Stimulated Phonon Emission by Supersonic Electrons and Collective Phonon Propagations

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The emission of phonons by electrons with supersonic drift velocities is examined, and the crystal momentum transferred to a piezoelectric phonon band is determined. This momentum is transferred principally to a band of phonons which are in a cone centered about the direction of electron drift and of frequencies approximately 10^{10}–10^{11} cps. This momentum-transfer process is stimulated in the sense that it amplifies any variations in the phonon momentum distribution. In particular, it will amplify a collective wave in the "phonon gas" which corresponds to a sound wave in a particle gas. The collective wave is described in terms of a local particle drift velocity and a local temperature. In a phonon gas, temperature plays the role that pressure plays in a particle gas. By the use of energy and momentum conservation equations, it is shown that such a collective wave is possible when the usual large losses are overcome. The derived gain and expected loss of the collective wave are shown to be comparable in CdS at room temperature. The current saturation observed when the electron drift velocity exceeds the sound velocity appears in the self-consistent solutions of the electron and phonon systems.

I. INTRODUCTION

THE observation of an anomalously slow phonon propagation has recently been reported in CdS crystals under conditions of acoustic gain.[1] This has been interpreted as a collective phonon propagation similar to the collective phonon wave which describes second-sound propagation in super-fluid helium. In nonconducting solids the collective phonon wave in a "phonon gas" could be considered analogous to a sound wave in a molecular gas. The collective wave would then be a density fluctuation in the phonon distribution, much like a sound wave is a particle density fluctuation in a particle gas. In a macroscopic description an increase in phonons gives rise to an increase in temperature. Temperature plays a role in phonon transport similar to pressure in particle gas transport. The collective phonon wave could be described equally well as a temperature fluctuation wave.

The propagation of such a collective phonon wave in a solid has been examined in some detail.[2] It was found that umklapp collisions (which have no counterpart in molecular gases) would destroy the crystal momentum of the phonons and damp the collective propagation. In solids, at all but lowest temperatures, this damping term is much larger than the harmonic term, and diffusion, (i.e., normal thermal conductivity) rather than harmonic propagation, occurs. At very low temperatures the umklapp collision rate is greatly reduced and collective waves may be possible, but the reduction in normal collisions would then tend to disrupt the waves. Normal collisions are necessary to maintain the collective nature of the wave, just as particle collisions are necessary for sound propagation in particle gases. These two conditions, long umklapp relaxation time and short normal relaxation time, are extremely difficult to satisfy simultaneously.

In a semiconducting solid, particularly those with piezoelectric phonon modes, one can transfer energy and crystal momentum from electrons to the phonons. It has been shown by Hutson, McFee, and White[3] that for ultrasonic waves, i.e., low-frequency phonons, the internal losses of the phonons can be more than compensated for when $v_d > v_s$. Here, v_d is the electron drift velocity and v_s the sound velocity. If such a mechanism could transfer crystal momentum to phonons capable of participating in a collective wave, which would overcome the internal losses, it would be possible to propagate the type of collective phonon wave discussed. Whether or not a particular phonon mode could participate in a collective wave would depend on how strongly coupled the mode was to the bulk of the collective wave by the mixing collisions.

The transfer of momentum or energy to phonons has been examined by White[4] for low frequencies, i.e., $q\Lambda < 1$, where q is the phonon wave vector and Λ is the electron free path. In this range, the electrons are able to equilibrate to their local surroundings. The best description of this system is in terms of the classical fields and currents used by White. For higher frequency phonons, where $q\Lambda > 1$, a better description of the system involves quantum mechanical electron and phonon states which undergo collisions. Pippard[5] has discussed the interaction mechanism for high-frequency phonons from a degenerate electron band with imposed drift velocity, $v_d > v_s$. Spector[6] has derived the amplification of high- and low-frequency phonons using a free-electron gas model for conduction of electrons in metals. Neither of these approaches is ideally suited to examine the highly nondegenerate-electron distribution which exists

[1] H. Kroger, E. W. Prohofsky, and R. W. Damon, Phys. Rev. Letters **11**, 246 (1963).

[2] E. W. Prohofsky and J. A. Krumhansl, Phys. Rev. **133**, A1403 (1964). E. W. Prohofsky, thesis, Cornell University, 1963 (unpublished).

[3] A. R. Hutson, J. H. McFee, and D. L. White, Phys. Rev. Letters **7**, 237 (1961).

[4] D. L. White, J. Appl. Phys. **33**, 2547 (1962).

[5] A. B. Pippard, Phil. Mag. **8**, 161 (1963).

in CdS under conditions of acoustic gain. Paranjape[7] has discussed the amplification of high-frequency sound waves using a quantum mechanical approach suitable to nondegenerate electron distributions.

This paper will examine the transfer of momentum to high-frequency phonons using a golden-rule scattering integral technique suitable to nondegenerate-electron distributions in direct semiconductors. This momentum-transfer mechanism will, in turn, be used to evaluate the momentum transferred to collective phonon disturbances. It is believed that the high-frequency phonons will be most important in the collective wave behavior for two reasons: The number of higher frequency modes is much greater than that of low-frequency modes, and the mixing collision rate should be much less for low frequencies. This drop-off of mixing collision rate will supply a lower limit to the phonon frequencies involved in the collective wave. Those modes which have a lifetime longer than the period of the collective wave cannot be equilibrated within the collective wave.

In Sec. II, the first-order stimulated phonon emission and momentum-transfer rates are evaluated for a model in which there is a piezoelectric phonon mode and a nearly free, nondegenerate-electron band.

The results of Sec. II show that the electrons are strongly coupled to phonons of moderately high frequencies, but the interaction drops very rapidly beyond a certain cutoff frequency due to the mass mismatch between electrons and high-energy phonons. Energy and momentum transfer, upon collision, is most efficient for particles of roughly the same mass. An apparent phonon mass is

$$m_p = h\nu/v_s^2, \qquad (1.1)$$

where h is Planck's constant, ν the frequency, and v_s the velocity of sound. The frequency most strongly coupled to electrons is that for which $m_p = m_e$, where m_e is the electron mass. This frequency is roughly $\nu = 10^{11}$ cps. The actual peak frequency found in Sec. II is very close to this value, but varies with temperature as it is effected by such factors as the available phonon and electron phase space and the average electron energy. The frequency dependence of the efficiency of energy and momentum transfer is greatly modified when the coupling has a frequency dependence, as is the case in deformation potential scattering.

The cutoff of the interaction for high-phonon frequencies enhances the probability of collective wave propagation. The phonon momentum losses rise sharply with frequency. Pumping momentum into a low-frequency band can give rise to a situation similar to that encountered in anomalous thermoelectric power.[8] The situation is illustrated in Fig. 1. A low-frequency band of phonons becomes "hot" with respect to the bulk of

[6] H. N. Spector, Phys. Rev. 127, 1084 (1962).
[7] B. V. Paranjape, Phys. Letters 5, 32 (1963).
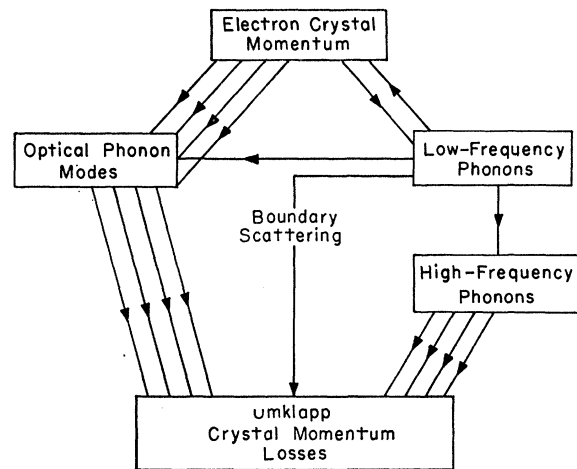[8] C. Herring, Phys. Rev. 96, 1163 (1954).

FIG. 1. Crystal momentum relaxation flow chart. Electrons accelerated by an electric field lose crystal momentum in collisions with optical and low-frequency acoustic phonons. For low electron drift velocities the relaxation to optical modes dominates. When $v_d > v_s$ the coupling to low-frequency acoustic modes becomes dominant. This low-frequency band can become hot since the momentum flowing into it can be greater than its losses to the high-frequency modes.

the thermal phonons. The losses from this band are slow because of a bottleneck which keeps these phonons from equilibrating with higher frequencies. The high-frequency umklapp relaxation time is very fast, and the thermal phonons remain uneffected in the propagation of the lower-frequency band.

The emission process for $v_d > v_s$ is somewhat similar to Čerenkov radiation. As described in Sec. II, it, too, is limited to a forward cone centered about the direction of electron drift. The active phonon band which may support collective oscillations is a medium-frequency band of phonons occupying a cone in phonon-phase space centered about v_d.

The expression for the crystal-momentum transfer, found in Sec. II, is only valid in a region where $v_d \approx v_s$. The use of perturbation theory appears valid, as the dimensionless coupling parameter described by Froelich[9] is

$$\alpha = \left[\frac{\hbar}{2\rho\nu}\right]^{1/2} \left(\frac{ep}{d}\right)[\hbar\nu]^{-1} = \left[\frac{ep}{d}\right]\left[\frac{\pi}{\rho\hbar}\right]^{1/2} \nu^{-3/2}, \quad (1.2)$$

where ρ is the lattice density, e the electronic charge, p the proper piezoelectric constant, and d a dielectric constant. The parameter is certainly small for frequencies of the order of 10^{10}–10^{11} cps. For low values of electric field, the electron relaxation is mainly due to collisions with optical modes. This relaxation time is roughly 10^{-11}–10^{-12} sec, and the inequality $q\Lambda > 1$ is also satisfied for phonon frequencies of the order of 10^{11} cps.

The problem of the normal collision rate is discussed in Sec. III. The collisions between phonon states in the active band are primarily due to higher-order electron

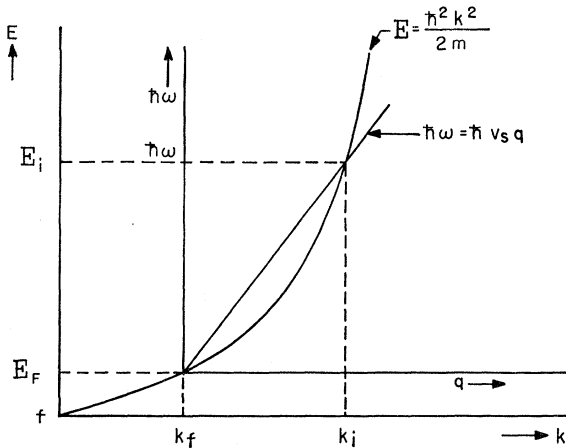[9] H. Froehlich, Advan. Phys. 3, 325 (1954).

FIG. 2. Energy and momentum conserving phonon emission. The phonon dispersion relation $\hbar\omega = \hbar v_s q$ is superimposed on the electron dispersion relation $E = \hbar^2 k^2/2m$. A possible phonon (ω, q) can be emitted in an energy and momentum conserving process if (1) $E_i = \hbar\omega + E_F$, and (2) $k_i = q + k_F$, where i and F refer to the initial and final electron states. All such double intersections insure that these two conditions are satisfied. The double intersections cease when the line is tangent to the curve when $E_i = E_F$ and the velocity of this state is equal to the vleocity of sound.

phonon collisions. The electrons, in this case, play a role very similar to that of the rotons in liquid helium. The same conditions which bring about large momentum transfer $(v_d > v_s)$ cause this second-order collision mechanism to have a focusing effect on the phonons. That is, it tends to make the final phonon state more likely to be within the gain cone than the initial phonon state. The inability of the electron to transfer large amounts of energy to the phonons tends to keep the final state within the frequency limit of the active phonon band.

In Sec. IV, the equations for energy and crystal momentum conservation are applied to the active phonon band; the equation of conservation of momentum is applied to the electrons; and the interaction developed in Sec. II is used to couple the equations. A zero-order solution exists which describes the equilibrium distortion in the electron distribution (current), and the phonon distribution (heat flow) under the high electric field conditions. This solution shows a marked current saturation similar to that first reported by Smith.[10] In addition, solutions for displacements about this equilibrium exist which correspond to the collective propagation discussed. Throughout this section the excess energy stored in the electron distribution is ignored compared to that stored in the phonon distribution. Plasma effects are ignored because the electron density is assumed very small.

In Sec. V, numbers such as electron mass, piezoelectric constant, etc., are substituted into the equations that have been developed. The gain of the system is then compared to the expected phonon crystal momentum losses.

[10] R. W. Smith, Phys. Rev. Letters 9, 87 (1962).

II. STIMULATED PHONON EMISSION BY ELECTRONS IN PIEZOELECTRIC MATERIALS

In the quantum limit, i.e., $q \cdot \Lambda > 1$, interactions involving electrons and phonons must conserve energy and crystal momentum modulo $\hbar\mathbf{G}$ ($\mathbf{G}$ being a reciprocal lattice vector). The possible initial and final states in which an electron emits or absorbs a single phonon in a normal collision (i.e., $\hbar\mathbf{G} = 0$) can be determined by using a diagram similar to the one in Fig. 2. The curved line represents a dispersion relation for an electron in one dimension and is assumed to be a quadratic function of the momentum. The curve in Fig. 2 could be related to excited electrons in a direct material such as CdS. The straight line represents the acoustic phonon-dispersion relation; the slope equals the velocity of sound.

If the origin of the phonon line were placed at any point on the electron curve, a second intersection by the phonon line would imply conservation of energy and momentum between the initial and final electron states determined by the two intersections. All possible interactions correspond to some possible placements of the curves that result in two intersections. It can be seen that such intersections cease when the phonon line is just tangent to the electron curve. At this point, the energy and momentum of the emitted phonon have decreased to zero. The velocity of this limiting electron state is equal to the velocity of sound. For emission, the initial electron state must be on the high-energy side of the tangent point (i.e., $v_e > v_s$), and the final state must be on the low-energy side (i.e., $v_e < v_s$). For absorption, the initial and final states are reversed. In the case of a quadratic-electron-dispersion relation, the initial and final $\mathbf{k}$ values are equidistant from $\mathbf{k}'$, the location of the tangent point.

In a three-dimensional solid, the dispersion relation for electrons is a bowl-shaped hypersurface in four-
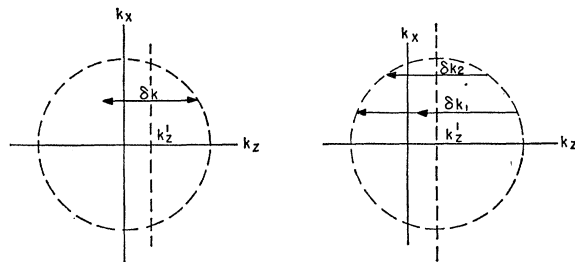

FIG. 3. Electron population inversion with respect to phonon emission. (This mechanism has been discussed by Pippard, Ref. 5.) The dashed circle represents phenomenologically the electron distribution in k space. The dashed line at k_z' represents those states whose velocity is equal to the sound velocity in the z direction. Emission or absorption of a phonon with $q = -\delta k$ involves initial and final states which are symmetrically spaced about the plane k_z'. In the figure on the left the electron distribution is at rest and there are fewer electrons on the high-energy side of k_z'. In the figure on the right the electron drift velocity is equal to the sound velocity and there are as many electrons on the high-energy side of k_z'. For greater drift velocities there will be more electrons on the high-energy side giving rise to a population inversion with respect to emission of phonons in the z direction.
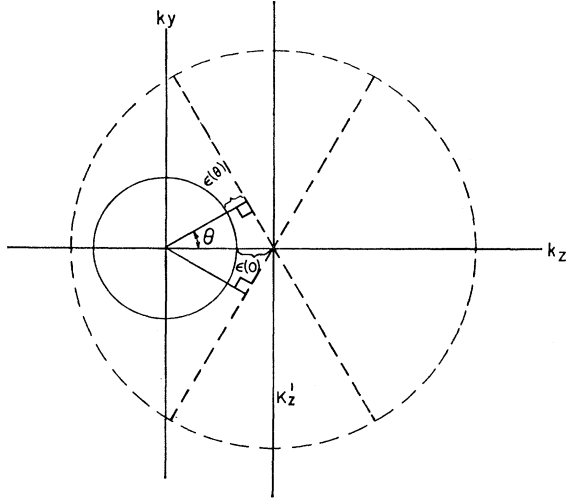
FIG. 4. The extent of population inversion for phonons not parallel to v_d. The solid-line sphere corresponds to the velocity of sound in any direction. The dashed sphere represents the electron distribution with a large drift velocity $v_d = v_s + \epsilon_0$. Phonons oriented at an angle θ will see a population inversion for $\epsilon(\theta) > 0$, where $\epsilon(\theta) = v_d \cos\theta - v_s(\theta)$.

dimensional space. Since only one phonon is involved in these processes, we may still use a one-dimensional phonon-dispersion line to examine the emission of phonons with one orientation. The origin of the phonon line is placed at a point on the electron-dispersion surface, and a second intersection implies a possible momentum and energy conserving process. The line and hypersurface will have a tangent point which will play a role similar to the tangent point in the one-dimensional analysis.

Based on the above discussion, it can be seen that a surface could be drawn in electron **k** space perpendicular to the direction of the emitted or absorbed phonon wave vector **q** at **k'**, where $v_e(\mathbf{k'}) = v_s$. All emission processes have the initial electron state on the high-energy side of this surface and the final state on the low-energy side. The change in electron wave vector, $\Delta\mathbf{k} = -\mathbf{q}$, is exactly bisected by this surface when the electron dispersion relation is quadratic in the momentum.

Pippard[5] has pointed out, in arguments based on a diagram similar to that of Fig. 3, that the phonon-emission rate could be greatly enhanced if the electron-drift velocity v_d were made greater than the sound velocity. In such a case, the centroid of the electron distribution in **k** space is outside the surface described in the preceding paragraph. Therefore, more electrons in states of high energy are opposite holes on the other side of this surface, and electron transition to the lower states is very probable. One can speak of a population inversion in **k** space with respect to phonon emission.

The emission of phonons for a given orientation in space will be shown as a first approximation to be proportional to a parameter ϵ, where

$$\epsilon = v_d - v_s, \tag{2.1}$$

where v_d is the electron drift velocity and v_s the sound velocity. For different phonon orientations, as those whose wave vector make an angle θ with the direction of electron flow, the emission is proportional to

$$\epsilon(\theta) = v_d \cos\theta - v_s. \tag{2.2}$$

One can define an angular cone in phonon q space in which phonon emission will be enhanced. This is demonstrated in Fig. 4.

All the collisions discussed are normal, and the effect of such collisions is a transfer of momentum from one system to the other. The population inversion, when $v_d > v_s$, favors momentum transfer to the phonons in the "gain cone" from the electrons. For well-defined electron and phonon states, the rate of phonon emission is

$$\left.\frac{\partial N_q}{\partial t}\right|_{\text{col}} = \frac{2\pi}{\hbar} \sum_{kk'} g^2 \{[N(q)+1]f(k)[1-f(k')]$$
$$- N(q)f(k')[1-f(k)]\}$$
$$\times \delta(\mathbf{k'}-\mathbf{q}-\mathbf{k})\delta(\mathscr{E}_{k'}-\hbar\omega-\mathscr{E}_k), \tag{2.3}$$

where $N(q)$ is the occupation number of phonon mode q, $f(k)$ is the occupation number of the initial electron state, and $f(k')$ is the occupation number of the final electron state. The factor g is the coupling constant, or matrix element, for the coupling of the electron transition and the phonon. This energy can be written as

$$g = \frac{xep}{d} = \left(\frac{\hbar}{2\rho\omega_q}\right)^{1/2} \frac{ep}{d}, \tag{2.4}$$

where x is a normalized phonon displacement, e the charge on the electron, p the appropriate component of the piezoelectric tensor parallel to the phonon displacement, d the appropriate dielectric constant, ρ the lattice density, and ω_q the frequency of the phonon mode **q**.

For high temperatures we assume $N(q) > 1$. This is equivalent to ignoring the spontaneous emission in comparison to the stimulated emission.

$$\left.\frac{\partial N}{\partial t}\right|_{\text{col}} \approx \frac{2\pi}{\hbar} \sum_{kk'} g^2 N[f(k) - f(k')]$$
$$\times \delta(\mathbf{k'}-\mathbf{q}-\mathbf{k})\delta(\mathscr{E}_{k'}-\hbar\omega-\mathscr{E}_k). \tag{2.5}$$

For a quadratic electron dispersion relation,

$$k = (m^*/\hbar)v_s + (q/2) + k_{x,y}, \tag{2.6}$$

$$k' = (m^*/\hbar)v_s - (q/2) + k_{x,y}. \tag{2.7}$$

For drift velocities near the velocity of sound let

$$v_d = v_s + \epsilon. \tag{2.8}$$

One may assume that the electron distribution under high field is a Fermi distribution centered at $k_z = (m^*/\hbar)v_d$. That is,

$$f[(m^*/\hbar)v_d] = f_0(0), \tag{2.9}$$

where $f_0(k)$ is the equilibrium Fermi distribution. Substituting Eq. (2.8) into Eq. (2.5), and summing over k_x', k_y', k_z' to eliminate the $\delta(\mathbf{k})$ function,

$$\frac{\partial N(q)}{\partial t}\bigg|_{\text{col}} = \frac{2\cdot 2\pi}{8\pi^3\hbar}\sum_k g^2 N(q)$$

$$\times\left[f_0\left(\frac{q}{2}-\frac{m^*}{\hbar}\epsilon+k_{x,y}\right)-f_0\left(\frac{q}{2}+\frac{m^*}{\hbar}\epsilon+k_{x,y}\right)\right]$$

$$\times\delta(\mathcal{E}_{k'}-\hbar\omega-\mathcal{E}_k)\approx\frac{-1}{2\pi^2\hbar}\sum_k g^2 N(q)$$

$$\times\frac{\partial f_0}{\partial E}\,\text{grad}_k E\cdot\frac{m^*}{\hbar}\cdot 2\epsilon\delta(\mathcal{E}_{k'}-\hbar\omega-\mathcal{E}_k) \quad (2.10)$$

as f_0 has reflection symmetry.

Summing over k_z to eliminate $\delta(\mathcal{E})$,

$$\frac{\partial N(q)}{\partial t}\bigg|_{\text{col}} = \frac{1}{2\pi^2\hbar}\int_{k_{x,y}} g^2 N(q) f_0\left(\frac{q}{2}+k_{x,y}\right)$$

$$\times\left[1-f_0\left(\frac{q}{2}+k_{x,y}\right)\right]\frac{1}{KT}\frac{m^*}{\hbar}2\epsilon dk_x dk_y. \quad (2.11)$$

After replacing the remaining sums by integrals and carrying out the integration over $dk_x dk_y$ we obtain

$$\frac{\partial N(q)}{\partial t}\bigg|_{\text{col}} = \frac{1}{2\pi}\frac{1}{\hbar^4}m^{*2}g^2 N(q)\epsilon$$

$$\times\left[1-\tanh\left(\frac{\hbar^2 q^2}{16m^*KT}-\frac{E_F}{2KT}\right)\right]. \quad (2.12)$$

The total crystal momentum transferred via this process is

$$\frac{\partial}{\partial t}\langle\hbar\mathbf{q}\rangle\bigg|_{\text{col}} = \sum_q \hbar\mathbf{q}\frac{\partial N(q)}{\partial t}\bigg|_{\text{col}}. \quad (2.13)$$

Replacing the summation by an integral, Eq. (2.13) becomes

$$\frac{\partial\langle\hbar\mathbf{q}\rangle}{\partial t}\bigg|_{\text{col}} = \frac{1}{16\pi^4\hbar^4}\int m^2 g^2 \hbar q\,\cos\theta N(q)\epsilon(\theta)$$

$$\times\left[1-\tanh\left(\frac{\hbar^2 q^2}{16m^*KT}-\frac{E_F}{2KT}\right)\right]2\pi q^2\sin\theta d\theta dq. \quad (2.14)$$

The interesting range of θ runs from zero to θ', where θ' describes a gain cone in $\mathbf{q}$ space, and

$$\epsilon(\theta')=(v_s+\epsilon)\cos\theta'-v_s=0,$$
$$\cos\theta'=\frac{1}{1+\epsilon/v_s}. \quad (2.15)$$

It is assumed that the value $\epsilon/v_s\ll 1$ to be consistent with previous approximations, therefore

$$\theta'\approx(2\epsilon/v_s)^{1/2}<1, \quad (2.16)$$

and the phonons involved in the interaction do not deviate greatly in the direction of propagation. Therefore, to first order, one can use a single component of the piezoelectric tensor in Eq. (2.4) for all the phonons in the cone.

One can express the effects of the angular integration of Eq. (2.14) in terms of an averaged value of $\epsilon(\theta)$.

$$\bar{\epsilon}=2\pi\int_0^{\theta'}\cos\theta\epsilon(\theta)\,\sin\theta d\theta\approx 2\pi\frac{\epsilon^2}{v_s}, \quad (2.17)$$

to lowest order in ϵ/v_s. Equation (2.14) then becomes

$$\frac{\partial\langle\hbar q\rangle}{\partial t}\bigg|_{\text{col}} = \frac{1}{8\pi^2}\frac{1}{\hbar^4}m^{*2}g^2\frac{\epsilon^2}{v_s}\int N(q)\hbar q$$

$$\times\left[1-\tanh\left(\frac{\hbar^2 q^2}{16m^*KT}-\frac{E_F}{KT}\right)\right]q^2 dq. \quad (2.18)$$

For a nondegenerate electron gas the term in the square brackets can be simplified as

$$1-\tanh\left(\frac{\hbar^2 q^2}{16m^*KT}-\frac{E_F}{2KT}\right)$$

$$\approx\frac{\hbar^3 n}{2(2m^*KT)^{3/2}}\frac{1-\tanh(\hbar^2 q^2/16m^*KT)}{1+\tanh(\hbar^2 q^2/16m^*KT)}, \quad (2.19)$$

where n is the electron density. A further simplification occurs if we define a temperature T_s by

$$KT_s=\tfrac{1}{2}m^*v_s^2; \quad (2.20)$$

and Eq. (2.18) becomes

$$\frac{\partial\langle\hbar q\rangle}{\partial t}\bigg|_{\text{col}} \approx\frac{\pi}{8}\frac{1}{\hbar}m^{*2}g^2 v_s^3(KT_s KT)^{-3/2}\frac{\epsilon^2}{v_s}n\int \hbar q N(q)$$

$$\times\frac{[1-\tanh(\hbar^2 v_s^2 q^2/32KT_s KT)]q^2 dq}{1+\tanh(\hbar^2 v_s^2 q^2/32KT_s KT)}. \quad (2.21)$$

We now allow for a distortion in the phonon field characterized by the phonons within the gain cone having a net drift velocity λ. In this case, the distorted phonon probability distribution becomes

$$N(q)=N_0(q)+\lambda\cdot\hbar\mathbf{q}\frac{1}{KT}N_0(N_0+1), \quad (2.22)$$

where N_0 refers to the equilibrium Bose distribution. It is now possible to separate the momentum transfer process into two parts, i.e.,

$$\frac{\partial\langle\hbar q\rangle}{\partial t}\bigg|_{\text{col}} = R_1\frac{\epsilon^2}{v_s}+R_2\frac{\epsilon^2\lambda}{v_s^2}, \quad (2.23)$$

where

$$R_1 = \frac{\pi^2}{2} \frac{m^{*2}}{\rho v_s} \left(\frac{ep}{d\hbar}\right)^2 \left(\frac{T}{T_s}\right)^{3/2} n$$

$$\times \int x^2 \frac{1}{e^x-1} \frac{1-\tanh[(1/32)(T/T_s)x^2]dx}{1+\tanh[(1/32)(T/T_s)x^2]}$$

$$= \frac{\pi^2}{2} \frac{m^{*2}}{\rho v_s} \left(\frac{ep}{d\hbar}\right)^2 \left(\frac{T}{T_s}\right)^{3/2} n \int r_1(x)dx, \qquad (2.24)$$

$$R_2 = \frac{\pi^2}{8} \left(\frac{ep}{d\hbar}\right)^2 \frac{m^{*2}}{\rho v_s} \left(\frac{T}{T_s}\right)^{3/2} n$$

$$\times \int x^3 \operatorname{csch}^2 \frac{x}{2} \frac{1-\tanh[(1/32)(T/T_s)x^2]dx}{1+\tanh[(1/32)(T/T_s)x^2]}$$

$$= \frac{\pi^2}{2} \left(\frac{ep}{d\hbar}\right)^2 \frac{m^{*2}}{\rho v_s} \left(\frac{T}{T_s}\right)^{3/2} n \int r_2(x)dx, \quad (2.25)$$

where

$$x = \hbar v_s q / KT. \qquad (2.26)$$

The first term on the right-hand side of Eq. (2.23) describes spontaneous emission—spontaneous in the sense that the drifting electron distribution loses momentum to a nondrifting phonon distribution. The second term is a stimulated-emission term—stimulated in the sense that momentum is transferred at a rate determined by the net momentum present in the phonon system. This stimulated term describes a process whose net effect is a linear amplification of the net local momentum of the phonon distribution. The function $r_2(x)$ is greater than $r_1(x)$, and the "stimulated" emission term can be comparable to the "spontaneous" one.

The function, $(T/T_s)^{3/2} r_2(\nu)$, is plotted in Fig. 5 for several values of T/T_s. The values of T/T_s correspond to helium, nitrogen, and room temperature for free electron masses, and room temperature for an effective mass of $0.2m_0$. At low-phonon frequencies $q\Lambda < 1$ this quantum approach is not valid. A more valid approach at these low frequencies is White's,[4] as previously discussed. The increase in the gain for phonons (as T/T_s increases) reflects the increased stimulation due to the presence of greater numbers of phonons. The shift in the frequency of maximum gain reflects the fact that the electrons have more energy and are more capable of emitting higher energy phonons.

III. NORMAL PHONON COLLISIONS AND HIGHER ORDER ELECTRON PHONON COLLISIONS

Phonons undergo collisions with other phonons via the anharmonic terms of the interatomic forces. Collisions in which no net crystal momentum is lost are called normal collisions. Such normal collisions are necessary for the propagation of a collective wave and establishing a local temperature. The collective wave
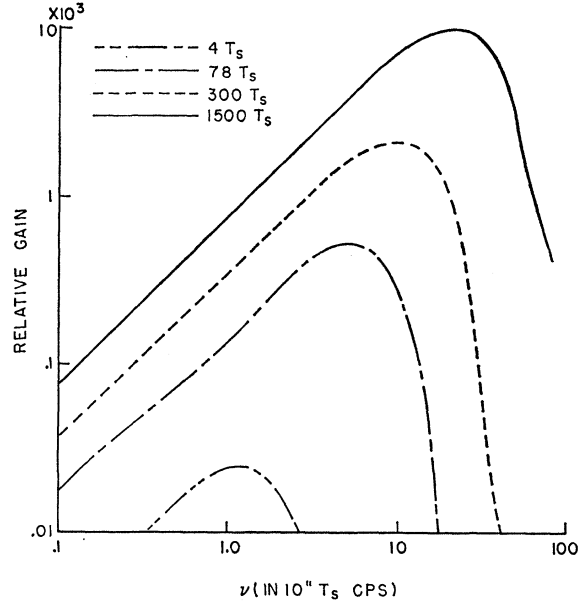

FIG. 5. Relative stimulated momentum gain as a function of phonon frequency. The function $r_2(\nu)$ is plotted for $T=4T_s$, $78T_s$, $300T_s$, and $1500T_s$. These temperatures correspond roughly to liquid helium, liquid nitrogen, and room temperature for $m^* \approx m_e$, and room temperature for $m^* \approx 0.2m_e$.

examined in this paper, however, must be propagated within a restricted band of lower-than-thermal frequencies, since the momentum loss of thermal phonons is much greater than the momentum gain available to the system. As pointed out in Sec. I, only the slow rate of equilibration of these low frequencies to the thermal reservoir makes the collective wave possible. Normal collisions in which momentum is transferred to phonon states outside of this restricted band will dissipate the momentum of the collective wave and give rise to damping. Instead of colliding among themselves, the low-frequency phonons within the restricted band are more likely to collide via the anharmonic perturbations with thermal-frequency phonons due to the much larger number of available states at higher frequencies. These collisions play the same role in this restricted-band collective wave as do the umklapp collisions in thermal phonon collective waves. The rate of these processes proceeding in the absence of carriers, i.e., dark attenuation in photoconductors, will be an estimate of the momentum losses from the restricted band to the thermal reservoir.

The slow rate of relaxation of the hot phonon band gives rise to a relaxation bottleneck. The phonons within this hot band are more likely to undergo further collisions with electrons before relaxation can occur. Successive electron-phonon collisions or higher order electron-phonon collisions are the fastest mixing collisions in the band and will set up a somewhat distorted equilibrium within the hot phonon band. The essential mixing collisions necessary for the propagation of the
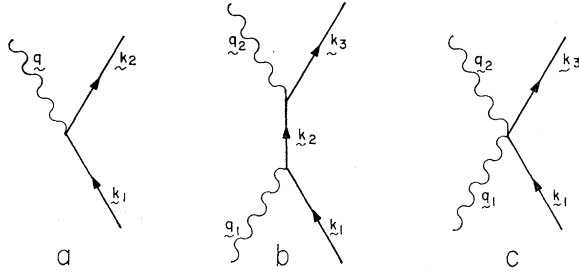
FIG. 6. (a) First-order phonon emission. (b) Phonon absorption followed by emission. The intermediate state need not conserve energy if it is short-lived. (c) Phonon emission and absorption. The existence of an intermediate state is not specified.

collective wave will be those diagrammed in Figs. 6(b) and 6(c). There will be some contribution from collisions [Fig. 6(b)] in which the intermediate state need not conserve energy if it is short lived.

The effect of large electronic drift velocities on these second-order electron-phonon processes can be examined qualitatively by describing the processes in a manner analogous to the emission problem of Sec. II. The change in electronic configuration in these second-order processes is the same as it would be if the process were an emission or absorption of an apparent phonon with energy and momentum equal to the differences in energy and momentum between the initial and final phonon states. The enhancement of a particular inter-action occurs where $v_d > v_a$, where v_a is the apparent emitted phonon velocity and is

$$v_a = \frac{\Delta\omega}{\Delta q} = \frac{v_s}{1+(q_1/\Delta q)[1-\cos(\varphi-\theta)]}, \quad (3.1)$$

where q_1, Δq, φ, and θ are illustrated in Fig. 7. A gain cone extending over an angle φ' for these second-order processes can be defined where

$$\cos\varphi' = \cos\theta' \frac{1}{1+(q_1/\Delta q)[1-\cos(\varphi'-\theta)]}, \quad (3.2)$$

where $\cos\theta'$ is given in Eq. (2.15). Because of the elec-tron population inversion, all interactions involving states q_2 whose vectors terminate on the ω_2 line inside φ' will tend to be apparent emissions, since the net interaction is more probable if q_2 is the final state and q_1 the initial state. For q_2 terminating outside φ' the net interaction favors transitions with q_2' as initial state and q_1 as final state. On the basis of this analysis it is seen that the mixing collisions tend to keep the excess phonon population within the gain cone where it is amplified, rather than outside the gain cone where it could be quickly damped. Furthermore, the focusing effect of the second-order collisions contribute to the net momentum gain of the phonon distribution.

One can easily obtain a lower limit of the second-order scattering from those intermediate states k_2 [Fig. 6(b)] in which energy is conserved. The second-order matrix

element from second-order perturbation theory is

$$(k_3 q_2 | V^2 | k_1 q_1) = \sum_{k_2} \frac{(k_2 | V^1 | k_1 q_1)(k_3 q_2 | V^1 | k_2)}{E(k_1)+\hbar\omega_1-E(k_3)-i/2\Gamma}, \quad (3.3)$$

where Γ is an energy related to the finite lifetime of the intermediate state. That group of states k_2 in which energy is conserved has a resonant energy denomina tor, and the matrix element takes on a Breit-Wigner form. In the case of strongly enhanced first-order processes, the main part of the broadening will come from processes like "a" in Fig. 6. In this situation, the contribution to the scattering by the energy-conserving intermediate states will be approximately equal to the rate of first-order absorption processes. The final states of these higher order processes will lie in the same frequency band as the first-order emission processes. The contribu-tion to mixing, from processes in which the intermediate state is not an energy conserving one, is expected to be small. This is because of the small size of the parameter α defined in Eq. (1.2) which determines the relative size of terms in this perturbation expansion.

A better understanding of the relative magnitudes of the emission and absorption rates can be achieved by examination of Fig. 3. When $v_d = v_s$ the electron distribu-tion is symmetrically distributed such that the proba-bility of absorption and emission are equal. The rate of both processes is very large because the energy and momentum in both the electron and phonon systems are ideally distributed for such interactions. The net trans-fer of momentum or energy is zero as the two processes cancel each other. An increase in drift velocity gives rise
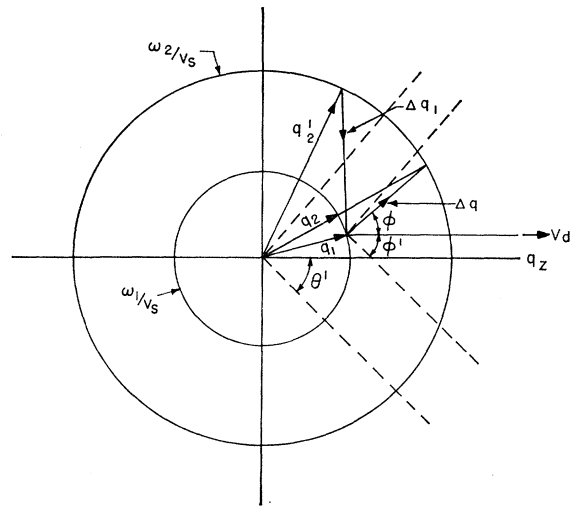

FIG. 7. Higher order electron phonon processes. These processes can be described in terms of the emission or absorption of apparent phonons. Two such processes are shown which have different changes in wave vector $q_2 - q_1$ or $q_2' - q_1$ for a fixed energy differ-ence $\omega_2 - \omega_1$. This is equivalent to a variable apparent velocity. The cone φ' is the gain cone for these apparent phonons. Emission or absorption of these apparent phonons will be more probable depending on whether q_2 is inside or outside φ'. θ' is the gain cone for first-order processes.

to a net phonon gain by increasing the emission rate relative to the absorption rate. The mixing rate, which is limited by the slower of the two processes (absorption or emission), would then be decreased due to the decrease in the absorption rate. If successive first-order processes are the dominant contribution to the mixing collisions, an increase in net gain will be accompanied by a decrease in the mixing rate. The ability to propagate a collective wave will depend on achieving a delicate balance between the gain and the mixing rate.

Higher temperatures will broaden the electron distribution and therefore reduce the sharpness of the population inversion for a given electron drift velocity. The phonon momentum gain per phonon mode actually decreases with increasing temperature [see Eq. (4.35)]. Higher temperatures will therefore tend to increase the mixing collision rate; lower temperatures will increase the gain. For a given drift velocity there is a temperature in which the two conflicting requirements for collective propagations may be simultaneously satisfied. Higher temperature will reduce the gain, and lower temperatures will reduce the mixing rate.

IV. CURRENT SATURATION AND COLLECTIVE WAVES

An equation for conservation of electron momentum can be derived from the Boltzmann transport equation and written as

$$\frac{\partial}{\partial t}\langle \hbar k_i\rangle + \frac{\partial}{\partial x_j}\langle v_j \hbar k_i\rangle - \frac{Ee}{\hbar}\left\langle \frac{\partial f}{\partial k_i}\right\rangle = \Delta \hbar q\Big|_{\text{col}}, \quad (4.1)$$

where the brackets refer to expectation values over the electron distribution. For a spatially uniform electron distribution, the second term is zero. When the drift velocity is less than the velocity of sound, the collision term can be simplified by a relaxation time approximation and becomes

$$\Delta \hbar q|_{\text{col}} = (-nmv_d/\tau_0), \quad (4.2)$$

where nm is the total electron mass per unit volume and τ_0 is the electron relaxation time. For materials such as CdS at room temperature, the electron relaxation is mainly due to collision with optical phonon modes. That part of the relaxation which is to optical modes, should remain relatively unaffected when v_d exceeds v_s. If we add the collision terms of Eq. (2.23) to represent the greatly enhanced part of the relaxation, Eq. (4.1) becomes

$$\frac{\partial}{\partial t}nmv_d - \frac{Ee}{\hbar}\left\langle\frac{\partial f}{\partial k}\right\rangle = -\frac{nmv_d}{\tau_0} - R_1\frac{\epsilon^2}{v_s} - R_2\frac{\epsilon^2\lambda}{v_s^2}; \quad (4.3)$$

and, using the usual definition of mobility μ, this becomes

$$\frac{\partial}{\partial t}nmv_d - \frac{nm\mu E}{\tau_0} = -\frac{nmv_d}{\tau_0} - R_1\frac{\epsilon^2}{v_s} - R_2\frac{\epsilon^2\lambda}{v_s^2}. \quad (4.4)$$

Equations for the conservation of energy and crystal momentum for the "hot" phonon band can be similarly derived. The conservation of energy is needed in this case, as the phonons are the thermal and inertial reservoir of the combined electron-phonon system. The conservation equations are

$$\frac{\partial}{\partial t}\langle \hbar q_i\rangle + \frac{\partial}{\partial x_j}\langle v_j \hbar q_i\rangle = \langle\Delta \hbar q_i\rangle_{\text{col}}, \quad (4.5)$$

$$\frac{\partial}{\partial t}\langle \hbar v_s q\rangle + \frac{\partial}{\partial x_j}\langle v_j \hbar v_s q\rangle = \langle\Delta \hbar v_s q\rangle_{\text{col}}. \quad (4.6)$$

The variation of energy in the restricted phonon band can be specified by an effective temperature

$$T = T_0 + T_1, \quad (4.7)$$

where T_1 may fluctuate. The distribution function for this group of modes then becomes

$$N(q) = [\exp|\hbar v_s q - \hbar\boldsymbol{\lambda}\cdot\mathbf{q}/K(T_0+T_1)| - 1]^{-1}, \quad (4.8)$$

where λ is the phonon drift described in Sec. II. Using Eq. (4.7) and (4.8), Eqs. (4.5) and (4.6) become

$$\frac{\partial}{\partial t}M\lambda + \frac{\partial}{\partial z}\gamma CT_1 = R_1\frac{\epsilon^2}{v_s} + R_2\frac{\lambda\epsilon^2}{v_s^2} - \frac{M}{\tau_\lambda}\lambda, \quad (4.9)$$

$$\frac{\partial}{\partial t}CT_1 + \frac{\partial}{\partial z}D\lambda_1 = v_s(\Delta \hbar q)_{\text{col}}. \quad (4.10)$$

Here, τ_λ refers to collisions which transfer momentum out of the active phonon band, C is the specific heat of the phonon band, γ is the average of $\cos^2\theta$ over the cone of phonons making up the phonon band, M and D are constants which depend on the ambient temperature as

$$M = \frac{1}{8\pi^3}\int \hbar q \cos\theta\frac{\cos\theta \hbar q}{KT}N_0(q)[N_0(q)+1]q^2 \times \sin\theta dq d\theta d\varphi$$

$$= \frac{1}{4\pi^2}\frac{\gamma(KT_0)^4}{\hbar^3 v_s^5}\int x^4 \operatorname{csch}^2\frac{x}{2}dx\int_0^{\theta'}\sin\theta d\theta, \quad (4.11)$$

$$D = \frac{1}{4\pi^2}\frac{\gamma(KT_0)^4}{\hbar^3 v_s^3}\int x^4 \operatorname{csch}^2\frac{x}{2}dx\int_0^{\theta'}\sin\theta d\theta. \quad (4.12)$$

It is understood that the integral over x includes only those frequencies in the "hot" phonon band.

A static equilibrium solution can be found by setting time and space derivatives equal to zero and solving for ϵ_0 and λ_0. Equations (4.4) and (4.9) become

$$\frac{-nm}{\tau_0}\mu E = \frac{-nm}{\tau_0}(v_s+\epsilon_0) - \frac{R_1\epsilon_0^2}{v_s} - \frac{R_2\epsilon_0^2\lambda_0}{v_s^2}, \quad (4.13)$$

$$0 = \frac{R_1\epsilon_0^2}{v_s} + R_2\frac{\epsilon_0^2\lambda_0}{v_s^2} - \frac{M\lambda_0}{\tau_\lambda}. \quad (4.14)$$

The sum of these equations is

$$\frac{nm}{\tau_0}\beta = \frac{nm}{\tau_0}\frac{\epsilon_0}{v_s} + \frac{M}{\tau_\lambda}\frac{\lambda_0}{v_s}, \quad (4.15)$$

where

$$\beta = \frac{\mu E}{v_s} - 1 \quad (4.16)$$

and is the effective excess field driving the system. The righ-hand side of Eq. (4.15) consists of the net loss from the combined electron-phonon system due to electrons and phonons colliding outside the system. The resulting cubic equation for ϵ_0 or λ_0 can be easily solved in two limits: for $0 < \beta \ll 1$

$$\epsilon_0/v_s = \tfrac{1}{2}\beta, \quad (4.17)$$

and for $\beta \approx 1$

$$\epsilon_0/v_s = (R_2/2R_1)^{1/2}\beta^{1/2}. \quad (4.18)$$

The current saturation, observed in CdS crystals[10] when the electron-drift velocity is comparable to or exceeds the sound velocity, is explained by these equations. At low electric field ($\alpha < 0$) the coupling between Eqs. (4.13) and (4.14) is negligible. The primary electron damping is due to the optical phonon modes and normal resistive behavior results. When $v_d \approx v_s$ the further current increases Δj for increases in field are given by

$$\Delta j = ne\Delta\epsilon_0 = ne\tfrac{1}{2}\mu\Delta E. \quad (4.19)$$

This causes a knee in the current voltage curves where the slope drops to one-half its original value. At very high electric fields the saturation increases and Δj becomes proportional to the square root of E. Accompanying this saturation, a net drift is set up in the phonon distribution described by λ_0, where

$$\frac{\lambda_0}{v_s} = \frac{nm\tau_\lambda}{M\tau_0}\left(\beta - \frac{\epsilon_0}{v_s}\right). \quad (4.20)$$

Although the electrons are capable of conforming to the instantaneous phonon distribution very quickly, it will take a time, roughly equal to the phonon relaxation time, for the phonons to achieve their equilibrium value. Such a delay in achieving equilibrium can explain the observed transient behavior in CdS.[10]

It is emphasized that these solutions are valid only in bulk material. No attempt has been made to introduce the effect of boundary conditions or spatial variations. The size of the sample should certainly effect transient time, particularly for small crystals. Also, no attempt has been made to include the effects of the low-frequency $q\Lambda < 1$ phonons on saturation. It is felt, however, because of the greater number of available high-frequency modes, that the effect of the low-frequency phonons will not be too important.

The electron-phonon system can be examined for its behavior under slight departures from equilibrium. The parameters ϵ, λ, and T can be allowed to vary in both time and space about their equilibrium values, i.e., $\epsilon = \epsilon_0 + \epsilon_1$, $\lambda = \lambda_0 + \lambda_1$, $T = T_0 + T_1$, where

$$\epsilon_1 = \epsilon_1{}^0 e^{i(\Omega t - lz)},$$
$$\lambda_1 = \lambda_1{}^0 e^{i(\Omega t - lz)}, \quad (4.21)$$
$$T_1 = T_1{}^0 e^{i(\Omega t - lz)}.$$

Equations (4.4, 4.9, and 4.10) become

$$i\Omega nm\epsilon_1{}^0 = -A\lambda_1{}^0 - B\epsilon_1{}^0 - (nm/\tau_0)\epsilon_1{}^0, \quad (4.22)$$

$$i\Omega M\lambda_1{}^0 - il\gamma CT_1{}^0 = B\epsilon_1{}^0 + A\lambda_1{}^0 - (M/\tau_\lambda)\lambda_1{}^0, \quad (4.23)$$

$$i\Omega CT_1{}^0 - ilD\lambda_1{}^0 = v_s[B\epsilon_1{}^0 + A\lambda_1{}^0 - (M/\tau_\lambda)\lambda_1{}^0], \quad (4.24)$$

where

$$A = R_2(\epsilon_0{}^2/v_s{}^2) \quad (4.25)$$

$$B = 2[R_1(\epsilon_0/v_s) + R_2(\epsilon_0\lambda_0/v_s{}^2)]. \quad (4.26)$$

The dispersion relation for simultaneous solution of Eqs. (4.22)–(4.24) has roots which describe the collective propagation discussed. This dispersion relation is

$$-\Omega^2 - 2i\Omega[A' - (1/\tau_u)] + \gamma\bar{v}_s{}^2 l^2 = 0, \quad (4.27)$$

where

$$A' = A/M, \quad (4.28)$$

$$B' = B/nm, \quad (4.29)$$

and

$$\bar{v}_s{}^2 = D/M. \quad (4.30)$$

Substituting in Eqs. (4.25), (4.11), and (2.26), this becomes

$$A' = 8\pi^4\left(\frac{ep}{d}\right)^2 \frac{\hbar v_s{}^4 m^{*2} n}{(KT)^4\rho}\left(\frac{T}{T_s}\right)^{3/2} n$$

$$\times\frac{\epsilon_0}{v_s}\frac{\displaystyle\int x^3 \operatorname{csch}^2\frac{x}{2}\frac{[1 - \tanh(1/32)(T/T_s)x^2]dx}{1 + \tanh[(1/32)(T/T_s)x^2]}}{\displaystyle\int x^4 \operatorname{csch}^2\frac{x}{2}dx}, \quad (4.31)$$

where the approximation

$$\int \cos^2\theta \sin\theta d\theta \approx \epsilon/v_s \quad (4.32)$$

has been used. Equation (4.32) holds for small ϵ/v_s. A simplifying approximation can be made in Eq. (5.1) by assuming

$$[1 - \tanh y] = \begin{cases} 1 & y < 0.5 \\ 0, & y > 0.5 \end{cases}, \quad (4.33)$$

and expanding the $\operatorname{csch}x/2$ functions for small values of x. The maximum value of x in the integrals is then given by

$$x_{\max} = 4(T_s/T)^{1/2}. \quad (4.34)$$

Carrying out the integral to x_{max}, Eq. (4.31) becomes

$$A' = 3\pi^4 \left(\frac{ep}{d}\right)^2 \frac{\hbar v_s{}^4 m^{*2}}{(KT)^4 \rho} \left(\frac{T}{T_s}\right)^2 n\frac{\epsilon_0}{v_s}. \qquad (4.35)$$

The damping of the collective wave is

$$\omega_I = \tfrac{1}{2}(1/\tau_\lambda - A'), \qquad (4.36)$$

where A is the stimulated momentum gain rate for a phonon distortion due to phonon emission by the electrons. When $\omega_I < 0$ a phonon distortion will grow rather than be damped.

In a more detailed treatment of this problem, one would retain the q dependence of both A' and $1/\tau_\lambda$. The gain band would be defined as those phonon modes for which

$$(1/\tau_\lambda)(\mathbf{q}) - A'(\mathbf{q}) < 0. \qquad (4.37)$$

The loss due to $1/\tau_\lambda(\mathbf{q})$ will modify the frequency distribution of the gain band. The value of gain at various frequencies shown in Fig. 5 is greater than the net gain by an amount equal to $1/\tau_\lambda(\omega)$, since the losses increase with increasing frequency and the density of frequencies participating in collective waves will be shifted to lower frequencies. The angular distribution of the hot phonons will also be affected since the gain cone should be defined by that angle at which the net gain is zero.

An exact determination of the collective wave velocity would require a more thorough investigation of the role of the mixing collisions as well as detailed knowledge of the loss mechanisms which damp the wave. The problem is in knowing the extent of the cone over which $\cos^2\theta$ must be averaged in the determination of γ of Eq. (4.9). The mixing collisions bring about transfers of momentum which involve phonon modes well outside the gain cone. The effect of the drag of this additional specific heat on the modes involved in the transport of momentum is not as yet understood. If the entire forward half of the phonon distribution is involved in the local thermalization process [i.e., determining T_1 of Eq. (4.9)], the value of γ will be $\tfrac{1}{3}$. A restriction to some smaller cone would increase this value. The maximum value of γ is $\gamma = 1$, and this would occur when the cone is limited to a line in the direction of current flow. The absence of mixing collisions would also give rise to an apparent value of $\gamma = 1$.

The factor $\bar{v}_s{}^2$ of Eq. (4.9) should be a weighted average over all the excitations participating in the collective wave. This is important in the case of second-sound propagation in He II as the admixture of rotons slows the collective wave velocity.[11] In the wave discussed in this paper, the electrons participate in the collective wave and should be averaged in. The effect of the electrons should be very small due to their relatively small numbers and small mass. The phonons involved in the wave also have varying velocities. This is because

[11] K. R. Atkins, *Liquid Helium* (Cambridge University Press, Cambridge, England, 1959), p. 148.

the phonons having piezoelectric fields become impure modes in some directions. In CdS the piezoelectric mode along the "C" axis is a longitudinal mode. The effect of the different velocities of phonons for large angles, off the direction of propagation, is reduced by the factors of $\cos\theta$ in the constants M and D of Eqs. (4.11) and (4.12). This change in velocity with orientation would also effect the extent of the gain cone, etc., with orientation.

V. CONCLUSION

The observation of an anomalously slow acoustic propagation in CdS has been recently reported.[1] The effect is directly related to the passage of a piezoelectrically active shear wave through the crystals under conditions of acoustic gain. The anomaly could only be explained as a wave traveling in the CdS with a velocity just slightly faster than $1/\sqrt{3}v_s$, where v_s is the velocity of the piezoelectric shear wave. This shear wave is the slowest of the possible sonic waves in CdS. The pulses were observable only with applied voltages such that the electrons had drift velocities roughly 50 percent greater than the velocity of sound. For drift velocities, roughly twice the velocity of sound, the pulses were damped out. Similarly, the pulses were observed only over a small range in temperature near 300°K. Higher and lower temperatures caused the wave to be damped. At low temperatures the velocity of the wave was observed to increase as the wave was damped. These observations can be explained on the basis of the theoretical development of a collective-wave propagation presented above.

The collective wave consists of a harmonic fluctuation in the density of phonons within an active phonon band (phonon frequencies $\sim 10^{10}$–10^{11} sec^{-1}). This could also be described as a harmonic temperature fluctuation in the active band. The phonon density (temperature) fluctuations in turn, have a frequency equal to that of the shear wave which generated them (shear wave frequency $\sim 10^7$ sec^{-1}). The role of the initial shear wave was to introduce a single-frequency perturbation, which when amplified, could be coherently detected.

The propagation of a collective wave requires that two conditions be satisfied: (1) The gain in crystal momentum of the active phonon band must exceed the losses, and (2) mixing collisions among phonons within the band must be sufficiently frequent that a local temperature can be defined within a collective wavelength.

In Sec. II of this paper, the momentum gain for piezoelectrically active phonons has been evaluated. The gain mechanism is such that the electrons couple only to a band of phonons of frequencies 10^{10}–10^{11} cps. In CdS, under conditions approximating those of the experimental observation, (i.e., at room temperature with $n = 10^{13}$ and $p \approx 10^7$ D/esu)[12] the gain in the band given by Eq. (4.35) is

$$A' \approx 10^8(\epsilon/v_s) \text{ sec}^{-1}. \qquad (5.1)$$

That is, this gain is great enough, at large electric field strengths, to overcome active phonon-thermal relaxation times of 10^{-8} sec.

The losses which this gain would have to overcome to satisfy condition (1) are the losses from the active phonon band to thermal-phonon reservoir. These processes are due to the anharmonic perturbations and do not involve electrons. The phonon-thermal phonon relaxation time for the phonon frequencies expected to be of importance is difficult to estimate, and direct measurements at these frequencies and temperatures in CdS are not available. The difficulty arises because the active band lies between very high ultrasonic frequencies and very low thermal frequencies. Woodruff and Ehrenreich[13] have shown that the phonon-phonon damping increases roughly as ω^2 in the ultrasonic region, and as ω for higher frequencies. The extrapolation to the frequencies important in collective wave propagation could be approximated by a variation as ω^n with $1 < n < 2$.

The dark relaxation time (i.e., no carriers) measured by Kroger[14] in CdS crystals, which have shown collective wave propagation, is 10^{-3}–10^{-4} sec^{-1} at 10^7 cps. If one extrapolates from 10^7 cps to the higher frequencies of the active phonon band, a very large part of the phonon band which is capable of supporting collective waves [satisfying condition (2)] will have stimulated gain greater than the losses. The net gain would be the difference between two large numbers, roughly the order of 10^8 sec^{-1}. From Eq. (4.35) it can be seen that the momentum gain decreases for decreasing electric field and for higher temperatures. This decrease in gain would explain the observed damping of the collective wave at lower electric fields and higher temperatures.

As pointed out in Sec. III, the major contribution to mixing collisions probably comes from successive absorption and emission processes. This combined rate is limited by the slower of these two processes. When the electron drift velocity is exactly v_s the rates of the two processes are expected to be equal. For higher drift velocities (i.e., higher external fields) the absorption rate, and therefore the mixing rate decreases. Higher temperatures tend to enlarge the spread in the electron distribution and would bring the absorption and emission rates closer together. Lower temperatures, on the other hand, tend to make the electron distribution more compact, which would decrease the absorption rate relative to the emission rate. The rate of mixing collisions then has opposite temperature and voltage dependence than the net gain. This loss of coherence, or second viscosity damping, would account for the damping of the collective wave at high electric fields and low temperatures. The increase in velocity with onset of loss of coherence or second viscosity is also observed in second-sound propagation.[11] It comes about because for few mixing collisions a large part of the collective wave energy travels straight through as unhindered phonons. With no normal collisions the energy transfer velocity would be v_s.

Another cause for decrease in the observability of the collective waves at low temperature is the decrease in the energy and momentum content of the effective band. Figure 5 shows the total crystal momentum transferred to the entire band for various temperatures. It can be seen that the total magnitude of the entire collective wave decreases with temperature.

We expect the mixing collision rate to be comparable to the rate of Eq. (5.1). Therefore, the highest collective frequencies possible at room temperature would be roughly 10^8 cps. For frequencies of 10^6 cps and lower, the collective wavelength would be as large or larger than the size of readily available crystals. The experimentally observed wave was observed at a frequency of 10^7 cps.

ACKNOWLEDGMENTS

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