example considered in the text, the following numbers result for $L=9$: $c/(\omega)^{3/2}\cong 1.60$, $\omega/c\cong 0.33$ F, and for distances to the right of the turning point ($r>r_T$) of 0, 0.33, 0.66, and 1 F, the corresponding error in the JWKB expression is found from Eq. (A8) to be equal to 17%, 10%, 5%, and 3%, respectively. It may be interesting to note that for $W\neq 0$ the quantity in (A8) does not become infinite at $r=r_T$, showing that the presence of W improves the validity of the JWKB approximation near the turning point.

A more general estimate of the error in the JWKB approximation can be obtained by utilizing the procedure outlined in the book on quantum mechanics by Kemble,[14] in which the difference between the actual potential and the potential for which the JWKB expression for $\mathfrak{F}$ is an exact solution, is treated as a perturbation.

[14] E. C. Kemble, *The Fundamental Principle of Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1937), Sec. 21.

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Nuclear Pair-Correlation Function via Electron Scattering

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A theoretical analysis is made of the differential cross section for the process of inelastic scattering of electrons by a nucleus accompanied by the emission of two nucleons. The analysis is twofold. First, assuming a purely electromagnetic interaction (of the semiclassical Möller type), and invoking closure and impulse approximations, the above cross section is shown to be simply proportional to the Fourier transform of the nucleon-nucleon correlation function in the nucleus (at a momentum determined experimentally). Secondly, following a physical idea due to Gottfried and employed in a similar context, the nuclear cross section is compared to that of the electrodisintegration of a deuteron. This has the advantage in that it eliminates the need to introduce an explicit Hamiltonian (as in the first case), and also it allows us to include some very important mesonic effects due to the virtual pion exchange between the two outgoing nucleons. This approach, therefore, shifts the problem to a theoretical understanding of the connection between the deuteron wave function and the pair-correlation function in a nucleus. A computation of the cross section is made, which is rather low, yet is seen to be within the reach of the present experimental techniques. Detailed kinematical questions are explored and the optimal experimental setup indicated. The corrections due to (1) the final-state interactions and (2) real-meson-production channels are also discussed. In conclusion, we surmise that there is a definite need for detailed experimental study of two or more nucleon emission phenomena for the determination of the correlation function at high momentum (or small distances). Beyond fixing some qualitative guidelines, the no-nucleon and one-nucleon emission cross sections seem to be less than adequate.

1. INTRODUCTION

We would like to make a proposal regarding the measurement of the 2-nucleon correlation function at very small distances, in light/medium nuclei via inelastic electron-nucleus scattering accompanied by the emission of two nucleons. Some of the physical ideas presented here are due to Gottfried employed in a similar context.

First, before entering into the main theme, a few words about the pair-correlation function and the need for its evaluation are in order. Let $\psi_0(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_A)$ be the nuclear ground state in configuration space (assumed properly normalized to unity) then the pair-correlation function $C(\mathbf{r}_1,\mathbf{r}_2)$ is defined as

$$C(\mathbf{r}_1,\mathbf{r}_2)=\int|\psi_0(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_A)|^2(d\mathbf{r}_3)(d\mathbf{r}_4),\cdots,(d\mathbf{r}_A),$$

and thus gives the conditional probability density for finding a particle at $\mathbf{r}_2$ if another one is known to exist at $\mathbf{r}_1$. Of course, in any realistic situation, one can define spin- and isospin-dependent functions (which we consider later on), but for the time being let us disregard such inessential complications.

Fundamental theories of nuclear matter to date (for example, that due to Brueckner), are all based on the 2-nucleon interaction. From these nuclear theories one can say something directly about the properties of the infinite nuclear medium. One would also hope to be able to predict properties of the models of low-energy nuclear physics. However, this last step has not been very successful as yet. To consider an example, in Brueckner's theory it is difficult simply to justify the existence of various phenomenological models (like the shell model) much less to "predict" their properties in detail.

Now in so far as more than 2-body correlations be negligible, the pair-correlation function describes the nuclear matter and such associated properties as the

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energy and particle densities of the nuclear medium, and is only distantly connected with shell and collective models, i.e., models describing *low-energy* nuclear level structure and transition probabilities. Thus, an accurate enough experimental determination of the pair-correlation function (or more specifically, its high-momentum components) ought to provide a direct, and we hope, a fruitful test of the fundamental theories, which the low-energy experiments in nuclear physics cannot. Its evaluation could probably verify the accuracy of the basic assumption involved in the nuclear many-body problem, viz., that the forces between two nucleons inside a nucleus are essentially the same as in free space, i.e., the neglect of intrinsically many-body forces.

The above discussion, we hope, would give a rough idea of the usefulness of a rather accurate knowledge of the pair-correlation function.

We shall assume, in the present work, that

$$C(\mathbf{r}_1,\mathbf{r}_2)\equiv C_s(\mathbf{r}_1,\mathbf{r}_2)|g(|\mathbf{r}_1-\mathbf{r}_2|)|^2,$$

where C_s is the correlation function of the shell model and thus contains correlations due to the Pauli principle and the finiteness of the nuclear system. The more "violent" (and interesting) correlations due to direct nucleon-nucleon forces are incorporated in $g(r)$.

The gross effects of C_s may be extracted out in the shape of a form factor $F(P)$ (defined and discussed later on) which is related to the probability for finding two nucleons very near each other with total momentum P in the *Slater determinant*.[1] We would like to reassert that this is relatively insensitive to the dynamical structure of the 2-nucleon direct interaction and hence is a less interesting part of the problem.

It was realized long ago that the total cross section for scattering a weakly interacting projectile through a *fixed* momentum transfer is simply proportional to the Fourier transform of the target's pair-correlation function. Electrons meet all the specifications required in the derivation of the above result, and being readily available become the most natural choice. Thus, quite a few theoretical analyses[2] have been made to compute sum rules for inelastic electron-nucleus scattering with moderately large ($\lesssim$2–2.5 F^{-1}) momentum transfers. However, so far, in deriving such sum rules, use has essentially been made of a *small* energy transfer (in order to effect closure) and a moderately *large* momentum transfer. There are at least two distinct disadvantages in these analyses. First, on the kinematical level, reflection shows that the combination of a large momentum transfer coupled with a small energy transfer emphasizes *single* nucleon excitations. This does not necessarily preclude two or more nucleon excitations; however, their effect is lessened. From the experimental cross section, therefore, one has to obtain the pair-correlation function which is but a small ripple over the main effect. The second objection is on the dynamical and somewhat deeper level. McVoy and Van Hove[2] obtain the correlation function, or more specifically its Fourier transform $C(q)$, which is effectively integrated out over *all* the internal momenta of the nucleons in the nucleus. This, unfortunately, has the disadvantage of averaging out the effect of the correlation. We may enlarge this argument as follows: By our basic ansatz, above, the interesting short-range correlations are presumably due to very closely lying nucleons, i.e., nucleons having high relative momenta and thus $g(r)$ is born of and is affected by such configurations *alone*. However, if one considers a cross section which includes *all* nucleon configurations, the effect gets washed out. This makes the theoretical interpretation of the data in terms of a pair-correlation function rather confused. Instead, we propose looking at particular values of the argument of the Fourier transform of g by observing the two outgoing nucleons' momenta.

This is partly remedied in recent theoretical analyses of the 1-nucleon (proton) ejection process by Jacob and Maris[3] and by Sitenko and Gur'ev.[4] But here again one has to average over the other "spectator" nucleon and then relate the cross section to the *single* nucleon momentum distribution. This, therefore, negates the possibility of a valid determination of the correlation function.

It should be obvious then that the most suitable choice for the determination of the pair-correlation function is the process in which two nucleons are ejected from the nucleus. And in so far as the outcoming nucleons carry off most of the energy-momentum imparted by the projectile and the basic premise that more than 2-body excitations (with high momenta) in the nucleus being small be true, one can sum over the residual nuclear states invoking closure, which is much more palatable here, and obtain a rather clear cut connection between the experimental cross section and the correlation function.

Now instead of electrons, one may think of employing nucleons, pions or photons as incident projectiles. Admittedly, the nucleons and pions would render a larger cross section. However, processes initiated by nucleons (and the same argument applies to pions as well) have some strong disadvantages. All the analyses[5] made so

[1] $F(P)$ is simply related to the Fourier transform of C_s (suppressing, for the moment, the spin and isospin labels), for the particular case of completely closed-shell nuclei. The assumption involved in defining $g(r)$ is that it is possible (and fruitful) to separate out the two types of correlations: $F(P)$ (or C_s) which incorporates the correlations for small momenta, $p\lesssim k_F(\approx 1\ \mathrm{F}^{-1})$, and $g(K)$, which contains the dynamical structure for $p\gtrsim 1\ \mathrm{F}^{-1}$. Thus, the shell (or independent-particle) model part of the function involves distances larger than the range of nuclear forces.

[2] K. W. McVoy and L. Van Hove, Phys. Rev. **125**, 1034 (1962). Detailed list of references to earlier work on the subject may be found here.

[3] G. Jacob and Th. A. J. Maris, Nucl. Phys. **31**, 139 (1962); **31**, 152 (1962).

[4] A. G. Sitenko and V. N. Gur'ev, Zh. Eksperim i Teor. Fiz. **39**, 1760 (1960) [English transl.: Soviet Phys. JETP **12**, 1228 (1961)].

[5] See for instance, K. A. Brueckner, R. J. Eden, and N. C. Francis, Phys. Rev. **98**, 1445 (1955).

far of nucleon-initiated processes have been on the basis of the Chew-Wick impulse approximation so that one may be able to relate it to the "free" nucleon cross section. One has to invoke it for fear of bringing in the actual nucleon-nucleon interactions which are highly model-dependent. As has been noticed before,[6] the use of the impulse approximation is suspect for such reactions initiated by nucleons (or pions). A naive way to visualize it is as follows: When one finds a nucleon with a very high momentum ($\gg k_F$, the Fermi momentum) in the nucleus, it is obvious that it must be due to some violent interaction with another close nucleon. Hence, one must take into account an interaction involving the incident nucleon (or pion) and the two nucleons which are very close together in the nucleus. Of course, we have no theory which can handle a system of three strongly interacting particles. The use of a weakly interacting projectile (like electron) offers hope because one "knows" the electromagnetic interactions between the incident particle and the 2-nucleon system (considered as a "quasideuteron"). One could, in principle, argue the use of the quasideuteron model for the incident nucleon (and pion) case also. However, there is little hope of fully describing it due to the apparent difficulty of describing reactions of three strongly interacting particles.

There is another disadvantage: the fact that nucleons have an extremely high probability of being absorbed in the nucleus. To quote a figure,[3] the attenuation factor for the emission of one nucleon is of the order of 0.005, compared to about 0.6–0.7 for the corresponding process initiated by electrons (i.e., at the same 3-momentum but much higher energy). (See also Sec. 5.3.)

The use of the photons has been realized before.[7] It is expected that the use of electrons as probes would be advantageous in that one can maneuver the momentum and energy at the same time instead of dealing with the fixed momentum-energy relation for the photons. Also, this relatively difficult experiment seems much more promising with monoenergetic photons transferred by the electron than with a bremsstrahlung beam. So, we consider the process

$$e+A \rightarrow e'+(A-2)+p+n$$

and obtain the differential cross section $[d\sigma/(d\mathbf{k}_e')\times(d\mathbf{k}_p)(d\mathbf{k}_n)]$. Of course, one has the disadvantage in the case of electrons in that the cross section is small and that one has to do coincidence work on three particles in the final state.

However, the considerable enhancement of the cross section for forward scattered electrons offers a ray of hope. The emission and the proposed detection of the (two) particles is of crucial importance because it would tend to distinguish this process from the overwhelmingly large elastic peak (for forward electrons), the latter of which, as is well known, has been largely responsible for the lack of experimental information on the forward inelastic electron scattering.

Now there are two ways to calculate the T-matrix element for the above process. One is to introduce, as in the past, a purely electromagnetic interaction Hamiltonian (say, of the Möller type) for the electron-nucleon system with measured form factors, etc. The other is to relate the transition matrix element for the nuclear case to that of the electrodisintegration of the deuteron. This comparison of the above-mentioned nuclear cross section to that of deuteron electrodisintegration allows us to get rid of an explicit reliance on an assumed interaction between the electron and the nuclear system. Also, it allows us to include very important mesonic effects due to the virtual pion exchange between the two outgoing nucleons. This approach is thus designed to obtain the least ambiguous interpretation of the experiments, which, however, will depend on a theoretical understanding of the deuteron problem.

After these qualitative introductory remarks we would like to derive a few quantitative results for the relevant cross section. As alluded to above, we would make two sets of (formally different) calculations. In Sec. 2, we give a computation of the cross section on the basis of an assumed electromagnetic interaction. Then, in Sec. 3, we formally derive the matrix element for transition on the basis of the quasideuteron model, *not* assuming a form for the interaction beyond the fact that it depends only upon 2 nucleons. This is then related to an analogously derived deuteron disintegration cross section in Sec. 4. In Sec. 5, we discuss the final-state interaction corrections. The detailed kinematical questions and the proposed experimental setup for the determination of $g(r)$ are described in Sec. 6. The mesonic interference and other problems involved in a "clean" determination of the correlation function are discussed in Sec. 7. Lastly, we discuss the whole program, its consequences and limitations in conclusion.

Our notation will be as follows: We use natural units, $\hbar=c=1$. The electron momenta in the initial and final states are $\mathbf{k}_e$ and $\mathbf{k}_e'$ with energies $E_e=(k_e^2+m_e^2)^{1/2}$ and $E_e'=(k_e'^2+m_e^2)^{1/2}$, where m_e is the mass of the electron. Thus, $\mathbf{q}=\mathbf{k}_e-\mathbf{k}_e'$ is the momentum transferred to the target; $\omega=E_e-E_e'$ is the energy transferred to the target. The target is assumed to be at rest initially. $q_\mu^2=\mathbf{q}^2-\omega^2$ is the square of the 4-momentum transfer. $\boldsymbol{\alpha}$ is the electron's Dirac operator. $|u_i\rangle$ and $|u_f\rangle$ are free-electron spinors. $\mathbf{p}$ and $\boldsymbol{\sigma}$ are momentum and spin operators in the nucleon's space. $F_1(q_\mu^2)$ and $F_2(q_\mu^2)$ are the nucleon's charge and magnetic moment form factors. $|0\rangle$ and $|n\rangle$ are the ground state and the nth excited state of a nucleus with A-nucleons. The subscript j refers to the jth nucleon. $e_j=\frac{1}{2}(1+\tau_{3j})$ gives the charge or proton projection operator. $\mu_j=\frac{1}{2}(1+\tau_{3j})+\tau_{3j}\kappa$ gives the magnetic moment projection operator. κ is the anomalous magnetic moment of the nucleon, in nuclear

[6] K. Gottfried, Ann. Phys. (N.Y.) **21**, 29 (1963).

[7] K. Gottfried, Nucl. Phys. **5**, 557 (1958). Most of the notation in this reference is closely followed for ease of comparison.

magnetons. θ is the electron scattering angle, i.e., $\cos\theta=\hat{k}_e\cdot\hat{k}_e'$. Other symbols used are defined at appropriate places.

2. CROSS SECTION: ON THE BASIS OF THE ELECTROMAGNETIC INTERACTION

2.1. Electron-Nucleon Interaction

We treat the electrons relativistically and the nucleons nonrelativistically. The covariant electron-nucleon interaction is well known and its reduction to the 2-component form for the nucleon has been made by McVoy and Van Hove.[2] In the plane-wave approximation for the electrons, they obtained the interaction, correct through order q^2/M^2 as

$$\begin{aligned}H'=&(4\pi e^2/q_\mu^2)\langle u_f|F_1e^{iq_\mu x_\mu}\\&-(F_1/2M)[(\mathbf{p}\cdot\boldsymbol{\alpha})e^{iq_\mu x_\mu}+e^{iq_\mu x_\mu}(\mathbf{p}\cdot\boldsymbol{\alpha})]\\&-[(F_1+\kappa F_2)/2M]i\boldsymbol{\sigma}\cdot(\mathbf{q}x\boldsymbol{\alpha})e^{iq_\mu x_\mu}\\&-(q_\mu^2/8M^2)(F_1+2\kappa F_2)e^{iq_\mu x_\mu}\\&+[(F_1+2\kappa F_2)/8M^2]i\boldsymbol{\sigma}\cdot\{\mathbf{p}x(\omega\boldsymbol{\alpha}-\mathbf{q})e^{iq_\mu x_\mu}\\&\qquad-e^{-iq_\mu x_\mu}(\omega\boldsymbol{\alpha}-\mathbf{q})x\mathbf{p}\}|u_i\rangle.\quad(1)\end{aligned}$$

The symbols have meaning as defined in the notation.

We use the following approximations for the nucleon form factors (which are supposed to be valid up to 2.5 F^{-1}):

$$F_{1p}(q_\mu^2)=F_{2p}(q_\mu^2)=f(q_\mu^2)\,,\quad\text{for the proton}$$

and

$$F_{1n}(q_\mu^2)=0;\quad F_{2n}(q_\mu^2)=f(q_\mu^2),\quad\text{for the neutron.}$$

We may remark that one can evaluate the cross section without invoking this equality of the form factors—only the formulas become lengthier and more complicated. Thus, if needed, calculations may be made without this approximation, so as to hold even for higher nucleon momenta. However, then the interaction may have to be modified (i.e., inclusion of terms higher than q^2/M^2 may be necessary).

Using (1) and the above approximation on the form factors one obtains the transition matrix element from the ground state (E_0) to the excited state (E_n) of the nucleus, in the first Born approximation, as[2]

$$\begin{aligned}|M_{n0}|^2=&\delta(\omega-E_n+E_0)(4\pi e^2/q_\mu^2)f^2(q_\mu^2)\\&\times|\langle u_f|u_i\rangle Q_{n0}-\langle u_f|\boldsymbol{\alpha}|u_i\rangle\cdot\mathbf{J}_{n0}|^2,\quad(2)\end{aligned}$$

where

$$Q_{n0}=\langle n|Q|0\rangle=\langle n|\sum_{j=1}^{A}[e_j+(q^2/8M^2)(e_j-2\mu_j)]e^{i\mathbf{q}\cdot\mathbf{r}_j}|0\rangle$$

$$\begin{aligned}\mathbf{J}_{n0}=\langle n|\mathbf{J}|0\rangle=&\langle n|\sum_{j=1}^{A}[(e_j/2M)(\mathbf{p}_je^{i\mathbf{q}\cdot\mathbf{r}_j}+e^{i\mathbf{q}\cdot\mathbf{r}_j}\mathbf{p}_j)\\&+(\mu_j/2M)i\boldsymbol{\sigma}_j\times\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{r}_j}]|0\rangle.\quad(3)\end{aligned}$$

Now we shall use the extreme relativistic limit for the electrons: $K_e=k_e$ and $E_e'=k_e'$. If we square M_{n0}, sum and average over final and initial electron spin states, we get[2]

$$\tfrac{1}{2}\sum_{\text{spins}}|M_{n0}|^2=\delta(\omega+E_0-E_n)(4\pi e^2/q_\mu^2)^2f^2(q_\mu^2)R_n\,,\quad(4)$$

where

$$\begin{aligned}&(1+\cos\theta)^{-1}R_n\\&=|Q_{n0}|^2-\frac{\omega}{q}[(\hat{q}\cdot\mathbf{J}_{n0}^*)Q_{n0}+\text{c.c.}]\\&\qquad+\tfrac{1}{3}(1+2\tan^2\theta/2)|J_{n0}|^2\quad(5)\\&-\frac{1}{2}\left(2\tan^2\theta\Big/2-\frac{3\omega^2}{q^2}+1\right)[|\hat{q}\cdot J_{n0}|^2-\tfrac{1}{3}|\mathbf{J}_{n0}|^2].\end{aligned}$$

This result is in the lab frame. One can make a transformation to the center-of-mass frame of the recoil nucleus by making the so-called Gartenhaus-Schwartz transformations.[8]

2.2. Validity of the Born Approximation

We may need to consider high-Z nuclei for which the distortion of the electron becomes considerable, and so plane waves for electrons would not suffice.

Even at the lowest energies under consideration, say $\gtrsim300$ MeV, the Born approximation is an accurate approximation[9] for $1p$ shell and beginning of $2s$ and $1d$ shells; rather accurate throughout the $2s$ and $1d$ shells ($Z\alpha<0.15$) and useful through the $2p$ and $1p$ shells ($Z\alpha<0.3$). For O^{16}: $Z\alpha\approx0.11$, Ca^{40}: $Z\alpha<0.3$.

However, this may be remedied by using the so-called Schiff approximation[10] so that even heavier nuclei, e.g., Pb^{82} may be used. On the other hand, the experimentalists may not like to employ very high-Z nuclei because of multiple scattering.[11]

The validity of the first Born approximation reduces down to the fact that there occurs only one "hard" $e-N$ scattering. As discussed by Barber,[12] the Born approximation treatment is based upon two important assumptions: (i) that the first-order perturbation theory is adequate, and (ii) that we can describe the initial and final electron states as plane waves. He quotes a work of Rodenberg[13] in which he has investigated the second-order corrections to the assumption (i) and finds that they are very small compared to the Coulomb correction [breakdown of assumption (ii)]. This latter screening correction is never more than a few percent.

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[8] S. Gartenhaus and C. L. Schwartz, Phys. Rev. **108**, 482 (1957). See also Ref. 2.

[9] R. S. Willey, Nucl. Phys. **40**, 529 (1963).

[10] L. I. Schiff, Phys. Rev. **103**, 443 (1956); **104**, 1481 (1956); R. J. Glauber (unpublished); W. E. Drummond, Phys. Rev. **116**, 183 (1959); W. Czyz and K. Gottfried, Ann. Phys. (N.Y.) **21**, 47 (1963).

[11] M. Ross (private communication).

[12] W. C. Barber, Ann. Rev. Nucl. Sci. **12**, 1 (1962).

[13] R. Rodenberg, Z. Naturforsch. **16a**, 1242 (1961).

To get an idea, the screening correction should be about 6% for Pb^{82} and about 1% for O^{16}. Consequently our basic postulate regarding the fact that there occurs only one hard $e-N$ scattering ought to hold very well.

We have dealt at length on this point because it is quite important, as it eliminates the possibility of a cascade process, i.e., to say that the electron first knocks out a nucleon and then another nucleon (both nucleons not necessarily being in close vicinity of each other). This would then spoil the effort to obtain 2-nucleon correlation function for very small distances (very high momenta).

2.3. Partial Closure over Residual Nuclear States

Now we would like to put the nuclear state $|n\rangle$ into a more tractable form. Following the general route[14] (see Appendix A for derivation) one has

$$|n\rangle=(1+G_E{}^+W)|\chi_f{}^{(-)}\Xi_s\rangle,$$

where

$$G_E=(E-H_N+i\epsilon)^{-1}, \tag{6}$$

where $\chi_f{}^{(-)}$ is a 2-nucleon scattering state, Ξ_s is an antisymmetrized $(A-2)$-body state for the residual nucleus in a state s. W is the interaction between the pair (outgoing nucleons) and the remaining $(A-2)$ nucleons. Then, formally one can write down the cross-section as

$$d\sigma=2\pi\tfrac{1}{2}\sum_{\substack{\text{electron}\\ \text{spins}}}|M_{n0}|^2\frac{d\mathbf{k}_e{}'}{(2\pi)^3}\frac{d\mathbf{k}_p}{(2\pi)^3}\frac{d\mathbf{k}_n}{(2\pi)^3}\frac{d\mathbf{P}_r}{(2\pi)^3}, \tag{7}$$

$\mathbf{P}_r$ gives the recoil momentum of the residual nucleus.

At this stage, in the analysis of the inelastic electron-nucleon scattering, one usually invokes partial closure over the recoiling nuclear states. Thus, we discuss, somewhat in detail, the qualitative aspects of the validity of the same argument in our case.

In order that partial closure over the residual nuclear states $|\Xi_s\rangle$ be applicable here, it must be true that for fixed q, ω, θ, $\mathbf{k}_p$, $\mathbf{k}_n$ there will only be a "small" region (in an absolute sense) in E_s, the excitation energy of $|\Xi_s\rangle$, that will yield a large matrix element. But of course, if we fix *all* of these momenta and energies, we won't be allowed to sum over $|\Xi_s\rangle$. Thus, the condition for the applicability of the closure condition reduces to the question: whether the residual states confine themselves to a narrow band so that one may be justified in talking of a well-defined average energy $\bar{\epsilon}\ll\omega$. Put differently, our approximation works better the closer the outgoing nucleons get to carrying off all the energy transferred by the electron.

In our case, since the two outgoing nucleons would probably emerge from the nucleus carrying with it most of the energy imparted to the target nucleus, the (residual) nucleus must then be effectively confined to within a small energy interval. This depends on the assumptions that no other nucleons were strongly correlated with these two and that there is no strong final-state scattering. So, for the process under consideration, the partial closure argument is seen to be much more palatable than in the case of simple inelastic electron-nucleon scattering (with *no observation* of nucleon emission) considered by many authors in the past, e.g., see McVoy and Van Hove.[2] Consequently their restriction regarding the maximum value of the momentum transfer $q\lesssim 2.5\ \mathrm{F}^{-1}$ (below which the closure approximation supposedly holds) probably applies to the *recoil nucleus alone;* i.e., in our case to the quantity P [defined later via Eq. (25)].

On the basis of the above reasoning, the operators in the matrix element ought not to be sensitive to relatively small percentage changes in some of the variables q, ω, θ, k_n, k_p, etc. So that we can vary these a little to obtain our sum over $|\Xi_s\rangle$. It is suspected that at these high energies, large q, etc., averaging over any variable which appears in an insensitive fashion (in order to obtain a sum over E_s), would give us closure. For example, for small θ, $q_\mu{}^2$ is the only rapidly varying quantity in the cross section except perhaps the phase space factors for the nucleons. Thus in this case one could average over a small region in q, keeping $q_\mu{}^2$ fixed. Or, one could just use poor resolution on E_n, E_p (final nucleon energies). Indeed the experimentalists probably won't be able to avoid poor resolution on E_n.

Of course, one cannot, at this stage, say anything about the validity of the above arguments on experimental grounds. Let

$$\epsilon=\omega-(E_p+E_n), \tag{8}$$

where p, n stand for emitted proton and neutron. Let E_s be the excitation energy of the residual nucleus; E_t and E_r the binding energy of the ground state and the recoil nucleus.

The difference, therefore, gives the binding energy of the pair; E_R is the kinetic energy of the recoil nucleus. Also, as discussed above, we assume that the residual states which are populated come from a narrow band, characterized by a well-defined average energy.

$$\bar{\epsilon}=\langle E_s+E_R+E_t-E_r\rangle_{\mathrm{av}}\ll\omega. \tag{9}$$

An estimate of $\bar{\epsilon}(\approx 50\ \mathrm{MeV})$ has been made in Appendix B.

Now one can sum over the residual states to obtain

$$d\sigma=2\pi\delta(\epsilon-\bar{\epsilon})\left(\frac{4\pi e^2}{q_\mu{}^2}\right)^2 f^2(q_\mu{}^2)(1+\cos\theta)\times\tfrac{1}{2}A(A-1)\mathcal{A}_f\frac{d\mathbf{k}_e{}'}{(2\pi)^3}\frac{d\mathbf{k}_p}{(2\pi)^3}\frac{d\mathbf{k}_n}{(2\pi)^3}, \tag{10}$$

the factor $\frac{1}{2}A(A-1)$ appears, because we are for simplicity considering nuclei with equal numbers of neu-

[14] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **91**, 398 (1953).

trons and protons. Also,

$$\mathcal{A}_f=\langle 0|\{Q_1{}^+\Lambda_f{}^{(-)}Q_1-(\omega/q)(Q_2{}'^+\Lambda_f{}^{(-)}Q_1+Q_1{}^+\Lambda_f{}^{(-)}Q_2{}')$$
$$+\tfrac{1}{3}(2\tan^2\theta/2+1)(\mathbf{Q}_2{}^+\Lambda_f{}^{(-)}\cdot\mathbf{Q}_2)$$
$$-\tfrac{1}{2}[2\tan^2\theta/2+1-(3\omega^2/q^2)]$$
$$\times(Q_2{}'^+\Lambda_f{}^{(-)}Q_2{}'-\tfrac{1}{3}\mathbf{Q}_2{}^+\Lambda_f{}^{(-)}\cdot\mathbf{Q}_2)\}|0\rangle,\quad(11)$$

where

$$\begin{pmatrix}Q_1\\ \mathbf{Q}_2\\ Q_2{}'\end{pmatrix}=(1+WG_E)\begin{pmatrix}\Theta_1\\ \mathbf{\Theta}_2\\ \Theta_2{}'\end{pmatrix};\quad \Theta_2{}'=\hat{q}\cdot\mathbf{\Theta}_2 \qquad \Lambda_f{}^{(-)}=|\chi_f{}^{(-)}\rangle\langle\chi_f{}^{(-)}|.\quad(12)$$

Thus, $\Lambda_f{}^{(-)}$ is the projection operator onto the two-body scattering state of nucleon pair. The Θ's are also 2-body (nucleon) operators.[15]

$$\Theta_1=\Theta_{1p}+\Theta_{1n}$$
$$=\left[e_p+\frac{q^2}{8M^2}(e_p-2\mu_p)\right]e^{i\mathbf{q}\cdot\mathbf{r}_p}+\left[e_n+\frac{q^2}{8M^2}(e_n-2\mu_n)\right]e^{i\mathbf{q}\cdot\mathbf{r}_n},\quad(13)$$

$$\mathbf{\Theta}_2=\frac{e_p}{2M}[\mathbf{p}_pe^{i\mathbf{q}\cdot\mathbf{r}_p}+e^{i\mathbf{q}\cdot\mathbf{r}_p}\mathbf{p}_p]+\frac{\mu_p}{2M}i(\boldsymbol{\sigma}_p\cdot\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}_p}+[p\rightarrow n].$$

2.4. The Correlation Function

The operator W incorporates the effect of all the other nucleons on the pair. So a neglect of W is akin to the neglect of final-state interactions. Putting $W=0$, the operator Q's reduce to Θ's and thus the corresponding expressions for the matrix element $\mathcal{A}_f$ remain the same save for replacing Q by Θ.

Later on, we consider the final-state interactions and give a treatment based on the optical model.

Let us, therefore, consider the following transition matrix element:

$$\mathcal{A}'=\tfrac{1}{2}A(A-1)\langle 0|\Theta^+\Lambda_f{}^{(-)}\Theta|0\rangle.\quad(14)$$

Define

$$\psi_0(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_A)\equiv\langle\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_A;\xi_1,\xi_2,\cdots,\xi_A|0\rangle\quad(15)$$

as the nuclear ground state in configuration space; ξ's denote all the quantum numbers necessary to specify the complete state. We can thus expand (14) on this basis, as follows:

$$\mathcal{A}'=\tfrac{1}{2}A(A-1)\sum_{(\xi)(\xi')(\xi'')}\int(d\mathbf{r})(d\mathbf{r}')(d\mathbf{r}'')(d\mathbf{r}''')$$
$$\cdot\psi_0{}^*(\mathbf{r},\xi)\langle\mathbf{r}\xi|\Theta^+|\mathbf{r}'\xi'\rangle\langle\mathbf{r}'\xi'|\chi_f{}^{(-)}\rangle\langle\chi_f{}^{(-)}|\mathbf{r}''\xi'\rangle$$
$$\cdot\langle\mathbf{r}''\xi'|\Theta|\mathbf{r}'''\xi''\rangle\psi_0(\mathbf{r}'''\xi'').\quad(16)$$

Here $(\mathbf{r})$, $(\mathbf{r}')$, $\cdots$; (ξ), (ξ'), $\cdots$, etc., denote the set $\{\mathbf{r}_1,\mathbf{r}_2\cdots\mathbf{r}_A\}$, $\cdots\{\xi_1,\xi_2\cdots\xi_A\}$, etc. Now remembering that the operators Θ are only 2-nucleon operators, i.e., they depend only on nucleons 1 and 2, we obtain

$$\mathcal{A}'=\tfrac{1}{2}A(A-1)\sum_{\xi,\xi',\xi''}\int d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_1{}'d\mathbf{r}_2{}'d\mathbf{r}_1{}''d\mathbf{r}_2{}''d\mathbf{r}_1{}'''d\mathbf{r}_2{}'''$$
$$\cdot\langle\mathbf{r}_1\mathbf{r}_2\xi|\Theta^+|\mathbf{r}_1{}'\mathbf{r}_2{}'\xi'\rangle\langle\mathbf{r}_1{}'\mathbf{r}_2{}'\xi'|\chi_f{}^{(-)}\rangle$$
$$\cdot\langle\chi_f{}^{(-)}|\mathbf{r}_1{}''\mathbf{r}_2{}''\xi'\rangle\langle\mathbf{r}_1{}''\mathbf{r}_2{}''\xi'|\Theta|\mathbf{r}_1{}'''\mathbf{r}_2{}'''\xi''\rangle$$
$$\cdot\sum_{(\xi_3\cdots\xi_A)(\xi_3{}'\cdots\xi_A{}')}\int(d\mathbf{r}_3\cdots d\mathbf{r}_A)\psi_0{}^*(\mathbf{r}_1\mathbf{r}_2\cdots\mathbf{r}_A;\xi)$$
$$\cdot\psi_0(\mathbf{r}_1{}'''\mathbf{r}_2{}'''\cdots\mathbf{r}_A;\xi'').\quad(17)$$

Here in Eq. (17), the meaning of ξ is a little different: It just signifies the spin and I-spin states of a 2-nucleon system (ST,M_SM_T).

Now in order to be able to compute the matrix element, we must assume some functional form for the nuclear ground state. We assume the well-known function which has been extensively used in the past:

$$\psi_0(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_A)=\prod_{i>j=1}^{A}C_{ij}(r_{ij})\Phi_s(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_A),\quad(18)$$

where

$$C_{ij}(r_{ij})=\sum_\xi g_\xi(r_{ij})\Lambda_{ij}{}^S\Lambda_{ij}{}^T.\quad(19)$$

Here Φ_s is the independent-particle Slater determinant and C_{ij} denote the pairwise correlations introduced in order to maintain the boundary condition that the wave function vanish whenever any two nucleons approach to within the repulsive core radius. This function is the 2-particle correlation function, introduced earlier, *for a given spin and isospin state.* Λ^S and Λ^T are the spin and I-spin projection operators, respectively.

Equation (18) is roughly based on three assumptions: (i) the neglect of nuclear surface effects in the correlation C_{ij} (which implies treating the nucleus as an infinite medium of average particle density), (ii) the neglect of Coulomb interactions between protons, and (iii) the maintenance of the orthogonality of the independent-particle states. These assumptions have been critically examined in detail by Drell and Huang[16] and by Jastrow,[17] who first proposed them. The interested

[15] Even though we call Q's (or, Θ's) 2-body operators, from the form of the interaction Hamiltonian postulated, Eq. (13) is actually only a sum of 1-*nucleon* operators. This is a very good approximation for moderately large energy-momentum transfers. However, for extremely large energy-momentum range, one has to incorporate the pion-connected additions to the purely electromagnetic interactions and thus Θ's must in fact be at least 2-nucleon operators.

[16] S. D. Drell and K. Huang, Phys. Rev. **91**, **1527** (1953).

[17] R. Jastrow, Phys. Rev. **98**, 1479 (1955).

reader is referred to these papers for detail; we do not examine it here.

With the nuclear ground state of Eq. (18), it is a simple task to obtain, for completely closed-shell nuclei, after a little algebra:

$$\sum_{\xi_{A-2}}\int(d\mathbf{r}_3\cdots d\mathbf{r}_A)$$
$$\cdot\psi_0^*(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\cdots\mathbf{r}_A;\xi,\xi)_{A-2}\psi_0(\mathbf{r}_1'\mathbf{r}_2'\mathbf{r}_3'\cdots\mathbf{r}_A;\xi',\xi_{A-2})$$
$$=\delta_{\xi\xi'}\frac{(A-2)!}{A!}\sum_{\alpha,\beta}\phi_\alpha^*(\mathbf{r}_1)\phi_\beta(\mathbf{r}_2)[\phi_\alpha(\mathbf{r}_1')\phi_\beta(\mathbf{r}_2')$$
$$-(-)^{S+T}\phi_\alpha(\mathbf{r}_2')\phi_\beta(\mathbf{r}_1')]g_\xi^*(r)g_\xi(r'). \quad (20)$$

The result (20) follows from the orthonormal character of the single-particle states $\phi_i(\mathbf{r}_j)$. Such a simple form is not obtained for nuclei with extra nucleons outside of closed shells. In the applications to follow, the deviation between the two cases is however small.[16]

We need one more approximation to bring (20) to a specially simple form. Our whole approach is based on the fact that the two outgoing nucleons must have been extremely close in the nucleus in order to have high momenta. So, for the *shell-model part*, we can effectively say that the main contribution comes in when $\mathbf{r}_1\approx\mathbf{r}_2\approx\frac{1}{2}\mathbf{R}$ and $\mathbf{r}_1'\approx\mathbf{r}_2'\approx\frac{1}{2}\mathbf{R}'$. In other words, we are limiting to *initial* S states. With this approximation, (20) reduces to:

$$\approx\delta_{\xi\xi'}\delta_{S,1-T}\frac{(A-2)!}{A!}2\sum_{\alpha,\beta}\phi_\alpha^*(\mathbf{R})\phi_\beta^*(\mathbf{R})$$
$$\cdot\phi_\alpha(\mathbf{R}')\phi_\beta(\mathbf{R}')g_\xi^*(r)g_\xi(r'). \quad (21)$$

Thus, from (17) and (21), one obtains, after making the usual lab to center-of-mass transformations, viz., $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$; $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$, etc.:

$$\mathcal{A}'=\sum_{\xi,\xi'}\delta_{S,1-T}\int d\mathbf{r}d\mathbf{r}'\cdots d\mathbf{r}'''$$
$$\cdot g_\xi^*(r)g_\xi(r''')\int d\mathbf{R}\cdots d\mathbf{R}'''\langle\mathbf{r}\mathbf{R}\xi|\Theta^+|\mathbf{r}'\mathbf{R}'\xi'\rangle$$
$$\cdot\langle\mathbf{r}'\mathbf{R}'\xi'|\chi_f^{(-)}\rangle\langle\chi_f^{(-)}|\mathbf{r}''\mathbf{R}''\xi'\rangle$$
$$\cdot\langle\mathbf{r}''\mathbf{R}''\xi'|\Theta|\mathbf{r}'''\mathbf{R}'''\xi''\rangle\sum_{\alpha,\beta}\phi_\alpha^*(\mathbf{R})\cdots\phi_\beta(\mathbf{R}'''). \quad (22)$$

Now,

$$\langle\mathbf{r}\mathbf{R}\xi|\Theta|\mathbf{r}'\mathbf{R}'\xi'\rangle=\delta(\mathbf{R}-\mathbf{R}')e^{i\mathbf{q}\cdot\mathbf{R}}\langle\mathbf{r}\xi|\Theta'|\mathbf{r}'\xi'\rangle$$

and (23)

$$\langle\mathbf{r}'\mathbf{R}'\xi'|\chi_f^{(-)}\rangle=\delta_{\xi'\xi_f}e^{i\mathbf{P}_f\cdot\mathbf{R}}\chi(\mathbf{r}'\xi_f),$$

where $\mathbf{P}_f=\mathbf{k}_p+\mathbf{k}_n$, is the final outgoing nucleon momentum, and Θ' is the relative part of the operator Θ; $\chi(\mathbf{r}\xi_f)$ is the relative 2-nucleon final wave function. With these, (22) simply reduces to

$$\mathcal{A}'=\sum_{\xi,\xi_f}\delta_{S,1-T}\int d\mathbf{R}d\mathbf{R}'e^{i(\mathbf{P}_f-\mathbf{q})\cdot(\mathbf{R}-\mathbf{R}')}\sum_{\alpha,\beta}\phi_\alpha^*(\mathbf{R})\phi_\beta^*(\mathbf{R})$$
$$\times\phi_\alpha(\mathbf{R}')\phi_\beta(\mathbf{R}')\int d\mathbf{r}\cdots d\mathbf{r}'''g_\xi^*(r)g_\xi(r''')$$
$$\times\langle\mathbf{r}\xi|\Theta^+|\mathbf{r}'\xi_f\rangle\chi(\mathbf{r}'\xi_f)\chi^*(\mathbf{r}''\xi_f)\langle\mathbf{r}''\xi_f|\Theta'|\mathbf{r}'''\xi\rangle. \quad (24)$$

Introducing $\mathbf{P}=\mathbf{P}_f-\mathbf{q}$, the total 2-nucleon momentum *initially*, and the form factor

$$F(P)=\delta_{S,1-T}\sum_{\alpha,\beta}\left|\int d\mathbf{R}e^{-i\mathbf{P}\cdot\mathbf{R}}\phi_\alpha(\mathbf{R})\phi_\beta(\mathbf{R})\right|^2, \quad (25)$$

we can simplify (24) to:

$$\mathcal{A}'\equiv F(P)S_{fi}, \quad (26)$$

where S_{fi} is the relevant transition matrix element

$$S_{fi}=\sum_{\xi,\xi_f}\left|\int(d\mathbf{r})(d\mathbf{r}')\chi^*(\mathbf{r}\xi_f)\langle\mathbf{r}\xi_f|\Theta'|\mathbf{r}'\xi\rangle g_\xi(r')\right|^2. \quad (27)$$

This is our formal expression.

We have explicit expressions for the operators Θ from Eq. (13). One can also substitute for the final 2-nucleon scattering state, $\chi(\mathbf{r}\xi_f)$, the phenomenological phase-shift expression.[18] Thus, in principle, one can reduce S_{fi} to an expression involving $g(r)$ and various nucleon phase shifts. Because of the partial-wave expansion of χ, it would be impossible to *determine* $g(r)$ from an experimental knowledge of S_{fi} (through the cross section). However, one can employ various suggested functional forms for $g(r)$ and try to fit the experimental data. If the cross section were even moderately well determined, one may try, say, a 2-parameter representation for $g(r)$.

The lack of any experimental activity on this process inhibits us from making a detailed phase-shift analysis. However, we have left the expressions in a form amenable to that effect, if that were needed in future.

For the present, in the next subsection, we console ourselves with using plane waves for χ, hoping to be able to extract the important gross features of the problem and studying its behavior.

It must be borne in mind that the actual result is not quite Eq. (27), since if we look back into the original matrix element $\mathcal{A}_f$ of Eq. (11) we see that there are interference terms like $\langle 0|\Theta_2'^+\Lambda^{(-)}\Theta_1|0\rangle$, etc., whereas the expression (27) has been derived by using $\mathcal{A}_f$ of the form $\langle 0|\Theta^+\Lambda_f^{(-)}\Theta|0\rangle$ as in Eq. (14). This, however, is not a serious drawback and the consequence of this is just to make S_{fi} more involved. In an actual computation of the cross section using the interaction [as in (28) below] we have taken account of this fact.

[18] L. Durand, Phys. Rev. **115**, 1020 (1959).

The formal result has been left as such in its compact form of Eq. (27).

The form factor $F(P)$ depends solely upon the Slater determinant states and has been computed by Gottfried (see Fig. 1 of Ref. 7). In principle, $F(P)$ can be defined for an arbitrary shell model state or, if preferred, one may not even bring in the shell model. This is because all that $F(P)$ expresses is the momentum distribution of the nucleons in the nucleus when they move independently, devoid of any short-range correlations. Then, the factor $g(r)$ expresses the average small distance correlations.

This qualitative feature is partially brought out by a numerical calculation made by Gottfried, i.e., that $F(P)$ is almost model-independent. Hence it confirms our rather naive remarks in the last paragraph.

Now the physics beyond the shell model is embodied in S_{fi}. Unfortunately, however, $F(P)$ is a rapidly varying function and S_{fi} which depends upon the short-range correlations is not. Hence, care must be taken to extract information about the slowly varying function S_{fi} and not have it masked by $F(P)$. It is hoped that by properly choosing the kinematics etc. (discussed later) there would be some detectable effect due to the variation of S_{fi}.

2.5. Plane Waves for χ_f

Pending the solution of the problem of determining the exact final scattering state, we have computed the matrix element S_{fi} on the basis of the given Hamiltonian, using plane waves for the final nucleons. The actual computation is quite lengthy yet straightforward. One obtains

$$S_{fi}=\left[\left(1-\frac{\omega}{2M}\right)^2+\frac{q^2}{4M^2}(1-2\mu_1)\right]\{|g_0^-|^2+3|g_1^-|^2\}$$
$$-\frac{q^2\mu_2}{2M^2}\,\mathrm{Re}\{g_0^-(g_0^+)^*+3g_1^-(g_1^+)^*\}$$
$$+\frac{q^2}{4M^2}\left(\tan^2\theta/2+\tfrac{1}{2}-\frac{\omega^2}{2q^2}\right)\{|\mu_1g_0^--\mu_2g_0^+|^2$$
$$+3|\mu_1g_1^--\mu_2g_1^+|^2+4|\mu_1g_1^-+\mu_2g_1^+|^2\}\,,\quad(28)$$

where

$$g_{0,1}^{\mp}(K)\equiv g_{0,1}(|\mathbf{k}\mp\tfrac{1}{2}\mathbf{q}|)=\int(d\mathbf{r})e^{-i\mathbf{K}\cdot\mathbf{r}}g_{0,1}(r)\,,\quad(29)$$

and $\mathbf{k}=\frac{1}{2}(\mathbf{k}_p-\mathbf{k}_n)$ gives the relative momentum of the outgoing pair. The lower indices (0 and 1) on g denote the two spin states that a 2-nucleon system can inhabit. The $(\mp)$ superscript, of course, relates to the two cases $\mathbf{K}^{\mp}=\mathbf{k}\mp\frac{1}{2}\mathbf{q}$.

It is seen from (28) then that the cross section is indeed related to the squares of some linear combinations of the Fourier transform of the correlation functions, as expected from our discussion in Sec. 1.

Also, to get some idea as to the behavior of S_{fi}, we have employed a few well-known types of functions for $g(r)$. Calculations have been performed for a pure Fermi gas and for Brueckner-type functions. For completeness, these are listed in Appendix C.

As stated earlier, if experimental results were shortly available, one would be able to compare them with these estimates of the behavior of the differential cross-section for various models.

Actually, the rather powerful result

$$d\sigma\sim|g(K)|^2,$$

where K may be fixed experimentally, is rendered partially invalid by the final state interactions which are large for low energies and by the initial pion-exchange effects which are substantial at higher energies.

3. CROSS SECTION FROM QUASIDEUTERON MODEL

3.1. Quasideuteron Model

Now as discussed earlier in Sec. 1, we would like to obtain an analogous expression to that in the last section, for the transition matrix element without assuming a particular form for the operators Θ, beyond the fact that they are 2-nucleon operators.

The formalism goes through exactly as before and one ends up with the same expression. We derive the matrix element a little differently from the last section, although quite a few steps are duplicated. It has been rederived for clarity and completeness. Also, some of the arguments involved in invoking various approximations are amplified. Those with faith may omit till Eq. (49).

The cross section may again be cast into a form similar to Eq. (10), after invoking closure over the residual $(A-2)$-body states. Thus, we again are led to consider a matrix element of the form of Eq. (14). A manipulation of this element would form the subject of this subsection.

Consider again a matrix element

$$\mathcal{A}_f'=\tfrac{1}{2}A(A-1)\langle 0|\Theta^+\Lambda_f^{(-)}\Theta|0\rangle\,,\quad(30)$$

where Θ is the 2-nucleon operator responsible for the transition.

Let us define the second-order density matrix Γ as[19]

$$\Gamma=\langle\mathbf{r}_1\mathbf{r}_2SM_STM_T|\rho|\mathbf{r}_1'\mathbf{r}_2'S'M_S'T'M_T'\rangle\quad(31)$$
$$=\tfrac{1}{2}A(A-1)\int\langle 0|\eta;\mathbf{r}_1'\mathbf{r}_2'S'M_S'T'M_T'\rangle$$
$$\times\langle\eta;\mathbf{r}_1\mathbf{r}_2SM_STM_T|0\rangle d\eta\,,\quad(32)$$

[19] The use of the density matrix in a similar context was first employed in Ref. 7. Of course, the results obtained herein can be obtained even otherwise, using standard methods (see Sec. 2, for instance). However, this formalism is more elegant and explicit. See also S. Fujii, Nuovo Cimento **25**, 995 (1962).

where $\xi\equiv(SM_S,TM_T)$ specifies the spin and isotopic spin states of the nucleon-nucleon pair, and η denotes all the coordinates of the rest of the nucleons in the ground state. Thus, using the density matrix, Eqs. (30) and (31) give us

$$\mathcal{A}_f'=\tfrac{1}{2}A(A-1)\langle 0|\Theta^+\Lambda_f^{(-)}\Theta|0\rangle$$
$$=\sum_{\xi,\xi'}\int\int\int\int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_1' d\mathbf{r}_2'\langle\mathbf{r}_1\mathbf{r}_2\xi|\rho|\mathbf{r}_1'\mathbf{r}_2'\xi'\rangle \times\langle\mathbf{r}_1'\mathbf{r}_2'\xi'|\Theta^+\Lambda_f^{(-)}\Theta|\mathbf{r}_1\mathbf{r}_2\xi\rangle. \quad (33)$$

Now since we want to investigate the behavior of the 2-particle correlation function beyond the shell model, we must somehow extract out explicitly the contribution due to direct internucleon forces. As in the past, we also make the simplifying ansatz that the pair correlation function is given by $\rho_s(\mathbf{r}_1,\mathbf{r}_2)|g(r)|^2$, where ρ_s is the shell-model correlation function and $g(r)$ contains those correlations due to direct nucleon-nucleon forces. Written in detail in terms of the density matrix, our ansatz asserts that

$$\langle\mathbf{r}_1\mathbf{r}_2\xi|\rho|\mathbf{r}_1'\mathbf{r}_2'\xi'\rangle\equiv g_\xi(r)\langle\mathbf{r}_1\mathbf{r}_2\xi|\rho_s|\mathbf{r}_1'\mathbf{r}_2'\xi'\rangle g_{\xi'}^*(r'), \quad (34)$$

where $\langle|\rho_s|\rangle$ is the density matrix of a Slater determinant constructed from shell-model spin orbitals, and $g_\xi(r)=g_{ST}(|\mathbf{r}_1-\mathbf{r}_2|)$ describes the presumably short-range modifications due to direct nuclear forces. Now due to the Pauli principle, $g_{ST}(r)\rightarrow 1$ as $r>d$, where d is a characteristic correlation length (the "healing distance" of Gomes, Walecka, and Weisscopf) assumed to be small compared with the nuclear radius.

Then, using (34), Eq. (33) can be written as

$$\mathcal{A}_f'=\tfrac{1}{2}A(A-1)\langle 0|\Theta^+\Lambda_f^{(-)}\Theta|0\rangle$$
$$=\sum_{\xi,\xi'}\int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_1' d\mathbf{r}_2' g_\xi(r)\langle\mathbf{r}_1\mathbf{r}_2\xi|\rho_s|\mathbf{r}_1'\mathbf{r}_2'\xi'\rangle g_{\xi'}^*(r')\langle\mathbf{r}_1'\mathbf{r}_2'\xi'|\Theta^+\Lambda_f^{(-)}\Theta|\mathbf{r}_1\mathbf{r}_2\xi\rangle$$
$$=\sum_{\xi,\xi',\xi''}\int d\mathbf{r}_1\cdots d\mathbf{r}_2''' g_\xi(r''')g_{\xi'}^*(r'')\langle\mathbf{r}_1'''\mathbf{r}_2'''\xi''|\rho_s|\mathbf{r}_1''\mathbf{r}_2''\xi'\rangle$$
$$\times\langle\mathbf{r}_1''\mathbf{r}_2''\xi'|\Theta^+|\mathbf{r}_1'\mathbf{r}_2'\xi\rangle\langle\mathbf{r}_1'\mathbf{r}_2'\xi|\chi_f^{(-)}\rangle\langle\chi_f^{(-)}|\mathbf{r}_1\mathbf{r}_2\xi\rangle\langle\mathbf{r}_1\mathbf{r}_2\xi|\Theta|\mathbf{r}_1'''\mathbf{r}_2'''\xi''\rangle. \quad (35)$$

Now let us decompose the above equation into center of mass and relative coordinates:

$$\mathbf{R}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2),\quad \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\ \text{etc.},$$
$$\langle\mathbf{r}_1''\mathbf{r}_2''\xi'|\Theta^+|\mathbf{r}_1'\mathbf{r}_2'\xi\rangle =\delta(\mathbf{R}''-\mathbf{R}')e^{-i\mathbf{q}\cdot\mathbf{R}'}\langle\mathbf{r}''\xi'|\Theta^+|\mathbf{r}'\xi\rangle, \quad (36)$$

where now it must be remembered that Θ refers to the same operator as before, except that it is rid of its center-of-mass part. Similarly,

$$\langle\mathbf{r}_1\mathbf{r}_2\xi|\chi_f^{(-)}\rangle=e^{i\mathbf{P}_f\cdot\mathbf{R}}\chi_f(\mathbf{r}\xi_f)\delta_{\xi\xi f}, \quad (37)$$

where

$$\mathbf{P}_f=\mathbf{k}_p+\mathbf{k}_n$$

is the total momentum of the outgoing pair of nucleons.

Putting all these factors in (35), one obtains

$$\mathcal{A}_f'=\sum_{\xi',\xi''}\int d\mathbf{r}d\mathbf{r}'d\mathbf{r}''d\mathbf{r}''' g_\xi^*(r''')g_{\xi'}(r'')$$
$$\times\int d\mathbf{R}''\delta(\mathbf{R}''-\mathbf{R}')\int d\mathbf{R}'''\delta(\mathbf{R}'''-\mathbf{R})$$
$$\times\langle\mathbf{r}''\xi'|\Theta^+|\mathbf{r}\xi\rangle\chi(\mathbf{r}\xi_f)\chi^*(\mathbf{r}'\xi_f)\langle\mathbf{r}'\xi|\Theta|\mathbf{r}'''\xi'\rangle$$
$$\times\int d\mathbf{R}d\mathbf{R}'e^{-i(\mathbf{P}_f-\mathbf{q})\cdot\mathbf{R}}\langle\mathbf{R}\mathbf{r}'''\xi''|\rho_s|\mathbf{R}'\mathbf{r}''\xi'\rangle e^{i(\mathbf{P}_f-\mathbf{q})\cdot\mathbf{R}'}. \quad (38)$$

3.2. Quasideuteron Approximation

Due to the short wavelength (high-energy transfer) the initial interaction must affect only a small part of the nucleus, not the system as a whole. (Multiple scattering is improbable.) In fact, this has already been assumed, viz., that the electron interacts "hard" just once (see Sec. 2.2). Thus we surmise that the emitted neutron-proton pair must have come from within the nucleus when the two must have been extremely near, in order that both may be knocked off simultaneously.

In other words, the absorption of the virtual photon by the two nucleons must occur when the two are in near proximity of each other. This has been used extensively to study the photonuclear disintegration by Levinger *et al.*, and it goes by the brand name of "quasideuteron model." It is precisely this fact that makes the ejection of the pair an excellent tool for studying the short-range pair correlations in the nuclei.

Thus, the main contribution to the matrix element comes from very small interparticle distances. Hence, in order to simplify the computation, we take $\mathbf{r}_1\approx\mathbf{r}_2$ and $\mathbf{r}_1'\approx\mathbf{r}_2'$ in $\langle|\rho_s|\rangle$. In effect, therefore, we reduce ourselves to considering only the *initial* relative S state of the n, p pair in the nucleus.

Let us define

$$\mathbf{P}=\mathbf{P}_f-\mathbf{q}, \quad (39)$$

which gives the total momentum of the pair in the nucleus. Now the Fourier transform,

$$\langle \mathbf{P}0\xi''|\rho|\mathbf{P}0\xi'\rangle$$
$$\equiv(2\pi)^{-3}\int d\mathbf{R}d\mathbf{R}'e^{-i\mathbf{P}\cdot\mathbf{R}}\langle \mathbf{R}0\xi''|\rho|\mathbf{R}'0\xi'\rangle e^{i\mathbf{P}\cdot\mathbf{R}'},\quad(40)$$

which is equal to $\frac{1}{2}A(A-1)$ times the probability of finding two nucleons of total momentum $\mathbf{P}$ and 0 separation in the nuclear ground state $|0\rangle$. We must now investigate the ξ, ξ' dependence.

For simplicity we shall consider *completely closed-shell nuclei*, e.g., O^{16} or Ca^{40}, etc. Then, we have

$$\langle \mathbf{r}_1\mathbf{r}_2\xi|\rho_s|\mathbf{r}_1'\mathbf{r}_2'\xi'\rangle =\delta_{\xi\xi'}\tfrac{1}{2}[\langle \mathbf{r}_1|\mathfrak{D}|\mathbf{r}_1'\rangle\langle \mathbf{r}_2|\mathfrak{D}|\mathbf{R}_2'\rangle -(-)^{S+T}\langle \mathbf{r}_1|\mathfrak{D}|\mathbf{r}_2'\rangle\langle \mathbf{r}_2|\mathfrak{D}|\mathbf{r}_1'\rangle],\quad(41)$$

with

$$\langle \mathbf{r}_1|\mathfrak{D}|\mathbf{r}'\rangle=\sum_{nlm}\phi_{nlm}(\mathbf{r}_1)\phi_{nlm}^*(\mathbf{r}'),\quad(42)$$

where ϕ_{nlm}'s are the spatial parts of the shell-model one-body wave functions. In this approximation then, the matrix element (41) reduces to

$$\langle \mathbf{r}_1\mathbf{r}_2\xi|\rho_s|\mathbf{r}_1'\mathbf{r}_2'\xi'\rangle=\delta_{\xi\xi'}\delta_{S,1-T}\langle \mathbf{r}_1|\mathfrak{D}|\mathbf{r}_1'\rangle^2.$$

Now one can define

$$\int\int d\mathbf{R}d\mathbf{R}'e^{-i\mathbf{P}\cdot\mathbf{R}}\langle \mathbf{R}0\xi|\rho_s|\mathbf{R}'0\xi\rangle e^{i\mathbf{P}\cdot\mathbf{R}'}\equiv\delta_{S,1-T}F(P),\quad(44)$$

so that the form factor $F(P)$ is proportional to the probability for finding two nucleons of total momentum P and 0 separation in the Slater determinant. We feel that only a small error is made by adopting the same expression (44) for nonclosed-shell nuclei.

Then, from (38) and the following equations one gets

$$\mathfrak{A}_f'=\sum_{\xi'\xi''}\int d\mathbf{r}\cdots d\mathbf{r}'''g_{\xi'}(r''')g_{\xi''}^*(r'')\langle \mathbf{r}''\xi'|\Theta^+|\mathbf{r}\xi_f\rangle \times\chi_f(\mathbf{r}\xi_f)\chi_f^*(\mathbf{r}'\xi_f)\langle \mathbf{r}'\xi_f|\Theta|\mathbf{r}'''\xi''\rangle\delta_{S,1-T}F(P)\delta_{\xi'\xi''}\quad(45)$$
$$=F(P)\sum_{\xi}\left|\int d\mathbf{r}d\mathbf{r}'\chi_f^*(\mathbf{r}\xi_f)\langle \mathbf{r}\xi_f|\Theta|\mathbf{r}'\xi\rangle g_\xi(r')\right|^2,\quad(46)$$

where $\xi\equiv(S,T)\equiv(S,1-S)$ for nonvanishing matrix elements. Let us define

$$M_f(\xi\to\xi_f)\equiv\int d\mathbf{r}d\mathbf{r}'\chi_f^*(\mathbf{r}\xi_f)\langle \mathbf{r}\xi_f|\Theta|\mathbf{r}'\xi\rangle g_\xi(r').\quad(47)$$

Since in the final state the spins and the isospins of the nucleons are not measured, one must sum over it. So, define another quantity

$$S_{fi}\equiv\sum_{\xi_f}|M(\xi\to\xi_f)|^2,\quad(48)$$

so that the cross section can be written compactly with the neglect of final-state interactions as follows:

$$d\sigma=B\cdot\delta(\epsilon-\bar{\epsilon})F(P)S_{fi}(d\mathbf{k}_e')(d\mathbf{k}_p)(d\mathbf{k}_n),\quad(49)$$

where B contains certain kinematical factors.

This result may be compared formally with analogous expressions derived for the photodisintegration processes. It has purposely been put into such a form to bring out explicitly the apparent similarity between the electro- and photodisintegration processes (at least for almost-forward electrons)—a fact which has been exploited thoroughly in the past in various other contexts.

4. DEUTERON ELECTRODISINTEGRATION

The purpose of considering the deuteron breakup and relating it to the complex nuclei case, as mentioned before in Sec. 1 (see also Ref. 7), is twofold: (i) to include the pair interaction in the final state i.e., right after the absorption of the virtual photon, and (ii) in order to be able to include the pion or exchange contributions to the purely electromagnetic interaction (at least between the two nucleons in question). The goal at first would be to measure a single number γ (defined below). For the process, $e+d\to e'+n+p$, we can write

$$d\sigma_D=2\pi\tfrac{1}{2}\sum_{\text{spins}}|M^D|^2d^3(dN/dE) =2\pi\tfrac{1}{2}\sum_{\text{spins}}|M^D|^2(2\pi)^{-6}(d\mathbf{k}_e')(d\Omega_p)\cdot\frac{E_pk_p^2(\omega-E_p)}{k_p\omega-E_p\hat{k}_p\cdot\mathbf{q}},\quad(50)$$

where $\mathbf{q}$ and ω retain their old definitions and $E_p=(k_p^2+M^2)^{1/2}$; Ω_p gives the solid angle of the proton. Retracing the old steps of Sec. 3, one obtains simply

$$\tfrac{1}{2}\sum_{\text{spins}}|M^D|^2=\tfrac{1}{2}\sum_{\text{spins}}|\langle f|H'|i\rangle^2 \sim\langle\phi_D|\Theta^+\Lambda_f^{(-)}\Theta|\phi_D\rangle\equiv\mathfrak{A}^D,\quad(51)$$

where $\mathfrak{A}^D$ is exactly the same as $\mathfrak{A}_f'$ for the complex nuclei case, except that the nuclear ground state is to be replaced by the deuteron ground state, $\phi_D(\mathbf{r}\xi)$.

Now here we have to make an assumption. That either transition from $^1S_0$ states may be neglected or that the value of $g(r)$ is the same for $S=0$ and $S=1$ states. We assume the latter. Then, following Gottfried, let us assume that

$$|g_{10}(r)|^2\equiv\gamma^3|\phi_D(r)|^2,\quad(52)$$

for $r\lesssim 1f$, where γ is a certain constant. Then

$$\mathfrak{A}_f'=3\gamma^3\mathfrak{A}_f^D.\quad(53)$$

And so we have

$$\frac{d\sigma_A}{(d\mathbf{k}_e')(d\mathbf{k}_p)(d\mathbf{k}_n)}=B\delta(\epsilon-\bar{\epsilon})F(P)\mathfrak{A}_f\Gamma f_a,$$
$$\frac{d\sigma_D}{(d\mathbf{k}_p')(d\Omega_p)}=(2\pi)^{-3}B\frac{E_p(\omega-E_p)k_p^2}{k_p\omega-E_p\hat{k}_p\cdot\mathbf{q}}\mathfrak{A}_f^D.\quad(54)$$

Here (Γf_a) denotes the final-state corrections (formally derived in Sec. 5).

As long as one can take the deuteron breakup to be on the mass shell of the complex nuclei case, the two expressions can be related to obtain

$$\left(\frac{d\sigma_A/(d\mathbf{k}_e')(d\Omega_p)}{d\sigma_D/(d\mathbf{k}_e')(d\Omega_p)}\right) = \frac{3\gamma^3}{(2\pi)^3}\delta(\epsilon-\bar{\epsilon})F(P)k_pE_p\left(\frac{k_p\omega-E_p\hat{k}_p\cdot\mathbf{q}}{E_pk_p^2(\omega-E_p)}\right) \times f_a\Gamma(\mathbf{k}_p,\mathbf{k}_n)dE_p(d\mathbf{k}_n). \quad (55)$$

5. FINAL-STATE INTERACTIONS AND OTHER PROCESSES

This is based on Gottfried's treatment. We just sketch his arguments—the detailed logic and computations may be found in Ref. 7.

Here one retains W and assumes that the rest of the $(A-2)$ nucleons serve to define a static optical-model potential well $\mathcal{W}$ seen by the pair. Since elastic and inelastic (refraction and absorption) scattering both occur, $\mathcal{W}$ would in general be complex. Let us now make the approximation that V, i.e., the interaction between the outgoing pair of nucleons can be neglected during the multiple scattering.

Then, it is found that the expressions involve off-energy-shell matrix elements. We make the simple assumption that these are the same as on the energy shell. A plausible justification of this assumption is given by Gottfried.

5.1. Refraction at the Nuclear Surface

Real part of $\mathcal{W}$ is rather shallow (≈ 10 MeV) for high-energy nucleons ($\gtrsim 150$ MeV), so we can treat Re$\mathcal{W}$ in Born approximation. The cross section is then seen to be proportional to

$$|\mathcal{T}|^2 \equiv |\mathcal{T}_0|^2\Gamma(\mathbf{k}_p,\mathbf{k}_n), \quad (56)$$

where $\mathcal{T}_0$ is the amplitude in the absence of final-state interactions, and $\Gamma(\mathbf{k}_p,\mathbf{k}_n)$ is the correction factor due to refraction at the nuclear surface.

$$\Gamma(\mathbf{k}_p,\mathbf{k}_n) = [1-(1/\sqrt{2})\mathcal{W}_0 MD^2 e^{P^2D^2/8}](1/K_nD) \times\{\mathcal{J}(K_n+k_n)+\mathcal{T}(K_n-k_n)+(n\leftrightarrow p)\}, \quad (57)$$

where

$$\mathcal{J}(s) = D^{-1}\int_0^\infty \sin sx\, e^{-\frac{1}{2}x^2D^2}dx. \quad (58)$$

This has been obtained simply by assuming that all the nucleons are in the lowest state of a harmonic oscillator with spatial wave function proportional to $e^{-1/2r^2D^2}$, and that the optical-model potential is a Gaussian

$$\mathcal{W}(r) = \mathcal{W}_0 e^{-r^2D^2}. \quad (59)$$

5.2. Attenuation

A simple minded mean free path type calculation[7] made on the basis of the work of Serber *et al.*, says that the cross section is depleted by a factor of $f_a=0.30$ (for O^{16}).

This factor may be compared with the one derived by Jacob and Maris.[3] By the simple WKB method they obtain a reduction factor of 0.59 for 1-nucleon emission from 1*s* states. Assuming that the two nucleons may be absorbed independently one obtains a factor of 0.35 for 2-nucleon cases. This is thus quite close to the estimate made by Gottfried. In passing, it may be mentioned that for the corresponding process induced by protons (instead of electrons as over here) the attenuation factor for the emission of one nucleon is of the order of 0.005. This, as mentioned in Sec. 1, suggests a strong motivation for using a weakly interacting projectile like the electron for inducing the 2-nucleon emission.

5.3. Discussion of Final-State Correction

Now a qualitative discussion of the final-state correction made above is in order. It is quite obvious that the above estimates of refraction at the surface and the attenuation are extreme oversimplifications made on the basis of some brutal assumptions. For instance, in Eq. (56) one should actually have an integration over the directions and magnitudes of all intermediate 2-nucleon momenta, e.g.,

$$\int d\mathbf{k}_p'd\mathbf{k}_n'|\mathcal{T}_0(\mathbf{k}_p',\mathbf{k}_n')|^2\Gamma(\mathbf{k}_p',\mathbf{k}_n';\mathbf{k}_p,\mathbf{k}_n).$$

This is due to the fact that the nucleons are deflected and lose kinetic energy passing through the nucleus. Thus, $(\mathbf{k}_n-\mathbf{k}_p)$ and $(\mathbf{k}_n+\mathbf{k}_p)$ values *in* the nucleus are not the same as *outside* because of the momentum transfers to the recoil nucleons on passing through the nuclear surface.

However, we have made the approximation that the matrix element in the intermediate state is the same as on the mass shell. If this were not done, comparison with the deuteron would not be allowed. Also, the refraction is weak because of shallow potential for high-energy nucleons. Consequently, the energy loss in the refraction process is neglected. The absorption process which in a crude way is supposed to incorporate the energy loss, etc., is done via a mean-free-path treatment.

A more satisfactory and complete final state treatment is possible and can be effected. However, the present experimental situation does not warrant such a detailed exposition. If the projected and proposed experiments were to be performed, the above calculations could be done and therefore, we have left the expressions in a form that is available to that effect. In our present treatment, we shall incorporate the effects of final-state interactions by multiplying the cross section computed with the impulse approximation by the factor $f_a\Gamma(\mathbf{k}_p,\mathbf{k}_n)$.

5.4. Meson Production

Now it is relevant to ask if at such high-energy momenta under consideration, the mesic degrees of freedom can be ignored. And what estimates can be made about the amount of interference from the process of producing a *real* pion, and what can be done to minimize this process?

Here we are alluding, of course, to the process of the production of a (real) pion by the electron on a nucleon and its subsequent absorption by any other nucleon in the nucleus. One must remember that this mesic effect is different from the absorption of a virtually exchanged pion by the neighboring nucleon (which as remarked in Sec. 1, we hope to account for by deuteron comparison). This latter is a more important effect since it is a quick process and can thus occur way off the mass shell.

A simple order-of-magnitude estimate of the interference from the process in which a (real) pion is produced by the electron on a nucleon and is subsequently reabsorbed by another nucleon in the nucleus is sketched below and is found to contribute $\lesssim$5–10% of the main process. There may also occur the case in which a pion is produced which is absorbed by two nucleons ("quasi-deuteron"). This process may also contribute $\lesssim$10%.

The estimate for the process in which first the incident electron hits one nucleon producing a pion which then gets reabsorbed by another nucleon can simply be made on the basis of probability considerations. The estimate is admittedly crude and serves only to give an indication of the size of the process. The total cross section for this process may be written as

$$\sigma_\pi=\int(d\mathbf{P}_1)(d\mathbf{P}_2)(d\mathbf{k}_\pi)\cdot\sigma(eN\rightarrow e'N'\pi)\cdot N(P_1)\cdot W_N\cdot N(P_2)\cdot\delta(\text{momentum-conservation}),$$

where $\sigma(eN\rightarrow e'N'\pi)$ gives the cross section for producing a pion of momentum $\mathbf{k}_\pi$ from $e-N$ scattering; $N(P_1)$ gives the probability of finding a nucleon of momentum P_1 in the nucleus; W_N is the probability of absorbing a real meson by a nucleon in the nucleus.

Of course, in order to be able to proceed we must make certain approximations. We assume the electroproduction of pion cross section to be at its peak. Also, from Wilson's work[20] on the deuteron, we have $W_N=\frac{1}{5}$.

The above assumptions, we hope would give a maximum limit on the total cross section. Assuming as usual, that $N(P)$, the probability of finding a nucleon with momentum P in the nucleus is of the form $\sim e^{-P^2/P_0^2}$ with $P_0\approx0.9$ F^{-1}, we obtain

$$\sigma_\pi\sim W_NN(|\mathbf{k}_1+\mathbf{k}_\pi^{(0)}-\mathbf{q}|)N(|\mathbf{k}_2-\mathbf{k}_\pi^{(0)}|)\cdot\sigma_{\max}{}^{e\pi}.$$

[20] R. R. Wilson, Phys. Rev. **86**, 125 (1952); **104**, 218 (1956).

With geometry for the nucleon and electron counters considered in the estimation of the cross section for main process in Sec. 6.5, we obtain that this cross section is $\lesssim$5–10% of the main channel.

Similarly, the interference from the other channel (2-nucleon absorption) may be computed. Analogously, one finds it to be $\lesssim$10%.

6. KINEMATICS AND CROSS-SECTION ESTIMATE

6.1. A Dependence of the Cross Section

The cross section depends on A through two factors: the form factor $F(P)$ and the attenuation factor f_a. We see below that the maximum of $F(P)$ is $F(0)=\frac{1}{4}A$. Thus, taking f_a as a constant, the maximum cross section varies as A. $F(P)$ goes down rather rapidly as P increases. Hence, to keep the cross section appreciable we must try for $P\rightarrow0$, i.e., $|\mathbf{P}|=|\mathbf{k}_p+\mathbf{k}_n-\mathbf{q}|\rightarrow0$. This also agrees with the intuitive picture that the cross section should be peaked around $\mathbf{k}_p+\mathbf{k}_n\approx\mathbf{q}$.

From our expression of $F(P)$, for an infinite square well, say, one can obtain

$$F(P)=\tfrac{1}{4}A[1+(P^4/16\alpha^4)]e^{-P^2/2\alpha^2},$$

where α is adjusted to yield the rms charge found in electron scattering. Following Fujii,[19] $\alpha=0.622$ F^{-1}. Anyway, one finds $F(0)=\frac{1}{4}A$.

Also, we have made a rough estimate of the average, $\langle F(P)\rangle_{\text{av}}$, for P lying below the Fermi momentum. We obtain $\langle F(P)\rangle_{\text{av}}$ for $0<P<200$ MeV/c to be $\sim\frac{1}{6}A$. In other words, $\langle F(P)\rangle_{\text{av}}$ is down by some 25–30% of its peak value $\frac{1}{4}A$ at $P=0$. This question is of practical interest in getting a better estimate of experimental cross section with imperfect resolution.

6.2. Kinematics to Exclude Single-Nucleon Emission

It would be advantageous to arrange the kinematics such that *single*-nucleon emission is excluded (or at least greatly reduced). As shown below, in order to effect this we must have large ω and small $\mathbf{q}$ (of course within the physical limits) in order that the virtual photon may not be absorbed by a single nucleon alone. For extremely relativistic electrons,

$$\mathbf{q}=\mathbf{k}_e-\mathbf{k}_e',$$
$$\omega=k_e-k_e'.$$

Thus, $|\mathbf{q}|\geq\omega$ in the physical region. Now

$$\mathbf{q}=\mathbf{P}_f-\mathbf{P}=\mathbf{k}_1+\mathbf{k}_2-\mathbf{P}.$$

For the maximum cross section case $\mathbf{P}=0$. So let us consider the case of $\mathbf{P}=0$.

Treating the nucleons nonrelativistically then, we have for single-nucleon emission

$$\omega=(k_1^2/2M)+\bar{\epsilon}.$$

(An estimate of $\bar{\epsilon}\approx 50$ MeV has already been made in Appendix B.) Also, assuming that the initial nucleon has Fermi momentum k_F,

$$\mathbf{q}=\mathbf{k}_1+\mathbf{k}_F.$$

Then,

$$|\mathbf{q}-\mathbf{k}_F|\leq k_1\leq q+k_F.$$

So, in order that a single-nucleon emission may *not* occur, we must have

$$q\geq\omega>[(q+k_F)^2/2M]+\bar{\epsilon}. \qquad (60)$$

As an example, let us consider $q=200$ MeV$/c=k_F$. Then, ω must be between 130 and 200 MeV.

Now there may be nucleons in the nucleus with several times the Fermi momentum, which may yet be emitted singly; actually it is the high-momentum tail that we are interested in. Let us set a maximum limit of $2k_F\approx 400$ MeV$/c$. Then our criterion becomes instead of (60)

$$q\geq\omega>[(q+2k_F)^2/2M]+\bar{\epsilon}. \qquad (61)$$

As examples let us consider two cases: (i) for $q=400$ MeV$/c$, we must have $\omega>370$ MeV and (ii) for $q=800$ MeV$/c$, $\omega>770$ MeV.

This would then exclude the possibility of a single-nucleon emission save those having momenta in the nucleus larger than twice that of Fermi momentum.

When q and ω are almost equal (as is seen to be required for the exclusion of single-particle emission) the scattering becomes almost forward, i.e., $\theta\approx 0°$.

Now for low-momentum transfers, i.e., small $|\mathbf{q}|$, Czyż and Gottfried[10] have pointed out that the transverse electron-nucleon interaction is not important, and that one would therefore expect the Coulomb interaction to dominate (which would probably accentuate the single-proton emission).

However, hopefully, this is *not* so for large q, as shown by Willey[9] and by Czyż.[21] They point out that the transverse interactions *are* very important even for small θ, provided the momentum transfer is of the order of (or, of course, if greater than) k_F, the Fermi momentum. Thus, one has to *include* the transverse interactions in any realistic calculation dealing with short-range correlations (which dominate for large ω's) even in the case of small-angle inelastic scattering.

6.3. Proposed Kinematics for the Process

First, we must note that both $\mathbf{q}$ and ω must be kept constant to effect closure over the $(A-2)$-body states. We consider *forward* scattered electrons. The cross section is then the largest because of the small q_μ^2 in the denominator. We also require the energy of the individual nucleons emitted to be $\gtrsim 100$ MeV in order that there be small final-state corrections.

In Sec. 2.5, we obtained the cross section as a linear combination of the squares of the correlation functions, $g(K_+)$ and $g(K_-)$, where the arguments are

$$\mathbf{K}_\pm=\mathbf{k}\pm\tfrac{1}{2}\mathbf{q}=\tfrac{1}{2}(\mathbf{k}_p-\mathbf{k}_n\pm\mathbf{q}). \qquad (62)$$

Thus, for ease of interpretation, it would be very desirable to have the relative momentum of the outgoing nucleons $\mathbf{k}\perp\mathbf{q}$, so that the arguments $K_+=K_-$. As can easily be ascertained we cannot entertain the other possibilities, viz., $\mathbf{k}=0$ or $\mathbf{q}=0$.

So, we present here the various kinematical values obtained (with $\mathbf{k}\perp\mathbf{q}$ and $\mathbf{P}\approx 0$) for two sets of $|\mathbf{q}|$ values, 200 and 400 MeV$/c$.

Set I:

$$\left.\begin{array}{l} k_e=500\text{ MeV}/c \\ k_e'=300\text{ MeV}/c \\ E_n=E_p=75\text{ MeV} \end{array}\right\} \rightarrow \left\{\begin{array}{l} q=200\text{ MeV}/c \\ \omega\lesssim 200\text{ MeV}/c \\ \omega-\bar{\epsilon}\approx 150\text{ MeV} \\ k_n=k_p\approx 400\text{ MeV}/c \end{array}\right. \quad \text{Then, the argument of } g \text{ is } K=K_+=K_-\approx 400\text{ MeV}/c. \qquad (63)$$

Set II:

$$\left.\begin{array}{l} k_e=500\text{ MeV}/c \\ k_e'=100\text{ MeV}/c \\ E_n=E_p=175\text{ MeV} \end{array}\right\} \rightarrow \left\{\begin{array}{l} q=400\text{ MeV}/c \\ \omega\approx 400\text{ MeV}/c \\ \omega-\bar{\epsilon}\approx 350\text{ MeV} \\ k_n=k_p\approx 600\text{ MeV}/c \end{array}\right. \quad \text{and } K\approx 600\text{ MeV}/c. \qquad (64)$$

Thus, we see that it is rather easy to satisfy the various conditions. From the above examples it is clear that one can explore the correlation function from $K\approx 300$ to 700 MeV$/c$ with 500-MeV electrons.

6.4. Experimental Setup Proposed

Thus on the basis of all the kinematical questions discussed in this section, our proposed experimental setup would be (say, for 500-MeV incident electrons) as follows.

One would have an electron-proton-neutron coincidence setup measuring the direction and energy of electrons and protons, and the direction and a (suitable) low-energy cutoff of neutrons (as discussed below). Since the cross section is peaked for forward electrons, one will have to look at forward electrons, and admit neutrons and protons of approximately equal and opposite transverse momentum. The counters may be designed to insure counting of particles only from configurations such that P is small.

The experiment can be done at a synchrotron or a linear accelerator, either using an external beam or inside of a synchrotron as has been done in some recent Cornell experiments.

Now, a word about the outgoing neutrons. From our expressions for the cross section, it seems that one may circumvent the task of determining the energy of the outgoing neutrons using the energy delta function. However, we obtained these expressions on the basis

[21] W. Czyz, Phys. Rev. (to be published).

of closure which necessitated assuming little energy taken by the recoil nucleus. So, in order to avoid counting nonsensical events in which the recoil nucleus carries a large chunk of the energy transfer we must have at least a low-energy cutoff on the neutron counter, such that let us say P (i.e., the recoil momentum of the residual nucleus) up to 200 MeV/c may occur. A poor resolution of the neutron energy (which probably cannot be helped anyway) is needed to include a (small) band of P and a band of residual excitation energies. (The latter probably to justify closure.)

6.5. Estimate of the Cross Section

With the kinematics proposed, we would now like to get an estimate of the size of the cross section. As an accurate cross-section value is not required presently, we shall restrict ourselves to a crude probability and order of magnitude estimate.

Consider the scattering of an electron by a proton of momentum k_p. Let the relative probability of finding a proton in the nucleus with the magnitude of momentum k_p in Δk_p be $P(k_p)\Delta k_p$. Also, with respect to the incident electron, the proton has a relative probability of being in the solid angle$=\Delta\Omega_p/4\pi$ (where $\Delta\Omega_p$ is the solid angle of the proton counter).

Let k_n be the observed neutron momentum if $\mathbf{P}=0$ (i.e., the recoil momentum be 0). Now $P\lesssim 200$ MeV/c defines $\Delta\theta_n\approx 200/k_n$. The neutron will thus come out in a solid angle $\Omega'=\frac{1}{2}(\Delta\theta_n)^2\Delta\theta_n=(200)^3/2k_n^3$. But the neutron counter sees only a fraction of these. So the relative probability for seeing the neutron is $\Delta\Omega_n/\Omega'$, where $\Delta\Omega_n$ is the size of the neutron counter.

Also, the "elastic" electron scattering (in the forward direction) by a proton is

$$\frac{d\sigma}{d\Omega}\approx\alpha^2\theta_e^2k_e'^2\bigg/k_e^2k_e'^2\left(\theta_e^2+\frac{m_e^2q^2}{k_e^2k_e'^2}\right)^2.$$

The observed cross section will then be

$$\Delta\sigma\approx\left[\int d\Omega_e'\alpha^2\theta_e^2k_e'^2\bigg/k_e^2k_e'^2\left(\theta_e^2+\frac{m_e^2q^2}{k_e^2k_e'^2}\right)^2\right]\times\left(\frac{\Delta\Omega_p}{4\pi}\right)\left(\frac{\Delta\Omega_n}{\Omega'}\right)P(k_1)\Delta k_1. \quad (65)$$

Let us take the geometry of the proton and neutron counters to be such that $\Delta\Omega_{p,n}=10^{-2}$ sr. Also, let $k_e=500$ MeV/c; $k_e'=300$ MeV/c; $q\approx 200$ MeV/c and accept for the scattered electrons, $\theta_{\max}\approx 10°$. Lastly, taking the probability of finding the nucleon with the requisite momenta $k_p\approx 400$ MeV, and with Δk_p about 50 MeV, to be $\approx 10^{-2}$, we obtain

$$\Delta\sigma\approx 5\times 10^{-6}\mu\text{b}. \quad (66)$$

7. CONCLUSION

We would like to emphasize that the previously proposed setups with single-nucleon excitation or single-nucleon emission are virtually useless in determining the dynamically interesting very short-range part of the pair-correlation function. Even the qualitative characteristics of $g(r)$ are blurred in these experiments due to the lack of knowledge about the "other" nucleon in question. Consequently, it may be surmised with some degree of confidence that one needs (at least) 2-nucleon emission experiments for the study of $g(r)$. The choice of projectiles and targets may be argued upon for specific purposes and specific kinematical setups, etc.

Next, some remarks about closure are in order. The closure argument is so far a largely (experimentally) untested postulate underlying virtually all investigations in this field. The whole idea of closure is based on the fact that there is a "natural" narrow band of excitation that the residual states confine themselves to. Thus, it seems very important to have an experimental check on closure. It is in this respect that the electron-induced experiment offers an advantage over the photon-induced process.

Let us return once again to the (real) photon induced reaction, alluded to before. Consider first the deuteron photodisintegration process: $\gamma+d\rightarrow p+n$. If one measures the outgoing proton and neutron momenta completely the incident photon energy may be "deduced" easily. Also, in the process $\gamma+A\rightarrow(A-2)+p+n$, to the extent that closure is valid, i.e., the residual nucleus partakes in only a small portion of the total energy-momentum transfer, one can again determine the incident bremsstrahlung energy, if one has a complete knowledge of the outgoing nucleons' momenta. In the case of electrons, we are advocating a complete direction and momentum measurement on electrons and protons and a direction measurement on neutrons coupled with a "weak" determination of neutron energy (the latter of which can hardly be avoided). Consequently, in the case of electrons one has the advantage of being able to experimentally comment on the excitation energy spectrum of the residual nucleus. Thus, *a posteriori*, one may be able to support or refute the earlier assumption about the validity of the closure approximation. This, we assert again, is not possible in case of the processes induced by bremsstrahlung photons.

It is relevant also to estimate how large the pp production channel is compared to the np channel under consideration. This, of course, is related to the problem of estimating the amount of error incurred in neglecting the initial P states.

According to Gottfried's estimate[7] for the photo-case pp production should be about 2% of the np production, as also found experimentally. Also, the neglect of the initial P state in that case involves an error of 7%.

This latter estimate is based on an order of magnitude argument and hence is suspect quantitatively. Anyway, we can probably quite safely take both the pp production and the error involved in neglecting the initial P states to be small and hence negligible. We do not investigate this point in our work.

It is, of course, hoped that an experimental *a posteriori* confirmation of the above neglect shall be forthcoming.

For the energies under consideration, which are above the threshold for isobar production, the meson disturbance for forward scattered electrons is probably just as serious as in the photo-case. However, we would like to point out the following.[22] The main contribution to the matrix element for the electron production of pions comes from small-angle scattering and the transverse interactions. The rest is due to the large electron scattering angles. A simple computation shows that for incident electrons of 600 MeV/c momentum and with outgoing electrons corresponding to a photon of 200-MeV energy, about 95% of the above cross section comes from scattering angles less than 6%. Thus, it is easy to infer that since the longitudinal fields seem to be weakly coupled to the pions, scattering the electrons through larger angles would reduce the meson production processes considerably. For such configurations, therefore, electrons would decidedly be preferable over photons to study the nucleon correlations.

Now from our estimates of the cross section, we believe that it is within the reach of the present experimental techniques. Also, in principle, the various corrections due to meson production, final-state interactions, and the bremsstrahlung contamination of the true cross section, can separately be made. However, considerable care and accuracy is required for a reliable determination of the correlation function, due to the above corrections.

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APPENDIX A

We would like to sketch the derivation of the formula—the method used here is standard.

$|n\rangle$ is an exact eigenstate of the nuclear Hamiltonian H_N having $|\Phi_f\Xi_s\rangle$ as its incident plane-wave part, and has incoming spherical waves for infinite separation between the nucleon-pair and the residual system. H_N is comprised of the kinetic energy operator K, the potential energy of the residual system U, the interaction between the nucleon pair V, and W, the interaction between the pair and the remaining $(A-2)$ nucleons. Thus,

$$(K+U-E)|\Phi_f\Xi_s\rangle=0 \qquad \text{(A1)}$$

and

$$|n\rangle=|\Phi_f\Xi_s\rangle+\frac{1}{E-K-U-i\epsilon}(V+W)|n\rangle. \qquad \text{(A2)}$$

Here we must point out that it is an excellent approximation to neglect the antisymmetrization of the pair with the $(A-2)$ residual nucleons at such high energies, provided the reaction products have momenta much greater than those present in the target.[7]

Let us introduce a new state vector

$$|\chi_f^{(-)}\Xi_s\rangle=\left(1+\frac{1}{E-K-U-V-i\epsilon}V\right)|\Phi_f\Xi_s\rangle \qquad \text{(A3)}$$

$$=|\Phi_f\Xi_s\rangle+\frac{1}{E-K-U-i\epsilon}V|\chi_f^{(-)}\Xi_s\rangle, \qquad \text{(A4)}$$

where $\chi_f^{(-)}$ is the 2-body scattering state corresponding to Φ_f as its incident wave.

From (A2) and (A4) on eliminating Φ_f, one gets

$$|n\rangle=\left(1+\frac{1}{E-K-U-V-W-i\epsilon}W\right)|\chi_f^{(-)}\Xi_s\rangle$$
$$=(1+G_E^+W)|\chi_f^{(-)}\Xi_s\rangle, \qquad \text{(A5)}$$

where

$$G_E=(E-H_N-i\epsilon)^{-1}.$$

Thus, Eq. (6) of the text is shown.

APPENDIX B

Estimate of $\bar{\epsilon}$: Equation (9) of the text defines $\bar{\epsilon}$ as the sum of the (i) excitation energy, (ii) recoil energy, and (iii) the difference in binding energy for the residual nucleus. For the particular case of O^{16}, say, losing two nucleons to become N^{14}, the difference in binding energy is 23 MeV.[7] The recoil energy should be of the order of 10 MeV. The excitation energy (or the so-called rearrangement energy) occurs because, due to correlations between nucleons, a quasifree event in general does not result in a corresponding "hole state," but often in an excited state. The average value of this excitation is of the order of 15 MeV for an infinite nucleus.[3]

[22] R. H. Dalitz and D. R. Yennie, Phys. Rev. **105**, 1598 (1957); R. B. Curtis, *ibid.* **104**, 211 (1956).

Thus, we take $\bar{\epsilon}\lesssim 50$ MeV. Actually, if the closure approximation is valid at all, small fractional changes in $\bar{\epsilon}$ should be unimportant.

APPENDIX C

g(K) for Fermi gas and Brueckner function: Let us assume here that the correlation for both $S=0$ and $S=1$ states is the same.

(a) Fermi gas: Here

$$g_p(r)=\sin pr/pr\,, \tag{A6}$$

and one has to average over all values of p in the Fermi sea. Then,

$$g(K)=\int d\mathbf{r}e^{-i\mathbf{K}\cdot\mathbf{r}}g(r) \tag{A7}$$

for this model is simply obtained as a step function.

$$\begin{aligned} g_F(K)&=0 && \text{for}\quad K>k_F\,,\\ &=3\pi^2/k_F{}^3 && \text{for}\quad K=k_F\,,\\ &=6\pi^2/k_F{}^3 && \text{for}\quad K<k_F\,. \end{aligned} \tag{A8}$$

(b) Now we employ a little more realistic function, which is due to Brueckner and Gammel.[23] For the S state,

$$\begin{aligned} g_p(r)&=0\,, && r<r_c\\ &=1/\sqrt{\Omega}\left(\frac{\sin pr}{pr}-\frac{\sin pr_c}{pr}e^{-\lambda(r-r_c)}\right), && r>r_c\,, \end{aligned} \tag{A9}$$

where $\lambda^2=\frac{2}{5}k_F{}^2$ and Ω is the nuclear volume.

[23] K. Brueckner and J. Gammel, Phys. Rev. **109**, 1023 (1958).

The integration is straightforward. We obtain for

(i) $K>k_F$:

$$g_B(K)=\frac{4\pi}{K^3}(Kr_c\cos Kr_c-\sin Kr_c)+\frac{12\pi}{Kk_F}\,\frac{k_Fr_c\cos k_Fr_c-\sin k_Fr_c}{k_F{}^2r_c{}^2}\times\frac{(\lambda\sin Kr_c+K\cos Kr_c)}{(\lambda^2+K^2)}, \tag{A10}$$

(ii) $K=k_F$:

$$g_B(K)=\frac{3\pi^2}{k_F{}^3}-\frac{4\pi}{3}r_c{}^3-\frac{12\pi}{k_F{}^2}\,\frac{\sin k_Fr_c-k_Fr_c\cos k_Fr_c}{k_F{}^2r_c{}^2}\times\frac{(\lambda\sin k_Fr_c+k_F\cos k_Fr_c)}{(\lambda^2+k_F{}^2)} \tag{A11}$$

(iii) $K<k_F$:

$$g_B(K)=\frac{6\pi^2}{k_F{}^3}-\frac{4\pi^2}{3}r_c{}^3-\frac{12\pi}{Kk_F}\,\frac{\sin k_Fr_c-k_Fr_c\cos k_Fr_c}{k_F{}^2r_c{}^2}\times\frac{(\lambda\sin Kr_c+K\cos Kr_c)}{(\lambda^2+K^2)}. \tag{A12}$$

For the case of $K>k_F$, which is the region we are interested in, if we put $k_Fr_c\ll 1$, Eq. (A10) gives

$$g_B(K)\approx 4\pi r_c\left[\frac{1}{K^2}\left(\cos Kr_c-\frac{\sin Kr_c}{Kr_c}\right)-\frac{K^{-1}\lambda\sin Kr_c+\cos Kr_c}{(\lambda^2+K^2)}\right]. \tag{A13}$$