

tion, since the increase in $|\Delta E|$ gives rise to the increase in electrons and that in holes by equal numbers.

It seems interesting to compare the present results on Co_{1-x}Fe_xSi with our recent work on Co_{1-x}Ni_xSi ($0.14 \geq x \geq 0$) solid solutions. In the latter case, the experimental data are qualitatively interpreted on the assumptions that (1) the solid solutions are semi-metallic, (2) m_p , η , and $\alpha (= \frac{5}{2})$ remain unchanged with x , and (3) one Ni atom substituting for one Co atom produces approximately one electron. Since Ni has one more 3d electron while Fe has one less 3d electron in

comparison with Co, the results on these two solid solutions seem to be quite reasonable. Details of the work on Co_{1-x}Ni_xSi will be published elsewhere in the near future.

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Galvanomagnetic Effects in *n*-Type Germanium

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The galvanomagnetic coefficients of *n*-type Ge have been calculated theoretically and compared with previously published experimental data. Calculations have been made for material containing from 10^{14} to 10^{18} impurity centers per cm³, and comparisons were made with samples having free-carrier concentrations from 5×10^{13} to 2×10^{17} per cm³. The calculations were made primarily at 77 and 300°K, but also as functions of temperature. A solution of the Boltzmann transport equation with a collision time which was anisotropic, energy-dependent, and which depended on the impurity content was used. In the expressions Herring and Vogt's equations were used for the acoustic lattice scattering, the Brooks-Herring equation modified by Ham's calculation of the anisotropy was used for the ionized impurity scattering, and the Erginsoy formula for neutral impurity scattering was used to account for the energy-independent scattering. It is shown that even though a constant mean collision time can be used to qualitatively predict the dependence of the coefficients on the magnetic field, the quantitative agreement with experiment is considerably improved when a more realistic form of the collision time is used for the calculations. Scattering functions which take the temperature dependence of the scattering and the presence of ionized impurities into account enable quantitative predictions of the behavior of the galvanomagnetic coefficients to be made quite successfully.

INTRODUCTION

A LARGE amount of accurate experimental data has been published on the galvanomagnetic effects, the Hall effect and the magnetoresistance, for the elemental semiconductors germanium and silicon. Qualitatively, the theoretical calculations are in good agreement with the experimental data as functions of the orientation and field dependence of the effects. However, quantitatively the dependence on temperature and impurity content of the semiconductor is not in as good agreement as might be hoped for.¹⁻⁹ Primarily, this is due to the difficulty of calculating the conductivity tensor when the relaxation time equations

are complicated because of the presence of more than one effective scattering mechanism. With the present relatively easy access to large computers, this calculation problem is greatly eased. Such a calculation has been made for the case of *n*-type silicon by one of the authors³ with fairly good agreement between experiment and theory. Therefore, it seems appropriate to perform these calculations for *n*-type germanium and to compare the results with the extensive previously published experimental data.

THEORY

The formal theory of conductivity was used in the usual manner to evaluate the conductivity for a single ellipsoidal energy surface with an anisotropic energy-dependent relaxation time.^{1-3,10} For this development

* Operated with support from the U. S. Air Force.

¹ W. M. Bullis and W. E. Krag, *Phys. Rev.* **101**, 580 (1956).

² W. M. Bullis, *Phys. Rev.* **109**, 292 (1958).

³ W. E. Krag, *Phys. Rev.* **118**, 435 (1960).

⁴ C. Goldberg, *Phys. Rev.* **109**, 331 (1958).

⁵ C. Goldberg and W. E. Howard, *Phys. Rev.* **110**, 1035 (1958).

⁶ R. A. Laff and H. Y. Fan, *Phys. Rev.* **112**, 317 (1958).

⁷ D. Long and J. Myers, *Phys. Rev.* **115**, 1107 (1959).

⁸ D. Long and J. Myers, *Phys. Rev.* **120**, 39 (1960).

⁹ H. Miyazawa and H. Maeda, *J. Phys. Soc. Japan* **15**, 1924 (1960).

¹⁰ The development for a single-energy minimum is exactly the same as that followed in Ref. 3. It is done in considerable detail in W. E. Krag, PhD. thesis, Massachusetts Institute of Technology, June 1959 (unpublished). The procedure for combining the conductivities of the several [111] oriented ellipses is also the same as in the above references, modified of course by the different orientations of the ellipsoidal energy surfaces.

it is required that the material be homogeneous and isothermal so that the Boltzmann equations do not have a positional dependence, the electric field be strong enough to produce only first-order perturbations in the electron distribution in energy and the distribution in energy of the electrons be Maxwellian. The assumption that the relaxation-time tensor is expressible as a tensor diagonal in the same space coordinates as the mass tensor is also made.

After expanding the distribution function to first order the Boltzmann equation can be solved for the

distribution function itself which can then be used to solve the current density equation and hence to obtain the conductivity tensor for a single ellipsoid. For germanium, the energy surface of the conduction band is such that there are four equivalent ellipsoids of revolution oriented with their axes along the four [111] axes of the cubic germanium crystal.¹¹ When the components of the conductivity tensor for each ellipsoid are transformed into a common cubic coordinate system and summed over all the ellipsoids, the total conductivity tensor is

$$\sigma_{ij} = \frac{4en}{3\sqrt{\pi}(kT)^{5/2}} \int []_{ij} \mathcal{E}^{3/2} e^{-\epsilon/kT} d\mathcal{E},$$

where

$$\begin{aligned} []_{11} = & \frac{\frac{1}{3}[2a_t + a_l] + a_l^2 a_l B_1^2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} - 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 - B_2 B_3 - B_1 B_3) \right]} + \frac{\frac{1}{3}[2a_t + a_l] + a_l^2 a_l B_1^2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} + 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 - B_2 B_3 + B_1 B_3) \right]} \\ & + \frac{\frac{1}{3}[2a_t + a_l] + a_l^2 a_l B_1^2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} - 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 - B_2 B_3 + B_1 B_3) \right]} + \frac{\frac{1}{3}[2a_t + a_l] + a_l^2 a_l B_1^2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} + 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 + B_2 B_3 - B_1 B_3) \right]}, \\ []_{12} = & \frac{\frac{1}{3}[a_l - a_t + a_l^2 (B_1 + B_2 - B_3) - a_l a_l (B_1 + B_2 + 2B_3)] + a_l^2 a_l B_1 B_2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} - 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 - B_2 B_3 - B_1 B_3) \right]} \\ & + \frac{\frac{1}{3}[a_l - a_t + a_l^2 (B_1 - B_2 - B_3) - a_l a_l (B_1 - B_2 + 2B_3)] + a_l^2 a_l B_1 B_2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} + 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 - B_2 B_3 + B_1 B_3) \right]} \\ & + \frac{\frac{1}{3}[a_l - a_t - a_l^2 (B_1 + B_2 + B_3) + a_l a_l (B_1 + B_2 - 2B_3)] + a_l^2 a_l B_1 B_2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} - 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 + B_2 B_3 + B_1 B_3) \right]} \\ & + \frac{\frac{1}{3}[a_l - a_t - a_l^2 (B_1 - B_2 + B_3) + a_l a_l (B_1 - B_2 - 2B_3)] + a_l^2 a_l B_1 B_2}{1 + \frac{a_l a_l}{3} \left[2 + \frac{a_t}{a_l} + 2 \left(1 - \frac{a_t}{a_l} \right) (B_1 B_2 + B_2 B_3 - B_1 B_3) \right]}, \quad (1) \end{aligned}$$

and by permuting the subscripts 1, 2, 3 and using the Onsager relation, $\sigma_{ij}(\mathbf{B}) = \sigma_{ji}(-\mathbf{B})$, the other components of the conductivity tensor are easily obtained. In these expressions n is the electron concentration in each of the ellipsoids (assumed to be the same in each ellipsoid), k the Boltzmann constant, T the absolute temperature in °Kelvin, B_i the component of the magnetic field along the i th cubic axis in the cubic coordinate system and $a_{i(l)} = e[\tau_{i(l)}/m_{i(l)}]$, where $\tau_{i(l)} = 1/\nu_{i(l)}$ is

the relaxation time transverse (parallel) to the major axis of the ellipsoid. τ_t and τ_l are not necessarily identical either in magnitude or energy dependence. $m_{i(l)}$ is the electron mass transverse (parallel) to the major axis of the ellipsoidal energy surface.

The galvanomagnetic coefficients are expressed in

¹¹ J. H. Crawford, Jr., H. C. Schweinler, and D. K. Stevens, Phys. Rev. **99**, 1330 (1955).

the following manner^{2,3}: the magnetoresistance is

$$\frac{\Delta\rho}{\rho_0} = \frac{\rho - \rho_0}{\rho_0} = \frac{(\mathbf{E} \cdot \mathbf{J})_B}{(\mathbf{E} \cdot \mathbf{J})_{B=0}} - 1, \quad (2)$$

and the Hall coefficient is

$$R = \frac{(\mathbf{E} \cdot \mathbf{J} \times \mathbf{B})}{(\mathbf{J} \times \mathbf{B})^2}. \quad (3)$$

Using the tensor form of Ohm's Law $\mathbf{E} = \boldsymbol{\rho} \cdot \mathbf{J}$ the right-hand side of these equations can be written in terms of the components of the resistivity tensor ρ_{ij} and the direction cosines of the magnetic field. The ρ_{ij} will, of course, be functions of the magnitude and direction of the magnetic field.¹² It is difficult to obtain sufficient experimental accuracy to be able to invert the measured resistivity tensor. Usually the conductivity tensor is calculated and inverted numerically for comparison with the experimental data.

In the literature on the galvanomagnetic effects several rather useful approximations to the relaxation time have been used. The obvious approximation to make is that of a constant mean free time. In this case the entire expression inside the brackets in Eq. (1) can be brought outside the integral, which then becomes a gamma function, and the conductivity tensor can be easily calculated. This approximation has proved to be useful in the analysis of several types of data.^{2,13,14} Another approximation which has been widely used is the assumption that the relaxation time is proportional to a simple power expression in energy.^{2,4-9,11,13-17} Even when the relaxation time is expressed as a simple power expression over one range of energy and by another power expression over another range of energy,¹⁸ e.g.,

$$\begin{aligned} \tau_1 &= l_1 \mathcal{E}^a, & 0 \leq \mathcal{E} \leq \mathcal{E}_1, \\ \tau_2 &= l_2 \mathcal{E}^b, & \mathcal{E}_1 \leq \mathcal{E} \leq \mathcal{E}_2, \end{aligned} \quad (4)$$

the conductivity integrals can be integrated analytically and the conductivity tensor directly evaluated. This approximation has been useful in cases where a single scattering mechanism is dominant.¹⁹ However, when the relaxation time is expressed in a more complex manner the integrals must be calculated numerically, and they are complicated enough to require the use of a rather large computer, such as the IBM 7090 which was used for these calculations.

¹² It will be noted that the Hall coefficient is indeterminate when \mathbf{J} and \mathbf{B} are parallel. This is unfortunate but the Hall voltage is experimentally zero for this case.

¹³ B. Lax, H. Zeiger, and R. Dexter, *Physica* **20**, 818 (1954).

¹⁴ L. Gold and L. Roth, *Phys. Rev.* **107**, 358 (1957).

¹⁵ F. Seitz, *Phys. Rev.* **73**, 549 (1948).

¹⁶ B. Abeles and S. Meiboom, *Phys. Rev.* **95**, 31 (1954).

¹⁷ F. Blatt, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1957), Vol. 4, p. 200.

¹⁸ G. B. Benedek, W. Paul, and H. Brooks, *Phys. Rev.* **100**, 1129 (1955).

¹⁹ C. Herring, T. H. Geballe, and J. E. Kunzler, *Bell System Tech. J.* **38**, 657 (1959).

In Eq. (1) the only unknowns are the concentration n , the masses m_t and m_l and the relaxation times τ_t and τ_l . n , of course, can be rather closely estimated from the measured Hall coefficient of the sample. We have used the cyclotron resonance values of the effective mass for m_l and m_t since it has been shown that the effective electron masses do not change appreciably with temperature, at least up to 100°K.²⁰ It is also reasonable to suspect that they do not change appreciably up to 300°K.^{2,5,6} For the relaxation time equations we have used previously published results, taking into account three types of scattering; neutral impurity scattering, ionized impurity scattering, and lattice scattering by acoustic phonons.

Neutral impurity scattering has been shown to be relatively temperature-independent, with a collision frequency²¹

$$\nu_N = \frac{1}{\tau_N} = \frac{20\kappa N_n h^3}{8\pi^3 m^{*2} e^2}, \quad (5)$$

where $\kappa = 16$ is the dielectric constant for Ge, N_n is the number of neutral impurities which is, unfortunately, unknown. h is Planck's constant, m^* is the effective mass $m^* = (m_t^2 m_l)^{1/3}$ and e is the electron charge.

For ionized impurity scattering we have used the Brooks-Herring formula,^{17,22}

$$\begin{aligned} \frac{1}{\tau_I} &= \frac{1}{\tau_I} = N_I \pi e^4 (2m^*)^{-1/2} (\mathcal{E})^{-3/2\kappa-2} \\ &\times \left[\ln(1+b) - \frac{b}{1+b} \right], \end{aligned} \quad (6)$$

where

$$b = 8\pi m^* \kappa k T \mathcal{E} / n' e^2 h^2.$$

N_I is the number of ionized impurities and we have taken $n' = N_I$. The term in brackets takes account of the screening of one ionized impurity by another. Since Eq. (6) was developed for an isotropic energy band minimum it is necessary to modify the equation to allow for the anisotropy of the energy minima of the conduction band in Ge. Ham²³ evaluated the anisotropy of the ionized impurity scattering and showed that for small angle scattering $\tau_l/\tau_t = 12$ which was later shown experimentally to be reasonably accurate.⁶ Then $[1/\tau_{IL}(1/\tau_{IT})^2]^{1/3} = \nu_I = 1/\tau_I$, where ν_I is the ν_I of Eq. (6), $1/\tau_{IL}$ is the collision frequency for electrons moving along the major ellipsoidal axis, and $1/\tau_{IT}$ is the relaxation time for electrons moving in a direction perpendicular to the major axis.

For acoustical lattice phonons Herring and Vogt²⁴

²⁰ D. M. S. Bagguley, R. A. Stradling, and J. S. S. Whiting, *Proc. Roy. Soc. (London)* **A262**, 340 (1961).

²¹ C. Erginsoy, *Phys. Rev.* **79**, 1013 (1950).

²² The formula as written in Ref. 17 is incorrect by a factor of 2 [D. M. Brown and R. Bray, *Phys. Rev.* **127**, 1593 (1962)].

²³ F. S. Ham, *Phys. Rev.* **100**, 1251 (1955).

²⁴ C. Herring and E. Vogt, *Phys. Rev.* **101**, 944 (1956).

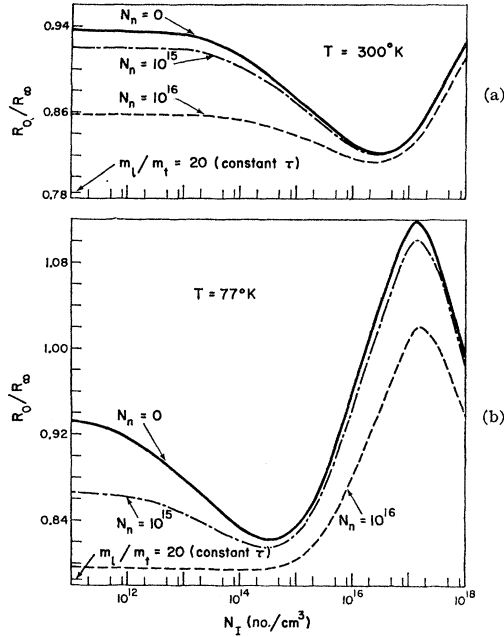


FIG. 1. Calculated ratio of the zero field to the infinite field Hall coefficient R_0/R_∞ as a function of the ionized impurity concentration. The curves are drawn for three values of the neutral impurity concentration, N_n . (a) $T=300^\circ\text{K}$, (b) $T=77^\circ\text{K}$. The value of R_0/R_∞ for a constant mean-free time and $m_l/m_t=20$ is indicated on the left-hand margins.

obtained the expressions

$$\frac{1}{\tau_l} = \frac{3\pi CkT\mathcal{E}^{1/2}}{Vc_l} [\xi_l \Xi_d^2 + \eta_l \Xi_d \Xi_u + \zeta_l \Xi_u^2],$$

$$\frac{1}{\tau_t} = \frac{3\pi CkT\mathcal{E}^{1/2}}{Vc_l} [\xi_t \Xi_d^2 + \eta_t \Xi_d \Xi_u + \zeta_t \Xi_u^2], \quad (7)$$

where

$$C = \frac{(m_l^2 m_t)^{1/2} V}{2^{3/2} \pi^2 \hbar^4},$$

and c_l is the average elastic constant for longitudinal acoustic waves. ξ , η , and ζ are numerical factors which depend on the mass anisotropy ratio, and Ξ_d and Ξ_u are the deformation potentials due to dilation and shear respectively. Experimental values for all of the numbers in these equations are available. In the calculations we used the cyclotron resonance values²⁵ for m_l and m_t and the anisotropy ratio m_l/m_t ; or $m_l=0.0819m_0$, $m_t=1.64m_0$, and $m_l/m_t=20$. For the sound velocity we have used McSkimin's²⁶ results at 300°K and extrapolated his results slightly to obtain the 77°K data. At 300°K, $c_l=1.50 \times 10^{12}$ dyn/cm² and at 77°K, $c_l=1.53 \times 10^{12}$ dyn/cm². The ratio $\tau_l/\tau_t=1.22$ for acous-

tic scattering was obtained from magnetoresistance data¹⁹ using the cyclotron resonance values for m_l and m_t . Using Herring and Vogt's²⁴ Fig. 5 the ratio $\Xi_d/\Xi_u = -0.438$; $\Xi_u = -17.3$ eV was obtained from piezo-resistance data.²⁷ There is also a contribution to the lattice scattering from optical phonons around 300°K. We neglected this contribution since it appeared to be small, and because there are no available numbers for the various constants used in the theory.²⁸

To combine the effects of the various scattering mechanisms one sums the scattering frequencies $\nu_i=1/\tau_i$ to get a total scattering frequency $\nu=1/\tau$. Summing the frequencies of the three scattering mechanisms described above leads to equations of the following form:

$$\nu_l = L_1 x^{1/2} + I_1 x^{-3/2} \left[\ln(1+ax) - \frac{ax}{1+ax} \right] + N,$$

$$\nu_t = L_2 x^{1/2} + I_2 x^{-3/2} \left[\ln(1+ax) - \frac{ax}{1+ax} \right] + N, \quad (8)$$

where ν_l and ν_t are the collision frequencies for electrons moving parallel and perpendicular to the major axes of the constant energy ellipsoids respectively. L_1 , L_2 , I_1 , I_2 , a , and N are constants which can be evaluated from Eqs. (5)–(7). x is the reduced energy \mathcal{E}/kT .

With these expressions the value of the relaxation time can be calculated on an absolute basis. The acoustic scattering does not depend on the number of impurities, at least to first order, and can be considered to be the same for all of the samples. The ionized impurity scattering depends, of course, on the number of impurities. In the absence of any data on compensation² we have used the Hall coefficient to get the number of free carriers and assumed that this is equal to the number of ionized impurities in the particular samples.

Calculating the conductivity tensor is then simply a matter of introducing a_l and a_t ($a_{l(t)} = e[\nu_{l(t)} m_{l(t)}]^{-1}$) and B into Eq. (1) and evaluating the integrals.

RESULTS

Figure 1 shows the ratio of the zero field to the infinite field Hall coefficient R_0/R_∞ as a function of the ionized impurity concentration for different neutral impurity concentrations at 300 and 77°K. It should be noted that this ratio changes rapidly with concentration at both 300 and 77°K in the often used impurity concentration range of $N_I=10^{16}/\text{cm}^3$.²⁹ Included in the figure is the value of R_0/R_∞ for the constant mean-free-time approximation with $m_l/m_t=20$. Throughout the calculations the amount of lattice scattering is considered to

²⁷ F. J. Morin, T. H. Geballe, and C. Herring, Phys. Rev. **105**, 525 (1957).

²⁸ C. Herring, Bell System Tech. J. **34**, 238 (1955).

²⁹ We do not understand the sharp decrease R_0/R_∞ at high-impurity concentrations at 77°K. It does however appear to be real.

²⁵ R. N. Dexter, H. J. Zeiger, and B. Lax, Phys. Rev. **104**, 637 (1956).

²⁶ H. J. McSkimin, J. Appl. Phys. **24**, 988 (1953).

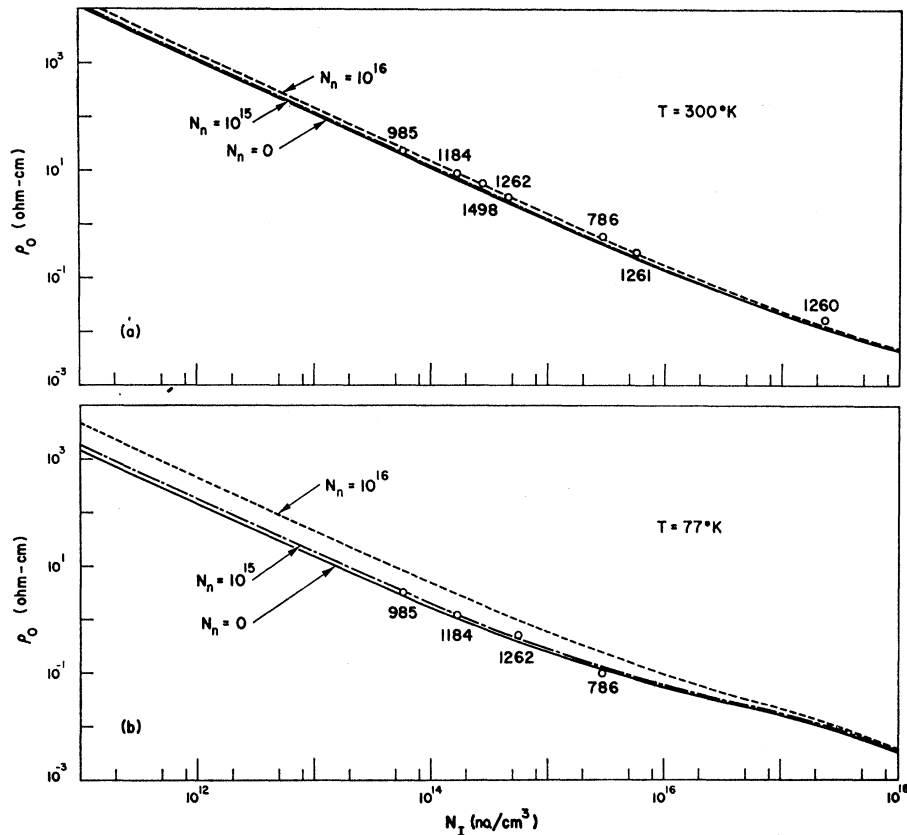


FIG. 2. Calculated zero-field resistivity as a function of the ionized impurity concentration. The curves are drawn for three values of the neutral impurity concentration N_n . The available measurements of the resistivities of the samples used in the comparisons of the data are also indicated, where the values of N_I were obtained by the method indicated in the text. (a) $T=300^\circ\text{K}$, (b) $T=77^\circ\text{K}$.

be dependent on temperature only. R_∞ has been shown to be independent of all the crystal parameters, such as scattering, orientation, and temperature, and dependent only on the number of free carriers.³⁰ R_0 , on the other hand, depends on all of these factors except the orientation of the current and magnetic field with respect to each other and to the crystal axes.² Figure 1(a) was used iteratively to get an accurate value of the carrier concentration from the measured R_0 .

Figure 2 shows the resistivity versus carrier concentration at 300 and 77°K for several values of the neutral impurity concentration. Also shown in the figure are the measured values of the resistivity for many of the samples we used for comparison with theory. Numbers were not available for the samples not included on these figures. In general, the calculated Hall mobility when there were no neutral impurities was higher than the experimental Hall mobility. We therefore chose a value of the neutral impurity concentration which was sufficient to bring the calculated resistivity close to its measured value at the measured carrier concentration. This is a plausible assumption since it is well known that there were often rather large concentrations of neutral oxygen in grown germanium crystals.³¹

Figure 3 shows the field dependence of the Hall

TABLE I. Measured and calculated characteristics of samples considered in preparing Figs. 2-5.

Sample number	Resistivity at 300°K (ohm-cm) measured	Resistivity at 77°K (ohm-cm) measured	R_0 at 300°K (cm²/coul) measured	N_I (#/cm³) calculated	N_n (#/cm³) calculated
985	23.6	3.2	100	5.74×10^{13}	10^{14}
1184	8.89	1.2	33.4	1.69×10^{14}	10^{15}
786	0.6	0.1	1.8	2.96×10^{15}	10^{15}
1261	0.293		0.90	5.83×10^{15}	10^{15}
1260	0.016		0.023	2.34×10^{17}	10^{16}
1262	3.2	0.5	12	4.6×10^{14}	10^{15}
1498	5.6		20	2.8×10^{14}	10^{15}

coefficient normalized to zero magnetic field for several samples with different impurity concentrations for two different orientations at 77 and 300°K. The concentrations of impurities used for the calculations are given in Table I. Figure 4 does the same for the magnetoresistance versus magnetic field data. As indicated by the figure, the calculated variation with orientation also agrees with experiment. The failure to saturate in the high-field region of the magnetoresistance seen in Fig. 4(c) is probably due to sample inhomogeneities.² Several of the low-field magnetoresistance coefficients as a function of impurity concentration are presented in Fig. 5. As explained by Bullis² the upturn at the right of the

³⁰ M. Shibuya, Phys. Rev. **95**, 1385 (1954).

³¹ W. Kaiser, Phys. Rev. **105**, 1751 (1957).

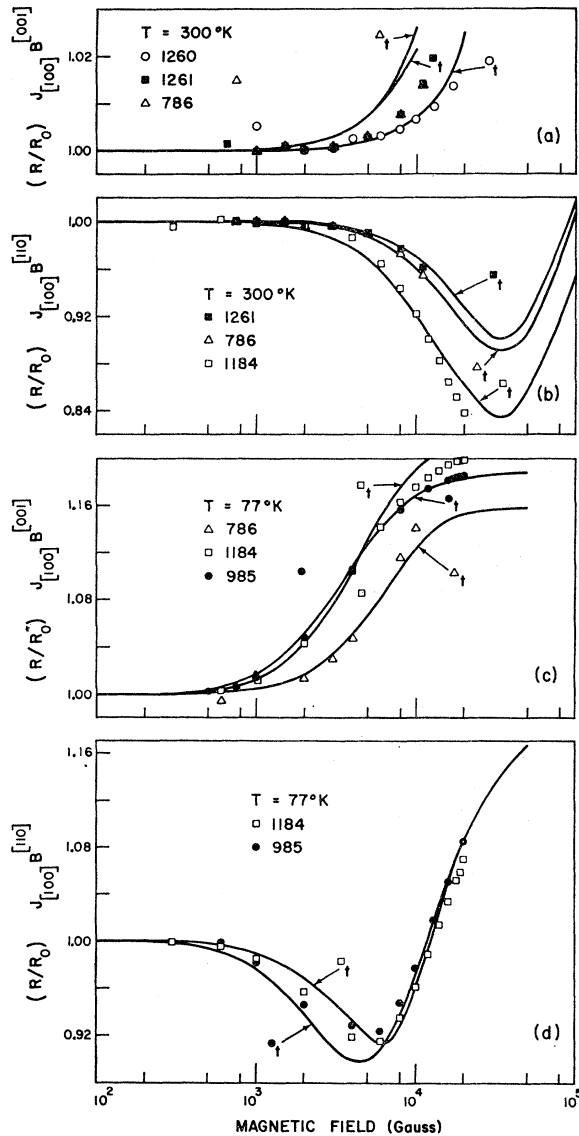


FIG. 3. Calculated and experimental field dependence of the normalized Hall coefficient R_H/R_0 . The solid lines are theoretical, the points experimental. Each theoretical line is labeled with the same symbol with a subscript t as the experimental points for the corresponding sample. See Table I for sample information. (a) $T = 300^\circ\text{K}$, $J_{[100]}B^{[001]}$; (b) $T = 300^\circ\text{K}$, $J_{[100]}B^{[110]}$; (c) $T = 77^\circ\text{K}$, $J_{[100]}B^{[001]}$; (d) $T = 77^\circ\text{K}$, $J_{[100]}B^{[110]}$.

figure is due to the presence of a small number of holes in the pure samples.

The high-field magnetoresistance limit for B and J in certain crystallographic directions has often been used^{2,19} to make an estimate of the mass anisotropy factor $K = m_l/m_t$. Unfortunately it can be seen from Fig. 4(e) that the magnetoresistance even in the high-field limit depends on the number of impurities or rather the amount of ionized impurity scattering as well as the mass anisotropy. However, assuming no gross anisotropy in the lattice scattering, very pure

samples, and taking due precautions in the measurements, in the absence of any direct information on the mass anisotropy, such as from cyclotron resonance experiments,²⁰ the high-field magnetoresistance should lead to a good approximation to the mass anisotropy.

Figure 6 shows a comparison of some experimental data taken by Goldberg and Howard⁵ with our calculations. Figure 6 is concerned with the magnetoresistance symmetry relations, which they showed to hold in their samples. Our concern here is with the changes in the various coefficients with temperature and impurity concentration. The notation used in Figs. 6 and 7 is

$$\nu_2 \equiv \lim_{H \rightarrow 0} \frac{\frac{\Delta\rho}{\rho_0 H^2} \Big|_{[110]}}{\frac{\Delta\rho}{\rho_0 H^2} \Big|_{[001]}} = \frac{(b+c+\frac{1}{2}d)}{b}, \quad (9)$$

$$\nu_1 \equiv \lim_{H \rightarrow 0} \frac{\frac{\Delta\rho}{\rho_0 H^2} \Big|_{[\bar{1}10]}}{\frac{\Delta\rho}{\rho_0 H^2} \Big|_{[110]}} = \frac{(b+\frac{1}{2}d)}{b}, \quad (10)$$

$$\nu_3 \equiv \left(\frac{R_0}{\rho_0}\right)^2 / \lim_{H \rightarrow 0} \frac{\Delta\rho}{\rho_0 H^2} \Big|_{[001]} \Big|_{[110]}, \quad (11)$$

and finally

$$W = 2\nu_2/(1+\nu_3), \quad (12)$$

where b , c , and d are the low-field magnetoresistance coefficients,²² a subscript indicates the current direction and the superscript the magnetic field direction. Figures 6 and 7 show that the low-field coefficients are as expected from theory. Figure 7 in particular shows experimental data of another type which is sometimes used^{5,6} to estimate the value of the anisotropy $K = (m_l/m_t)(\tau_l/\tau_t)$. However, when the relaxation-time equations are of the form Eq. (8) the anisotropy factor τ_l/τ_t changes as a function of the electron energy depending on the impurity concentration and temperature as well as the anisotropy of ionized impurities. Therefore, estimating the value of K_m from measurements of this kind on samples with mixed scattering seems risky at best. However, if the mass ratio is known from other measurements one can at least learn whether τ_l/τ_t is greater or less than one.

Laff and Fan⁶ used measurements of the Hall mobility and magnetoresistance to show that a value of $\tau_l/\tau_t = 12$ was reasonable by measuring a factor

$$W = \lim_{H \rightarrow 0} \frac{\frac{\Delta\rho}{\rho_0 H^2} \Big|_{[100]}}{\frac{\Delta\rho}{\rho_0 H^2} \Big|_{[010]} + \mu_H^2}, \quad (13)$$

²² G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).

which is identical to the factor *W* used by Goldberg *et al.* [see Eq. (12)], versus temperature down to temperatures low enough so that the ionized impurities become neutral impurities. Their experimental results

and our calculations are shown in Fig. 7(b). At 20°K we have calculated two points. Point A was calculated for the data given by Laff and Fan indicating that the electrically active impurities were 80% deionized. Point

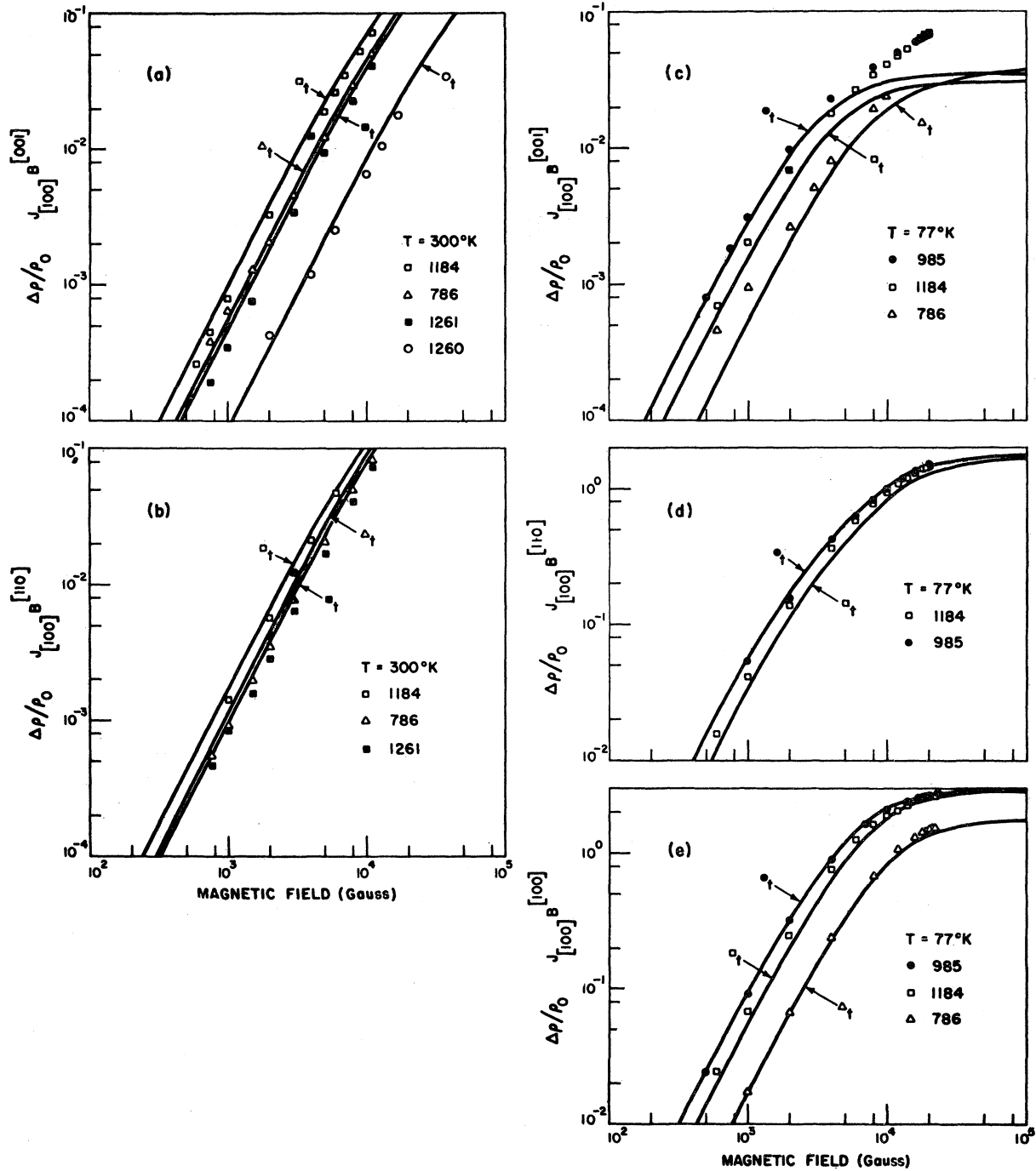


FIG. 4. Calculated and experimental field dependence of the magnetoresistance $\Delta\rho/\rho_0$. The solid lines are theoretical, the points experimental. Each theoretical line is labeled with the same symbol with a subscript *t* as the experimental points for the corresponding sample. See Table I for sample information. (a) $T = 300^\circ\text{K}$, $J_{[100]}B^{[001]}$; (b) $T = 300^\circ\text{K}$, $J_{[100]}B^{[110]}$; (c) $T = 77^\circ\text{K}$, $J_{[100]}B^{[001]}$; (d) $T = 77^\circ\text{K}$, $J_{[100]}B^{[110]}$; (e) $T = 77^\circ\text{K}$, $J_{[100]}B^{[100]}$.

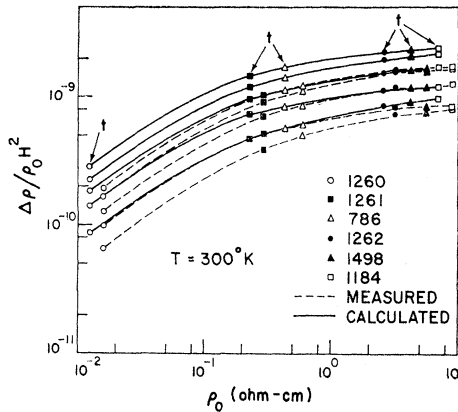


FIG. 5. Calculated and experimental values of several of the low-field magnetoresistance coefficients as a function of the sample resistivity. Each symbol represents a particular sample, the columns of points labeled t represent the calculated values, the other columns of points are experimental. In decreasing order of the magnetoresistance coefficients the current and magnetic directions are: (1) $J_{[100]}B^{[001]}$; (2) $J_{[1\bar{1}0]}B^{[110]}$; (3) $J_{[100]}B^{[110]}$; (4) $J_{[110]}B^{[110]}$; (5) $J_{[100]}B^{[001]}$, respectively, for both the experimental and calculated curves. See Table I for the sample information.

B was calculated considering the electrically active impurities to be only 20% deionized. It is to be noted that the experimental point is between the two results. These calculations indicate that the neutral impurity

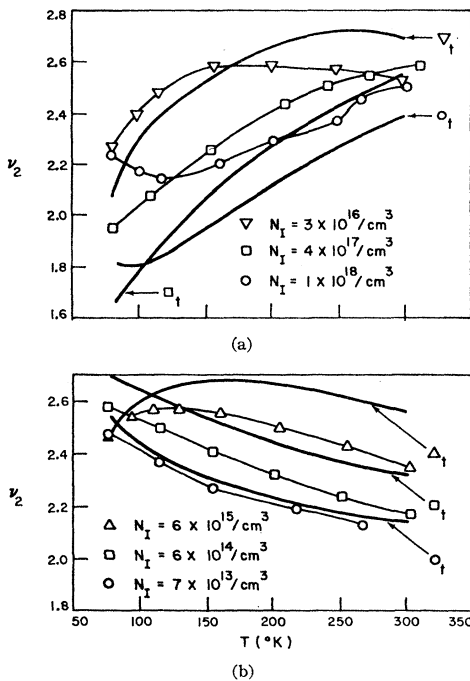


FIG. 6. ν_2 (see text) versus temperature for samples with various ionized impurity concentrations. The points are the experimental data from Goldberg and Howard (Ref. 5). The solid curves are calculated using the values measured by Goldberg *et al.* for the ionized impurity concentrations and considering $N_n=0$. The N_I for each of the samples is indicated on the figure.

scattering is overestimated by the Erginsoy formula [Eq. (5)]. It is apparent that the temperature region between 15 and 25°K would be useful for studying the relative effectiveness of ionized and neutral impurity scattering in Ge. This figure indicates that $\tau_l/\tau_t=12$ for ionized impurities is a reasonable value and that neutral impurity scattering is approximately isotropic. Dakhovskii³³ has recently published calculations using the experimental data of Fig. 7(b) and a theory put forth by Samoilovich *et al.*³⁴ for the ionized impurity scattering. Samoilovich *et al.* conclude that the scattering anisotropy τ_l/τ_t falls off at low-electron energies. Our calculations indicated however that this effect

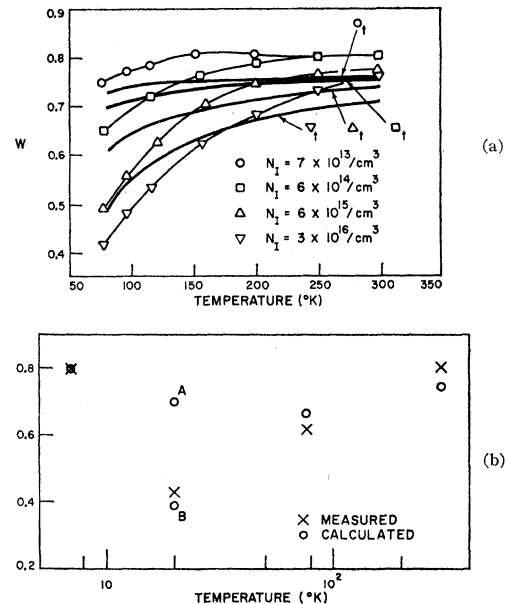


FIG. 7. The ratio W (see text) versus temperature. (a) From Goldberg and Howard (Ref. 5). The points are their experimental values. The solid curves are calculated using their measured values for the ionized impurity concentrations and considering $N_n=0$. The N_I for each of the samples is indicated in the figure. (b) From Laff and Fan (Ref. 6). The crosses are their experimental values, the circles are calculated points (see text).

occurred at such a low energy that it could be neglected. Supporting the neglect of this effect is the fact that their curve of W versus T did not drop anywhere near as low as point B in this figure.

That it is difficult to estimate the anisotropy from high-field magnetoresistance data, which is sometimes done,² is shown again in Fig. 8. Here we have plotted the saturation values of the longitudinal magnetoresistance for the [100] and [111] directions as a function of ionized impurity concentration. Consideration

³³ I. V. Dakhovskii, *Fiz. Tverd. Tela* 5, 2332 (1963) [English transl.: *Soviet Phys.—Solid State* 5, 1695 (1964)].

³⁴ A. G. Samoilovich, I. Y. Korenblit and I. V. Dakhovskii, *Dokl. Akad. Nauk SSSR* 139, 355 (1961) [English transl.: *Soviet Phys.—Dokl.* 6, 606 (1962)].

of Fig. 8 would indicate that any value of the mass anisotropy from longitudinal magnetoresistance measurements should be considered as only a rough guide, especially when one realizes that neutral impurities would raise and any compensation of the ionized impurities would lower the calculated saturation values of the magnetoresistance shown in the figure. A different τ_l/τ_t for the lattice scattering would also affect the results.

CONCLUSIONS

It is apparent that calculations using a sum of energy-dependent anisotropic collision mechanisms can be used to fit galvanomagnetic data in *n*-type germanium with good accuracy. This is true with regard to temperature dependences, orientation effects, magnetic field effects and the effects of varying the impurity concentrations in the material. Particularly, it does not yet seem necessary to invoke such exotic effects as a failure in the relaxation time approximation or failures of the symmetry conditions. Discrepancies between theory and experiment in the most commonly used regions of temperature, impurity concentration and field, are more plausibly regarded as due to inexact values for the constants which go into the calculations or nonuniformities in the material. It has been pointed out that at high magnetic fields nonuniformities in impurity concentrations can lead to some rather strange phenomena.³⁵ In addition at high temperatures (perhaps even at 300°K) it is well known that optical phonons can be expected to have a significant effect on the mobility. And at high-impurity concentrations we

³⁵ C. Herring, J. Appl. Phys. **31**, 1939 (1960).

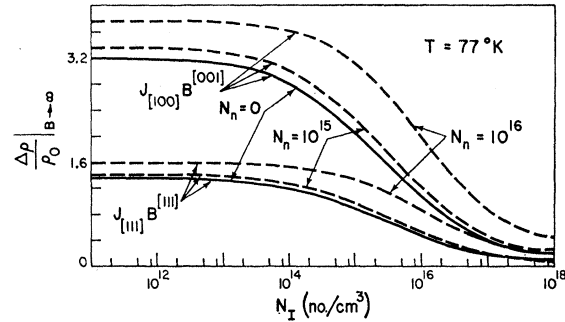


FIG. 8. Calculated longitudinal magnetoresistance as a function of the ionized impurity concentration at 77°K for two directions of the sample current.

expect that electron-electron collisions would begin to play a significant role.

It has been suggested²⁸ that measurements of the galvanomagnetic phenomena under extreme conditions such that a single type of scattering mechanism is dominant would be useful. Calculations of the type we have performed here are not difficult or particularly time consuming using a computer and we believe should be used for further comparisons of theory and experiment in the classical region of galvanomagnetic phenomena.

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