ground state is possible when λ is very large and fixed, we conclude that a power-series expansion in λ^2 of the ground-state energy has a finite radius of convergence in the weak-coupling limit. This suggests strongly that the modified LLP variational method employed in this paper, even if carried out to all orders in λ, would fail for $\lambda \gg 1$. We should note, however, that in typical ionic crystals $\lambda = 1$ when the applied fields are in the hundreds of kilogauss. At such field strengths the validity of the Fröhlich Hamiltonian, given by (6)–(8), is doubtful.

The question of how the size of α affects the radius of convergence of an expansion of the ground-state energy in powers of λ^2 obtained from carrying out the modified LLP variational method to all orders in λ^2, remains unanswered.

In the limit of very weak field, namely, the limit (5), one can show, using a method due to Platzman[11] for performing the summation on n in (42), that the weak-coupling energy to order α is correctly given by the eigenvalues of the effective Hamiltonian obtained by replacing p^2 by $\mathbf{\Pi}^2$ in the field-free weak-coupling polaron energy, given by[1]

$$E_{\mathrm{wc}}(p^2)=\left[p^2-\alpha\frac{\sin^{-1}[(p^2)^{1/2}]}{(p^2)^{1/2}}\right].$$

The essential step in the proof of this result is to neglect the commutator $[p_y,y]$ wherever it appears. This neglect can be rigorously justified in the limit (5). The effective Hamiltonian $E_{\mathrm{wc}}(\mathbf{\Pi}^2)$ is applicable only when the condition

$$n\lambda^2+p_z^2<1$$

is satisfied.

The validity of replacing the field-free polaron energy spectrum $E(p^2)$ by $E(n\lambda^2+p_z^2)$ in the presence of a sufficiently weak magnetic field is undoubtedly not restricted to the weak coupling limit, but the author has not yet found a rigorous proof of this for the intermediate coupling case.

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Volume Magnetostriction in Gadolinium Single Crystals*

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The magnetostriction of single-crystal gadolinium has been measured from 77 to 325°K in magnetic fields up to 20 kOe. A preliminary result of this work is the behavior of the forced volume magnetostriction. These results have been employed to obtain the partial differential coefficient of Curie temperature as a function of pressure over the above temperature range by means of the well-known thermodynamic expression due to Kornetzki. It is found that $\partial\theta/\partial P = -1.26\pm0.10$ °K/1000 atm at 290°K. These results are compared with the results obtained by direct measurement and discussed further.

THE forced volume magnetostriction above technical saturation in ferromagnetics is related to the pressure dependence of the magnetization through the thermodynamic relation

$$(\partial\omega/\partial H)_P=-(\partial I/\partial P)_H, \tag{1}$$

where $\omega=\Delta V/V$. Kornetzki,[1] assuming that a change in the spontaneous magnetization with a change in the volume at a definite temperature and at a definite magnetic field can occur only through a volume dependence of the Curie temperature or the exchange interaction energy and an arbitrary form for $M_{H,T}=f(T/\theta,H)$, where θ is the Curie temperature, finds that the forced volume magnetostriction and the pressure dependence of the Curie temperature are related through[2]

$$1/\theta(\partial\theta/\partial P)$$
$$=1/T(\partial\omega/\partial H)\bigg/\left[\rho\frac{\partial\sigma}{\partial T}-\beta/K(\partial\omega/\partial H)\right], \tag{2}$$

where ρ is the density, σ the specific magnetization, β the volume thermal expansion coefficient, and K the volume compressibility.

The magnetostriction of single crystals of gadolinium has been measured from 77 to 325°K in magnetic fields up to 20 kOe. A preliminary result from this study

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[2] The right band of Eq. (2) should be multiplied by $(1+H/NI)$, where N is the Weiss molecular field factor. Calculating N from the susceptibility measurements in the paramagnetic region H/NI was found to be less than 0.03 and consequently was neglected.

is the behavior of the forced volume magnetostriction. The results are in qualitative agreement with the work of Bozorth and Wakiyama[3] and more complete. The forced magnetostriction in the **c** and **a** crystallographic directions are found to peak in the region of the Curie temperature. The peaks shift toward higher temperatures with increasing H while their magnitudes decrease with increasing H. Figure 1 presents the average value of $\partial\omega/\partial H$ for a change in H from 11 to 20 kOe as a function of temperature.

Using the $\partial\omega/\partial H$ data of Fig. 1, the known value of ρ and $\partial\sigma/\partial T$ shown in Fig. 1 obtained from the work of Nigh *et al.*,[4] $\partial\theta/\partial p$ can be calculated. In performing these calculations, the second term in the denominator of Eq. (2) has been ignored since it is less than 3% of the first term. It is seen (in Fig. 2) that $\partial\theta/\partial p$ is negative and presents a rather violent variation with temperature in the vicinity of the Curie temperature.

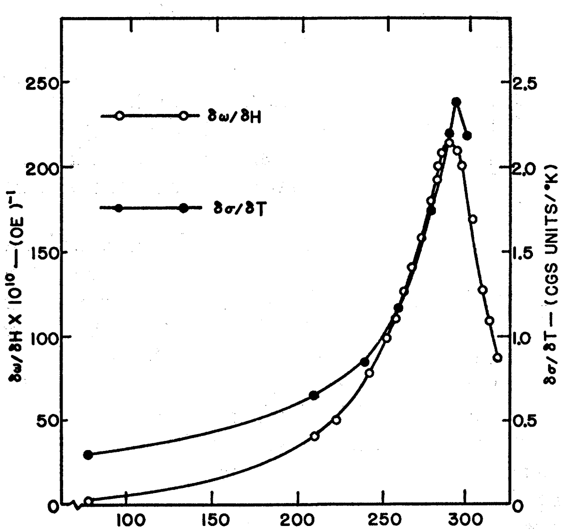

FIG. 1. Plots of forced volume magnetostriction of gadolinium for a magnetic field change from 11 to 20 kOe and the slope of the specific magnetization of gadolinium at 17 kOe as a function of temperature.

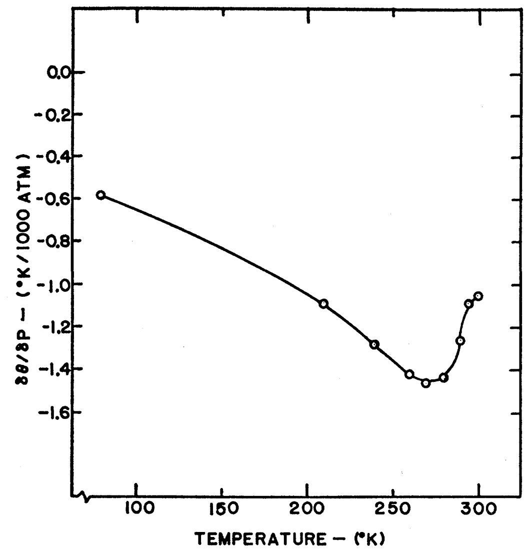

FIG. 2. The calculated value of the change in the Curie temperature with pressure as a function of temperature.

Patrick[5] has measured directly the change in Curie temperature with change in pressure. He found that at 290°K, $\partial\theta/\partial p = -1.2 \pm 0.05$°K/1000 atm. More recent measurements by Robinson *et al.*[6] found θ to be depressed linearly with pressure at a rate of -1.58°K/1000 atm up to 21.2×10^3 atm. This implies that $\partial\theta/\partial p$ is constant over a temperature range from the Curie point to 33°K below the Curie point. The calculated $\partial\theta/\partial p$ at 290°K from the forced volume magnetostriction data is -1.26 ± 0.10°K/1000 atm. This is in fair agreement with the direct pressure measurements of Patrick and Robinson. Our greatest error occurs in the determination of $\partial\sigma/\partial T$ in the region of the Curie temperature. However, the constancy of $\partial\theta/\partial p$ as reported by pressure measurements is not corroborated by our results. This may indicate the inadequacy of the assumptions in deriving Eq. (2). Other parameters besides the interatomic distance may influence the exchange interaction in gadolinium.

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