# Experimental Consequences of $\phi - \omega$ Mixing

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In this paper a simple model is proposed in which the observed  $\phi$  and  $\omega$  resonant states are considered as mixtures of "pure" states  $|Y\rangle$  and  $|B\rangle$  corresponding to hypercharge and baryonic mesons. The implications of this model for the isoscalar nucleon form factor; the decays of the  $\phi$ ,  $\omega$ , and  $\pi^0$  mesons; the role of the  $\omega$  and  $\phi$  mesons in nuclear forces; the mass distribution of Dalitz pairs in the decay  $\pi^0 \rightarrow \gamma + e^+ + e^-$ ; and the photoproduction of  $\eta$  mesons are briefly considered.

(1)

### I. INTRODUCTION

E shall consider two  $T=0, J=1^-, G=-1$  vector mesons, B and Y. The B meson we shall assume is coupled universally to the conserved baryon current and in the unitary symmetry scheme<sup>1-3</sup> belongs to the singlet representation, while Y is coupled universally to the conserved hypercharge current and is a member of the unitary symmetry octet representation.

The states  $|B\rangle$  and  $|Y\rangle$  are eigenstates of a Hamiltonian  $H_0$  which describes their interactions when unitary symmetry is not violated. We shall suppose here that unitary symmetry is broken by adding a "small" perturbing potential V to  $H_0$ . The complete Hamiltonian  $H = H_0 + V$  then has eigenstates  $|\phi\rangle$  and  $|\omega\rangle$  which can be written as

$$\begin{aligned} |\phi\rangle &= a | Y\rangle + b | B\rangle, \\ |\omega\rangle &= a | B\rangle - b | Y\rangle, \end{aligned}$$

$$a^2 + b^2 = 1$$
, (2)

where the phases are chosen so that a and b are real and positive. We shall identify  $|\omega\rangle$  with the observed<sup>4</sup>  $T=0, J=1^-, G=-1$   $3\pi$  resonance at 780 MeV, and  $|\phi\rangle$  with the observed<sup>5,6</sup> T=0, J=1<sup>-</sup>, G=-1 K\bar{K} resonance at 1020 MeV.

In this note we shall study some experimental consequences of a simple model in which the  $|\phi\rangle$  and  $|\omega\rangle$ states are described as mixtures of pure  $|Y\rangle$  and  $|B\rangle$ states. We retain the universality hypothesis and neglect those violations of unitary symmetry which are not directly implied by Y, B (or  $\phi$ ,  $\omega$ ) mixing.

To determine the mixing parameters, a and b, we write the Hamiltonian H in the form

$$H = H_0 + V = \begin{pmatrix} m_{0Y} + V_{YY} & V_{YB} \\ V_{BY} & m_{0B} + V_{BB} \end{pmatrix}, \quad V_{YB} = V_{BY}. \quad (3)$$

Presumably the Okubo mass formula<sup>7</sup> fails to work well for the vector-meson octet because it takes into account only the diagonal elements of the matrix (3). However, we assume that the diagonal element  $m_{0Y} + V_{YY}$  is the mass predicted for the Y meson by the Okubo relation

$$m_Y \approx m_{0Y} + V_{YY} \approx \left(\frac{4m_{K^*}^2 - m_{\rho}^2}{3}\right)^{1/2} \approx 925 \text{ MeV}.$$

Then the fact that  $|\phi\rangle$  and  $|\omega\rangle$  are eigenstates of H with known eigenvalues  $m_{\phi}$  and  $m_{\omega}$  allows us to determine the mixing parameters a and b. We find:

$$a \approx 0.78$$
 and  $b \approx 0.62$ . (4)

These values are in good agreement with those recently found by several other workers each using somewhat different methods and each concerned with somewhat different consequences of  $\phi - \omega$  mixing from those considered here.8-11

In the introduction and throughout this paper we use freely the vector-meson description of the  $\phi$  and  $\omega$ resonances. This is not to imply that we necessarily take very seriously the usual field theoretic formulations which underlie such a description. In particular, all of the important results obtained or used here can also be obtained as approximations in a purely dispersion theoretic calculation, a point emphasized by Gell-Mann and Zachariasen.<sup>3,12</sup> We regard the vector-meson

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<sup>&</sup>lt;sup>1</sup> M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20, 1961 (unpublished). <sup>2</sup> Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

<sup>&</sup>lt;sup>3</sup> M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

 <sup>&</sup>lt;sup>4</sup> B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters 7, 178 (1961).
 <sup>5</sup> P. Schlein, W. E. Slater, L. T. Smith, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 10, 368 (1963).

<sup>&</sup>lt;sup>6</sup> P. L. Connolly, E. L. Hart, K. W. Lai, G. London, C. G. Moneti *et al.*, Phys. Rev. Letters **10**, 371 (1963).

<sup>&</sup>lt;sup>7</sup> S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).
<sup>8</sup> S. L. Glashow, Phys. Rev. Letters 11, 48 (1963).
<sup>9</sup> J. J. Sakurai, University of Chicago preprint EF INS 63-28 (unpublished).

<sup>&</sup>lt;sup>10</sup> S. Okubo, Phys. Letters **5**, 165 (1963). <sup>11</sup> V. Singh (private communication).

<sup>&</sup>lt;sup>12</sup> M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 965 (1961).

language as a convenient and intuitive way of presenting an approximate dispersion theoretic calculation.

# II. INFLUENCE OF THE $\phi$ AND $\omega$ MESONS ON THE ISOSCALAR CHARGE FORM FACTOR OF THE NUCLEON. APPLICATIONS.

First, we must determine the couplings of the  $\phi$  and  $\omega$  mesons to the photon and to nucleons.

Following Sakurai,<sup>13</sup> we introduce the coupling strength  $f_Y$  of the Y meson to the hypercharge current  $j_Y$  and the coupling strength  $f_B$  of the B meson to the baryon current  $j_B$ , both defined in analogy with the electric charge e. The photon is coupled to the hypercharge current with strength e/2 and the Y meson is coupled to this current with strength  $f_Y$ . Then, according to Gell-Mann and Zachariasen,<sup>12</sup> the coupling of a Y meson with mass  $m_Y$  to a photon is given by  $\gamma_{Y\gamma} = -em_Y^2/2f_Y$ . On the other hand, the B is coupled to the independent baryon current and can therefore not couple directly to the photon. Intuitively, one might think of  $\omega$  as a superposition of Y and B mesons with masses  $m_{\omega}$ , and the  $\phi$  as a superposition of Y and B mesons with masses  $m_{\phi}$ . Then Eq. (1) implies that

$$\gamma_{\phi\gamma} = -\left(aem_{\phi}^2/2f_Y\right), \quad \gamma_{\omega\gamma} = +bem_{\omega}^2/2f_Y. \quad (5)$$

As these relations play an important role in extracting information about coupling constants from processes like  $\pi^0 \rightarrow 2\gamma$ , it may be of interest to note that if the bare mass of the Y meson is zero they can be derived from field theory.<sup>14</sup>

It would seem likely that the general features of Eq. (5) are correct. But whether the masses  $m_{\phi}^2$  and  $m_{\omega}^2$ , rather than some average mass such as  $m_Y^2$ , should appear in these equations seems to us a delicate point, which we shall not try to settle here. Instead we note that a direct experimental test of Eq. (5) is obtained by observing the decays  $\omega \rightarrow e^+ + e^-$  and  $\phi \rightarrow e^+ + e^-$ . Neglecting the mass of the electron and using the couplings of Eq. (5), one finds<sup>16,16</sup>

$$\Gamma(\omega \rightarrow e^+ + e^-) \approx b^2 [\alpha^2/12] [f_Y^2/4\pi]^{-1} m_\omega \approx 0.89 \text{ keV},$$

Then we have:

(a) 
$$\Box^{2}\langle 0 | Y(x) | \phi \rangle = \Box^{2}\langle 0 | Y(0) | \phi \rangle e^{-i(P_{\phi} \cdot x)}$$
$$= -m_{\phi}^{2}\langle 0 | Y(0) | \phi \rangle e^{-i(P_{\phi} \cdot x)}$$

and

(b) 
$$\Box^{2}\langle 0 | Y(x) | \phi \rangle = f_{Y}\langle 0 | j_{Y}(x) | \phi \rangle$$
$$= f_{Y}\langle 0 | j_{Y}(0) | \phi \rangle e^{-i(P\phi \cdot x)}.$$

By definition,  $\gamma_{\phi\gamma} = (e/2)\langle 0 | j_Y(0) | \phi \rangle$  and we also note that  $\langle 0 | Y(0) | \phi \rangle$  is simply the mixing coefficient *a*. Hence, (a) and (b) together give

$$\gamma_{\phi\gamma} = -aem_{\phi^2}/2f_Y.$$

<sup>15</sup> Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 8, 79 (1962). <sup>16</sup> M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962). and

$$\Gamma(\phi \to e^+ + e^-) \approx a^2 [\alpha^2/12] [f_Y^2/4\pi]^{-1} m_\phi \approx 1.8 \text{ keV.} \quad (6)$$

In arriving at the numerical values quoted above, we have taken the coupling<sup>17</sup>  $f_Y \approx (\frac{3}{4})^{1/2} f_{\rho}$  from unitary symmetry, and estimated  $f_{\rho}$  from the measured width  $\Gamma(\rho \rightarrow 2\pi)$ . For a  $\rho$  width of 100 MeV, one finds  $f_{\rho}^2/4\pi \approx 2$  and  $f_X^2/4\pi \approx 1.5$ . An accurate experimental determination of the ratio  $\Gamma(\omega \rightarrow e^+ + e^-)/\Gamma(\phi \rightarrow e^+ + e^-) \approx (b^2/a^2)(m_{\omega}/m_{\phi}) \approx 0.48$  would also provide a critical test of this model and in particular of Eq. (5).

An experimental study of the decays  $\omega \to \pi^+ + \pi^$ and  $\phi \to \pi^+ + \pi^-$  could provide interesting information about the pion form factor  $F_{\pi}(t)$ . In this model the partial widths are

$$\begin{split} &\Gamma(\omega \to \pi^+ + \pi^-) = b^2 [\alpha^2/48] \\ &\times [f_{Y^2}/4\pi]^{-1} m_\omega [1 - (4m_\pi^2/m_\omega^2)]^{3/2} |F_\pi(m_\omega^2)|^2, \end{split}$$

and

$$\begin{split} \Gamma(\phi &\to \pi^+ + \pi^-) = a^2 [\alpha^2/48] \\ &\times [f_Y^2/4\pi]^{-1} m_\phi [1 - (4m_\pi^2/m_\phi^2)]^{3/2} |F_\pi(m_\phi^2)|^2 \end{split}$$

Now let us consider the coupling of the  $\phi$  and  $\omega$  mesons to nucleons. These are given by Eq. (1) as

 $f_{\phi NN} = a f_{YNN} + b f_{BNN}$ ,

and

 $f_{\omega NN} = a f_{BNN} - b f_{YNN}.$ 

In accord with our universality hypothesis we will set  $f_{YNN} \approx f_Y$  and  $f_{BNN} \approx f_B$ .

We suppose that the isoscalar charge form factor of the nucleon is dominated by the  $\phi$  and  $\omega$  resonant states. Then one writes:

$$F_{1S}(t) \approx \left(1 - \frac{f_{YNN}}{f_Y}\right) + \frac{a}{f_Y} \frac{f_{\phi NN} m_{\phi}^2}{m_{\phi}^2 - t} - \frac{b}{f_Y} \frac{f_{\omega NN} m_{\omega}^2}{m_{\omega}^2 - t}.$$
 (9)

Note that since  $F_{1S}(0)=1$ , complete  $\phi-\omega$  dominance would require that  $f_{YNN}=f_Y$ , in close analogy with the case of no mixing.

Since there is only one free parameter in (9) one could, in principle, determine  $f_{\phi NN}$  and  $f_{\omega NN}$  by fitting the experimental form factors. However, given the present uncertainty in the experimental data and the theoretical uncertainty as to the effect of the higher mass states on Eq. (9), we have concluded that it would only be misleading to attempt to use the form factors to estimate  $f_{\phi NN}$  and  $f_{\omega NN}$ .

On the other hand, some rather definite qualitative statements can be made. At low t,  $F_{1S}$  appears to fall off faster than either the  $\phi$  or  $\omega$  terms separately.<sup>18</sup> This

(8)

(7)

<sup>&</sup>lt;sup>18</sup> J. J. Sakurai, Ann. Phys. (N. Y.) 11, 1 (1960).

<sup>&</sup>lt;sup>14</sup> The derivation is as follows. Let Y'(x) be the renormalized Heisenberg field operator of a Y meson of bare mass zero. It satisfies the field equation  $\Box^2 Y(x) = f_Y j_Y(x).$ 

 $<sup>^{17}</sup>$  Our coupling is  $f_{\rho}{=}2\gamma_{\rho};\,\gamma_{\rho}$  being the coupling of Gell-Mann et al. (see Refs. 3, 12, and 16).

<sup>&</sup>lt;sup>18</sup> L.<sup>4</sup>Hand, D. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).

more rapid decrease can only be obtained (i), if  $f_{\omega NN}$ and  $f_{\phi NN}$  are of the same sign and (ii), if  $|f_{\omega NN}|$  is considerably larger than  $|f_{\phi NN}|$ . Preliminary data<sup>19</sup> from the Cambridge Electron Accelerator indicates that a fit to  $F_{1 \text{ proton}}$  can be obtained with expressions of the form  $m_{\omega}^2/(m_{\omega}^2-t)$  or  $m_{\rho}^2/(m_{\rho}^2-t)$ , for t ranging from  $\approx 1$  to 5 (BeV)<sup>2</sup>. This would seem to indicate that the neutron form factor is negligible compared to the proton form factor even at high-momentum transfers and that the isovector and isoscalar charge form factors are, to a fair approximation, dominated by the  $\rho$  and  $\omega$ poles. In particular, the high-energy data suggest that  $F_{1S}$  is at most 40%  $\phi$ .

Moreover, if one assumes that  $F_{1 neutron}$  is always small compared to  $F_{1 proton}$ , the experimental data indicate that

$$-1 \leq (a/f_Y) f_{\phi NN} \leq 0, \quad -2 \leq (b/f_Y) f_{\omega NN} \leq -1.$$
 (10)

If as before we take  $f_Y^2/4\pi \sim 1.5$ , then we find

$$0 \leq f_{\phi NN^2}/4\pi \leq 2.5, \quad 4 \leq f_{\omega NN^2}/4\pi \leq 16.$$
 (11)

Sakurai<sup>13</sup> has pointed out that an isoscalar vector meson could account for the hard core and spin-orbit interactions which are essential features of the nuclear forces. Judging from Eqs. (10) or (11) it seems that the  $\omega$  meson must be a major contributor to the nuclear forces while the  $\phi$  meson with its higher mass and weaker coupling should be less important.

A number of authors<sup>20–22</sup> have attempted to determine the vector meson-nucleon couplings from nucleonnucleon scattering data. The details of the results vary considerably, but all the fits, whether they are to the low-energy phase shifts,20 the phenomenological potentials,<sup>21</sup> or the high-energy pp cross sections,<sup>22</sup> seem to require that  $f_{\omega NN^2}/4\pi$  be considerably larger than  $f_{\rho NN}^2/4\pi$ , in agreement with Eqs. (10) and (11).

Arnold and Sakurai<sup>23</sup> have pointed out that a vector meson coupled to the hypercharge current, which has the property that the product of its couplings to nucleons and kaons is  $f_{\omega KK} f_{\omega NN}/4\pi \sim 5$  could explain in a rough way the low-energy Kp scattering data including the  $Y_0^*$ . It is interesting to observe that Eqs. (1), (4), and (11) and the assumed universality of the Y-meson couplings give  $1.5 \leq f_{\omega KK} f_{\omega NN}/4\pi \leq 3.0$ , which may perhaps be considered reasonably good agreement in view of the crude approximations involved.

## III. DECAY RATES OF THE $\phi$ , $\omega$ , AND 70 MESONS

In this section we shall try to give a coherent description of the decays of the  $\phi$ ,  $\omega$  and  $\pi^0$  mesons on the basis of the present model.

The decay  $\phi \rightarrow K\bar{K}$  will proceed through the coupling of the Y-component of the  $\phi$  to the hypercharge carried by the K mesons. The width is obtained simply by multiplying Sakurai's result<sup>24</sup> by  $a^2$ , which gives

$$\Gamma(\phi \to K\bar{K}; \text{ both charge modes}) \approx 2.1 \text{ MeV.}$$
 (12)

Since the experimental value<sup>25</sup> of the branching ratio  $\Gamma(\phi \to \rho + \pi) / \Gamma(\phi \to K\bar{K})$  now appears to be  $\approx 0.10$  $\pm 0.10$ , this estimate for  $\Gamma(\phi \rightarrow K\bar{K})$  indicates a total  $\phi$  width of  $\approx 2.3 \pm 0.2$  MeV, which is comparable with the experimental width<sup>6,25,26</sup> of  $\approx 3.1 \pm 0.8$  MeV.

The  $\omega$  width has recently been measured<sup>27</sup> to be  $9.5 \pm 2.1$  MeV. For an experimental branching ratio<sup>27</sup>  $\Gamma(\omega \rightarrow \text{neutrals})/\Gamma(\omega \rightarrow 3\pi) \approx 12\%$  one finds  $\Gamma(\omega \rightarrow 3\pi)$  $\approx 8.5 \pm 1.9$  MeV. With the assumption that this decay is dominated<sup>16</sup> by  $\omega \rightarrow \rho + \pi \rightarrow 3\pi$ , one finds<sup>28</sup>

$$f_{\omega\rho\pi}^2/4\pi \approx (0.41 \pm 0.09)/m_{\pi}^2.$$
 (13)

The width  $\Gamma(\phi \rightarrow \rho + \pi)$  and the branching ratio  $\Gamma(\phi \rightarrow \rho + \pi) / \Gamma(\phi \rightarrow K\bar{K})$  can be predicted once  $f_{\phi\rho\pi}^2/4\pi$ is known. Lacking this information, we may note instead that the present experimental value of the branching ratio  $\Gamma(\phi \rightarrow \rho + \pi) / \Gamma(\phi \rightarrow K\bar{K})$  ( $\approx 0.10 \pm 0.10$ ), our estimate for  $\Gamma(\phi \to K\bar{K})$  and Eq. (13) suggest that  $(f_{\omega\rho\pi^2}/4\pi)/(f_{\phi\rho\pi^2}/4\pi)\approx 850$ . On the other hand, a preliminary analysis<sup>29</sup> of  $\phi$  and  $\omega$  production in the reaction  $\pi + N \rightarrow N + 4\pi$  indicates that  $(f_{\omega \rho \pi^2}/4\pi)/$  $(f_{\phi\rho\pi^2}/4\pi) \approx 100$ . This corresponds to a branching ratio  $\Gamma(\phi \rightarrow \rho + \pi) / \Gamma(\phi \rightarrow K\bar{K}) \approx 0.8 \pm 0.2$ , which is <1 and in somewhat better agreement with the older determinations<sup>6</sup> of  $\Gamma(\phi \rightarrow \rho + \pi) / \Gamma(\phi \rightarrow K\bar{K})$  which gave  $0.35 \pm 0.20$ . Further experimental results bearing on the ratio  $f_{\omega\rho\pi^2}/f_{\phi\rho\pi^2}$  would clearly be of high interest.

Assuming that the decay  $\pi^0 \rightarrow 2\gamma$  is dominated by  $\pi^0 \rightarrow \rho + \phi \rightarrow 2\gamma$  plus  $\pi^0 \rightarrow \omega + \rho \rightarrow 2\gamma$ , we find for a coupling  $f_{\omega e \pi^2}/4\pi \approx (0.41 \pm 0.09)/m_{\pi^2}$ 

$$\Gamma(\pi^0 \to 2\gamma) \approx [1 - (a f_{\phi \rho \pi}/b f_{\omega \rho \pi})]^2 (24.8 \pm 5.3 \text{ eV}).$$
 (14)

If  $f_{\phi\rho\pi}/f_{\omega\rho\pi} \approx 1/10$ , Eq. (14) predicts  $\Gamma(\pi^0 \rightarrow 2\gamma) \approx 19 \pm 4.0$  eV, a value which is somewhat greater than the latest experimental value<sup>30</sup> of  $\approx 6.3 \pm 1.0$  eV. For a smaller or negative value of  $f_{\phi\rho\pi}/f_{\omega\rho\pi}$ , the discrepancy would be larger. Thus, for  $f_{\phi\rho\pi}/f_{\omega\rho\pi} \approx 1/(850)^{1/2}$ , the predicted width would be  $\approx 22.6 \pm 4.9$  eV. Considering the crudeness of the calculations and the uncertainty in the data, a factor of 3 or so difference between theory and experiment is perhaps not alarming. For example,

- - At an  $\omega \rho \pi$  vertex we write  $f_{\omega \rho \pi} \epsilon_{\mu \nu \sigma \tau} k_{\mu}^{\omega} \epsilon_{\nu}^{\omega} k_{\sigma}^{\rho} \epsilon_{\tau}^{\rho}$ .
- <sup>29</sup> N. Xuong (private communication).
   <sup>30</sup> G. Von Dardel, D. Dekkers, R. Mermod, J. Van Putten, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 4, 51 (1963).

<sup>&</sup>lt;sup>19</sup> Private communication from N. F. Ramsey to M. Gell-Mann. We wish to thank Professor Gell-Mann for bringing these results to our attention.

 <sup>&</sup>lt;sup>20</sup> R. S. McKean, Phys. Rev. 125, 1399 (1962).
 <sup>21</sup> D. Amati, E. Leader, and B. Vitale, Phys. Rev. 130, 750 (1963).

 <sup>&</sup>lt;sup>22</sup> Riazuddin and Fayyazuddin, Phys. Rev. **132**, 873 (1963).
 <sup>23</sup> R. Arnold and J. J. Sakurai, Phys. Rev. **128**, 2808 (1963).

J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962).
 P. L. Connolly, E. L. Hart, K. W. Lai, G. London, G. C. <sup>26</sup> N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz et al., Phys. Rev. Letters 11, 438 (1963).
<sup>27</sup> N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz et al., Phys. Rev. Letters 11, 436 (1963).
<sup>28</sup> A. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz et al., Phys. Rev. Letters 11, 436 (1963).

the  $\pi^0$  width is proportional to<sup>16</sup>  $f_{\rho}^{-4}$ , assuming  $f_Y = (\frac{3}{4})^{1/2} f_{\rho}$ . If we took the  $\rho$ -width to be 120 MeV instead of 100 MeV, then our prediction for the width would be reduced by a factor of 1.5.

Previous calculations<sup>12,16</sup> of the branching ratio  $\Gamma(\pi^0 \rightarrow 2\gamma)/\Gamma(\omega \rightarrow \pi^0 + \gamma)$  can easily be adapted to include the  $\phi$  meson. One finds

$$\frac{\Gamma(\pi^{0} \to 2\gamma)}{\Gamma(\omega \to \pi^{0} + \gamma)} = (1.7 \times 10^{-5}) \left[ 1 - \frac{a}{b} \frac{f_{\phi\rho\pi}}{f_{\omega\rho\pi}} \right]^{2},$$
  

$$\approx 1.3 \times 10^{-5} \text{ if } f_{\phi\rho\pi}^{2} / f_{\omega\rho\pi}^{2} = 1/100,$$
  

$$\approx 1.6 \times 10^{-5} \text{ if } f_{\phi\rho\pi}^{2} / f_{\omega\rho\pi}^{2} = 1/850. \quad (15)$$

The experimental  $\pi^0$  width, the experimental  $\omega$  width and the experimental branching ratio  $\Gamma(\omega \rightarrow \text{neutrals})/\Gamma(\omega \rightarrow 3\pi)$  of  $\approx 12\%$  suggest  $\Gamma(\pi^0 \rightarrow 2\gamma)/\Gamma(\omega \rightarrow \pi^0 + \gamma) \approx 0.63 \times 10^{-5}$ .

The distribution in mass of the Dalitz pairs in the decay  $\pi^0 \rightarrow \gamma + e^+ + e^-$  is of considerable interest in connection with models of  $\pi^0$  decay of the type discussed here. Applying the present model to this decay gives the form factor

$$\Gamma(t) = \frac{1}{2} \left\{ \frac{m_{\rho}^{2}}{m_{\rho}^{2} - t} + [f_{Y_{\rho\pi}}]^{-1} \times \left[ a f_{\phi\rho\pi} \frac{m_{\phi}^{2}}{m_{\phi}^{2} - t} - b f_{\omega\rho\pi} \frac{m_{\omega}^{2}}{m_{\omega}^{2} - t} \right] \right\}.$$
 (16)

Neglecting  $f_{\phi\rho\pi}$  compared to  $f_{\omega\rho\pi}$  in the derivative one finds  $[d\Gamma(t)/dt]_{t=0} \approx +0.03/m_{\pi}^2$ . Experiments indicate<sup>\$1,32</sup> a negative value for  $\Gamma'(0)$ , but do not rule out a small positive value.<sup>\$3</sup> It is of interest to check this point experimentally, because if it is definitely established that  $\Gamma'(0)$  is negative, it would seem very difficult indeed to escape the conclusion that the  $\phi$ ,  $\omega$  and  $\rho$ resonances do *not* dominate the form factor entering in  $\pi^0$  decay.

- <sup>32</sup> H. Kobrak, Nuovo Cimento 20, 1115 (1961).
- <sup>33</sup> H. Kobrak (private communication).

#### IV. APPLICATIONS TO PHOTOPRODUCTION OF $\pi^0$ AND $\eta$ MESONS

We shall estimate some coupling constants which may be of interest in photoproduction. Following the usual "pole dominance" method we set  $f_{\omega\pi\gamma} \approx e f_{\omega\rho\pi}/f_{\rho}$ ,  $f_{\phi\pi\gamma} \approx e f_{\phi\rho\pi}/f_{\rho}$ , and  $f_{\rho\pi\gamma} \approx e f_{Y\rho\pi}/2f_Y = e(af_{\phi\rho\pi} - bf_{\omega\rho\pi})/2f_Y$ . Since  $|f_{\omega\rho\pi}|^2 \gg |f_{\phi\rho\pi}|^2$ , one can probably neglect  $\phi$  exchange in analyzing  $\pi^0$  photoproduction. We have also seen that  $f_{\omega NN^2}$  is probably large compared to  $f_{\rho NN^2}$  and at any rate  $f_{\rho\pi\gamma}^2/f_{\omega\rho\gamma}^2 \approx 0.12$ , so the major vector-meson effects in  $\pi^0$  photoproduction should come from  $\omega$  exchange. The analogous couplings in the case of  $\eta$ -meson production can be obtained from unitary symmetry and we find [using  $f_{\phi\rho\pi}/f_{\omega\rho\pi} \approx 1/(850)^{1/2}$ ] that

$$\frac{f_{\omega\eta\gamma}}{f_{\omega\eta\gamma}} \approx \left(\frac{f_{\rho}}{2f_Y}\right) \left[a^2 - \frac{b^2}{2} + \frac{3}{2}ab\frac{f_{\phi\rho\pi}}{f_{\omega\rho\pi}}\right] \approx 0.26,$$
$$\frac{f_{\phi\eta\gamma}}{f_{\omega\pi\gamma}} \approx \left(\frac{f_{\rho}}{2f_Y}\right) \left[\frac{3}{2}ab + \left(b^2 - \frac{a^2}{2}\right)\frac{f_{\phi\rho\pi}}{f_{\omega\rho\pi}}\right] \approx 0.41,$$

and

$$\frac{f_{\rho\eta\gamma}}{f_{\omega\eta\gamma}} \approx \frac{1}{2} \left[ a \frac{f_{\phi\rho\pi}}{f_{\omega\rho\pi}} - b \right] \approx -0.30.$$
(17)

The  $\phi$  pole is considerably further from the physical region than the  $\omega$  pole and we expect the  $\rho$  to be less strongly coupled to nucleons than the  $\omega$ . These facts, plus the results of Eq. (17), mean that the vector-meson effects which appear to be present in  $\pi^0$  photoproduction<sup>34</sup> at high energies will be considerably suppressed in  $\eta$  photoproduction.

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