It is of interest to note that the $T=1$ $K^+\bar{K}^0$ effective masses from the final states $K^+p\bar{K}^0$, $K^+\pi^+\bar{K}^0n$, and $K^+p\bar{K}^0\pi^+$ showed no enhancement.

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Dynamical Model of the 1535-MeV Ξ^* Hyperon*

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An attempt is made to understand the dynamical origin of the 1535-MeV Ξ^* hyperon. We study the problem of scattering in the $J=\frac{3}{2}^+$-state of the $\bar{K}\Lambda$-channel by the N/D method. The influence of all other channels are ignored. The dynamical singularities of the partial-wave amplitude are assumed to arise mainly from the near by cut (due to the nucleon exchange in the crossed channel) and the far left-hand cut $(-\infty<S\leqslant 0)$. The contribution of the former is evaluated explicitly in terms of the ΛNK coupling constant and that of the latter, by the method of Balázs, through the introduction of the effective range pole terms. The analysis is found to be quite insensitive to the choice of the ΛNK coupling constant. The D function is found to have the desired behavior for the occurrence of a resonance or bound state in the energy region of interest. The main force for the existence of the resonance or bound state seems to arise from the singularities associated with the far left-hand cut. The best set of self-consistent solutions for the position and residue indicates the presence of a bound state in the $J=\frac{3}{2}^+$ state of $\bar{K}\Lambda$ system at $S_R\approx 109\, m_\pi^2$ with residue $K_{\Xi^*}\approx 10$.

I. INTRODUCTION

THE discovery of the Ξ^* hyperon[1] with $S=-2$ and $I=\frac{1}{2}$ at 1535 MeV fits beautifully with the scheme of SU(3) symmetry.[2] In order that it may be identified as a member of the tenfold representation,[3] to which the (3,3) πN resonance belongs, its spin and parity should be $\frac{3}{2}^+$, which also seems to be true from the recent UCLA experiment.[4] The present paper is an attempt to account dynamically for the existence of Ξ^*. In the present paper, we will confine our attention to the $J=\frac{3}{2}^+$ state only and examine whether in this state one should expect a resonance or bound state (depending upon which channel one is considering) at a mass around 1535 MeV with $I=\frac{1}{2}$, $S=-2$, and $Y=-1$. The criterion used to determine whether or not such a resonance or bound state is expected and if so with what mass and residue, is the vanashing of the D function together with approximate self-consistency between the input and output values of the position and residue of the resonance or bound state in question. It is the same criterion used recently in the dynamical explanations of the ρ meson,[5,6] the (3,3) N^* etc.[7,8]

In each one of the above problems, including the present one, some of the main difficulties from the point of view of practical calculations are:

(1) Inadequate knowledge of the far left-hand cut contribution to the N function.

(2) Inadequacy of the knowledge of the ratio of the total to the elastic partial-wave cross section (the so called R_l function) at higher energies, which through the unitarity condition is material for the evaluation of the D function.

(3) Presence of many channels of strongly interacting particles.

As regards the first difficulty, Balázs[6] introduced a trick by which one can approximately replace the far left-hand cut contribution to the N function, by a few

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[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962). Y. Neeman, Nucl. Phys. **26**, 222 (1961).

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[4] P. E. Schlein *et al.*, Phys. Rev. Letters **11**, 167 (1963).

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[6] L. A. P. Balázs, Phys. Rev. **126**, 1220 (1962).

[7] V. Singh and B. M. Udgaonkar, Phys. Rev. **130**, 1117 (1963).

[8] E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963); J. S. Ball and D. Y. Wong (to be published).

effective range pole terms, the positions of which are roughly determined by inspecting the behavior of the kernel of the N function, while the residues are determined by the use of the fixed energy dispersion relation. In spite of the approximate nature of such a procedure, there exist at least a prescription within this scheme to choose the positions and the residues of the effective range poles. Such a procedure has also been successfully applied to the problem of $\pi\pi$ scattering,[6] the isovector part[9] of the electromagnetic structure of the nucleon and the (3,3) N^* resonance[7] etc. We will, therefore, adopt this method[10] to evaluate the far left-hand cut contribution to the N function.

As regards the second difficulty, we will only mention that, since the D-function integral is highly convergent (the integrand of the D function behaves as $S^{-5/2}$ for large S), the bulk of the contribution to the integral is expected to come from the nearby region to the physical threshold. Over this region, the total cross section is hopefully well approximated by the elastic cross section, so that it may be reasonable to neglect the effect of the inelastic processes in the evaluation of the D function. In the present work, we will adopt this elastic approximation, i.e., we will put $R_l=1$. The effect of inelastic processes[11] may be considered in later work along these lines.

As regards the third difficulty, one is forced to consider only as few channels as possible, partly because of simplicity, and partly because the merit of the theory would be lost, if one has to introduce too many unknown parameters (like coupling constants) into the theory. Firstly, of course, one considers only the two-particle channels, hopefully, since the lack of phase space in three or multiparticle systems is expected to diminish their effects. The same argument applies against considering two-particle channels with very high thresholds as compared to those with lower thresholds, especially if one is examining the presence of resonant or bound states nearer the lower threshold.

In the N^* problem, therefore, one considers only the πN channel, since $K\Lambda$ and $K\Sigma$ channels are relatively far away. In the present problem, one may, hopefully, expect the most important channels to be (I) $\pi\Xi$ (threshold$\approx$1455 MeV), (II) $\bar{K}\Lambda$ (threshold$\approx$1610 MeV), and (III) $\bar{K}\Sigma$ (threshold$\approx$1690 MeV), omitting still higher mass systems like $\eta\Xi$ etc.

[9] V. Singh and B. M. Udgaonkar, Phys. Rev. **128**, 1820 (1962).

[10] It should be stressed that Zemach and Zachariasen (Ref. 5) and Abers and Zemach (Ref. 8) do not follow the Balázs method. They encounter divergent integrals, for which they introduce cutoffs, which are kept as arbitrary parameters in the theory. Even though they find that some of the results are not too sensitive to the choice of the cutoff, there is really no direct guiding principle to choose the cutoff in such a procedure.

[11] In a recent work (to be published) Balázs has considered the effect of inelastic processes on P wave $\pi\pi$ scattering by utilizing the idea that high-energy contribution is the s channel can be approximated by Regge poles in the t and u channels. He finds that this improves the result on the position and the width of the ρ meson. However, the change, especially in the position, due to the inclusion of the inelastic effects, is not too drastic. (It is found to be less than 16%.)

As regards the $\pi\Xi$ channel, it is well known that a Chew-Low type theory applied to this case with Ξ exchange in the crossed u channel gives rise to repulsion in the $I=\frac{1}{2}$ state of the direct channel. Secondly, the decay width of Ξ^* to $(\pi\Xi)$ system is found to be (7 ± 2) MeV.[4] This is a rather small width as compared to a width of about 100 MeV for the (3,3) N^* resonance (even taking account of the difference due to kinematic factors[12] in the two cases). This means that Ξ^* is rather weakly coupled to the $\pi\Xi$ channel. These two facts together suggest that the main part of the attraction, which is responsible for the formation of the observed Ξ^*, probably does not arise from the $\pi\Xi$ channel. It decays to the $\pi\Xi$ system, since that is the only open channel it can decay into. As regards the $\bar{K}\Sigma$ channel, here again, the nucleon exchange in the crossed u channel, leads to repulsion in the $I=\frac{1}{2}$ state of the direct channel. So, by the same token one may expect that the driving force for the existence of Ξ^* does not owe its origin to the $\bar{K}\Sigma$ channel either.[13] Without any further apology, we will therefore omit, as a first approximation, both the $\pi\Xi$ and the $\bar{K}\Sigma$ channels and confine our attention to the isolated model of scattering in $\bar{K}\Lambda$ channel only, uninfluenced by the presence of any other channel. As a first remark, let us note that in this case nucleon exchange in the crossed u channel does give rise to attraction.

In Sec. II we discuss the kinematics and the singularities of the $J=\frac{3}{2}^+$ partial-wave amplitude. In Sec. III we introduce the N/D equation for the partial-wave amplitude; the N function is assumed to receive its dominant contribution from the nearby cut (due to nucleon exchange) and the far left-hand cut $(-\infty<S\leqslant0)$; the former is denoted by $N_{(N)}(S)$ and the latter by $N_{(L)}(S)$. $N_{(N)}(S)$ is evaluated explicitly in terms of the ΛNK coupling constant, while $N_{(L)}(S)$ is approximated by the effective range pole-terms. The positions of the effective range poles are chosen by inspecting the behavior of the kernel of the N function, while the residues (called b_3 and b_4) are treated as unknown parameters. In Sec. IV we write down the partial-wave amplitude given by the fixed energy dispersion relation and in Sec. V we determine the unknown residues b_3 and b_4 through the use of the fixed energy dispersion relation. The D-function, which can then be evaluated exhibits desired behavior for the existence of a resonance or bound state in the system under consideration. The best set of self-consistent values for the position and residue is found to be $S_R\approx109m_\pi^2$ and $K_{\Xi^*}\approx10$, respectively. In Sec. VI we list a few possible improvements on the present calculation. In Appendix A we demonstrate the unimportance of ω and ϕ contributions to the fixed energy dispersion relation which was assumed in Sec. IV, and in Appendix

[12] The kinematic factors favor the N^* width over Ξ^* width by less than a factor of 4.

[13] In each one of these cases, we are assuming that the baryon exchange in the crossed channel primarily determines whether or not the channel is attractive, if one were to consider scattering in this channel as an isolated problem. The influence of other channels is, of course, expected to alter the situation.

B we discuss the results for a matching procedure, which is different from that introduced in Sec. V.

II. KINEMATICS AND SINGULARITIES OF THE PARTIAL-WAVE AMPLITUDE

The kinematical considerations for baryon-meson scattering has been developed extensively by many authors. Following the notations of Frautschi and Walecka,[14] we denote the four momenta of the incoming Λ and $\bar{K}$ by p_1 and q_1, and those of the outgoing Λ and $\bar{K}$ by p_2 and q_2, respectively. The Lorentz scalar T matrix defined as usual (with $\bar{u}u=1$) has the general structure

$$T=-A(S,t,u)+\tfrac{1}{2}(\boldsymbol{q}_1+\boldsymbol{q}_2)B(S,t,u)\,, \tag{1}$$

where A and B are the invariant amplitudes and S, t, u are the Mandelstam variables given by

$$\begin{aligned} S&=(p_1+q_1)^2=W^2\,,\\ t&=(q_1-q_2)^2=-2q^2(1-\cos\theta)\,,\\ u&=(p_2-q_1)^2=2(\Lambda^2+K^2)-W^2+2q^2(1-\cos\theta)\,, \end{aligned} \tag{2}$$

where W, q, and θ denote the total energy, the three-momentum and the scattering angle, respectively, all measured in the center-of-mass system. We have

$$q^2=\frac{\{S-(\Lambda+K)^2\}\{S-(\Lambda-K)^2\}}{4S}\,. \tag{3}$$

Following Frautschi and Walecka,[14] we choose the partial-wave amplitude to be

$$\begin{aligned} g_{1^+}&=W^2e^{i\delta_{1^+}}\sin\delta_{1^+}/q^3\\ &=\frac{1}{32\pi q^2}[\{(W+\Lambda)^2-K^2\}\{A_1+(W-\Lambda)B_1\}\\ &\qquad+\{(W-\Lambda)^2-K^2\}\{-A_2+(W+\Lambda)B_2\}]\,, \end{aligned} \tag{4}$$

where

$$(A_l,B_l)\equiv\int_{-1}^{+1}(A(S,t,u),B(S,t,u))P_l(\cos\theta)d(\cos\theta)\,. \tag{5}$$

The singularities of the partial-wave amplitude (shown in Fig. 1) arise as follows:

(i) *u-channel singularities.* The lowest mass intermediate state in the u channel is the nucleon, which gives rise to (as $\cos\theta$ varies from -1 to $+1$) two branch cuts along the real axis in the S plane.

$$-\infty<S\leqslant0$$

and

$$\begin{aligned} L_1=(\Lambda^2-K^2)^2/N^2\simeq58.2m_\pi^2\leqslant S\leqslant L_2\\ =2(\Lambda^2+K^2)-N^2\simeq107.5m_\pi^2\,. \end{aligned}$$

Higher mass exchanges in the u channel give rise to continuous cuts further to the left. For example, πN exchange in u channel gives a branch cut along the real axis

$$-\infty<S\leqslant2(\Lambda^2+K^2)-(N+\pi)^2$$

(these cuts are not shown in Fig. 1).

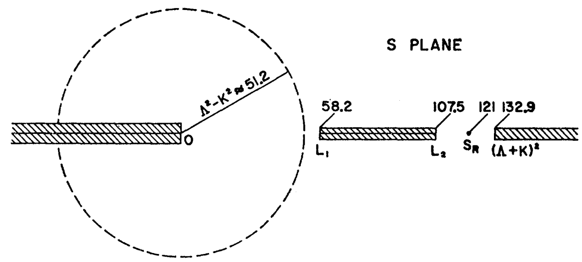

FIG. 1. Singularities of the partial-wave amplitude $g_{1^+}(S)$ in the S plane. The figures are given in units of m_π^2.

(ii) *t-channel singularities.* The lowest mass intermediate state in the t channel is the two-pion system, which gives rise to three branch cuts:

(a) A circular branch cut with radius $|S|=\Lambda^2-K^2$ and center at the origin.

(b) A cut on the real axis (not shown in Fig. 1):

$$\begin{aligned} \Lambda^2+K^2-2\pi^2-[(\Lambda^2-\pi^2)(K^2-\pi^2)]^{1/2}\leqslant S\\ \leqslant\Lambda^2+K^2-2\pi^2+[(\Lambda^2-\pi^2)(K^2-\pi^2)]^{1/2} \end{aligned}$$

and

(c) a cut from $-\infty<S\leqslant0$.

Higher mass states like ω or ϕ exchange (ρ exchange is forbidden in the present problem by isospin conservation) contribute to the discontinuities along the circular cut ($|S|=\Lambda^2-K^2$) and the cut from $-\infty<S\leqslant0$.

(iii) *S-channel singularities.* The S-channel intermediate states give rise to the usual physical cut along the real axis for $S\geqslant(\Lambda+K)^2$, as well as poles below the physical threshold, corresponding to presence of bound states. The only possibly known bound state in the $\bar{K}\Lambda$ system with $J=\frac{3}{2}^+$ is Ξ^*, which is located at

$$S=S_R\simeq121m_\pi^2\,.$$

III. *N/D* EQUATION

Given the singularities of the partial-wave amplitude, discussed in the previous section, one can write it as

$$g_{1^+}=ND^{-1}\,, \tag{6}$$

where the N function, as usual, has the left-hand unphysical cut and the D function, has only the right-hand physical cut.

As in the case of the dynamical explanation[7] of the (3,3) N^*, we will *assume*[15] that the most important

[14] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).

[15] In doing so, we are undoubtedly dropping the contributions from some of the other nearby singularities. For example, we are ignoring the contribution from the circular branch cut arising from the exchange of two pions, ω or ϕ mesons etc., in the t channel. The inclusion of such singularities would, of course, have introduced undesirably too many unknown parameters into the theory. Their omission may, however, be justified *a posteriori*, since one finds at the end of the calculation that the N function receives its dominant

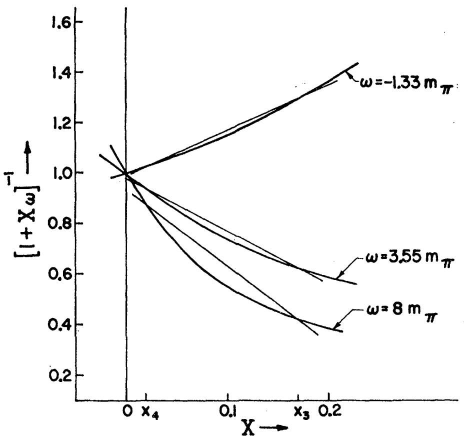

FIG. 2. Balázs—curves for the kernel of $N_{(L)}(S)$.

contribution to the N function comes from: (a) The discontinuity across the far left-hand cut (extending from $-\infty < S \leqslant 0$) which arises due to a variety of particles (N, ω, ϕ etc.) exchanged in both u and t channels, and (b), the nearby branch cut from $L_1 \leqslant S \leqslant L_2$, which arises solely due to the nucleon exchange in the u channel. Thus we have[16]

$$N(S)=N_{(L)}(S)+N_{(N)}(S)\,, \tag{7}$$

where

$$N_{(L)}(S)=\frac{1}{\pi}\int_{-\infty}^{0}\frac{\{\mathrm{Im}g_{1^+}(S')\}D(S')}{S'-S}dS'\,, \tag{8}$$

$$N_{(N)}(S)=\frac{1}{\pi}\int_{L_1}^{L_2}\frac{\{\mathrm{Im}g_{1^+}^{(N)}(S')\}D(S')}{S'-S}dS'\,. \tag{9}$$

$g_{1^+}^{(N)}$ is the contribution to the partial-wave amplitude g_{1^+} due to the nucleon exchange in the crossed u channel and is given by

$$\begin{aligned}g_{1^+}^{(N)}=&(g_{\Lambda NK}^2/32\pi q^4)\\&\times[\{(W+\Lambda)^2-K^2\}(W+N-2\Lambda)Q_1(a)\\&+\{(W-\Lambda)^2-K^2\}(W+2\Lambda-N)Q_2(a)]\,,\end{aligned} \tag{10}$$

where

$$a=\{2(\Lambda^2+K^2)-W^2-N^2\}/2q^2+1\,. \tag{11}$$

$Q_l(x)$ denotes the Legendre function of the second kind and $g_{\Lambda NK}$ stands for the ΛNK coupling constant.[17] From Eq. (10)

$$\begin{aligned}\mathrm{Im}g_{1^+}^{(N)}=&-(g_{\Lambda NK}^2/64q^4)\\&\times[\{(W+\Lambda)^2-K^2\}(W+N-2\Lambda)P_1(a)\\&+\{(W-\Lambda)^2-K^2\}(W+2\Lambda-N)P_2(a)]\,.\end{aligned} \tag{12}$$

We will write a once subtracted dispersion relation for the D function, normalizing it to unity at the subtraction point S_0. Thus, we have

$$\begin{aligned}D(S)&=1+\frac{S-S_0}{\pi}\int_{(\Lambda+K)^2}^{\infty}\frac{\{\mathrm{Im}g_{1^+}^{-1}(S')\}N(S')}{(S'-S)(S'-S_0)}dS'\\&=1-\frac{S-S_0}{\pi}\int_{(\Lambda+K)^2}^{\infty}\frac{(q'^3/S')N(S')}{(S'-S)(S'-S_0)}dS'\,,\end{aligned} \tag{13}$$

where we have used the unitarity condition in the elastic approximation to write the last step of Eq. (13).

Effective Range Pole Approximation for $N_{(L)}(S)$

Following the method of Balázs,[6] let us change the variable by putting

$$\begin{aligned}S&=\Lambda^2+K^2+2\Lambda\omega\,,\\ \omega&=-1/x\,,\end{aligned} \tag{14}$$

so that by Eq. (8), we may write

$$N_{(L)}(S)=\frac{1}{\pi}\int_0^{x_L}\frac{\Phi(x')}{1+x'\omega}dx'\,, \tag{15}$$

where $\Phi(x')$ stands for $[\mathrm{Im}g_{1^+}(S')D(S')\omega']$, and

$$x_L=2\Lambda/\Lambda^2+K^2\simeq 0.20m_\pi^{-1}\,. \tag{16}$$

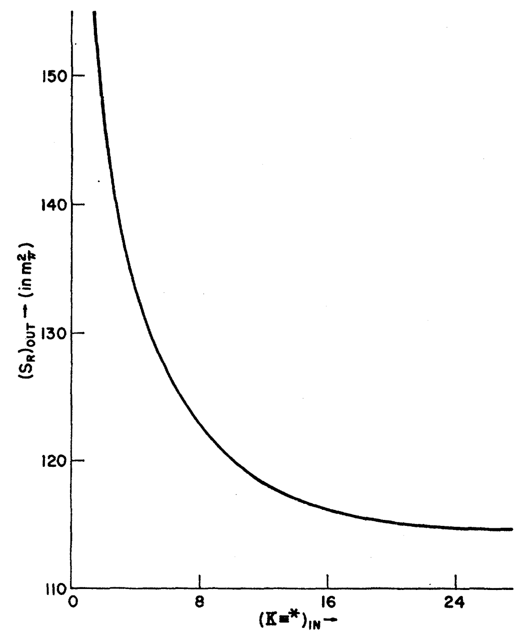

FIG. 3. A typical plot of the output value of the position of the bound state as a function of the input value of the residue $(K_{\Xi^*})_{\mathrm{in}}$. This particular plot is for $S_0=55m_\pi^2$, $(S_R)_{\mathrm{in}}=121m_\pi^2$ and $g_{\Lambda NK}^2/4\pi=1$. The form of the curve is roughly the same for other choice of S_0, $(S_R)_{\mathrm{in}}$ and $g_{\Lambda NK}^2/4\pi$.

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contribution from the far left-hand cut anyway. For example, the contribution from the nearby nucleon cut ($L_1 \leqslant S \leqslant L_2$) is found to be, at most, 5% of that from the far-left-hand cut ($-\infty < S \leqslant 0$), for $g_{\Lambda NK}^2/4\pi \approx 1$ (see Table II).

[16] Note that $N(s)$ does not receive any contribution from the bound state at S_R, since the D function is identically zero at S_R.

[17] In our convention the pion nucleon coupling constant is given by $g_{NN\pi}^2/4\pi \approx 15$.

If one now plots the kernel $y=1/(1+x\omega)$ versus $x(0\leqslant x\leqslant x_L)$ for various values of ω in the region of interest[18] ($m_K<\omega\lesssim 8m_\pi$, say) one notices (see Fig. 2) that the kernel can be approximated to a fair accuracy by straight lines passing through two points whose x coordinates are

$$x_3\simeq 0.17m_\pi^{-1} \quad\text{and}\quad x_4\simeq 0.02m_\pi^{-1}. \qquad (17)$$

The approximation, of course, continues to be reasonable for much higher values of ω. (It is also good for values of ω much below the physical threshold, for example, $\omega\simeq 2.1m_\pi$ or $\omega\simeq -1.3m_\pi$ corresponding to two of our matching points $S=SM_2=110m_\pi^2$ and $S=SM_1=55m_\pi^2$, respectively, to be introduced later.) With this approximation, therefore,

$$N_L(S)\simeq\frac{1}{\pi}\int_0^{x_L}dx'\Phi(x')\times\left\{\frac{(x'-x_4)/(x_3-x_4)}{1+x_3\omega}+\frac{(x'-x_3)/(x_4-x_3)}{1+x_4\omega}\right\}$$
$$=b_3/S-S_3+b_4/S-S_4, \qquad (18)$$

where b_3 and b_4 are unknown constants, independent of S. The positions[19] of the effective range poles S_3 and S_4, corresponding to x_3 and x_4 given by Eq. (17) are [by Eq. (14)],

$$S_3\simeq -17.6m_\pi^2 \quad\text{and}\quad S_4\simeq -722m_\pi^2. \qquad (19)$$

b_3 and b_4 will be treated as unknown parameters and will be determined by the use of the fixed energy dispersion relation (see Sec. V).

Two-Pole Approximation for $N_{(N)}(S)$ ($S\geqslant(\Lambda+K)^2$)

In the present problem $N_{(N)}(S)$ [given by Eq. (9)] involves the contribution from the discontinuity across the cut from $L_1\leqslant S\leqslant L_2$, which is rather long (about $50m_\pi^2$) compared to its mean distance from the physical region. It may be recalled that the analogous cut in the πN problem ($[(N^2-\pi^2)/N]^2\leqslant S\leqslant N^2+2\pi^2$) was so short, as compared to its mean distance from the physical region, that one could always replace it by a pole with fixed residue. Such a procedure is, of course, quite inadequate in the present case. The main problem is, firstly, that the integral running over the region from L_1 to L_2 involves the D function, which is not known, and secondly, for convenience in the calculation of the D function that involves $N_{(N)}(S)$, one needs a suitable form for $N_{(N)}(S)$. Thus, to avoid essentially the problem of solving coupled integral equations, we will proceed as follows: Insofar as we want to calculate $N_{(N)}(S)$ for evaluating the D function, we will assume a linear form for $D(S)$, which is unity at the subtraction point S_0 and zero at the input value of the position of the bound state $(S_R)_{\text{in}}$. Thus, for S in the physical region, we will put in Eq. (9),

$$D(S')=1-(S'-S_0)/[(S_R)_{\text{in}}-S_0]. \qquad (20)$$

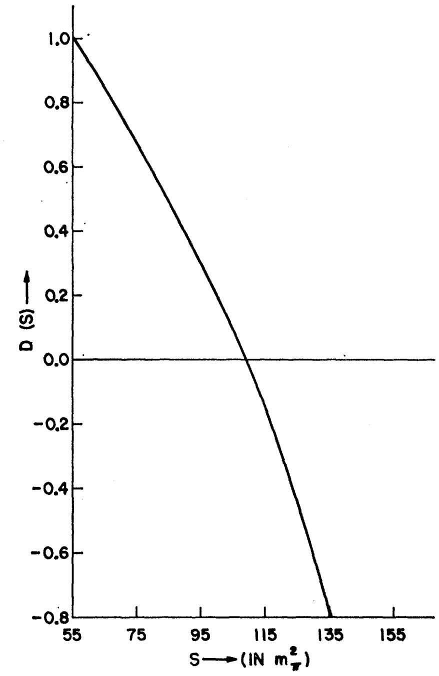

FIG. 4. A typical plot of $D(s)$ versus S. This particular plot is for $S_0=55m_\pi^2$, $(S_R)_{\text{in}}=109m_\pi^2$, $(K_{\Xi^*})_{\text{in}}=10$, and $g_{\Lambda NK}^2/4\pi=1$.

We do not expect to make an error by more than 10–20% in the evaluation of $N_{(N)}(S)$ due to such an approximation. The form of the D function (see Fig. 4) obtained at the end of the calculation for the self-consistent solution does not differ much from the above linear form [Eq. (20)], which is at least consistent with the initial assumption. In any case, an error of 10–20% in $N_{(N)}(S)$ is quite irrelevant for the over-all conclusion, since $N_{(L)}(S)$ involving the effective range pole terms is found to be much more important than $N_{(N)}(S)$.

We took two values for the subtraction point S_0, namely, 55 and $110m_\pi^2$, and used three different values[20] of $(S_R)_{\text{in}}=109$, 121, and $145m_\pi^2$ for each value of S_0.

[18] The dominant contribution to the D integral is expected to come from the region $(\Lambda+K)^2\approx 133m_\pi^2\leqslant S<200m_\pi^2$, which is, therefore, the range of values of S for which $N_{(L)}(S)$ needs to be evaluated as well as possible. This range corresponds to $m_K\leqslant\omega<8m_\pi$, say.

[19] It may be noted that, even though one has a certain range for the choice of the values of S_3 and S_4, this range is rather limited. This is because, while X_3 and X_4 should lie in the range $0\leqslant x\leqslant 0.2m_\pi^{-1}$, they cannot be chosen either too close or too far apart within this range, since in either case the accuracy becomes poorer as is clear by mere inspection of the curves in Fig. 2. As S_3 and S_4 are already so far to the left, small variations in their values are not expected to be felt in the physical region. This has been explicitly checked by Balázs and Singh and Udgaonkar in the $\pi\pi$ and N^* problems, respectively (private communication).

[20] This is to test the degree of self-consistency in the position and residue of the bound state or resonance in question as a function of the input value $(S_R)_{\text{in}}$.

TABLE I. Residues for nucleon-cut poles.

$(S_R)_{\text{in}}$ (in m_π^2)	$b_1(4\pi/g_{\Lambda NK}^2)$ $S_0=55m_\pi^2$	$b_1(4\pi/g_{\Lambda NK}^2)$ $S_0=110m_\pi^2$	$b_2(4\pi/g_{\Lambda NK}^2)$ $S_0=55m_\pi^2$	$b_2(4\pi/g_{\Lambda NK}^2)$ $S_0=110m_\pi^2$
109	1.22	−65.4	0.54	−29.0
121	1.34	8.04	1.14	6.82
145	1.48	3.82	1.86	4.8

Evaluating now $N_{(N)}(S)$ [by Eqs. (9), (12) and (20)], we find, rather surprisingly, that for $(\Lambda+K)^2<S\lesssim 300m_\pi^2$, and for each choice of S_0 and $(S_R)_{\text{in}}$, $N_{(N)}(S)$ can very well be represented (to better than 5% accuracy) by a two-pole formula of the form,

$$N_{(N)}(S)\simeq b_1/S-S_1+b_2/S-S_2\,, \tag{21}$$

where

$$S_1=70m_\pi^2 \quad\text{and}\quad S_2=102m_\pi^2\,. \tag{22}$$

The accuracy continues to be reasonable for higher values of S also. The residues b_1 and b_2 are proportional to $g_{\Lambda NK}^2/4\pi$ and depend[21] upon the particular choice of S_0 and $(S_R)_{\text{in}}$. Their values, evaluated for the various choices of S_0 and $(S_R)_{\text{in}}$ are given in Table I.

We will use Eq. (21) to evaluate $N_{(N)}(S)$ for S in the physical region.

$N_{(N)}(S)$ for $S<(\Lambda+K)^2$

As will be discussed later, we need the values of $N_{(N)}(S)$ at the two matching points $SM_1=55m_\pi^2$ and $SM_2=110m_\pi^2$ which are below the physical threshold. Since these two points are so close on either side of the nucleon cut ($L_1\leqslant S\leqslant L_2$), the two-pole formula [Eq. (21)] for $N_{(N)}(S)$ cannot be applied at these two points. We will, therefore, evaluate $N_{(N)}(S)$ at either of the two matching points $S=SM_1$ or $S=SM_2$ as follows:

By Eqs. (13), (18), and (21) we have

$$D(S)=1-\frac{S-S_0}{\pi}\sum_{i=1}^{4} b_iF(S,S_i,S_0)\,, \tag{23}$$

where

$$F(S,S_i,S_0)=\int_{(\Lambda+K)^2}^{\infty}\frac{(q'^3/S')}{(S'-S)(S'-S_0)(S'-S_i)}dS'\,. \tag{24}$$

Substituting Eq. (23) in Eq. (9) we have

$$N_{(N)}(S)=\left[K_0(S)+\sum_{i=1}^{4} b_iK_i(S,S_0)\right], \tag{25}$$

where

$$K_0(S)=\int_{L_1}^{L_2}\frac{\operatorname{Im}g_{1^+}{}^{(N)}(S')}{S'-S}dS' \tag{26}$$

and

$$K_i(S,S_0)=-\frac{1}{\pi}\int_{L_1}^{L_2}\operatorname{Im}g_{1^+}{}^{(N)}(S')F(S',S_i,S_0)\times\left(\frac{S'-S_0}{S'-S}\right)dS'\,. \tag{27}$$

Now, since we can evaluate[22] $F(S,S_i,S_0)$, $K_0(S)$, and $K_i(S,S_0)$ explicitly, we will use Eq. (25) to evaluate $N_{(N)}(S)$ at the matching points. This will give us $N_{(N)}(S)$ linearly in terms of b_i's. The unknowns b_3 and b_4 will be evaluated through the matching conditions (Sec. V) given by the N/D equation and the fixed-energy dispersion relation for the partial-wave amplitude, which we discuss below.

IV. FIXED-ENERGY DISPERSION RELATION

The fixed-energy dispersion relation for the invariant amplitudes (in our case A and B) is given by

$$A(S,t,u)=\frac{R_N}{u-m_N^2}+\frac{R_{\Xi^*}}{S-m_{\Xi^*}^2}+\frac{R_\omega}{t-m_\omega^2}+\frac{R_\phi}{t-m_\phi^2}+\frac{1}{\pi}\int_{(\Lambda+K)^2}^{\infty}du'\frac{A_u(u',S)}{u'-u}+\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}dt'\frac{A_t(t',S)}{t'-t}\,, \tag{28}$$

where R_N, R_{Ξ^*}, R_ω, and R_ϕ are the appropriate residues at the respective poles.[23] A similar relation holds for $B(S,t,u)$.

We will assume that the integral terms on the right hand side of Eq. (28) are not important as compared to the remaining terms. This is partly based on the observation that there are no known resonances (or bound states), at least for reasonably low values of u' and t' contributing to the said integrals. For example, the lowest mass resonance, which may contribute to the u' integral is the $N_{1/2}{}^*$ at 1512 MeV. Similarly, in the t channel, once we have taken out the ω and ϕ contributions, there are no known resonances having $I=0$, $S=0$ below the f^0 at 1250 MeV. Thus, insofar as one may hope that such high mass contributions to the dispersion integrals may not be important, it may be legitimate to drop them.

Furthermore, for reasons discussed in Appendix A, the ω and ϕ contributions to the $J=\frac{3}{2}^+$ partial-wave amplitude are found (for reasonable choice of coupling

[21] *A priori*, S_1 and S_2 are also expected to depend upon the particular choice of S_0 and $(S_R)_{\text{in}}$, quite apart from the possibility that a two-pole formula may not be adequate in each case. Luckily, however, they are found to be hardly sensitive to the choice of S_0 and $(S_R)_{\text{in}}$ and for the choice of S_0 and $(S_R)_{\text{in}}$, that we tried (i.e., $S_0=55$ and $110m_\pi^2$ and $S_R=109$, 121, $145m_\pi^2$), it was found that a two-pole formula is quite adequate in each case with the positions of the poles being nearly fixed at 70 and $102m_\pi^2$, respectively.

[22] We used an IBM-7090 computer to evaluate these integrals for various values of S, S_i, and S_0.

[23] Note that we have neglected the width of Ξ^*, since we are ignoring the $\pi\Xi$ channel. Insofar as the $\bar{K}\Lambda$ channel alone is concerned, Ξ^* has to be treated as a bound state and hence should have zero width. Note also that we have taken out the ω and ϕ contributions from the $\int dt'$ term in Eq. (28) and have neglected their widths since they are small compared to their masses.

constants) to be totally negligible[24] compared to the nucleon and Ξ* contributions. Therefore, in order to simplify the discussion in terms of fewer number of parameters, we will drop the ω and ϕ contributions altogether from the rest of the paper. Thus, finally, the $J=\frac{3}{2}^+$ partial-wave amplitude, given by the fixed-energy dispersion relation reads

$$g_{1^+}(S)\simeq g_{1^+}{}^{(N)}(S)+g_{1^+}{}^{(\Xi^*)}(S)\,, \qquad (29)$$

where $g_{1^+}{}^{(N)}(S)$ is given by Eq. (10) and

$$g_{1^+}{}^{(\Xi^*)}(S)=(-\mathrm{K}_{\Xi^*})\left\{\frac{(W+\Lambda)^2-K^2}{(W_R+\Lambda)^2-K^2}\right\}\frac{1}{W-W_R}\,. \qquad (30)$$

W_R stands for $S_R^{1/2}$ and denotes the mass of Ξ*, K_{Ξ^*} is a real positive number denoting the residue of the partial-wave amplitude at the bound-state pole term (corresponding to Ξ*) and is proportional to the square of $\Xi^*\bar{K}\Lambda$ coupling constant. K_{Ξ^*} is not an experimentally accessible quantity until one has a reliable relationship between binding energy and coupling constant of a bound state.

V. MATCHING CONDITIONS AND RESULTS

We first wish to choose two matching points at which we will equate the values[25] of $g_{1^+}(S)$ given by the fixed-energy dispersion relation with those given by the N/D equation. These two matching points are chosen so as to satisfy[26] roughly the following two criteria. (1) Since, in practice, one makes a partial-wave expansion of A_t and A_u, occurring in Eq. (28), the respective partial-wave expansion should be convergent at the matching points. However, this expansion diverges for $S>S_{Th}=(\Lambda+K)^2$ due to the unknown singularities in A_t and A_u. Thus, it is desirable to choose the matching points as far away from this region as possible. (2) The partial-wave amplitude given by the N/D equation contains unknown singularities for $S<S_L$, where S_L is the point to the left of which the singularities are approximated by the effective-range pole terms. It is thus desirable to choose the matching points as far above S_L as possible also.

With these two criteria, we find it convenient[27] to choose the two matching points at

$$SM_1=55m_\pi{}^2\,,\quad \text{and} \quad SM_2=110m_\pi{}^2\,. \qquad (31)$$

We will choose[28] the subtraction point at

$$S_0=55m_\pi{}^2\,. \qquad (32)$$

Our matching equations are now provided by equating the values of $g_{1^+}(S)$ given by the fixed-energy dispersion relation [Eq. (29)] at $S=SM_1$ and SM_2 with those given by the N/D equation [Eq. (6)], where the N function is evaluated by using Eqs. (7), (18), and (25), while the D function is evaluated[29] by using Eq. (23). These two equations determine the two unknown residues b_3 and b_4 in terms of the input values of $g_{\Lambda NK}{}^2/4\pi$, K_{Ξ^*}, and S_R, which, in turn, gives us the D function [by Eq. (23)]. The obtained behavior of the D function, then, tells us (a), if one should expect a resonance or bound state in a certain energy range in the system under consideration corresponding to the vanishing of the real part of the D function in this energy range, and (b) if so, then the output values of the position and residue of the resonance or bound state are given by[30]

$$\mathrm{Re}D[(S_R)_{\mathrm{out}}]=0\,, \qquad (33)$$

$$(\mathrm{K}_{\Xi^*})_{\mathrm{out}}=-\frac{N[(S_R)_{\mathrm{out}}]}{\mathrm{Re}D'[(S_R)_{\mathrm{out}}]}\left(\frac{1}{2(W_R)_{\mathrm{out}}}\right). \qquad (34)$$

The results obtained for the behavior of the D function and the corresponding solutions of Eqs. (33) and (34) for various input values of the parameters $g_{\Lambda NK}{}^2/4_\pi$, S_R, and K_{Ξ^*} can be summarized as follows:

(1) If we keep the input values of K_{Ξ^*} and S_R fixed at any reasonable values, the values of b_3 and b_4 and hence, the behavior of the D function as well as the output values of K_{Ξ^*} and S_R, are found to hardly depend upon the value of the ΛNK coupling constant. We varied the value of $g_{\Lambda NK}{}^2/4\pi$ from 0.25 to 8.0 and found almost no difference (less than 10%) in the results. This is a rather lucky situation in view of the fact that the ΛNK coupling constant is not so well determined experimentally. Thus, since the conclusion is quite insensitive

[24] It is worth noting that a similar situation is encountered (Ref. 7) in the case of N^*, where the ρ contribution to the fixed-energy dispersion relation is found to be unimportant.

[25] It may be noted that we are following a matching procedure, slightly different from that in the previous analogous calculations of the $\pi\pi$ and πN (Refs. 6 and 7) problems, where the two matching equations came from matching value and derivative at a single point rather than matching values at two points. We feel, since our scheme is approximate, that it is perhaps a slightly *better* procedure to match values at two points rather than the value and derivative at a single point. However, for the sake of comparison, we mention the results for matching the value and derivative at the same point (SM_1 or SM_2) in Appendix B.

[26] I am grateful to L. A. P. Balázs for communications regarding this point.

[27] We intentionally avoid the choice of matching point on the cut $L_1\leqslant S\leqslant L_2$ to eliminate the necessity of evaluating principal valued integrations for $N_{(N)}(S)$ and its derivative.

[28] Since S_0 is just a normalization point for $D(s)$, the results are not expected to depend sensitively upon the choice of S_0. We have examined explicitly how far this is true by taking a different value of $S_0=110m_\pi{}^2$, but keeping the matching procedure the same (i.e., matching values at $S=SM_1$ and SM_2). The results, qualitatively, are just the same as for $S_0=55m_\pi{}^2$. The best set of self-consistant solutions (for $S_0=110m_\pi{}^2$) for the position and residue of the bound state are $S_R\approx 124m_\pi{}^2$ and $\mathrm{K}_{\Xi^*}\approx 8.5$. These values may be compared with those obtained for a choice of $S_0=55m_\pi{}^2$ [see Eq. (36)]. The degree of self-consistency obtained in either case is nearly the same.

[29] At this stage, the D function is evaluated only in terms of the unknowns b_3 and b_4.

[30] $N_{(N)}[(S_R)_{\mathrm{out}}]$ occurring in Eq. (34) is evaluated by using either Eq. (25) or (21) depending upon whether $(S_R)_{\mathrm{out}}<(\Lambda+K)^2$ or $>(\Lambda+K)^2$.

TABLE II. Results for the output values of the position and residue of the Ξ^* hyperon.[a]

$(S_R)_{\text{in}}$ (in m_π^2)	$(K_{\Xi^*})_{\text{in}}$	$g_{1^+}^{(N)}(S)$ $S=SM_1$	$S=SM_2$	$g_{1^+}^{(\Xi^*)}(S)$ $S=SM_1$	$S=SM_2$	b_3	b_4	$(S_R)_{\text{out}}$ (in m_π^2)	$N_{(N)}$	$N_{(L)}$	$(K_{\Xi^*})_{\text{out}}$
109	10	−0.16	0.73	2.3	−210	−574	7832	109	0.13	4.9	10.3
121	10	−0.16	0.73	1.8	18.5	−432	5982	120	0.10	4.0	9.1
145	10	−0.16	0.73	1.25	5.5	−276	3923	140	0.08	2.8	7.0

[a] These results are for a choice of the two matching points at $S=SM_1=55m_\pi^2$ and $S=SM_2=110m_\pi^2$ with the subtraction point $S_0=55m_\pi^2$. The quantities $N_{(N)}$ and $N_{(L)}$ given above are evaluated at $S=(S_R)_{\text{out}}$. The above results correspond to the choice $g_{\Lambda NK}^2/4\pi=1$.

to the choice of this coupling constant, the quantitative results to be given below will be only for a given choice, namely

$$g_{\Lambda NK}^2/4\pi=1\,, \tag{35}$$

which seems to be roughly the experimental value.[31]

(2) Irrespective of the value of $K_{\Xi^*}(\geqslant 0)$ and for any reasonably finite value of $g_{\Lambda NK}^2/4\pi(>0.25$, say), the real part of the D function is found to decrease monotonically with S, being unity at $S_0=55m_\pi^2$ and zero at some point above $55m_\pi^2$. This itself, therefore, indicates the occurrence[32] of a resonance or bound state in the system under consideration. The position of the zero, or in other words, the rate of decrease of $\text{Re}D(S)$, of course, very much depends upon the input value of K_{Ξ^*}. In fact, as one should expect physically, it is found that the higher the input value of K_{Ξ^*}, the lower is the position of the zero. Thus, one can expect either a resonance $[(S_R)_{\text{out}}>(\Lambda+K)^2]$ or a bound state $[(S_R)_{\text{out}}<(\Lambda+K)^2]$ in the system under consideration depending upon the strength of the residue K_{Ξ^*}. [A typical plot of $(S_R)_{\text{out}}$ versus $(K_{\Xi^*})_{\text{in}}$ is shown in Figure 3.]

(3) Hoping that our model is not far from reality, we demand that the physical values of S_R and K_{Ξ^*} should be those for which there is at least approximate self-consistency between their input and output values. In the first place, it is rather interesting that even though the stabilizing condition[33] encountered in the N^* calculation[7] does not occur in the present case, the degree of self-consistency does vary depending upon the input values of S_R and K_{Ξ^*}. Table II gives the output values of S_R and K_{Ξ^*} for different sets of input values. It may be inferred from the table that with the demand of self-consistency, there is a relatively narrow range of values for choosing these parameters. The best set of self-consistent solution (self-consistency is better than 5%) for the position and residue is found to be

$$(S_R)_{\text{out}}\simeq 109m_\pi^2\,,\quad (K_{\Xi^*})_{\text{out}}\simeq 10\,, \tag{36}$$

which, therefore, indicates the existence of a bound state rather than a resonance. Figure 4 shows a plot of the D function for the above self consistent solution.

(4) It ought to be emphasized that the effective-range pole terms, denoting the far left-hand cut contribution, are found to be much more important than the nearby nucleon cut contribution at every stage of the calculation. For instance, $N_{(L)}(S)$ is found to be bigger than $N_{(N)}(S)$ at least by an order of magnitude (see Table II) and the contribution of the sum of b_3 and b_4 terms to the D function [Eq. (23)] is found to be at least two orders of magnitude bigger than that of the b_1 and b_2 terms. Thus, no resonance or bound state is obtained if one includes only the nearby nucleon cut contribution ignoring that of the far left-hand cut. These clearly indicate the importance of the singularities associated with the far left-hand cut for the dynamical understanding of the Ξ^* hyperon.

VI. CONCLUSIONS

We have studied the problem of scattering in $J=\frac{3}{2}^+$ state in the $\bar{K}\Lambda$ channel, ignoring the influence of all other channels. We have considered our dynamical singularities to be represented by the nearby nucleon cut $(L_1\leqslant S\leqslant L_2)$ and the far left-hand cut $(-\infty<S\leqslant 0)$. The contribution of the former was evaluated explicitly in terms of the ΛNK coupling constant and that of the latter, by the method of Balázs, through the introduction of effective-range pole terms. The analysis is found to be quite insensitive to the choice of the ΛNK coupling constant and one finds that the D function has the desired behavior for the occurance of a resonance or bound state in the energy region of interest. The main force for the existence of the resonance or bound state seems to arise from the singularities associated with the far left-hand cut.[34] The best set of self-consistent solutions for the position and residue indicates the presence of a bound state at $S_R\simeq 109m_\pi^2$ with residue $K_{\Xi^*}\simeq 10$.[35]

Indeed, the obtained position is remarkably close to the experimental value of $121m_\pi^2$. Furthermore, it is

[31] For experimental data on associated photoproduction see B. D. McDaniel *et al.*, Phys. Rev. Letters **1**, 109 (1958). For Polology-analysis of the data see, M. J. Moravcik, Phys. Rev. Letters **2**, 332 (1959).

[32] Hence a finite value for K_{Ξ^*}.

[33] This comes about due to the fact that N^* occurs both in the crossed and the direct channel, the former giving rise to attraction and the latter to repulsion.

[34] The discontinuity across the far left-hand cut, as mentioned before, arises due to a variety of exchanges in the crossed channels. The effective-range pole-terms method, while in a sense includes the contributions due to all such exchanges as well as possible, precludes the necessity of specifying the contributions from *individual* exchanges.

[35] It may be noted for comparison that in the unitary symmetry model, the observed width (100 MeV) of N^* gives a value of $K_{\Xi^*}\approx 4$. However, it is hard to draw any conclusion in view of the seemingly large violation of unitary symmetry with regard to YNK coupling constants (Y stands for Λ and Σ).

rather striking that there does exist a solution with excellent self-consistency (better than 5%) between the input and output values of the parameters of the bound state. Both of these may point to the success of the present dynamical model for Ξ*. However, neither of them can be taken too seriously in view of the many approximations adopted in this paper.

In our opinion, the major point which ought to be investigated carefully is the influence of other channels. The main handicap in such an attempt, however, is the presence of many unknown coupling constants. At this point, let us draw attention to the work of Martin and Wali,[36] who considered a grand bootstrap mechanism for the $J=\frac{3}{2}^+$ tenfold resonances N^*, Y_1^*, Ξ*, and Ω^- (yet to be discovered) as arising from the mutual influence of all possible two-particle baryon pseudoscalar-meson channels. They find that for a suitable choice of the unknown mixing parameter (f/d ratio) in the SU(3) symmetric coupling, they can qualitatively reproduce the gross features of all the above tenfold resonances. They, however, approximate the dynamical singularities by those arising from the single-baryon exchange graphs only. In view of the fact that in our calculation we do find that the far left-hand cut contribution is important (see end of Sec. V), it is not clear how the results (either of Martin and Wali, or ours) would change for a more complete calculation which considers the mutual influence of all the important channels and also includes[37] the dynamical singularities other than those arising from the baryon exchange only. Undoubtedly, such a calculation will be of considerable complexity. A second point which needs to be examined is the effect of inelastic processes on the D function. Furthermore, it is worth investigating how the results would change if one introduces three or more effective-range poles, instead of just two.

As a parenthetical remark, however, from the success of the present ($\bar{K}\Lambda$) model, one may be tempted to guess that somehow the net effect of other channels is not important.

In conclusion, while one ought to reserve one's opinion on the success of the present model until one can ascertain that the results are not very much affected by improving the approximations, as for instance mentioned above, we feel that the results obtained in the present model are sufficiently interesting in themselves; and together with those of $\pi\pi$ and πN problems point to the success of such dynamical methods, in general, for the understanding of bound states and resonances.

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APPENDIX A

In this Appendix we will mention an estimate of the order of magnitude of the ω and ϕ-contributions to the fixed-energy dispersion relation [Eq. (28)]. In particular, we try to justify the assumption made in Sec. IV that the ω and ϕ contributions to the partial-wave amplitude g_{1^+} is negligible compared to those of the nucleon and Ξ*.

The ω and ϕ contributions to the partial-wave amplitude are given by (we retain only the charge couplings of ω and ϕ to the $\Lambda\bar{\Lambda}$ system)

$$g_{1^+}{}^{(\omega)}(S)=-(g_{KK\omega}g_{\Lambda\Lambda\omega}/32\pi q^4)[\{(W+\Lambda)^2-K^2\}(W-\Lambda)\times Q_1(1+m_\omega^2/2q^2)+\{(W-\Lambda)^2-K^2\}\times(W+\Lambda)Q_2(1+m_\omega^2/2q^2)], \quad (A1)$$

$$g_{1^+}{}^{(\phi)}(S)=g_{1^+}{}^{(\omega)}(S)(\omega\rightarrow\phi), \quad (A2)$$

where $g_{KK\omega}$ and $g_{\Lambda\Lambda\omega}$ denote the coupling constants of the ω meson to the $K\bar{K}$ and $\Lambda\bar{\Lambda}$ systems, respectively, and similarly for the ϕ meson.

There is no experimental information on the products of coupling constants $g_{KK\omega}g_{\Lambda\Lambda\omega}$ and $g_{KK\phi}g_{\Lambda\Lambda\phi}$. If we partly appeal to the hypothesis of unitary symmetry[38] and consider bare ϕ_0 and ω_0 mesons, where ϕ_0 belongs to the unitary octet together with ρ and K^*, while ω_0 is the unitary singlet coupled to the baryon current and assume that the coupling of the unitary octet vector mesons to the baryons is of the F type,[39] then[40] $g_{\Lambda\Lambda\phi_0}=0$ and $g_{KK\omega_0}=0$. Thus, bare ϕ_0 and ω_0 mesons cannot contribute to $\bar{K}\Lambda$ scattering. It is easy to check that in this picture, physical $(\phi+\omega)$ also do not contribute to $\bar{K}\Lambda$ scattering if we put $m_\phi=m_\omega$. Without entering into this picture of $\phi-\omega$ mixing, we may adopt for a rough order-of-magnitude estimate the values

$$g_{KK\phi}g_{\Lambda\Lambda\phi}/4\pi\approx g_{KK\omega}g_{\Lambda\Lambda\omega}/4\pi\approx 1. \quad (A3)$$

As we shall see, the results are hardly affected, even if we are wrong by a factor of 2–5 (say) in the above

[36] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963). See also R. H. Capps, Nuovo Clmento **27**, 1208 (1963).

[37] This may be done, at least approximately, by the use of effective-range pole terms, as we do in the present calculation.

[38] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962). J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962). Enrico Fermi Institute (EFINS 63-28) (unpublished).

[39] The D type coupling bears no resemblance to the hypothesis that ρ is coupled to the conserved isospin current, etc.

[40] Since ϕ_0 is supposed to be coupled to the hypercharge current and ω_0 to the baryon current, the former is not coupled to $\Lambda\bar{\Lambda}$ and the latter is not coupled to $K\bar{K}$.

choice. In fact, we find that it makes no difference, even if we drop the ω and ϕ contributions altogether. The reason essentially is: Using the values given by Eq. (A3) for the ω and ϕ couplings and using $g_{\Lambda NK}^2/4\pi \approx 1$,[31] we find that the sum of ω and ϕ contributions to $g_{1^+}(S)$, evaluated at either of the two matching points (55 and $110m_\pi^2$) is less than 15% of the nucleon contribution [Eq. (10)], evaluated at the same points. However, the nucleon contribution, itself, is much less than the Ξ^* contribution [Eq. (30)] for any reasonable value of K_{Ξ^*}, and especially for the self-consistent value of K_{Ξ^*} ($\approx 8\sim 10$). The Ξ^* contribution is more or less an order of magnitude bigger than the nucleon contribution. Thus, the over-all contribution of ω and ϕ terms to $g_{1^+}(S)$ is of the order of 1–5% of the sum of the nucleon and Ξ^* contributions, and hence may be ignored.

APPENDIX B

In order to examine whether the results depend sensitively upon the choice of the matching procedure, we also matched the value and derivative of the partial-wave amplitude at a single point, given by the N/D equation with those given by the fixed-energy dispersion relation, instead of matching values at two points. In this procedure it is convenient to choose the subtraction point S_0 at the matching point SM. This simplifies the evaluation of the derivative of the D function. Using Eqs. (7), (9), (18), and (23), we have

$$D'(SM=S_0) = -\frac{1}{\pi}\left[\sum_{i=1}^{4} b_i F(SM, S_i, S_0=SM)\right], \quad \text{(B1)}$$

$$N'(SM=S_0) = -\sum_{i=3}^{4} \frac{b_i}{(SM-S_i)^2} + \frac{1}{\pi}\left[L_0(SM) + \sum_{i=1}^{4} b_i L_i(SM)\right], \quad \text{(B2)}$$

where

$$L_0(SM) = \int_{L_1}^{L_2} \frac{\mathrm{Im}g_{1^+}{}^{(N)}(S')}{(S'-SM)^2} dS', \quad \text{(B3)}$$

$$L_i(SM) = -\frac{1}{\pi}\int_{L_1}^{L_2} \frac{\{\mathrm{Im}g_{1^+}{}^{(N)}(S')\} F(S', S_i, S_0=SM)}{(S'-SM)} dS'. \quad \text{(B4)}$$

We evaluated the quantities $L_0(SM)$ and $L_i(SM)$ by IBM-7090 in the same way as $K_0(SM)$ and $K_i(SM)$. The unknown residues b_3 and b_4 are then evaluated straightforwardly by the use of the two matching equations obtained by matching the value and derivative of the partial wave amplitude at a single point ($SM = SM_1 = 55m_\pi^2$ or $SM = SM_2 = 110m_\pi^2$), given by the N/D equation and the fixed-energy dispersion relation.

The results obtained by this matching procedure are, qualitatively, just the same as those for the alternative procedure of matching values at two points. In fact, all the qualitative features of the results summarized at the end of Sec. V are common to both the procedures. There are, however, some quantitative differences in the results. The best set of self-consistent solutions obtained by matching value and derivative at a single point ($SM = SM_1$ or SM_2) are

$$S_R \approx 100m_\pi^2, \quad K_{\Xi^*} \approx 14 \text{ (for } SM=SM_1=S_0=55m_\pi^2)$$

and

$$S_R \approx 126m_\pi^2, \quad K_{\Xi^*} \approx 6 \text{ (for } SM=SM_2=S_0=110m_\pi^2). \quad \text{(B5)}$$

It is interesting that the results obtained by matching values at two points SM_1 and SM_2 [see Eq. (36)] are somewhat a mean between the above two results, which may be expected *a priori*. This further justifies our intuitive reasoning [see Ref. (25)] that, in general, in an approximate scheme, it may be better to match values at two points rather than at a single point.