[j] M. L. Wiedenbeck, Phys. Rev. **67**, 92 (1945); A. C. Helmholz, R. W. Hayward, and C. L. McGinnis, *ibid.* **75**, 1469A (1949); N. Hole, Arkiv Mat. Astron. Fysik **36A**, No. 9 (1948); J. J. Kraushaar and R. V. Pound, Phys. Rev. **92**, 523 (1953).

respectively. The principal parts of the configurations of the ground states of As, Br, and Rb nuclei are assigned as $p_{3/2}$, from experimental data on magnetic moments.[25] In the same way, configuration assignments of $d_{5/2}$ to the ground states of Pd nuclei are confirmed, rejecting the possibility of the $(g_{7/2}{}^{\pm3})_{5/2}$ configuration. The changes of the reduced transition probabilities from isotope to isotope are well explained on the basis of the present calculations. This evidence supports strongly the pairing correlation theory. Other unsystematic data on $E3$ transitions are also tabulated in Table III.

There are many hindered $E3$ transitions which are not considered to be direct evidence of the pairing correlation effect on gamma-ray transitions. For example, many $E3$ transitions of $7/2+ \rightleftarrows p_{1/2}$ in the region Z, $N=50$ are hindered ($10^{-2}\sim10^{-4}$ of single-proton transition scale).[26] In this case the $7/2+$ states can be ascribed to $(g_{9/2}{}^{\pm3,5})_{7/2}$ configurations and the transitions of $7/2+ \rightleftarrows p_{1/2}$ are forbidden as single-particle transitions. The residual interaction, however, causes an admixture of the configuration $g_{7/2}$ into the $7/2+$ states and mixtures of the configurations $p_{3/2}$ and/or $f_{5/2}$ plus one phonon of $E2$ type into the $1/2-$ states and gives rise to $E3$ transitions of $7/2 \rightleftarrows p_{1/2}$. As was shown previously (Sec. II), the pairing correlations reduce the collective coupling effect and this turns out to decrease the configuration mixing and to retard the transitions strongly. One may thus conclude that most of the $E3$ transitions between low-lying states in medium weight nuclei are retarded directly or indirectly by the pairing correlations.

In the heavier mass region, e.g., in Ir and Au nuclei, there are also many hindered $E3$ transitions between $h_{11/2}$ and $5/2+$ states. These $5/2+$ states are, however, better explained as states of $d_{3/2}$ plus one phonon of $E2$ type, because of the enhancement of the $E2$ transitions[27] from these states to $d_{3/2}$ ground states. In this case, the $E3$ transitions between $h_{11/2}$ and the $5/2+$ states are forbidden as either single-particle transitions or as one-phonon transitions.

C. $E4$ and $E5$ Transitions

Experimental data on $E4$ and $E5$ transitions are few; some examples are the 0.622- and 0.912-MeV $E5$ transitions in Pb^{204}, the 0.547- and 0.787-MeV $E5$

[25] H. Noya, A. Arima, and H. Horie, Progr. Theoret. Phys. (Kyoto) Suppl. **8**, 33 (1958).

[26] M. Goldhaber and A. W. Sunyar, *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. XVI. *Note added in proof.* A quantitative discussion on the hindered $E3$ transitions of $7/2+ \rightleftarrows 1/2-$ (in Z, $N\cong50$ nuclei), $11/2- \rightarrow 5/2+$ (in Au and Tl nuclei) and $9/2- \rightarrow 3/2+$ (in Tl nuclei) will be given later. H. Ikegami and S. A. Moszkowski, Phys. Rev. (to be published).

[27] A. de-Shalit, Phys. Rev. **122**, 1530 (1961).

transitions and the 0.129-MeV $E4$ transition in Pb^{202}, the 0.265-MeV $E5$ transition in Cd^{113}, the 0.18-MeV $E5$ transition in Cd^{115}, and so on. These examples have some ambiguities in configuration assignments. Moreover, the calculated lifetimes are too sensitive to the assumed nuclear radius to enable us to conclude anything about the effect of pairing correlations on $E4$ and $E5$ transitions.

V. SHELL-MODEL CALCULATIONS AND THE BLOCKING EFFECT

In the above discussion, we have assumed the quasiparticle approximation. To test the accuracy of this

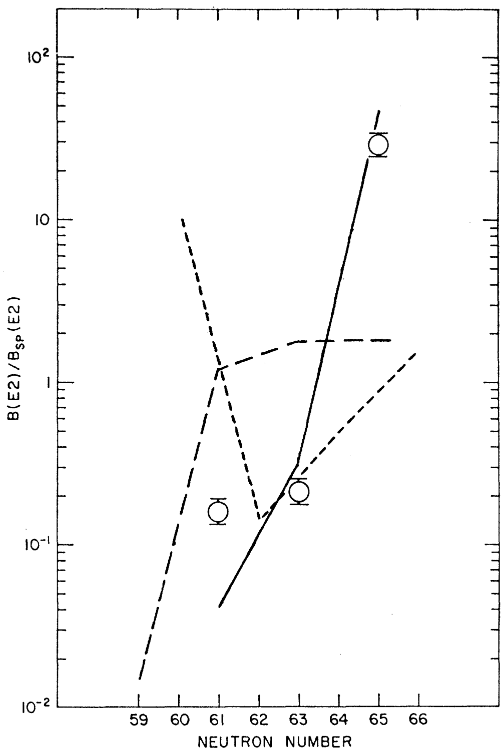

FIG. 6. Reduced $E2$ transition probabilities in single proton transition scale, $B(E2)/B_{sp}(E2)$, for neutron transitions $d_{5/2} \rightleftarrows s_{1/2}$ in Cd nuclei. Open circles and solid lines show experimental and calculated [from Eqs. (13) or (15), and (21)] results, respectively. Dotted and broken lines are obtained by taking parameter values λ and Δ of Refs. 12 and 13, respectively.

approximation, we take as examples the $E2$ transitions between $d_{3/2}$ and $s_{1/2}$ orbits in Sn^{118} and Sn^{117} and make a more exact calculation to compare with that of the quasiparticle approximation. If there is no pairing correlation, the configuration of the ground state of Sn^{118} may be $(d_{5/2})^6$, $(g_{7/2})^8(s_{1/2})^2(d_{3/2})^2$. The pairing correlation may, however, cause some configuration mixing. Because of large energy difference ($\sim$2.0 MeV) between the $g_{7/2}$ and $s_{1/2}$ orbits, particle jumps caused by pairing correlation from $d_{5/2}$ or $g_{7/2}$ orbits to $s_{1/2}$, $d_{3/2}$,

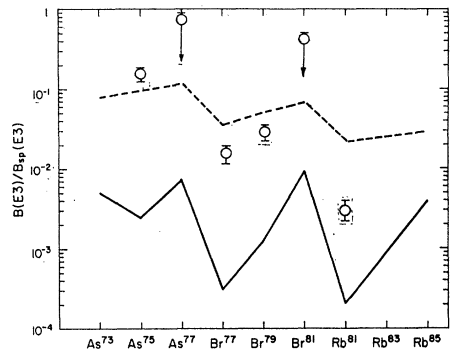

FIG. 7. Reduced $E3$ transition probabilities in the single proton transition scale, $B(E3)/B_{sp}(E3)$ for proton transitions $g_{9/2} \to p_{3/2}$ in As, Br, and Rb nuclei. Experimental results are represented by open circles. Calculated results from Eqs. (13) and (21) are shown by solid lines while dotted line is obtained by taking parameter values λ and Δ of Ref. 13.

and/or $h_{11/2}$ orbits may be neglected. Then it is sufficient to treat only four-particle configurations in the $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$ orbits. The 2.55- and 2.29-MeV states in Sn^{118} between which the $E2$ transition occurs

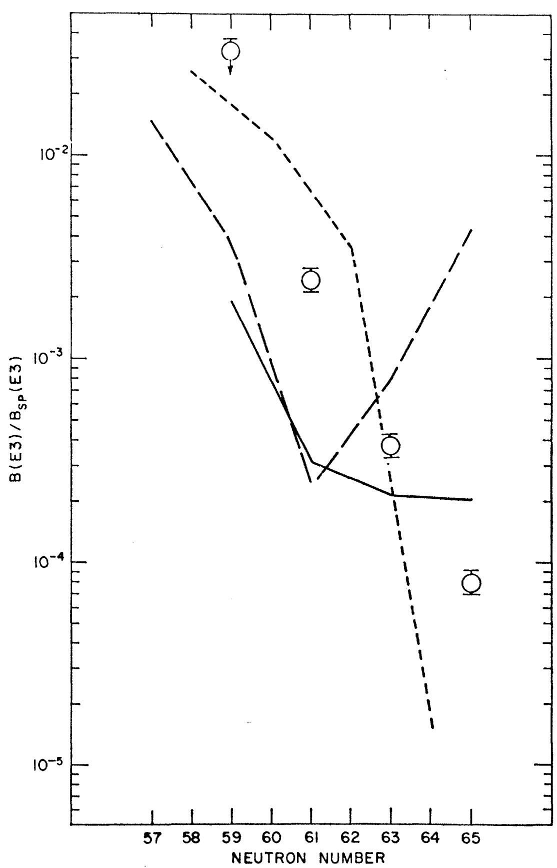

FIG. 8. Reduced $E3$ transition probabilities in the single proton transition scale $B(E3)/B_{sp}(E3)$ for neutron transitions $h_{11/2} \to d_{5/2}$ in Pd nuclei. Experimental results are represented by open circles, while calculated ones from Eqs. (13) and (21) are by solid lines. Dotted and broken lines are obtained by taking parameter values λ and Δ of Refs. 12 and 13, respectively.

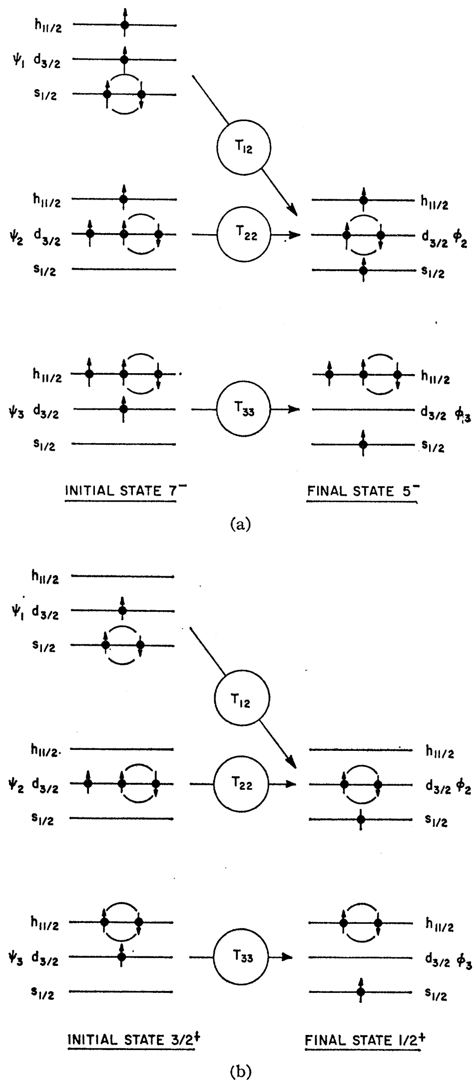

FIG. 9(a), (b). Schematic illustrations of configurations of 7− and 5− states in Sn^{118} and of $\frac{3}{2}+$ and $\frac{1}{2}+$ states in Sn^{117} and of the effect of pairing correlation on nuclear electric gamma-ray transition. The pair of particles coupled with zero angular momentum is indicated by the symbol ⟲. Possible particle transitions of $E2$ type between these configurations are indicated by arrows. The partial matrix element T_{12} corresponds to a particle transition of $s_{1/2}\rightarrow d_{3/2}$ while T_{22} and T_{33} correspond to particle transitions of $s_{1/2}\leftarrow d_{3/2}$. If there is no pairing correlation, $T_{22}=T_{33}=0$ and then T_{12} becomes the ordinary single-particle transition matrix element. When the pairing correlation is switched on, T_{22} and T_{33} become also finite and destructively interfere with T_{12}.

are considered to be $(h_{11/2},d_{3/2})_{7-}$ and $(h_{11/2},s_{1/2})_{5-}$ states, respectively, as in our previous work.[4,5,28] Thus, these states can be described by the following configuration mixing wave functions.

$$\Psi(J_i,M_i)=\sum_{m_2m_3}\langle j_2j_3m_2m_3|J_iM_i\rangle a_{j_2m_2}^\dagger a_{j_3m_3}^\dagger\Big(\sum_j \psi_jA_j^\dagger\Big)|0\rangle \quad \text{for the 7− state,} \quad (23a)$$

$$\Psi(J_fM_f)=\sum_{m_1m_3}\langle j_1j_3m_1m_3|J_fM_f\rangle a_{j_1m_1}^\dagger a_{j_3m_3}^\dagger\Big(\sum_j \phi_jA_j^\dagger\Big)|0\rangle \quad \text{for the 5− state,} \quad (23b)$$

$$j\equiv j_1,\, j_2,\, j_3\,.$$

Here $|0\rangle$ represents a particle vacuum state while $a_{jm}^\dagger$ and a_{jm} are creation and annihilation operators of a particle in a state $|jm\rangle$, respectively. The subscriptions 1, 2, and 3 refer to the $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$ orbits, respectively. $A_j^\dagger$ is an operator making a coupled pair in the orbit j and may be expressed as

$$A_j^\dagger=2^{-1/2}\sum_m\langle jjm-m|00\rangle a_{jm}^\dagger a_{j-m}^\dagger$$
$$=2^{-1/2}\sum_m\frac{(-)^{j-m}}{(2j+1)^{1/2}}a_{jm}^\dagger a_{j-m}^\dagger. \quad (24)$$

The wave function (23a) means that two neutrons, of spin $j_2(=3/2)$ and $j_3(=11/2)$, are coupled to give a spin $J_i(=7-)$, and the remaining two neutrons are coupled as a pair occupying the orbits $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$, with amplitudes ψ_1, ψ_2, and ψ_3, respectively. The wave function (23b) means that two neutrons, of spin $j_1(=1/2)$ and $j_3(=11/2)$, are coupled to give a spin $J_f(=5-)$, and the remaining two neutrons are coupled as a pair occupying the orbits $d_{3/2}$ and $h_{11/2}$, with amplitudes ϕ_2 and ϕ_3, respectively. (In this case, ϕ_1 vanishes because of the presence of an unpaired neutron in the $s_{1/2}$ orbit.) These configurations are schematically illustrated in Fig. 9. The amplitudes are determined by a diagonalization of the pairing correlation Hamiltonian and are expressed as

$$\psi_j=\frac{1}{N}\frac{(j+\frac{1}{2})^{1/2}}{E_j-E},\quad E_j\equiv\epsilon_{j_2}+\epsilon_{j_3}+2\epsilon_j; \quad \text{for the 7− state,} \quad (25a)$$

and

$$\phi_j=\frac{1}{N'}\frac{(j+\frac{1}{2})^{1/2}}{E_j'-E'},\quad E_j'\equiv\epsilon_{j_1}+\epsilon_{j_3}+2\epsilon_j; \quad \text{for the 5− state.} \quad (25b)$$

Here N and N' are normalization constants. The eigenvalues E and E' are obtained by solving the following dispersion equations

$$1-G\sum_j\frac{(j+\frac{1}{2})e_j}{E_j-E}=0,\quad e_j\equiv1-\frac{2(\delta_{jj_2}+\delta_{jj_3})}{2j+1}; \quad \text{for the 7− state,} \quad (26a)$$

$$1-G\sum_j\frac{(j+\frac{1}{2})e_j'}{E_j'-E'}=0,\quad e_j'\equiv1-\frac{2(\delta_{jj_1}+\delta_{jj_3})}{2j+1}; \quad \text{for the 5− state.} \quad (26b)$$

Here G is the coupling constant for the pairing inter-

[28] Later, the assignment was confirmed by the measurement of the magnetic moment of the 2.29-MeV state. E. Bodenstedt, H. J. Körner, E. Gerdau, J. Radeloff, K. Auerbach, L. Mayer, and A. Roggenbuch, Z. Physik 168, 370 (1962).

action, and the quantities e_j and e_j' represent a decrease of the pair multiplicity in the orbit j because the presence of an unpaired particle in that orbit for 7− and 5− states, respectively. Then the matrix element for the $E2$ transition between the 7− and 5− states in Sn^{118} is given as

$$\langle\Psi_{J_fM_f}|\mathfrak{M}(2,\mu)|\Psi_{J_iM_i}\rangle=\sum_{m_1m_3}\sum_{m_2'm_3'}\langle j_1j_3m_1m_3|J_fM_f\rangle\langle j_2j_3m_2'm_3'|J_iM_i\rangle \times\langle 0|\sum_{j'}\psi_{j'}A_{j'}a_{j_3m_3'}a_{j_2m_2'}|\mathfrak{M}(2,\mu)|a_{j_1m_1}{}^\dagger a_{j_3m_3}{}^\dagger\sum_j\phi_jA_j{}^\dagger|0\rangle\equiv T_{12}+T_{22}+T_{33}\,,\quad(27)$$

where

$$T_{12}=\frac{\langle J_i2M_i\mu|J_fM_f\rangle}{(2J_f+1)^{1/2}}\frac{-2\psi_1\phi_2}{(2j_1+1)^{1/2}(2j_2+1)^{1/2}}\langle\Psi_{J_f}\|\mathfrak{M}(2,\mu)\|\Psi_{J_i}\rangle.\quad(28a)$$

$$T_{22}=\frac{\langle J_i2M_i\mu|J_fM_f\rangle}{(2J_f+1)^{1/2}}\left(\frac{2j_2-1}{2j_2+1}\right)\psi_2\phi_2\langle\Psi_{J_f}\|\mathfrak{M}(2,\mu)\|\Psi_{J_i}\rangle.\quad(28b)$$

$$T_{33}=\frac{\langle J_i2M_i\mu|J_fM_f\rangle}{(2J_f+1)^{1/2}}\left(\frac{2j_3-1}{2j_3+1}\right)\psi_3\phi_3\langle\Psi_{J_f}\|\mathfrak{M}(2,\mu)\|\Psi_{J_i}\rangle.\quad(28c)$$

The meaning of partial matrix elements T_{12}, T_{22}, and T_{33} are schematically shown in Fig. 9. Here $\langle\Psi_{J_f}\|\mathfrak{M}(2,\mu)\|\Psi_{J_i}\rangle$ is a reduced matrix element of $E2$ transition between two-particle states $\Psi_{J_i}\to\Psi_{J_f}$ and is essentially given by Eq. (15), except for $(U_iU_f-V_iV_f)$ factor.[29] If there is no pairing correlation, $\psi_1=1$, $\psi_2=\psi_3=0$, $\phi_1=0$, $\phi_2=1$, and $\phi_3=0$, thus, $T_{22}=T_{33}=0$ and Eq. (27) becomes the ordinary single-particle transition matrix element. When the pairing interaction is switched on, the partial matrix elements T_{22} and T_{33} are also finite. As is seen from Eqs. (25), (26), and (28), the lowest eigenvalue E and E' are always smaller than E_j and $E_{j'}$, respectively, and thus $\psi_{j'}\cdot\phi_{j'}>0$ for any j and j', so the signs of both T_{22} and T_{33} are opposite to that of T_{12}. Therefore, partial matrix elements T_{22} and T_{33} destructively interfere with that of T_{12}. This means the configuration mixing caused by the pairing correlation gives rise to both particle transitions $s_{1/2}\to d_{3/2}$ and $d_{3/2}\to s_{1/2}$, resulting in a cancellation effect on the gamma-ray transition probability as is illustrated in Fig. 9.

From Eq. (27), one can obtain a quantity corresponding to the $(U_iU_f-V_iV_f)^2$ factor as

$$D\equiv\left[\frac{-2\psi_1\phi_2}{(2j_1+1)^{1/2}(2j_2+1)^{1/2}}+\frac{2j_2-1}{2j_2+1}\psi_2\phi_2+\frac{2j_3-1}{2j_3+1}\psi_3\phi_3\right]^2.\quad(29)$$

If we use the values of KS for orbit energies ϵ_{j_1}, ϵ_{j_2}, and ϵ_{j_3}, and for the coupling constant G, we obtain $D=0.009$ for the $E2$ transition between the 7− and 5− states in Sn^{118}. The value is considerably larger than that $D=0.0012$ obtained by the quasiparticle approximation using the same parameter values.

A similar treatment has also been carried out for the $E2$ transition between single-particle $s_{1/2}$ and $d_{3/2}$ states in Sn^{117}.[30] By putting $\delta_{jj_3}=0$ in Eq. (26) and by replacing E_j and E_j' in Eqs. (25) and (26) as

$$E_j=\epsilon_{j_2}+2\epsilon_j\,,\quad E_j'=\epsilon_{j_1}+2\epsilon_j\,,$$

the formula (29) is made applicable to this case. The numerical result is $D=2\times10^{-4}$ which is 250 times smaller than that $D=0.05$, obtained by the quasiparticle approximation.[31]

The discrepancies of the results evaluated by the two methods are fairly large. To improve this, further calculations have also been made for Sn^{117} by taking into account the effect of blocking[32] in the quasiparticle description. The result obtained for this case is $D=3.2\times10^{-2}$ which is slightly improved but is still different from the exact one by about a factor 100. It is, however, very interesting to note here that if the renormalized wave functions, obtained by projecting out only the terms that have the correct number of particles from the above BCS-type wave function taking into account

[29] In this case, j_0, j_i, and j_f in Eq. (15) are replaced by j_3, j_2, and j_1, respectively.

[30] The reduced matrix element $\langle\Psi_{J_i}\|\mathfrak{M}(2,\mu)\|\Psi_{J_f}\rangle$ in Eq. (28) is also replaced by $\langle\Psi_{j_1}\|\mathfrak{M}(2,\mu)\|\Psi_{j_2}\rangle$, ordinary matrix element of $E2$ transition between single-particle states $\Psi_{j_1}\to\Psi_{j_2}$. The reduced matrix element $\langle\Psi_{j_1}\|\mathfrak{M}(2,\mu)\|\Psi_{j_2}\rangle$ is essentially in agreement with Eq. (13) except for the $(U_iU_f-V_iV_f)$ factor.

[31] Since the calculated values of the parameters λ, Δ, and of the D factor seriously depend on the free-orbit energies, in this case one should not place important meaning on the discrepancy between these calculated D values and experimental ones. Most of this discrepancy is ascribable to an unsuitable choice of parameter values. In reality, the result is improved by the use of modified parameter values [Refs. 12, 13; see also Fig. 5(a)].

[32] V. G. Soloviev, Kgl. Danske. Videnskab. Selskab, Mat. Fys. Skrifter. **1**, No. 11 (1961).

the effect of blocking, are used, the D factor becomes $D=1.6\times10^{-3}$ which is closer to the exact value, though a discrepancy of about a factor of 10 still remains. In this connection, we have also investigated the overlap of the above projected wave functions with the exact ones; the overlap integrals are greater than 99% for both $s_{1/2}$ and $d_{3/2}$ states. This fact clearly shows that the hindrance factor, when it is very small, is very sensitive to the details of the wave function; i.e., a small difference of the wave function causes a large difference of the D factor. This behavior of the D factor is expected from the fact that it is obtained as a small difference of two large positive quantities.

It can be shown that the effect of blocking depends on the free-orbit energies, and in certain cases the effect can play a more important role than indicated above. For instance, if one would take a lower energy value[13] for the $s_{1/2}$ orbit, say $\epsilon_{d_{3/2}}-\epsilon_{s_{1/2}}\approx1.5$ MeV and $\epsilon_{h_{11/2}}-\epsilon_{s_{1/2}}\approx1.2$ MeV, then the exact calculation shows $D\sim0.02$ for the $E2$ transition between the $d_{3/2}$ and $s_{1/2}$ states in Sn^{117}. In this case, the quasiparticle calculation blocking included gives almost perfect agreement with the exact calculation. If one neglects the effect of blocking, the occupancy parameter value of the $s_{1/2}$ orbit for the $s_{1/2}$ state in Sn^{117} is fairly large ($V_{s_{1/2}}=0.924$, $U_{s_{1/2}}=0.383$), because of the low and isolated $s_{1/2}$ orbit. But when blocking is included, because of the presence of an unpaired $s_{1/2}$ quasiparticle, these values must be $V_{s_{1/2}}=0$, $U_{s_{1/2}}=1$. This large change is the reason why the effect of blocking plays an especially important role in this case.

VI. DISCUSSION

The effect of pairing correlations on nuclear gamma-ray transition probabilities has been discussed on the basis of the pairing correlation theory. It has been shown that even if collective coupling exists, gamma-ray transitions of electric multipole type between low-lying quasiparticle states whose shell-model orbit energies are symmetric related to the Fermi energy, i.e., $\epsilon_i+\epsilon_f\sim2\lambda$, may be extremely retarded. In this case the transition matrix element varies very sensitively with nucleon number through the variation of the Fermi energy. Some systematic data which allow an investigation of the variation of transition matrix element have been presented. These data supply strong evidence of the effect of pairing correlation on nuclear gamma-ray transitions of electric multipole type.

Some of the uncertainties in the theoretical estimates may be reduced if the hindrance factor $(U_iU_f-V_iV_f)^2$, which arises from the pairing correlation is estimated from empirical even-odd mass differences. The transition probabilities calculated in this way are more in agreement with the experimental data than ones calculated by using now available parameter values of λ, Δ, etc.

Shell-model calculations taking into account the pairing correlation were also made for $E2$ transitions in Sn^{117} and Sn^{118} and it was clarified that the hindrance of the transitions occurs as a result of destructive interference between particle transitions of $j_i\to j_f$ and $j_f\to j_i$. The numerical results were, however, different from those obtained on the basis of quasiparticle description. Taking into account the effect of blocking in the quasiparticle description improved the result especially for an isolated level with low angular momentum. The result was more improved when the renormalized wave functions obtained by projecting out only the terms that have the correct number of particles from the BCS-type wave functions were used, but a discrepancy still remained. Since the hindrance factor, when it is very small, is very sensitive to the details of the wave function, at the present stage, it may be better to estimate transition probabilities in the semi-empirical way described above.

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