potential depth; but a regorous explanation of a common reaction cross section for the two sets of curves is hindered by the present limited understanding of the physical significance of the optical model parameters.

The different sets of values of optical model parameters obtained by Perey and by Wilkins apparently originate in the basically different search programs used by each investigator to obtain best fits to elastic scattering and reaction cross section data. Since both sets of parameters give convergent fits for the reaction cross section data of this work, a more stringent test must await the availability of experimental elastic scattering angular distributions in the uranium region.

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Inelastic Scattering of Neutrons by Tritons*

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Assuming that the range of nuclear forces is small compared to the size of the triton and the wavelength of the incoming neutron ("zero-range" approximation), we derive a connection between the cross sections for elastic and inelastic n–t scattering by calculating the ratio of the first Born approximation for inelastic to that for elastic scattering. A calculation of the inelastic scattering cross section is made for an incoming neutron energy of 14 MeV. Since experimental elastic angular distributions are not available, we use values calculated by Bransden and Robertson for p–He^3 assuming Serber interaction. Inelastic angular distributions are calculated for the ejected deuteron, the ejected neutron, and the scattered neutron. Integrating over the distribution of the ejected deuteron, we obtain a value for the total n–t inelastic cross section of 343 mb. We realize that the calculations are very crude, but hope that the work will be helpful in planning future experiments.

I. TRITIUM WAVE FUNCTIONS

In this paper the triton will be treated as a deuteron-neutron bound pair. The ground-state wave function of tritium may be written as the product of a spatial part symmetrical in all three coordinates 123 times an antisymmetrical spin function

$$\psi=\varphi(123)\chi_a{}^m. \tag{1}$$

Since T is a spin-$\frac{1}{2}$ particle, the spin functions are

$$\chi_a{}^{1/2}=\frac{1}{\sqrt{2}}(\alpha_1\beta_2-\beta_1\alpha_2)\alpha_3 \tag{2}$$

and

$$\chi_a{}^{-1/2}=\frac{1}{\sqrt{2}}(\alpha_1\beta_2-\beta_1\alpha_2)\beta_3. \tag{3}$$

The space function after elastic scattering is

$$\Phi=\frac{\varphi(\mathbf{r})f(\mathbf{q})+\varphi(\mathbf{q}+\frac{1}{2}\mathbf{r})f(-(q/2)+\frac{3}{4}\mathbf{r})}{(3.344)^{1/2}}, \tag{4}$$

where $\varphi(\mathbf{r})$ is a function of the deuteron coordinates and $f(q)$ is a function of the relative coordinates of the triton. The factor $(3.344)^{1/2}$ is obtained by normalizing the integral

$$\int|\Phi|^2d\tau,$$

using

$$\varphi(r)=e^{-\alpha r}/r,\quad \alpha=0.23182F^{-1} \tag{5}$$

$$f(q)=e^{-\beta q}/q,\quad \beta=0.44743F^{-1} \tag{6}$$

and the triton is made up of neutrons at r_1 and r_2 and a proton at r_3. It is bombarded by a neutron at r_4. The following coordinates are used:

$$\mathbf{r}=\mathbf{r}_2-\mathbf{r}_3, \tag{7}$$

$$\mathbf{q}=\mathbf{r}_1-\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3), \tag{8}$$

$$\mathbf{Q}=\mathbf{r}_4-\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3), \tag{9}$$

$$\mathbf{s}=\tfrac{1}{4}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4). \tag{10}$$

With one of its neutrons excited, the triton could either have spin $\frac{3}{2}$ or $\frac{1}{2}$.

The $S=\frac{3}{2}$ Case

$$\psi=\varphi(\mathbf{r})f_{k'}{}^q(\mathbf{q})\chi_{3/2}{}^m - \varphi(\mathbf{q}+\tfrac{1}{2}\mathbf{r})f_{k'}{}^q\left(-\tfrac{1}{2}\mathbf{q}+\frac{3}{r}\mathbf{r}\right)\Gamma_{3/2}{}^m, \tag{11}$$

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where the superscript of $f_{k'}{}^{q}$ means the quartet case is present and the subscript refers to the triton momentum in center-mass. The $\Gamma_{3/2}{}^{m}$ takes the spin exchange of the two neutrons into account

$$\Gamma_{3/2}{}^{m}\equiv(12)_{\sigma}\chi_{3/2}{}^{m}=\chi_{3/2}{}^{m}. \tag{12}$$

The spin functions are

$$\chi_{3/2}{}^{3/2}=\alpha_1\alpha_2\alpha_3, \tag{13}$$

$$\chi_{3/2}{}^{1/2}=\frac{1}{\sqrt{3}}(\alpha_1\beta_2\alpha_3+\beta_1\alpha_2\alpha_3+\alpha_1\alpha_2\beta_3), \tag{14}$$

$$\chi_{3/2}{}^{-1/2}=\frac{1}{\sqrt{3}}(\beta_1\alpha_2\beta_3+\alpha_1\beta_2\beta_3+\beta_1\beta_2\alpha_3), \tag{15}$$

$$\chi_{3/2}{}^{-3/2}=\beta_1\beta_2\beta_3. \tag{16}$$

The $S=\frac{1}{2}$ Case

$$\psi=\varphi(\mathbf{r})f_{k'}{}^{d}(\mathbf{q})\chi_{1/2}{}^{m}-\varphi(\mathbf{q}+\tfrac{1}{2}\mathbf{r})f_{k'}{}^{d}(-\tfrac{1}{2}\mathbf{q}+\tfrac{3}{4}\mathbf{r})\Gamma_{1/2}{}^{m}. \tag{17}$$

Where the superscript of $f_{k'}{}^{d}$ refers to the doublet case.

$$\chi_{1/2}{}^{1/2}=6^{-1/2}(\alpha_1\alpha_2\beta_3+\alpha_1\beta_2\alpha_3-2\beta_1\alpha_2\alpha_3) \tag{18}$$

and

$$\chi_{1/2}{}^{-1/2}=-6^{-1/2}(\beta_1\beta_2\alpha_3+\beta_1\alpha_2\beta_3-2\alpha_1\beta_2\beta_3). \tag{19}$$

When two neutrons are excited, there are two possible cases, $S=\frac{3}{2}$ and $S=\frac{1}{2}$.

$S=\frac{3}{2}$: The deuteron must be in a triplet state and will be denoted by $\varphi_K{}^{t}(r)$. The subscript K refers to the deuteron momentum. Then the triton wave function may be written as

$$\psi=\varphi_K{}^{t}(\mathbf{r})f_{k',K}{}^{q,t}(\mathbf{q})\chi_{3/2}{}^{m}-\varphi_K{}^{t}(\mathbf{q}+\tfrac{1}{2}\mathbf{r})f_{k',K}{}^{q,t}\left(\frac{-\mathbf{q}}{2}+\tfrac{3}{4}\mathbf{r}\right)\Gamma_{3/2}{}^{m}. \tag{20}$$

$S=\frac{1}{2}$: Now the deuteron may be either triplet or singlet.

$$\psi=\varphi_K{}^{t}(\mathbf{r})f_{k',K}{}^{d,t}(\mathbf{q})\chi_{1/2}{}^{m}-\varphi_K{}^{t}(\mathbf{q}+\tfrac{1}{2}\mathbf{r})f_{k',K}{}^{d,t}\left(-\frac{\mathbf{q}}{2}+\tfrac{3}{4}\mathbf{r}\right)\Gamma_{1/2}{}^{m} \tag{21}$$

or

$$\psi=\varphi_K{}^{s}(\mathbf{r})f_{k',K}{}^{d,s}(\mathbf{q})(\chi_{1/2}{}^{m})'-\varphi_K{}^{s}(\mathbf{q}+\tfrac{1}{2}\mathbf{r})f_{k',K}{}^{d,s}\left(-\frac{\mathbf{q}}{2}+\tfrac{3}{4}\mathbf{r}\right)(\Gamma_{1/2}{}^{m})', \tag{22}$$

where the spin functions are

$$(\chi_{1/2}{}^{1/2})'=\frac{1}{\sqrt{2}}(\alpha_1\alpha_2\beta_3-\alpha_1\beta_2\alpha_3) \tag{23}$$

$$(\chi_{1/2}{}^{-1/2})'=\frac{1}{\sqrt{2}}(\beta_1\alpha_2\beta_3-\beta_1\beta_2\alpha_3) \tag{24}$$

$$(\Gamma_{1/2}{}^{m})'=(12)_{\sigma}(\chi_{1/2}{}^{m})'. \tag{25}$$

II. TOTAL SPIN FUNCTIONS

Since the ground state of tritium has $S=\frac{1}{2}$ and the incoming neutron has spin $\frac{1}{2}$, the triton-neutron system must have a total spin of either 0 or 1. In the ground state,

$$S_T=0,\quad (1/\sqrt{2})(\chi_a{}^{1/2}\beta_4-\chi_a{}^{-1/2}\alpha_4)\equiv\chi_i{}^{0} \tag{26}$$

$$S_T=1,\quad \chi_a{}^{1/2}\alpha_4\equiv\chi_i{}^{1}. \tag{27}$$

In the continuum states,

$$S_T=0,\quad \chi_0{}^{0}\equiv(1/\sqrt{2})(\chi_{1/2}{}^{1/2}\beta_4-\chi_{1/2}{}^{-1/2}\alpha_4), \tag{28}$$

$$\Gamma_0{}^{0}=(12)_{\sigma}\chi_0{}^{0}, \tag{29}$$

or

$$(\chi_0{}^{0})'\equiv\frac{1}{\sqrt{2}}[(\chi_{1/2}{}^{1/2})'\beta_4-(\chi_{1/2}{}^{-1/2})'\alpha_4] \tag{30}$$

$$(\Gamma_0{}^{0})'=(12)_{\sigma}(\chi_0{}^{0})' \tag{31}$$

$$S_T=1,\quad \chi_{1/2}{}^{1/2}\alpha_4\equiv\chi_1{}^{1} \tag{32}$$

$$\Gamma_1{}^{1}=(12)_{\sigma}\chi_1{}^{1} \tag{33}$$

or

$$(\chi_1{}^{1})'=(\chi_{1/2}{}^{1/2})'\alpha_4 \tag{34}$$

$$(\Gamma_1{}^{1})'=(12)_{\sigma}(\chi_1{}^{1})', \tag{35}$$

or for the case in which a state of tritium has spin $\frac{3}{2}$,

$$S_T=1,\quad \bar{\chi}_1{}^{1}=\frac{\sqrt{3}}{2}\chi_{3/2}{}^{3/2}\beta_4-\tfrac{1}{2}\chi_{3/2}{}^{1/2}\alpha_4 \tag{36}$$

$$\bar{\Gamma}_1{}^{1}=(12)_{\sigma}\bar{\chi}_1{}^{1}=\bar{\chi}_1{}^{1}. \tag{37}$$

III. CALCULATION OF n–$2n$ CROSS SECTION

Conservation of energy requires for the n–$2n$ process

$$k_f{}^{2}=k_i{}^{2}-(9/8)k'^{2}-(9/8)\alpha_T{}^{2}, \tag{38}$$

where

$$k_i{}^{2}=\frac{9}{8}\frac{M}{\hbar^2}E_{\text{lab}}. \tag{39}$$

E_{lab} is the energy of the incoming neutron in the lab system.

$$\alpha_T{}^{2}=\frac{4}{3}\frac{M}{\hbar^2}[(BE)_{\text{triton}}-(BE)_{\text{deuteron}}]. \tag{40}$$

$\mathbf{k}_i$ is the initial wave vector of the neutron 4 in the total center-of-mass system. $\mathbf{k}'$ is the final wave vector for relative motion of neutron 3 in the center-of-mass system defined by particles 1, 2, and 3. $\mathbf{k}_f$ is the final wave vector of the scattered neutron in the total center-of-mass system.

The differential cross section for the n–$2n$ process is

$$d\sigma_{\text{in}}=L^3\frac{2\pi}{\hbar v}|M_{\text{in}}|^2\rho_E(\mathbf{k}',\mathbf{k}_f)d\mathbf{k}'d\mathbf{k}_f, \tag{41}$$

where L^3 is the normalization volume and

$$\rho_E(\mathbf{k}',\mathbf{k}_f)d\mathbf{k}'d\mathbf{k}_f=\left(\frac{L}{2\pi}\right)^3k_f{}^{2}\left(\frac{\partial k_f}{\partial E}\right)_{k'}d\Omega_f\left(\frac{L}{2\pi}\right)^3d\mathbf{k}' \tag{42}$$

is the energy density of final states. E is the final energy in the lab system.

$$E=\frac{4}{3}\left(\frac{\hbar^2}{2M}\right)(-2\alpha^2+\tfrac{3}{2}k'^2+\tfrac{4}{3}k_f^2), \tag{43}$$

$$\alpha^2=\frac{M}{\hbar^2}(BE)_{\text{deuteron}}, \tag{44}$$

and $v=(4/3)\hbar k_i/m$ is the initial velocity of neutron 4 in the lab system.

The differential cross section for elastic scattering is

$$d\sigma_{\text{el}}=L^3\frac{2\pi}{\hbar v}|M_{\text{el}}|^2\rho_E(\mathbf{k}_f)d\mathbf{k}_f, \tag{45}$$

where

$$\rho_E(\mathbf{k}_f)d\mathbf{k}_f=\left(\frac{L}{2\pi}\right)^3k_f^2\left(\frac{\partial k_f}{\partial E}\right)d\Omega_f, \tag{46}$$

$$v=\frac{4}{3}\frac{\hbar k_i}{M}=\frac{4}{3}\frac{\hbar k_f}{M}, \tag{47}$$

and

$$E=\frac{8}{9}\frac{\hbar^2k_f^2}{M}. \tag{48}$$

The ratio of inelastic to elastic cross sections is

$$\frac{d\sigma_{\text{in}}}{\sigma_{\text{el}}}=\left|\frac{M_{\text{in}}}{M_{\text{el}}}\right|^2\frac{1}{(2\pi)^3}k'^2dk'd\Omega'd\Omega_f\frac{k_f}{k_i}, \tag{49}$$

where the matrix elements are to be calculated in Born approximation.

$$M=\{(1-P_{14}-P_{24})\psi_f\chi_f,\,[V_{nn}(r_1-r_4)+V_{nn}(r_2-r_4)+V_{np}(r_3-r_4)]\psi_i\chi_i\}, \tag{50}$$

where P_{ij} is an operator which exchanges space and spin coordinates of particles i and j. A Serber potential is assumed:

$$V_{\alpha\beta}(r_i-r_j)=U(r_i-r_j)\left[{}^3V_{\alpha\beta}{}^+\left(\frac{1+(ij)_x}{2}\right)\left(\frac{1+(ij)_\sigma}{2}\right)+{}^1V_{\alpha\beta}{}^+\left(\frac{1+(ij)_x}{2}\right)\left(\frac{1-(ij)_\sigma}{2}\right)\right]. \tag{51}$$

The initial wave function is

$$\psi_i\chi_i=\exp(i\mathbf{k}_i\cdot\mathbf{Q})\varphi(123)\chi_i^{S_T}, \tag{52}$$

and the final wave functions are, for elastic scattering,

$$\psi_f\chi_f=\exp(-i\mathbf{k}_f\cdot\mathbf{Q})\times\left[\frac{\varphi(\mathbf{r})f(\mathbf{q})+\varphi(\mathbf{q}+\frac{1}{2}\mathbf{r})f(-\frac{1}{2}\mathbf{q}+\frac{3}{4}\mathbf{r})}{(3.344)^{1/2}}\right]\chi_i^{S_T}, \tag{53}$$

and for inelastic n–$2n$ scattering,

$$\psi_f\chi_f=\exp(-i\mathbf{k}_f\cdot\mathbf{Q})\times[\varphi(\mathbf{r})f(\mathbf{q})\chi-\varphi(\mathbf{q}+\tfrac{1}{2}\mathbf{r})f(-\tfrac{1}{2}\mathbf{q}+\tfrac{3}{4}\mathbf{r})\Gamma]. \tag{54}$$

Applying the zero range approximation,[1] $\delta(\mathbf{r}_3-\mathbf{r}_4)$, in which it is assumed that the range of two-body nuclear forces is short compared to the size of the triton and the wavelength of the incident neutron, and neglecting all terms except those in which the arguments of φ and U overlap, we get for elastic scattering:

$$J=-\frac{6\pi}{(3.344)^{1/2}}\frac{M}{\hbar^2}({}^1V_{34}{}^+)r_0\varphi_f(0)r_0{}^2U_0\int\{\exp[-i\mathbf{k}_f\cdot(-(1/3)\mathbf{Q}+(8/9)\mathbf{q})]f(\mathbf{r})\exp(i\mathbf{k}_i\cdot\mathbf{Q})\varphi(123)\}\times\{[(14)_\sigma+(24)_\sigma]\chi_i{}^0,\chi_i{}^0\}d\mathbf{r}d\mathbf{q}\quad(S_T=0), \tag{55}$$

$$J=-\frac{6\pi}{(3.344)^{1/2}}\frac{M}{\hbar}({}^3V_{34}{}^+)r_0\varphi_f(0)r_0{}^2U_0\int\{\exp[-i\mathbf{k}_f\cdot(-(1/3)\mathbf{Q}+(8/9)\mathbf{q})]f(\mathbf{r})\exp(i\mathbf{k}_i\cdot\mathbf{Q})\varphi(123)\}\times\{[(14)_\sigma+(24)_\sigma]\chi_i{}^1,\chi_i{}^1\}d\mathbf{r}d\mathbf{q}\quad(S_T=1), \tag{56}$$

and for inelastic n–$2n$ scattering:

$$J=-6\pi\frac{M}{\hbar^2}({}^1V_{34}{}^+)r_0\varphi_f(0)r_0{}^2U_0\int\{\exp[-i\mathbf{k}_f\cdot(-(1/3)\mathbf{Q}+(8/9)\mathbf{q})]f(\mathbf{r})\exp(i\mathbf{k}_i\cdot\mathbf{Q})\varphi(123)\}\times\{[(14)_\sigma\Gamma-(24)_\sigma\chi],\chi_i{}^0\}d\mathbf{r}d\mathbf{q}\quad(S_T=0), \tag{57}$$

$$J=-6\pi\frac{M}{\hbar^2}({}^3V_{34}{}^+)r_0\varphi_f(0)r_0{}^2U_0\int\{\exp[-i\mathbf{k}_f\cdot(-(1/3)\mathbf{Q}+(8/9)\mathbf{q})]f(\mathbf{r})\exp(i\mathbf{k}_i\cdot\mathbf{Q})\varphi(123)\}\times\{[(14)_\sigma\Gamma-(24)_\sigma\chi],\chi_i{}^1\}d\mathbf{r}d\mathbf{q}\quad(S_T=1). \tag{58}$$

[1] R. M. Frank and J. L. Gammel, Phys. Rev. **93**, 463 (1954).

TABLE I. Initial- and final-spin functions before and after scattering, and the spin matrix elements obtained by performing the spin sums. In the notation $(A,B)_C$, A refers to the state of the triton, B to the state of the deuteron, and C to the total spin of the neutron-triton system.

	Spin function	Statistical weight	Initial spin function	Spin matrix element
Elastic Scattering:				
$(d,t)_0$	χ_i^0	$\frac{1}{3}$	χ_i^0	$(3.344)^{-1/2}$
$(d,t)_0$	χ_i^1	$\frac{2}{3}$	χ_i^1	$(3.344)^{-1/2}$
One neutron excited:				
$(d,t)_0$	χ_0^0	$\frac{1}{3}$	χ_i^0	0
$(q,t)_1$	$\bar{\chi}_1^1$	$\frac{2}{3}$	χ_i^1	$-2\sqrt{2}/\sqrt{3}$
$(d,t)_1$	χ_1^1	$\frac{2}{3}$	χ_i^1	$-2/\sqrt{3}$
Two neutrons excited:				
$(d,t^*)_0$	χ_0^0	$\frac{1}{3}$	χ_i^0	0
$(d,s^*)_0$	$(\chi_0^0)'$	$\frac{1}{3}$	χ_i^0	2
$(q,t^*)_1$	$\bar{\chi}_1^1$	$\frac{2}{3}$	χ_i^1	$-2\sqrt{2}/\sqrt{3}$
$(d,t^*)_1$	χ_1^1	$\frac{2}{3}$	χ_i^1	$-2/\sqrt{3}$
$(d,s^*)_1$	$(\chi_1^1)'$	$\frac{2}{3}$	χ_i^1	0

The spin sums are calculated and listed in Table I. After scattering we take as the wave function of the ground state of the triton

$$\Phi=\left(\frac{\alpha_T}{2\pi}\right)^{1/2}\frac{e^{-\alpha_T q}}{q} \tag{59}$$

and for the continuum state, $l=0$

$$\Phi=e^{i\mathbf{k}'\cdot\mathbf{q}}+f(\theta)\frac{e^{ik'q}}{q} \tag{60}$$

$$=e^{i\mathbf{k}'\cdot\mathbf{q}}+\left(\frac{e^{2i\delta_0(k')}-1}{2ik'}\right)\frac{e^{ik'q}}{q} \tag{61}$$

$$=\cos\delta_0(k')e^{i\delta_0(k')}\left[\frac{\sin k'q}{k'q}+\tan\delta_0(k')\frac{\cos k'q}{k'q}\right]. \tag{62}$$

The calculation will be performed for an incident neutron energy of 14 MeV. $\varphi(123)$, a symmetric function of particles 1, 2, and 3 is taken to be

$$\varphi(123)=e^{\beta^2(3/2r^2+2q^2)}. \tag{63}$$

For elastic scattering $|\mathbf{k}_f|=|\mathbf{k}_i|$ while for inelastic scattering $\mathbf{k}_f$ is determined by Eq. (38). However, using the method of the impulse approximation[1,2] the $\mathbf{k}_f$'s appearing in Eqs. (55)–(58) are taken to be the same. The ratio of the first Born approximation for inelastic scattering to that for elastic scattering is calculated in the following way: The coordinate $\mathbf{Q}$ is eliminated in terms of $\mathbf{q}$ and $\mathbf{r}$, and $\mathbf{q}$ is integrated analytically. The wave functions $f(\mathbf{r})$ which could be either quartet or doublet have been calculated previously for n–d scattering by Christian and Gammel.[3] Their doublet wave function did not vary much with relative n–d energy for the important region. A factor of

$$\left(-\frac{\tan\delta_0(k')}{k'a_4}\right)$$

is removed from $f_4(\mathbf{r})$, the quartet function, making it more nearly a constant as a function of energy. Since

$$\left[\frac{-\tan\delta_0(k')}{k'a_4}\right]^{-1}=-a_4k'\cot\delta_0=1-\tfrac{1}{2}\rho_4a_4k'^2, \tag{64}$$

with

$$a_4=6.2\text{ F},\quad a_2=0.8\text{ F},^3$$
$$\rho_4=3.582\text{ F},\quad \rho_2=45.1\text{ F},$$

we must then multiply $f_4(\mathbf{r})$ by $[1-0.23807E]$, where E is the energy of neutron 1 relative to the deuteron. The ratio of $|J_4/J_2|^2$ is integrated numerically on the IBM-7094 for several values of E and the following fit is obtained:

$$\left|\frac{J_4(\text{inel})}{J_2(\text{el})}\right|^2=\left(\frac{2\pi}{\alpha_T}\right)\left[\frac{\sin^2\delta_0(k')}{k'^2}\right]_4\frac{\lambda^2}{a_4^2}\times[(20.23)(1+20.52k'^2+105.3k'^4)] \tag{65}$$

$$\left|\frac{J_2(\text{inel})}{J_2(\text{el})}\right|^2=\left(\frac{2\pi}{\alpha_T}\right)[\cos^2\delta_0(k')]_2\lambda^2(3.344), \tag{66}$$

where $\lambda=1.332$ F appears because the Christian-Gammel functions are plotted in gauss range units. The resulting cross section for inelastic scattering is

$$d\sigma_{\text{inel}}=\left\{\frac{2}{3}\left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2\left(\frac{2\pi}{\alpha_T}\right)\left(\frac{\sin^2\delta_0(k')}{k'^2}\right)_4\frac{\lambda^2}{a_4^2}(20.23)(1+20.52k'^2+105.3k'^4)\right.$$
$$\left.+\frac{2}{3}\left(\frac{2}{\sqrt{3}}\right)^2(3.344)\left(\frac{2\pi}{\alpha_T}\right)\lambda^2[\cos^2\delta_0(k')]_2\right\}\sigma_{n-T}(\text{el})\frac{1}{(2\pi)^3}k'^2dk'd\Omega'd\Omega_f\frac{k_f}{k_i}. \tag{67}$$

The $\sin^2\delta_0(k')$ and $\cos^2\delta_0(k')$ terms are calculated from

$$k'\cot\delta_0(k')=-(1/a)+\tfrac{1}{2}\rho k'^2 \tag{68}$$

[2] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952).
[3] R. S. Christian and J. L. Gammel, Phys. Rev. **91**, 100 (1953).

and in place of the experimental n–T elastic angular distributions which are not available, calculated values by Bransdon and Robertson[4] assuming Serber interaction for p–He^3 are used. Their 14-MeV curve has not been verified experimentally, but results of Artemov, Kalinin, and Samoilov[5] for energies up to 9.6 MeV indicate that for the higher energies the theoretical curves fit the experimental angular distributions fairly well.

IV. ANGULAR DISTRIBUTION OF DEUTERONS EMITTED IN THE LAB SYSTEM

In order to calculate angular distributions for deuterons in the lab system, we make the following transformations:

$$\begin{aligned}\mathbf{P}_1&=-\tfrac{1}{3}\mathbf{k}_i+\tfrac{1}{4}\mathbf{P}_s &&\text{initial momentum of neutron 1,}\\ &=\mathbf{k}'-\tfrac{1}{3}\mathbf{k}_f+\tfrac{1}{4}\mathbf{P}_s &&\text{final;}\end{aligned}\tag{69}$$

$$\begin{aligned}\mathbf{P}_4&=\mathbf{k}_i+\tfrac{1}{4}\mathbf{P}_s &&\text{initial momentum of neutron 4,}\\ &=\mathbf{k}_f+\tfrac{1}{4}\mathbf{P}_s &&\text{final;}\end{aligned}\tag{70}$$

$$\begin{aligned}\mathbf{P}_d&=-\tfrac{2}{3}\mathbf{k}_i+\tfrac{1}{2}\mathbf{P}_s &&\text{initial momentum of the deuteron,}\\ &=-\mathbf{k}'-\tfrac{2}{3}\mathbf{k}_f+\tfrac{1}{2}\mathbf{P}_s &&\text{final;}\end{aligned}\tag{71}$$

$$k'^2dk'd\Omega'd\Omega_f\frac{k_f}{k_i}=\frac{d\mathbf{k}'d\mathbf{k}_f}{k_ik_f}\delta\left[k_f-\left(k_i^2-\frac{9}{8}k'^2-\frac{9}{8}\alpha_T^2\right)^{1/2}\right]$$
$$=\frac{dP_ddP_4}{k_ik_f}\left[k_f+\left(k_i^2-\frac{9}{8}k'^2-\frac{9}{8}\alpha_T^2\right)^{1/2}\right]\delta\left[k_f^2-\left(k_i^2-\frac{9}{8}k'^2-\frac{9}{8}\alpha_T^2\right)\right].\tag{72}$$

Substituting in k_f and k' in terms of P_d and P_4 according to Eqs. (70) and (71), we get

$$\frac{d\sigma_{\text{inel}}}{dE_dd\Omega_d}=\left\{(0.08353)[1+20.52(k')^2+105.3(k')^4]\left(\frac{\sin^2\delta_0}{k'^2}\right)_4+[0.26538(\cos^2\delta_0)_2]\right\}$$
$$\times(\sigma_{n-T})P_d^2\frac{(0.1553)}{E_d^{1/2}}\frac{P_4^2dP_4}{(P_s)(k_f)}\sin\theta_4d\theta_4d\varphi_4\left[\frac{1}{|P_4-r_1|}\delta(P_4-r_1)+\frac{1}{|P_4-r_2|}\delta(P_4-r_1)\right]$$
$$\times\left[k_f+\left(k_i^2-\frac{9}{8}k'^2-\frac{9}{8}\alpha_T^2\right)^{1/2}\right],\tag{73}$$

where

$$k'^2=P_d^2+\tfrac{4}{3}P_dP_4(\sin\theta_d\sin\theta_4\cos\varphi_4+\cos\theta_d\cos\theta_4)-\tfrac{4}{3}P_dP_s\cos\theta_d+\frac{4}{9}P_4^2-\frac{8}{9}P_4P_s\cos\theta_4+\frac{4}{9}P_s^2,\tag{74}$$

$$k_f=(P_4^2-\tfrac{1}{2}P_4P_s\cos\theta_4+\tfrac{1}{16}P_s^2)^{1/2},\tag{75}$$

$$k_i^2=0.37989,\quad \frac{9}{8}\alpha_T^2=0.22522,$$

$$A=P_s\cos\theta_4-P_d(\sin\theta_d\sin\theta_4\cos\varphi_4+\cos\theta_d\cos\theta_4),\tag{76}$$

$$B=(A^2+P_dP_s\cos\theta_d-0.150146-0.75P_d^2)^{1/2},\tag{77}$$

$$r_1=A+B,\tag{78}$$

$$r_2=A-B.\tag{79}$$

When integrating Eq. (73) numerically, we must be careful to take only the range of integration for which either r_1 or r_4 are real and positive. Integrating Eq. (73) using the data of Bransden and Robertson, we obtain a total inelastic cross section of 381 mb. This value is modified in order to be consistent with the known total n–t cross section of 980 mb. The elastic angular distributions were lowered by 10%, giving a total elastic cross section of 637 mb. We then obtain the angular distributions shown in Figs. 1 and 2, and a total inelastic cross section of 343 mb.

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[4] B. H. Bransden and H. H. Robertson, Proc. Phys. Soc. (London) **A72**, 770 (1958).

[5] K. P. Artemov, S. P. Kalinin, and L. N. Samoilov, Zh. Eksperim. i Teor. Fiz. **37**, 663 (1959) [English transl.: Soviet Phys.—JETP **10**, 474 (1960)].

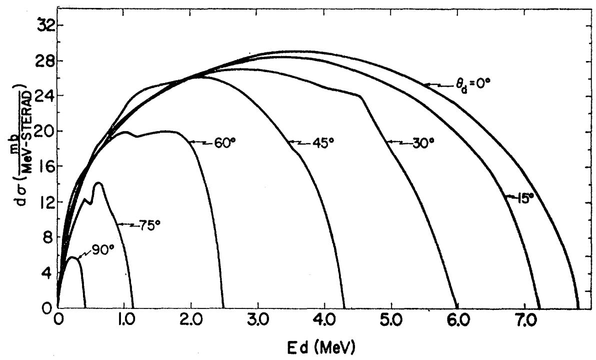

FIG. 1. Inelastic differential scattering cross section expressed in mb/MeV-sr as a function of energy of the ejected deuteron in the lab system plotted for several deuteron lab angles relative to the incoming neutron.

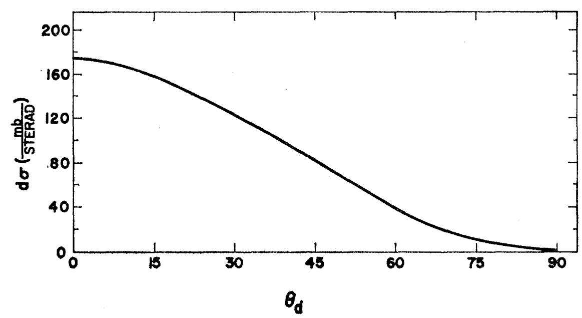

FIG. 2. Inelastic differential scattering cross section expressed in mb/sr as a function of lab angle of the scattered deuteron. This curve is obtained by integrating the curves in Fig. 1 over the deuteron energies.

V. ANGULAR DISTRIBUTIONS OF NEUTRON 4 AND NEUTRON 1

Using the same methods as the previous section, we obtain for neutron 4,

$$\frac{d\sigma_{\text{inel}}}{dE_4 d\Omega_4}=\left\{(0.002687)[1+20.52k'^2+105.3k'^4]\left[\frac{\sin^2\delta_0(k')}{k'^2}\right]_4+(0.008536)[\cos^2\delta_0(k')]_2\right\}$$
$$\times\left[\frac{(\sigma_{n-T})}{P_s k_f}P_4 P_1^2 dP_1 \sin\theta_1 d\theta_1 d\varphi_1\right]\left[\frac{\delta(P_1-r_2)}{|P_1-r_1|}+\frac{\delta(P_1-r_1)}{|P_1-r_2|}\right]\left[k_f+\left(k_i^2-\frac{9}{8}k'^2-\frac{9}{8}\alpha_T^2\right)^{1/2}\right], \quad (80)$$

where

$$k'^2=P_1^2+\tfrac{2}{3}P_1P_4(\sin\theta_4\sin\theta_1+\cos\theta_4\cos\theta_1)-\tfrac{2}{3}P_1P_s\cos\theta_1+\frac{1}{9}P_4^2-\frac{2}{9}P_4P_s\cos\theta_4+\frac{1}{9}P_s^2, \quad (81)$$

$$r_1=\frac{B+(B^2-C)^{1/2}}{2}, \quad (82)$$

$$r_2=\frac{B-(B^2-C)^{1/2}}{2}, \quad (83)$$

$$B=\tfrac{2}{3}[P_s\cos\theta_1-P_4(\sin\theta_1\sin\theta_4\cos\varphi_1+\cos\theta_1\cos\theta_4)], \quad (84)$$

$$C=4(P_4^2-\tfrac{2}{3}P_4P_s\cos\theta_4-\tfrac{1}{3}P_s^2+\alpha_T^2). \quad (85)$$

And for neutron 1, we obtain a similar expression

$$\frac{d\sigma_{\text{inel}}}{dE_1 d\Omega_1}=\left\{(0.002687)[1+20.52k'^2+105.3k'^4]\left[\frac{\sin^2\delta_0(k')}{k'^2}\right]_4+(0.008536)(\cos^2\delta_0(k'))_2\right\}$$
$$\times\frac{\sigma_{n-T}P_1P_4^2}{P_s k_f}dP_4\sin\theta_4 d\theta_4 d\varphi_4\left[\frac{\delta(P_4-r_2)}{|P_4-r_1|}+\frac{\delta(P_4-r_1)}{|P_4-r_2|}\right]\left[k_f+\left(k_i^2-\frac{9}{8}k'^2-\frac{9}{8}\alpha_T^2\right)^{1/2}\right], \quad (86)$$

$$r_1=\frac{B+(B^2-C)^{1/2}}{2}, \quad (87)$$

$$r_2=\frac{B-(B^2-C)^{1/2}}{2}, \quad (88)$$

$$B=\tfrac{2}{3}[P_s\cos\theta_4-P_1(\sin\theta_1\sin\theta_4\cos\varphi_4+\cos\theta_1\cos\theta_4)], \quad (89)$$

$$C=4(P_1^2-\tfrac{2}{3}P_1P_s\cos\theta_1-\tfrac{1}{3}P_s^2+\alpha_T^2). \quad (90)$$

Integrating Eqs. (80) and (86) numerically we get the angular distributions shown in Figs. 3, 4, and 5. The total n–T inelastic cross section obtained from the neutron 1 angular distributions is 349 mb which compares with 343 mb obtained from the deuteron angular distributions. The cross section obtained from the neutron 4 angular distributions will be the same since the neutron 1 and neutron 4 curves coincide except for small variations.

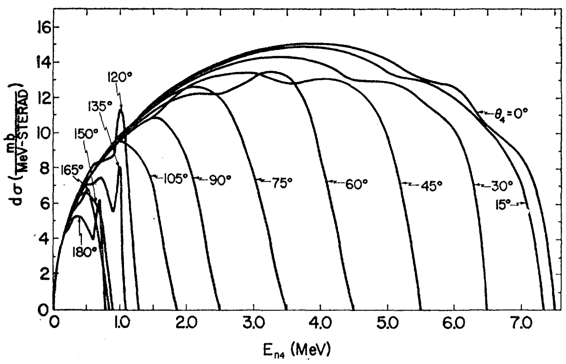

FIG. 3. Inelastic differential scattering cross section expressed in mb/MeV-sr as a function of energy of the scattered neutron 4 in the lab system plotted for several neutron 4 lab angles.

VI. ESTIMATE OF n–$3n$ CROSS SECTION AT 14 MeV

An order of magnitude estimate of the n–$3n$ cross section can be made by calculating

$$d\sigma_{\rm inel}=13.38\left(\frac{\sigma_{{\rm total}\,n-T}{}^{\rm (el)}}{4\pi}\right)\left(\frac{\sigma_{{\rm total}\,nd}}{2\alpha_T}\right)\left(\frac{\sigma_{{\rm total}\,np}}{2\alpha}\right)\times\frac{1}{(2\pi)^6}(4\pi)^3K^2dKk'^2dk'\frac{k_f}{k_i},\quad(91)$$

using the conservation of energy equation,

$$k_f{}^2=k_i{}^2-\frac{9}{8}k'^2-\frac{3}{2}K^2-\frac{9}{8}\alpha_T{}^2-\frac{3}{2}\alpha^2.\quad(92)$$

The answer obtained is 0.4 mb which indicates that the n–$3n$ inelastic scattering is insignificant. Therefore, a more careful evaluation will not be made. The same kind of estimate can be made for n–$2n$ scattering by

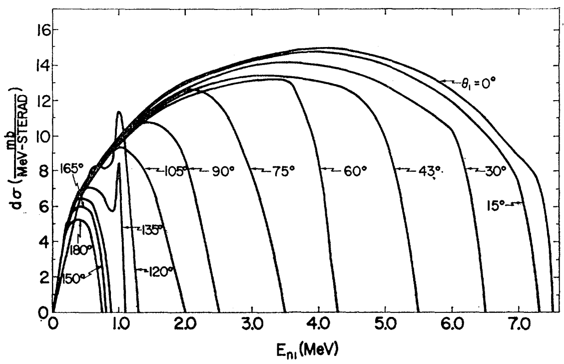

FIG. 4. Inelastic differential scattering cross section expressed in mb/MeV-sr as a function of energy of the ejected neutron 1 in the lab system plotted for several neutron 1 lab angles.

calculating

$$d\sigma_{\rm inel}=\frac{3.344}{3\pi}(\sigma_{{\rm total}\,n-T})\left(\frac{\sigma_{{\rm total}\,nd}}{\alpha_T}\right)\times\frac{1}{(2\pi)^3}(4\pi)^2k'^2dk'\frac{k_f}{k_i},\quad(93)$$

with the conservation of energy equation (38).

The result obtained is 217 mb which is a factor of 1.6 less than the more careful calculation but significantly greater than the n–$3n$ cross section.

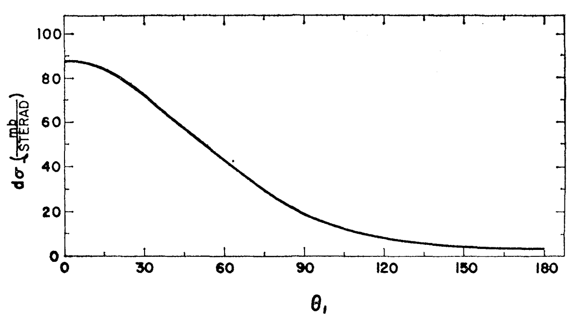

FIG. 5. Inelastic differential scattering cross section expressed in mb/sr as a function of lab angle of the scattered neutron 4. This curve is obtained by integrating the curves in Fig. 3 over energy.

VII. DISCUSSION

We realize that the calculations in this paper are very crude and many unjustifiable approximations have been made. We hope, however, that the work will be helpful in planning experiments and possibly serve as a guide for future theoretical work.