Model of Mesons and Baryons Based on SU_3 Symmetry*

YASUO HARA†
California Institute of Technology, Pasadena, California
(Received 13 November 1963)

A model of mesons and baryons which incorporates the octet scheme of SU_3 symmetry is constructed. In this model, the pseudoscalar mesons and a unitary singlet vector meson are regarded as bound states of baryons and antibaryons. This model is shown to be self-consistent and to explain the observed mass levels if we choose $\alpha_P \approx 3\,\beta_P$ and $\alpha_V = 0$, where α_P and β_P are the D- and F-coupling constants of the $B\bar{B}\pi$ Yukawa interaction, respectively, and α_V is the D-coupling constant of the $B\bar{B}V_8$ interaction. This set of coupling constants are shown to explain baryon resonances. The coupling of a unitary singlet vector meson with baryons is shown to be $\alpha\bar{B}^i[i\cos\theta\gamma_\mu V_\mu-(1/2m_B)\sin\theta\sigma_{\mu\nu}\partial_\nu V_\mu]B^i$ with $0<\theta\lesssim 40°$. It is also shown that the vector meson octet V_8 cannot be regarded as a bound state of the $\bar{B}B$ system.

I. INTRODUCTION

IT is an interesting fact that bosons, baryons, and their excited states can be classified by irreducible representations of the SU_3 group.[1–3] For example, three octets $(\eta,\pi,K,\bar{K})$, $(\varphi,\rho,K^*,\bar{K}^*)$, and (Λ,Σ,N,Ξ) have been found. The ω may be an SU_3 singlet.[4] $N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$, and the yet-to-be-discovered Ω_0^- may form a decuplet.[5–9]

The dynamical emergence of the decuplet has been successfully explained in terms of a Chew-Low-type theory by the author and Miyamoto[7] and by Martin and Wali.[8] The Born term has been found to be more attractive for the decuplet than for any other possible multiplets for a suitable choice of the ratio of the D- and F-coupling constants[1] of a $\pi B\bar{B}$ Yukawa interaction.

If we regard resonances and particles forming multiplets as composite particles, we have to show why certain particular multiplets are the lowest possible states by constructing a dynamical model incorporating the octet scheme of SU_3 symmetry. For example, we have to show why the octet pseudoscalar mesons are lighter than vector mesons, and why there are no unitary singlet pseudoscalar mesons, etc.

In this article we shall regard the pseudoscalar mesons and vector mesons as composite particles consisting of a baryon and an antibaryon, and we shall attempt to answer the above questions.[10] We shall also discuss baryon excited states and two-baryon states.

In the next section we shall explain our model and define our approximations. We shall discuss pseudoscalar mesons in Sec. III, vector mesons in Sec. IV, baryon and baryon excited states in Sec. V, and p-wave $B\bar{B}$ bound states corresponding to scalar mesons and axial vector mesons in Sec. VI.

II. MODEL

In this article we regard the pseudoscalar mesons and vector mesons as composite particles consisting of two-particle states. Three- and more-particle configurations of composite particles will be neglected for the sake of simplicity. The pseudoscalar mesons (π_8) will be considered as bound ${}^1S_0$ states of the baryon-antibaryon $(B\bar{B})$ system, and the unitary singlet vector meson (V_1) will be assumed to be a bound ${}^3S_1$ state of a $B\bar{B}$ pair. We neglect possible πV and $2V$ configurations, since the $\pi B\bar{B}$ Yukawa coupling constants and the $V_1B\bar{B}$ coupling constant are far bigger than the $\pi\pi V$ coupling constants in absolute value,[11–13] since it is complicated to consider them, and since π_8 and V_1 can be explained as $B\bar{B}$ bound states.

Since the ρ mesons were discovered as p-wave $\pi\pi$ resonances, it is clear that the vector mesons have considerable 2π configurations. The ρ mesons, however, may consist of $N\bar{N}$ too, as the ρNN coupling constant $g_{NN\rho}\neq 0$. Indeed, as has been shown by various authors, $g_{NN\rho}^2 \approx f_{\rho\pi\pi}^2$ $(g_{NN\rho}^2=\beta_V^2$ and $f_{\rho\pi\pi}^2=\frac{1}{4}f_{V\pi\pi}^2)$.[11] Therefore,

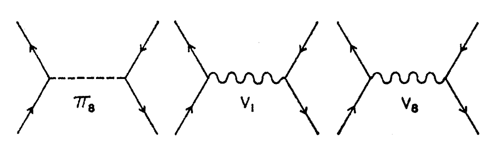

FIG. 1. Born diagrams for $B\bar{B}$ scattering.

* Work supported in part by the U. S. Atomic Energy Commission.

† On leave of absence from Physics Department, Tokyo University of Education, Tokyo, Japan.

1 M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20 (unpublished).

2 Y. Ne'eman, Nucl. Phys. **24**, 222 (1961).

3 S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

4 J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).

5 R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. **34**, 1 (1962).

6 S. L. Glashow and J. J. Sakurai, Nuovo Cimento **25**, 337 (1962).

7 Y. Hara and Y. Miyamoto, Progr. Theoret. Phys. (Kyoto) **29**, 466 (1963).

8 A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

9 R. Cutkosky, Ann. Phys. (N.Y.) **23**, 415 (1963).

10 Our model is a generalization of the one proposed by C. N. Yang and E. Fermi, Phys. Rev. **76**, 1739 (1946); and by Y. Miyamoto, Progr. Theoret. Phys. (Kyoto) **28**, 967 (1962).

11 K. Kawarabayashi and A. Sato, Nuovo Cimento **26**, 1017 (1962), and the papers cited therein.

12 Y. Hara, Progr. Theoret. Phys. (Kyoto) **28**, 1048 (1962).

13 According to Kawarabayashi and Sato (Ref. 11),

$$f_{\omega\rho\pi}^2/4\pi=0.65\times\Gamma(\omega\to\pi^+\pi^0\pi^-)/20\text{ MeV},$$

and

$$f_{1\omega NN}^2/4\pi=90\times\Gamma(\omega\to\pi^+\pi^0\pi^-)/20\text{ MeV}.$$

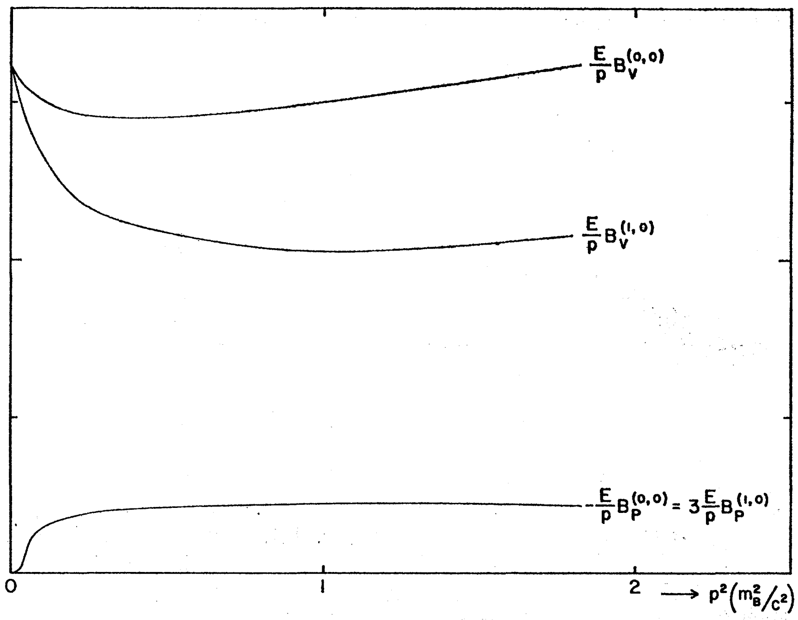

FIG. 2. Magnitudes of Born terms. Here V stands for "due to vector meson exchange," P stands for "due to pseudoscalar meson exchange," (0,0) stands for "$^1S_0$" and (1,0) stands for "$^3S_1 \to {}^3S_1$," with

$$m_B=\tfrac{1}{8}(2m_N+2m_\Xi+3m_\Sigma+m_\Lambda),$$
$$\mu_V^2=\tfrac{1}{8}(4\mu_{K^*}^2+3\mu_\rho^2+\mu_\varphi^2),$$
$$\mu_P^2=\tfrac{1}{8}(4\mu_K^2+3\mu_\pi^2+\mu_\eta^2).$$

we shall have to consider V_8 to consist of both $B\bar{B}$ and 2π.

It will next be assumed that the force which combines B and $\bar{B}$ is due to the exchange of the bosons, V_1, V_8, and π_8 (other yet-to-be-discovered bosons will not be considered[14]). (See Fig. 1.) Then we shall be able to calculate the Born terms corresponding to these processes using the following interaction Hamiltonian density,[15]

$$\alpha\bar{B}^i\left(i\cos\theta\cdot\gamma_\mu V_\mu-\frac{1}{2m_B}\sin\theta\cdot\sigma_{\mu\nu}\partial\;V_\mu\right)B^i$$
$$+\alpha_V\bar{B}^iD_{ij}{}^k\left(i\cos\phi'\cdot\gamma_\mu V_{k,\mu}-\frac{1}{2m_B}\sin\phi'\sigma_{\mu\nu}\partial_\nu V_{k,\mu}\right)B^j$$
$$+\beta_V\bar{B}^iF_{ij}{}^k\left(i\cos\phi\cdot\gamma_\mu V_{k,\mu}-\frac{1}{2m_B}\sin\phi\sigma_{\mu\nu}\partial_\nu V_{k,\mu}\right)B^j$$
$$+\bar{B}^i(\alpha_PD_{ij}{}^k+\beta_PF_{ij}{}^k)i\gamma_5B^j\pi_k. \quad (2.1)$$

Though it is desirable to iterate Born terms by solving suitable integral equations quantitatively, this will not be done in this article. Instead, we will assume that the ordering of mass levels is the same as the ordering of the Born terms according to their magnitude. Although this approximation may seem to be very rough, it turns out to be quite reasonable. This Born term is proportional to the one-boson exchange potential between a baryon and an antibaryon. An important part of the many-boson exchange potential is included in the potential due to the exchange of boson excited states. Therefore, our result may be trusted qualitatively.

If we solve the integral equations, the magnitudes of the coupling constants α, α_P, β_P, α_V, and β_V and those of the mixing angles θ, ϕ, and ϕ' can be calculated, and the consistency of our solution can be checked by comparing the output coupling constants with the input couplings. Although we will not be able to know the absolute values of the coupling constants (since we do not solve the equations), it will be seen that the ratio of the output coupling constants α_P/β_P may be estimated for a given set of input constants. By comparing the output with the input ratios, and by requiring that the order of the Born terms be the same as the order of the observed mass levels, we can determine the ratio α_P/α_P.

III. PSEUDOSCALAR MESONS

If we assume that the mesons are bound states of the $B\bar{B}$ system, the lightest mesons may be expected to be bound states in the $^1S_0$ and $^3S_1$ state, that is, pseudoscalar mesons and vector mesons. This is indeed in agreement with experiment. Then, we have only to explain why pseudoscalar mesons are lighter than vector mesons. This can be explained by assuming that the strongest interaction between B and $\bar{B}$ is due to the exchange of vector mesons. As is seen in Fig. 2, the interaction due to vector meson exchange is about $\frac{3}{2}$ times stronger in the $^1S_0$ state than in the $^3S_1$ state for pure vector coupling (θ, $\phi=0$). In fact, the interaction

[14] The existence of a unitary singlet scalar meson does not alter our conclusion (Sec. VI).

[15] $V_{k,\mu}$ and V_μ stand for the μth component of V_8 and V_1, respectively. m_B is the baryon mass, and $A_\mu B_\mu=\mathbf{A}\cdot\mathbf{B}-A_0B_0$. In the following, α_V will be assumed to be equal to zero. In this article, we assume that both the input and the resulting couplings of the $B\bar{B}\pi_8$ interaction are of pseudoscalar type instead of a pseudovector derivative coupling or of an arbitrary combination of both for the sake of simplicity.

in $^1S_0$ is always stronger than that in $^3S_1$ for any mixing ratio θ and ϕ, as is seen in Fig. 3 (for $p^2=\frac{1}{2}$).

In the following, it will be assumed that

$$\alpha^2/4\pi = 5\sim 10\,,\quad (\alpha_P+\beta_P)^2/4\pi = 15\,, \tag{3.1}$$

and

$$\beta_V^2/4\pi \approx 1\,,$$

which correspond to[11,12]

$$g_{NN\omega}^2/4\pi = 5\sim 10\,,\quad g_{NN\pi}^2/4\pi = 15\,,$$

and[11]

$$g_{NN\rho}^2/4\pi = 1\,.$$

Also, α_V will be assumed to be nearly equal to zero as this case is the most interesting one. Although the pseudoscalar coupling constant is the biggest one, it turns out that the force due to the exchange of pseudoscalar mesons is not particularly strong because of its pseudoscalar property (see Fig. 2).

The Born term for $B\bar{B}$ scattering due to the exchange of V_1, V_8, and π_8 can be written as[16]

$$T_{B,i}{}^{(0,0)} = \frac{\alpha^2}{4\pi} B_V{}^{(0,0)} + A_{V,i} B_V{}^{(0,0)} + A_{P,i} B_P{}^{(0,0)}\,, \tag{3.2}$$

where the A_i's are given in Table I and the B_i's are given in the Appendix and plotted in Figs. 2 and 3.

In order to compare the magnitudes of the Born terms for the six possible representations of the SU$_3$ group, we have to diagonalize the 2×2 matrix for the octet states:

$$\begin{vmatrix} 6\beta_V^2 B_V + (2\alpha_P^2 - 6\beta_P^2)(-B_P), & -4(5)^{1/2}\alpha_P\beta_P(-B_P) \\ -4(5)^{1/2}\alpha_P\beta_P(-B_P), & 6\beta_V^2 B_V + [-(10/3)\alpha_P^2 - 6\beta_P^2](-B_P) \end{vmatrix} = \begin{vmatrix} S, & -A \\ A, & S \end{vmatrix} \begin{vmatrix} B_1, & 0 \\ 0, & B_2 \end{vmatrix} \begin{vmatrix} S, & A \\ -A, & S \end{vmatrix} \quad (B_1 > B_2)\,, \tag{3.3}$$

where

$$B_1,\,B_2 = 6\beta_V^2 B_V + (-\tfrac{2}{3}\alpha_P^2 - 6\beta_P^2)(-B_P) \pm [(64/9)\alpha_P^4 + 80\alpha_P^2\beta_P^2]^{1/2}(-B_P)\,,$$

and

$$\left\{\frac{8}{3}\alpha_P^2 - \left[\left(\frac{64}{9}\right)\alpha_P^4 + 80\alpha_P^2\beta_P^2\right]^{1/2}\right\} S - 4(5)^{1/2}\alpha_P\beta_P A = 0\,, \tag{3.4}$$

and where α_V has been assumed to be equal to zero. It should be noted that the ratio S/A does not contain B_V or B_P.

Since the ratio S/A derived here is energy independent, it is the mixing ratio of the symmetric state $(-\sqrt{3}\bar{B}^i D_{ij}{}^k B^j/\sqrt{20})$ and the antisymmetric state $(\bar{B}^i F_{ij}{}^k B^j/\sqrt{12})$ of the octet pseudoscalar meson π^k, and is proportional to the ratio of the D- and F-couplings of $B\bar{B}\pi_8$ interaction. Specifically,

$$S/A = -[(5)^{1/2}/3](\alpha_P/\beta_P)\,. \tag{3.5}$$

This equality has been obtained without solving the integral equations. From Eqs. (3.4) and (3.5), two solutions

$$\alpha_P = 3\beta_P \quad\text{and}\quad \beta_P = 0 \tag{3.6}$$

are obtained. It is an interesting fact that $\alpha_P = 3\beta_P$ has been obtained elsewhere[7,8] as the ratio that makes the decuplet $p_{3/2}$ $B\pi$ resonances the lowest states.

Using the ratio (3.6) and the magnitudes of the coupling constants in (3.1), the relative magnitudes of the Born terms are shown in Fig. 4.[17] From Fig. 4, one of the octets of pseudoscalar mesons is seen to be the ground state.

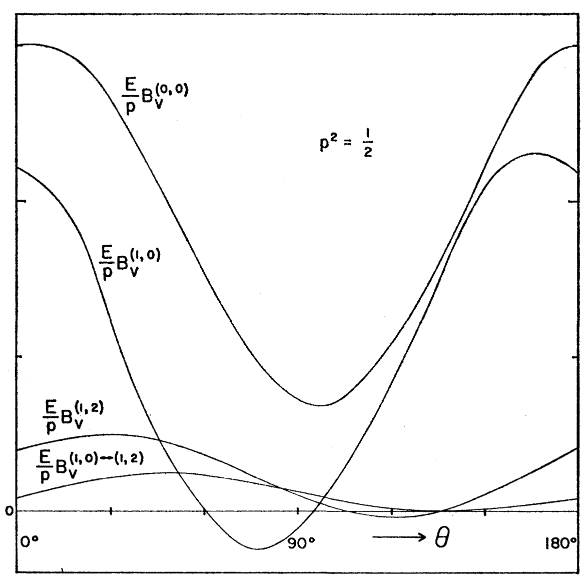

FIG. 3. Magnitudes of Born terms as functions of the mixing angle θ at $p^2=\frac{1}{2}$. In particular: $\theta=0°$: pure vector coupling, $\theta=90°$: pure tensor coupling. (1,2) stands for "$^3D_1 \to {}^3D_1$" and (1,0) $\leftrightarrow$ (1,2) stands for "$^3S_1 \leftrightarrow {}^3D_1$."

IV. VECTOR MESONS

1. V_1

The unitary singlet vector meson V_1 is assumed to be a bound state in the $J=1^-$ state of the $B\bar{B}$ system.

[16] Here,

$$T = T^{(J,L)}{}_B,\ \text{(dimension of the irreducible representation)},$$

and

$$B = B^{(J,L)}\ \text{(exchanged boson)}.$$

[17] In drawing Fig. 4, it is assumed that $\phi = \phi' = 0$.

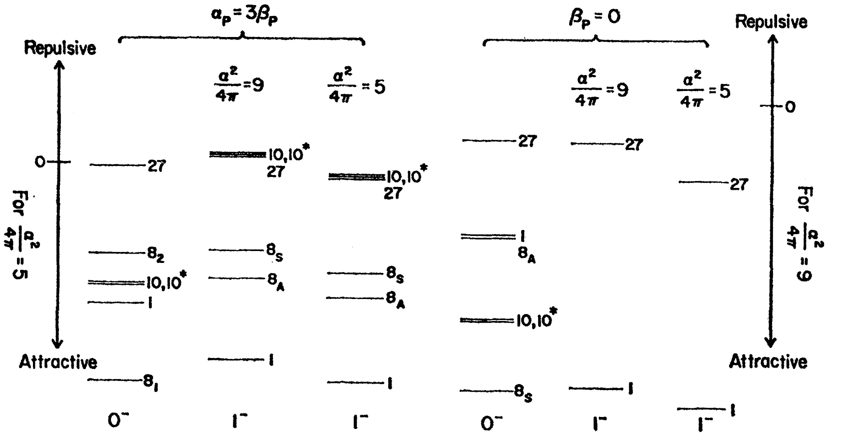

FIG. 4. Relative magnitudes of Born terms, with $\alpha^2/4\pi=5$ or 9, $(\alpha_P+\beta_P)^2/4\pi=15$, $\beta_V^2/4\pi=1$.

We neglect possible πV configurations, as has been explained in Sec. II. 2π states do not couple with V_1 (and V_{27} if it exists) due to the exclusion principle. In particular, p-wave $\pi\pi$ scattering occurs only in the 8_A, 10, and 10* states.

The Born term for $B\bar{B}$ scattering in the $J=1^-$ state due to the exchange of V_1, V_8, and π_8 can be written as

$$T_{B,i}{}^{(1,L)}=(\alpha^2/4\pi)B_V{}^{(1,L)}+A_{V,i}B_V{}^{(1,L)}+A_{P,i}B_P{}^{(1,L)}, \quad (4.1)$$

where the A_i's are given in Table I and the B_i's are given in the Appendix and plotted in Figs. 2 and 3. The relative magnitudes of the Born terms for V_1 and V_{27} are shown in Fig. 4. It will be seen that V_1 is heavier than π_8, but lighter than any other unobserved multiplets of pseudoscalar mesons if $\alpha^2/4\pi\gtrsim 9$. It will also be seen that there is no V_{27} in this model.

Next, let us consider the mixing of the vector and tensor couplings for the $V_1B\bar{B}$ interaction. The pole residues of the Born terms corresponding to Fig. 5 (at $s=\mu_V{}^2$) are as follows:

$$\begin{vmatrix} {}^3S_1\to{}^3S_1, & {}^3S_1\to{}^3D_1 \\ {}^3D_1\to{}^3S_1, & {}^3D_1\to{}^3D_1 \end{vmatrix} \propto \begin{vmatrix} A^2, & AB \\ AB, & B^2 \end{vmatrix}, \quad (4.2)$$

where

$$A=\left(1+\frac{\mu_V}{m_B}\right)\cos\theta+\left(1+\frac{\mu_V}{4m_B}\right)\frac{\mu_V}{m_B}\sin\theta \approx 1.7\cos\theta+0.9\sin\theta,$$

and

$$B=-\sqrt{2}\left(1-\frac{\mu_V}{2m_B}\right)\cos\theta+\sqrt{2}\frac{\mu_V}{2m_B}\left(1-\frac{\mu_V}{2m_B}\right)\sin\theta \approx -0.9\cos\theta+0.3\sin\theta.$$

As is seen in Fig. 3, the interaction which mixes the ${}^3S_1$ state and the ${}^3D_1$ state can be written approximately as ap^2 with $a>0$. Thus, $AB<0$ (for $p^2<0$), i.e.,

$$0°<\theta<70° \quad\text{or}\quad 160°<\theta<180°.$$

By looking at Fig. 3 and Eq. (4.2), the range

$$0°<\theta<40°$$

seems to be the most probable one. [If $\sin\theta<0, B/A \times({}^3D_1/{}^3S_1)$ is big. This is contradictory with Fig. 3.]

2. V_8

At first, let us assume that V_8 (V_{10} and V_{10^*}) consists only of $B\bar{B}$ pairs. If we assume $\alpha_P=\beta_P=0$, the potential (4.1) does not have consistent bound-state solutions in octet states [(4.3) and (4.4) are contradictory]. From

$$\begin{vmatrix} -2\alpha_V{}^2+6\beta_V{}^2+\alpha^2, & 4(5)^{1/2}\alpha_V\beta_V \\ 4(5)^{1/2}\alpha_V\beta_V, & (10/3)\alpha_V{}^2+6\beta_V{}^2+\alpha^2 \end{vmatrix} = \begin{vmatrix} S, & -A \\ A, & S \end{vmatrix} \begin{vmatrix} B_1, & 0 \\ 0, & B_2 \end{vmatrix} \begin{vmatrix} S, & A \\ -A, & S \end{vmatrix},$$

we obtain

$$\frac{S}{A}=\frac{4(5)^{1/2}\alpha_V\beta_V}{[(64/9)\alpha_V{}^4+80\alpha_V{}^2\beta_V{}^2]^{1/2}-(8/3)\alpha_V{}^2}, \quad (4.3)$$

while

$$S/A=-(5)^{1/2}\alpha_V/3\beta_V. \quad (4.4)$$

It is difficult to say whether the potential (4.1) has a consistent solution without solving integral equations, when the term due to an exchange of pseudoscalar

TABLE I. $A_{P,i}$ and $A_{V,i}$.

	27	10	10*	8_S	8_A	$8_S\leftrightarrow 8_A$	1
$4\pi A_i$	$\frac{4}{3}\alpha_i{}^2-4\beta_i{}^2$	$-(8/3)\alpha_i{}^2$	$-(8/3)\alpha_i{}^2$	$-2\alpha_i{}^2+6\beta_i{}^2$	$(10/3)\alpha_i{}^2+6\beta_i{}^2$	$4(5)^{1/2}\alpha_i\beta_i$	$(20/3)\alpha_i{}^2+12\beta_i{}^2$

TABLE II. B_i and C_i.

	27	10	10*	8_S	8_A	$8_S \leftrightarrow 8_A$	1
C_i	$-2\alpha_i^2-6\beta_i^2$	$-4\alpha_i^2-12\alpha_i\beta_i$	$-4\alpha_i^2+12\alpha_i\beta_i$	$3\alpha_i^2+9\beta_i^2$	$5\alpha_i^2-9\beta_i^2$	0	$-10\alpha_i^2+18\beta_i^2$
B_i	$\frac{1}{3}$	1	1	2	0	$\sqrt{5}$	-5

mesons is taken into account ($\beta_P=0$).[18] What we can say is that $|\alpha_V|$ must be much smaller than $|\beta_V|$ even if (4.1) has a consistent solution. In Fig. 4, the potentials for V_8 are drawn assuming $\alpha_V=0$. This solution is not satisfactory if it exists, since the strength of the Born term for V_1 is far more attractive than that for V_8, as is seen in Fig. 4. This is not in agreement with experiment. Thus, we have shown that the octet vector mesons V_8 cannot be regarded as bound states of the $B\bar{B}$ system.

Let us consider both $B\bar{B}$ and 2π configurations for V_8, V_{10}, and V_{10^*}. Now, p-wave scattering of 2π appears only in 8_A, 10, and 10* states. The Born terms for 2π scattering due to V_8 exchange for 10 and 10* are zero, and that for 8_A is attractive. Therefore, it is possible that the potential for V_8 becomes more attractive if we consider the coupling of 2π and $B\bar{B}$ states. Mixing matrices between the 2π and 8_A or 8_S states of the $B\bar{B}$ system corresponding to the Feynman diagrams of Fig. 6(a) can be written as

$$[-(10/3)\alpha_P^2-6\beta_P^2]C \quad \text{for} \quad 8_A \leftrightarrow 2\pi\,, \qquad (4.5\text{a})$$

and

$$4(5)^{1/2}\alpha_P\beta_P C \quad \text{for} \quad 8_S \leftrightarrow 2\pi\,, \qquad (4.5\text{b})$$

and those corresponding to Fig. 6(b) can be written as

$$0 \quad \text{for} \quad 8_A \leftrightarrow 2\pi\,, \qquad (4.6\text{a})$$

and

$$\tfrac{1}{4}(5)^{1/2}D \quad \text{for} \quad 8_S \leftrightarrow 2\pi\,, \qquad (4.6\text{b})$$

where C and D are functions of energy. We do not solve coupled integral equations in this article, however. This problem will be discussed elsewhere.

We have shown that π_8, V_1, and V_8 are the lowest possible states if we could show that the mass of V_1 is nearly equal to the masses of V_8 by solving the coupled integral equations of the 2π and $B\bar{B}$ states. As is seen in Fig. 4, V_{27} does not exist, while V_{10} and V_{10^*} are heavier since both the $\pi\pi$ and $B\bar{B}$ interactions are weak in the 10 and 10* states.[19]

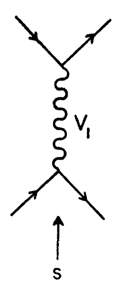

FIG. 5. The $B\bar{B}\to V_1\to B\bar{B}$ process.

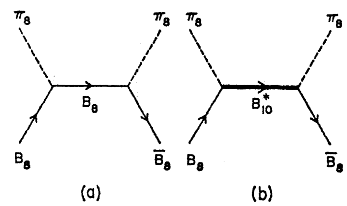

FIG. 6. The Born diagrams for $B\bar{B}\to 2\pi$ process: (a) B_8 exchange; (b) B_{10}^* exchange.

V. BARYONS AND THEIR EXCITED STATES

The p-wave πB scattering amplitude corresponding to Fig. 7(a) and 7(b) can be written as[7,9]

$$C_i\frac{q^2}{\omega}+B_i\frac{q^2}{\omega+\omega_r} \quad \text{for the } p_{1/2} \text{ state}, \qquad (5.1)$$

and

$$-2C_i\frac{q^2}{\omega}+\frac{B_i}{4}\frac{q^2}{\omega+\omega_r} \quad \text{for the } p_{3/2} \text{ state} \qquad (5.2)$$

in the static theory, where the C_i and B_i are given in Table II. (Common positive factors are omitted.)

1. ($N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$, Ω_0^-)

The $p_{3/2}\pi B$ scattering amplitudes have been analyzed by several authors. Miyamoto and the author have shown[7] that the resonances in the tenfold representation ($N_{3/2}^*$, Y_1^*, $\Xi_{1/2}^*$, Ω_0^-) are indeed the lowest states if

$$\alpha_P/\beta_P=1\sim 3\,.$$

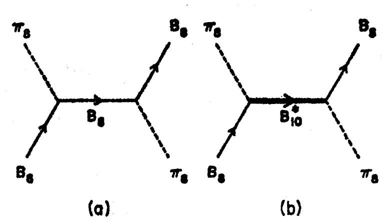

FIG. 7. The Born diagrams for $B\pi$ scattering: (a) B_8 exchange; (b) B_{10}^* exchange.

They used the Chew-Low static model. [If $\alpha_P=3\beta_P$, we have $m_{B(10)}=m_{B(1)}$ and $m_{B(10)}$ assumes its lowest value.] Martin and Wali[8] have shown that if the effects of the mass differences are taken into account and if the N/D method is used, resonances will be found in the tenfold representation for

$$\alpha_P/\beta_P=1\sim 5\,.$$

---

[18] If $\beta_P=0$, Eq. (3.1) has no solution.

[19] The mixing matrices are $-(8/3)\alpha_P^2C+\frac{1}{4}D$ both for $10\leftrightarrow 2\pi$ and for $10^*\leftrightarrow 2\pi$.

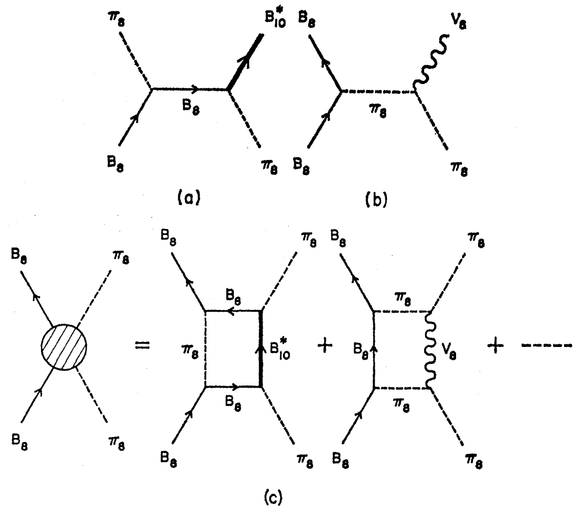

FIG. 8. The diagrams responsible for the $N_{1/2}^*$ resonances: (a) the $B\pi\to B^*\pi$ process; (b) the $B\pi\to BV$ process; and (c) their iterations.

The lowest possible positions of these resonances in the tenfold representation are attained when $\alpha_P/\beta_P=3$. Their results are in agreement with the results of the static model in which the effects of the mass differences are nelgected.

2. Octet Baryons

For the $p_{1/2}$ state, one of the octets is easily seen to be the ground state. If we neglect the first term in (5.1) compared with the second term, we obtain[9]

$$\alpha_P/\beta_P=[3(6)^{1/2}+3]/5\approx 2.07$$

by diagonalizing the matrix

$$\begin{vmatrix} 2, & \sqrt{5} \\ \sqrt{5}, & 0 \end{vmatrix} = \begin{vmatrix} S, & -A \\ A, & S \end{vmatrix} \begin{vmatrix} \lambda_1, & 0 \\ 0, & \lambda_2 \end{vmatrix} \begin{vmatrix} S, & A \\ -A, & S \end{vmatrix} \qquad (\lambda_1>\lambda_2)$$

and taking

$$S/A=[(5)^{1/2}/3](\alpha_P/\beta_P).$$

3. $N_{1/2}^*$

Let us assume that the πN second resonance, $N_{1/2}^*$, is due to the iteration of the Feynman diagrams in Figs. 8(a) and 8(b). Since $10\times 8=35+27+10+8$ and $8\times 8=27+10+10^*+8+8+1$, the process in Fig. 8(a) occurs only in 27, 10, and 8, and the corresponding matrix elements are

$$\left(-\frac{2}{\sqrt{3}}\alpha_P+\frac{2}{\sqrt{3}}\beta_P\right)V, \qquad \text{for} \quad 27,$$

$$\left(-\frac{2}{3}\right)^{1/2}\alpha_P-((6)^{1/2}\beta_P)V, \quad \text{for} \quad 10,$$

$$\left(-\frac{2}{\sqrt{3}}\alpha_P-\sqrt{3}\beta_P\right)V, \qquad \text{for} \quad 8_S,$$

and

$$-(5/3)^{1/2}\alpha_P V, \quad \text{for} \quad 8_A,$$

while the second process occurs in 27, 10, 10*, 8, 8, and 1, and the corresponding matrix elements are

$4\beta_P W$	for	27,
$4\alpha_P W$	for	10,
$-4\alpha_P W$	for	10^*,
$-6\beta_P W$	for	$8_S\leftrightarrow 8_S$ and $8_A\leftrightarrow 8_A$,
$-2(5)^{1/2}\alpha_P W$	for	$8_S\leftrightarrow 8_A$,
$-12\beta_P W$	for	1.

The force that produces resonances is approximately proportional to

$$(a_i^2V^2+b_i^2W^2)^{1/2}. \tag{5.3}$$

If $\alpha_P\approx 3\beta_P$, Eq. (5.3) is largest for an octet. Therefore, $N_{1/2}^*$ probably belongs to an octet together with Y_0^* (1520) and the yet-to-be-discovered Y_1^* and $\Xi_{1/2}^*$. Since the s-wave $B^*\pi$ system couples with the $d_{3/2}$ $B\pi$ state and the s-wave BV system couples with the $s_{1/2}$ and $d_{3/2}$ $B\pi$ states, the $B\pi$ resonances caused by this mechanism may have spin $d_{3/2}$. The wave functions of the octets in charge space are as follows:

$$N_{1/2}^*: \quad \frac{1}{(20(d^2+f^2))^{1/2}}[(d-(5)^{1/2}f)\eta N+(d+(5)^{1/2}f)K\Lambda-(3d+(5)^{1/2}f)\pi N-(3d-(5)^{1/2}f)K\Sigma],$$

$$Y_1^*: \quad \frac{1}{(30(d^2+f^2))^{1/2}}[-(6)^{1/2}d\eta\Sigma-(6)^{1/2}d\pi\Lambda-(20)^{1/2}f\pi\Sigma+(3d+(5)^{1/2}f)K\Xi+(3d-(5)^{1/2}f)\bar{K}N],$$

$$Y_0^*: \quad \frac{1}{(10(d^2+f^2))^{1/2}}[(6)^{1/2}d\pi\Sigma+\sqrt{2}d\eta\Lambda+(d-(5)^{1/2}f)K\Xi-(d+(5)^{1/2}f)\bar{K}N],$$

$$\Xi_{1/2}^*: \quad \frac{1}{(20(d^2+f^2))^{1/2}}[(d+(5)^{1/2}f)\eta\Xi+(d-(5)^{1/2}f)\bar{K}\Lambda+(3d+(5)^{1/2}f)\bar{K}\Sigma+(3d-(5)^{1/2}f)\pi\Xi],$$

TABLE III. $A_{P,i}'$ and $A_{V,i}'$.

	27	10	10*	8_S	8_A	$8_S \leftrightarrow 8_A$	1
$4\pi A_i'$	$\frac{4}{3}\alpha_i^2+4\beta_i^2$	$-(8/3)\alpha_i^2+8\alpha_i\beta_i$	$-(8/3)\alpha_i^2-8\alpha_i\beta_i$	$-2\alpha_i^2-6\beta_i^2$	$(10/3)\alpha_i^2-6\beta_i^2$	0	$(20/3)\alpha_i^2-12\beta_i^2$

where

$$d/f=1.4 \quad \text{if} \quad |V|\gg|W|$$

and

$$d/f=1 \quad \text{if} \quad |V|\ll|W|\,.$$

This result explains quite well the partial widths of Y_0^* (1520) and $N_{1/2}^*$ (1512).[3]

4. Deuteron

If our SU_3 symmetry is valid, the deuteron[20] must be a component of some representation of this group. It is easily seen that the deuteron belongs indeed to 10*. This decuplet consists of

$$I=0,\quad S=0:\quad NN,$$

$$I=\tfrac{1}{2},\quad S=-1:\quad (1/\sqrt{2})\Lambda N-(1/\sqrt{2})N\Sigma\,,$$

$$I=1,\quad S=-2:\quad -(1/\sqrt{2})\Lambda\Sigma+(1/\sqrt{3})\Xi N+(1/\sqrt{6})\,\Sigma\Sigma\,,$$

$$I=\tfrac{3}{2},\quad S=-3:\quad \Sigma\Xi\,.$$

Most of these states may appear as resonances. We will thus be able to check SU_3 symmetry by looking for ΛN resonances. SU_3 symmetry, however, will not be valid for many-baryon systems as charge independence does not hold for heavy nuclei.

Through charge conjugation, the $B\bar{B}$ scattering amplitude can be converted into the BB scattering amplitude. The Born terms can then be written as

$$T_B'^{(J,L)}=-\frac{\alpha^2}{4\pi}B_V^{(J,L)}-A_{V,i}'B_V^{(J,L)}+A_{P,i}'B_P'^{(J,L)}\,,$$

where the $A_{V,i}'$'s and $A_{P,i}'$'s are given in Table III, and where

$$B_P'^{(0,0)}=-3B_P'^{(1,0)}=B_P^{(0,0)}+\frac{\mu_P^2}{4Ep}Q_0\left(1+\frac{\mu_P^2}{2p^2}\right).$$

The difference between B_P' and B_P, namely $(\mu^2/4Ep)Q_0$, corresponds to a delta-function potential which does not contribute to BB scattering because of a hard core due to vector meson exchange in this case. Since $B_P'^{(1,0)}<0$ for $p^2>0$, the above decuplet is lower than the octet and the other decuplet.

VI. SCALAR MESONS AND AXIAL VECTOR MESONS

If mesons are $B\bar{B}$ bound states, the next excited states may be $^3P_0$, $^3P_1$, $^1P_1$, or $^3P_2$. If we assume that the interactions between B and $\bar{B}$ are due to the exchange of V_1, V_8, and π_8, the Born terms for these states, $T_{B,i}^{(J,L,S)}$, can be written as

$$T_{B,i}^{(J,L,S)}=(\alpha^2/4\pi)B_V^{(J,L,S)}+A_{V,i}B_V^{(J,L,S)}+A_{P,i}B_P^{(J,L,S)}\,,$$

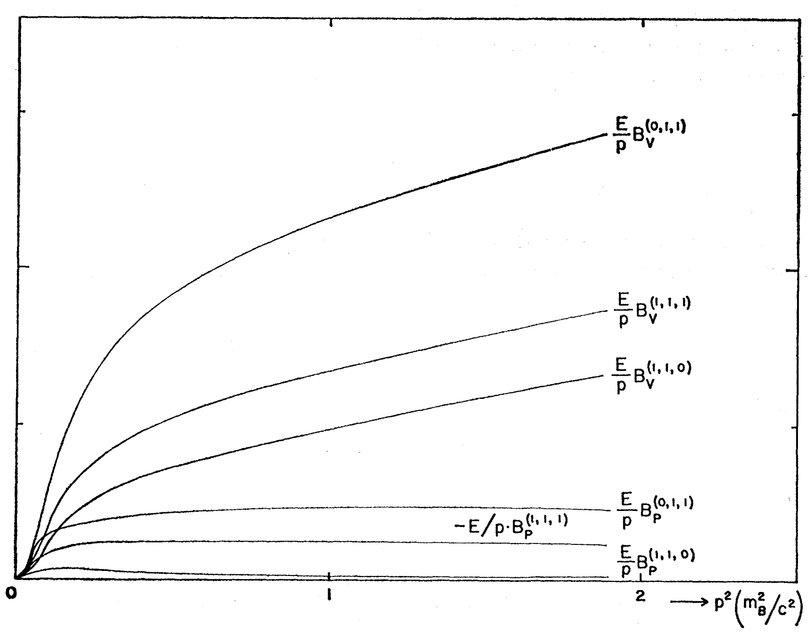

FIG. 9. Magnitudes of Born terms. Here, $B=B^{(J,L,S)}$.

[20] R. J. Oakes (to be published).

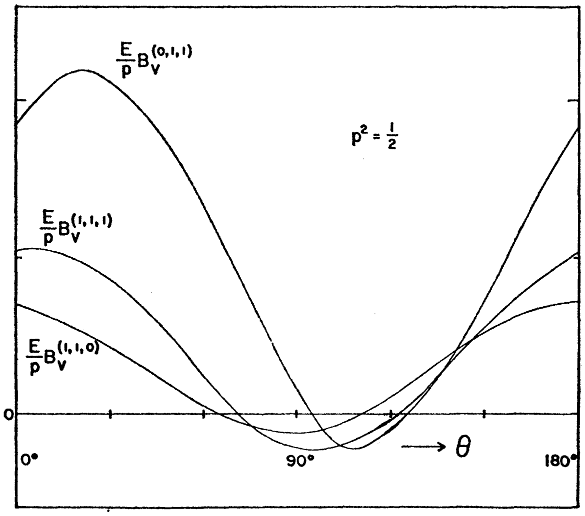

FIG. 10. Magnitudes of Born terms as functions of the mixing angle θ at $p^2=\frac{1}{2}$.

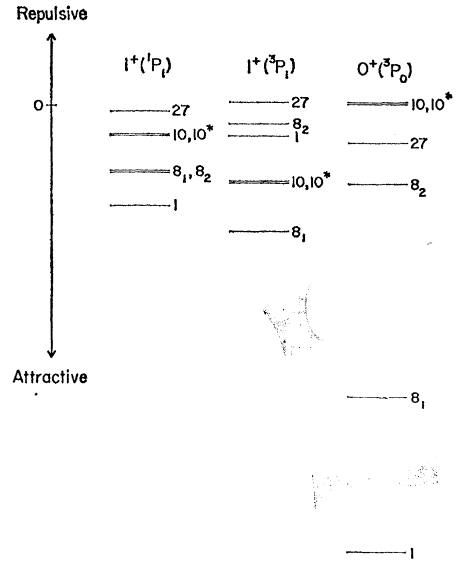

FIG. 11. Relative magnitudes of Born terms, with $\alpha^2/4\pi=5$ or 9, $(\alpha_P+\beta_P)^2/4\pi=15$, $\beta_V^2/4\pi=1$.

where the A_i's are given in Table I and the B_i's are given in the Appendix and plotted in Fig. 9 (for pure vector coupling) and Fig. 10 (for $p^2=\frac{1}{2}$). Using the ratio $\alpha_P=3\beta_P$ and the magnitudes of the coupling constants in (3.1), the relative magnitudes of the Born terms are shown in Fig. 11. The next state is seen to be a unitary singlet scalar meson ($^3P_0$). Since an s-wave 2π state couples with this meson, it is possible that it has a much smaller mass than would be expected from a $^3P_0$ bound state of the $B\bar{B}$ system. A discussion of this meson and octets of scalar mesons will be given elsewhere. Though we have not considered the exchange of this scalar meson between B and $\bar{B}$, its existence causes no difficulty, as its contribution is unitary spin independent and attractive both for $^3S_1$ and $^1S_0$ (slightly stronger for $^1S_0$ than for $^3S_1$).

Two types of axial vector mesons correspond to $^3P_1$ and $^1P_1$. The recently discovered $\pi^+\omega$ resonance[21] corresponds to a component of octet $^1P_1$ bound states if its spin is 1^+.

ACKNOWLEDGMENTS

The author would like to express his gratitude to Professor Murray Gell-Mann for helpful discussions. He is also grateful to Dr. Yuval Ne'eman and Dr. Louis Balazs for discussions and careful reading of the manuscript. A part of this work was done when the author was at the Summer Institute for Theoretical Physics held at the University of Wisconsin. It is a pleasure to thank Professor R. G. Sachs for the hospitality at the institute.

APPENDIX

The Born terms for $B\bar{B}$ scattering in various partial waves are derived as follows. According to Goldberger *et al.*,[22] partial-wave amplitudes in singlet spin states of $B\bar{B}$ systems (u channel), $\bar{f}_0{}^J(u)$, can be written as

$$f_0{}^J(u)=\frac{p_u}{2E_u}\int_{-1}^{1} f_1(u,z_u)P_l(z_u)dz_u\,, \tag{A1}$$

and[23]

$$\bar{f}_1(u,z_u)=E_u{}^2\bar{G}_1(u,s,t)-z_up_u{}^2\bar{G}_2(u,s,t)+m^2G_3(u,s,t)\,, \tag{A2}$$

where[12]

$$\bar{G}_i(u,s,t)=\tfrac{1}{2}\Delta_{ij}(-1)^{i+j}\bar{G}_i(t,s,u)\,, \tag{A3}$$

[21] M. Abolins, R. L. Lander, W. A. Mehlhop, N. Xuong, and P. M. Yager, Phys. Rev. Letters **11**, 381 (1963).
[22] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960).
[23] Here m is the baryon mass.

and[24]

$$\bar{G}_1(t,s,u)=-\frac{A_P}{\pi}\frac{1}{t-\mu_P{}^2}-\frac{A_V}{\pi}\left(\frac{u}{4m^2}-\frac{s}{4m^2}\right)(\sin\theta\cos\theta+\sin^2\theta)\frac{1}{t-\mu_V{}^2},$$
$$\bar{G}_2(t,s,u)=-\frac{A_V}{\pi}\frac{t}{4m^2}(\sin\theta\cos\theta+\sin^2\theta)\frac{1}{t-\mu_V{}^2},$$
$$\bar{G}_3(t,s,u)=-\frac{A_A}{\pi}\frac{1}{t-\mu_A{}^2}, \tag{A4}$$
$$\bar{G}_4(t,s,u)=-\frac{A_V}{\pi}(\cos^2\theta+\cos\theta\sin\theta)\frac{1}{t-\mu_V{}^2},$$
$$\bar{G}_5(t,s,u)=\frac{A_S}{\pi}\frac{1}{t-\mu_S{}^2}+\frac{A_V}{\pi}\left[\frac{s}{4m^2}-\frac{u}{4m^2}\right](\sin\theta\cos\theta)\frac{1}{t-\mu_V{}^2}.$$

Equation (A4) is derived by making use of the fact that the BB scattering amplitude can be written as

$$T=-\pi[P\bar{G}_1(t,s,u)+T\bar{G}_2+A\bar{G}_3+V\bar{G}_4+S\bar{G}_5].$$

Thus we obtain[25]

$$B_P{}^{(0,0)}=-\frac{p}{4E}(Q_0-Q_1),$$
$$B_S{}^{(0,0)}=\frac{1}{4Ep}[(E^2+m^2)Q_0-p^2Q_1],$$
$$B_V{}^{(0,0)}=\frac{1}{4Ep}\left\{\cos^2\theta(4E^2-2m^2)Q_0+\sin\theta\cos\theta\times2p^2(Q_0-Q_1)+\sin^2\theta\frac{p^2}{m^2}\left[(\tfrac{4}{3}p^2+2m^2)Q_0-(2m^2+p^2)Q_1-\frac{p^2}{3}Q_2\right]\right\}, \tag{A5}$$
$$B_A{}^{(0,0)}=\frac{1}{4Ep}(4E^2+2m^2)Q_0\ \text{(axial vector coupling)}.$$

The Born terms for triplet spin states can be obtained in a similar way:

$$B_P{}^{(1,0)}=\frac{p}{12E}(Q_0-Q_1),$$
$$B_P{}^{(1,0)\leftrightarrow(1,2)}=-\frac{\sqrt{2}p}{12E}(\tfrac{1}{3}Q_0+2Q_1-Q_2),$$
$$B_P{}^{(1,2)}=\frac{p}{12E}(Q_1-Q_2),$$
$$B_V{}^{(1,0)}=\frac{1}{18Ep}\left\{\cos^2\theta[(m+2E)^2Q_0+12p^2Q_1+2(E-m)^2Q_2]+\sin\theta\cos\theta\left[-(7m+8E)\frac{p^2}{m}Q_0+15p^2Q_1+8\frac{p^2}{m}(E-m)Q_2\right]+\frac{p^2}{m^2}\sin^2\theta\left[(-m^2+3p^2-2mE)Q_0+\left(\frac{9}{5}m^2-\frac{51}{10}p^2+\frac{6}{5}Em\right)Q_1+(-2m^2+\tfrac{3}{2}p^2+2Em)Q_2+\left(\frac{6}{5}m^2+\frac{3}{5}p^2-\frac{6}{5}Em\right)Q_3\right]\right\},$$

---
[24] Here P, V, A, and S stand for pseudoscalar, vector, axial vector, and scalar meson, respectively. A_i's are (coupling constant)^2.
[25] The arguments of Q_i's are $1+(\mu^2/2p^2)$.

$$B_V^{(1,0)\leftrightarrow(1,2)}=\frac{\sqrt{2}}{18Ep}\left\{\cos^2\theta[(E-m)(2E+m)Q_0-3p^2Q_1+(E-m)^2Q_2]+\sin\theta\cos\theta\left[(2E+m)\frac{p^2}{m}Q_0-6p^2Q_1+\left(3m^2-3mE+5p^2-2p^2\frac{E}{m}\right)Q_2\right]+\sin^2\theta\left[(m^2+\tfrac{3}{4}p^2+\tfrac{1}{2}mE)\frac{p^2}{m^2}Q_0+\left(-\frac{27}{10}m^2+\frac{3}{20}p^2-\frac{3}{10}Em\right)\frac{p^2}{m^2}Q_1+(2m^2-\tfrac{3}{4}p^2-\tfrac{1}{2}mE)\frac{p^2}{m^2}Q_2+\left(-\frac{3}{10}m^2-\frac{3}{20}p^2+\frac{3}{10}mE\right)\frac{p^2}{m^2}Q_3\right]\right\},$$

$$B_V^{(1,2)}=\frac{1}{18Ep}\left\{\cos^2\theta[2(E-m)^2Q_0+15p^2Q_1+(E+2m)^2Q_2]+\frac{p^2}{m^2}\sin\theta\cos\theta[8m(E-m)Q_0+21m^2Q_1+(-13m^2-8mE)Q_2]+\frac{p^2}{m^2}\sin^2\theta\left[\left(-2m^2+\frac{9}{4}p^2+2Em\right)Q_0+\left(\frac{9}{2}m^2-\frac{21}{4}p^2-\frac{6}{5}Em\right)Q_1+\left(-4m^2+\frac{9}{4}p^2-2Em\right)Q_2+\left(\frac{3}{2}m^2+\frac{3}{4}p^2+\frac{6}{5}Em\right)Q_3\right]\right\},$$

$$B_S^{(1,0)}=\frac{1}{12Ep}\left[\left(\frac{5}{3}m^2+\frac{5}{3}E^2+\frac{8}{3}mE\right)Q_0-3p^2Q_1+2(E-m)^2Q_2\right],$$

$$B_A^{(1,0)}=\frac{1}{12Ep}[-\tfrac{2}{3}(m+E)^2Q_0-\tfrac{4}{3}(E-m)^2Q_2]\quad\text{(axial vector coupling)}.$$

The Born terms for p-wave $B\bar{B}$ states, $B^{(J,L,S)}$, are

$$B_V^{(0,1,1)}=\frac{1}{4Ep}\left\{\cos^2\theta[4p^2Q_0+2m^2Q_1]+\frac{p^2}{m^2}\sin\theta\cos\theta[6m^2Q_0-(6m^2+2p^2)Q_1]+\frac{p^2}{3m^2}\sin^2\theta[(4m^2-4p^2)Q_0-(6m^2+3p^2)Q_1+(2m^2+p^2)Q_2]\right\},$$

$$B_P^{(0,1,0)}=\frac{p}{4E}(Q_0-Q_1),$$

$$B_V^{(1,1,1)}=\frac{1}{4Ep}\left\{\cos^2\theta[\tfrac{4}{3}p^2Q_0+2E^2Q_1+\tfrac{2}{3}p^2Q_2]+p^2\sin\theta\cos\theta[\tfrac{4}{3}Q_0-2Q_1+\tfrac{2}{3}Q_2]+\frac{p^2}{m^2}\sin^2\theta\left[(-\tfrac{1}{3}m^2-\tfrac{5}{6}p^2)Q_0+\frac{11}{10}p^2Q_1+(\tfrac{1}{3}m^2-\tfrac{1}{6}p^2)Q_2-\tfrac{1}{10}p^2Q_3\right]\right\},$$

$$B_P^{(1,1,1)}=-\frac{p}{4E}(\tfrac{2}{3}Q_0-Q_1+\tfrac{1}{3}Q_2),$$

$$B_V^{(1,1,0)}=\frac{1}{4Ep}\left\{\cos^2\theta(4E^2-2m^2)Q_1+p^2\sin\theta\cos\theta[-\tfrac{2}{3}Q_0+2Q_1-\tfrac{4}{3}Q_2]+\frac{p^2}{m^2}\sin^2\theta\left[-\tfrac{1}{3}(p^2+2m^2)Q_0+\left(2m^2+\frac{6}{5}p^2\right)Q_1-\tfrac{2}{3}(p^2+2m^2)Q_2-\tfrac{1}{5}p^2Q_3\right]\right\},$$

$$B_P^{(1,1,0)}=-\frac{\mathrm{p}}{4E}(-\tfrac{1}{3}Q_0+Q_1-\tfrac{2}{3}Q_2).$$