Consequences of a Weak Vector Boson for the Decay $K \to \mu+\nu+e^{+}+e^{-}$ †

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The effect of an intermediate vector boson, with arbitrary anomalous magnetic and quadrupole moments, has been investigated for the decay $K-\mu+\nu+e^{+}+e^{-}$. The presence of the boson can have a striking effect on the orientation of the plane of the Dalitz pair relative to the decay plane. This effect is greatest in a kinematic region where the Dalitz pair emerges at a large backward angle with respect to energetic muons. As in the radiative decay $K\to\mu+\nu+\gamma$, there is an enhancement in the differential transition probability in this region. Sample distributions in the momentum and energy separation of the Dalitz pair are given.

In a previous paper[1] we considered the consequences of the existence of a weak vector boson W for the decay

$$K\to\mu+\nu+\gamma. \quad (1)$$

We found that the emission of photons at large, backward angles with respect to the muon direction can be significantly enhanced, for energetic muons, by the presence of a (not too massive) W. In Table I we extend those calculations to cover the range of W mass of interest at present. We also remarked that the polarization vector of the photon which is favored by the presence of the W boson can be perpendicular to that resulting from inner bremsstrahlung alone. Therefore, the sign of the polarization will be reversed in regions where the contribution from the W boson dominates.

TABLE I. Ratio $R=\Gamma_D(m_W)/\Gamma_D(\infty)$ of decay rate for $K\to\mu\nu\gamma$ with a W boson to the decay rate for inner bremsstrahlung ($m_W=\infty$) in a kinematic region defined by $E_\gamma^{\exp}$, $E_\mu^{\exp}=137$ MeV, $\theta_1^{\exp}=180°$, $\theta_2^{\exp}=143°$ (cf. Ref. 1, Table II).

		$E_\gamma^{\exp}$ in MeV				
m_W	μ_W	24	47	71	94	118
2.5	−2	1.29	1.43	1.63	1.95	2.56
3.0	−2	1.11	1.17	1.25	1.39	1.66
3.5	−2	1.05	1.07	1.11	1.17	1.30
2.5	−1	1.14	1.22	1.32	1.49	1.82
3.0	−1	1.05	1.08	1.12	1.19	1.33
3.5	−1	1.02	1.03	1.04	1.08	1.14
2.5	0	1.05	1.07	1.11	1.18	1.31
3.0	0	1.01	1.02	1.03	1.06	1.11
3.5	0	1.00	1.00	1.00	1.01	1.04
2.5	+1	1.00	1.00	1.00	1.01	1.04
3.0	+1	0.99	0.99	0.99	0.99	1.00
3.5	+1	0.99	0.99	0.99	0.99	0.99
m_W	+2	1.00	1.00	1.00	1.00	1.00

In this paper we wish to discuss the related decay mode

$$K\to\mu+\nu+e^{-}+e^{+} \quad (2)$$

in which the photon is internally converted to an electron-positron pair. One reason for considering this Dalitz pair decay is that, experimentally, it is apparently very difficult to measure the correlation of the photon polarization with the decay plane, for reaction (1). On the other hand, it may be easier to measure the correlation of the plane of the Dalitz pair with the decay plane, for reaction (2). (As pointed out by Kroll and Wada,[2] there is a strong correlation between the plane of polarization of the photon and the plane of the pair into which it converts.) The Dalitz pair decay, of course, involves one higher order in the electromagnetic coupling, compared to the radiative decay, and is thus much rarer; the rate is reduced by approximately $2\alpha/\pi$. Another reason for looking at reaction (2) is that the detection efficiency for the radiative decay (1), under certain experimental arrangements, may be quite low, so that the counting rate may well be comparable to that for process (2).[3]

We will here assume that the correlations for the electron-positron pair in (2) follow from the well-known features of quantum electrodynamics.[4] We will make no new assumptions concerning the interactions involving the W boson. To lowest order in the fine-structure constant α, the Feynman diagrams for the process (2) are shown in Fig. 1. The charged W can interact with the electromagnetic field *via* its charge, magnetic moment, and quadrupole moment. The form of the coupling between the W and the electromagnetic field is given in Ref. 1 and elsewhere.[5,6] The W is assumed to have arbitrary anomalous magnetic and quadrupole moments, $(e/2m_W)(\mu_W-1)$ and (eQ_W/m_W^2), respectively. As far as the numerical calculations are concerned, we will

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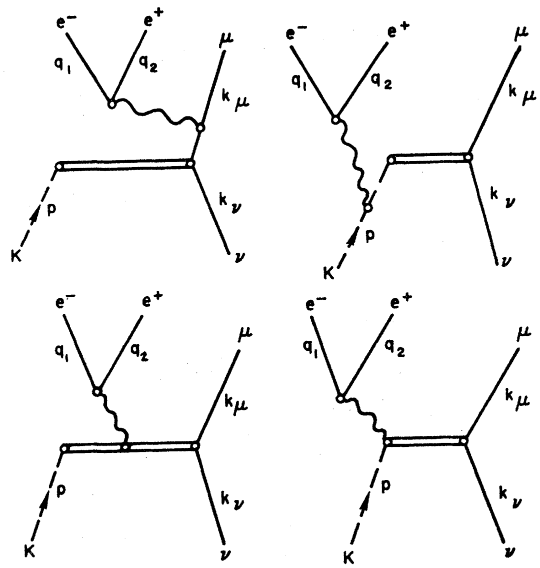

FIG. 1. Feynman diagrams for process (2).

assume that the intrinsic anomalous quadrupole moment is zero (i.e., $Q_W=0$). This is consistent with the ξ-limiting process constructed by Lee and Yang.[5] The W is coupled to the lepton current in the usual manner.[7]

The matrix element for process (2) is found by using standard techniques and can be conveniently written in the form

$$\mathfrak{M}=(ff'm_W^{-2})(2\pi)^{-3}e^2m_e(\pi E_KE_\mu E_\nu E_1E_2)^{-1/2} \times\delta^{(4)}(p-k_\mu-k_\nu-x)\tfrac{1}{2}J_\alpha{}^{(\text{pair})}J_\alpha x^{-2}, \quad (3)$$

where

$$J_\alpha{}^{(\text{pair})}=\bar{u}(\mathbf{p}_1)\gamma_\alpha\bar{v}(\mathbf{p}_2), \quad (4)$$

$$J_\alpha=(m_eE_\nu)^{1/2}\bar{\psi}_\mu(\mathbf{k}_\mu)(M_\alpha{}^{\mathrm{I}}+M_\alpha{}^{\mathrm{II}}+M_\alpha{}^{\mathrm{III}}) \times\tfrac{1}{2}(1+i\gamma_5)\psi_\nu(\mathbf{k}_\nu), \quad (5)$$

$$M_\alpha{}^{\mathrm{I}}=m_\mu\left[\frac{2p_\alpha-x_\alpha}{2p\cdot x-x^2}-\frac{\gamma_\alpha(m_\mu+p\cdot\gamma)}{2k_\mu\cdot x+x^2}\right], \quad (6)$$

$$M_\alpha{}^{\mathrm{II}}=\left(2-\mu_{\mathrm{W}}-\frac{Q_{\mathrm{W}}p\cdot x}{2m_{\mathrm{W}}^2}\right) \times\left[\frac{x\cdot\gamma(p-x)_\alpha-(p\cdot x-x^2)\gamma_\alpha}{m_{\mathrm{W}}^2-(p-x)^2}\right], \quad (7)$$

$$M_\alpha{}^{\mathrm{III}}=\left[\frac{1-\mu_{\mathrm{W}}}{m_{\mathrm{W}}^2-(p-x)^2}\right] \times\left[x_\alpha x\cdot\gamma-x^2\gamma_\alpha-\frac{m_\mu}{m_{\mathrm{W}}}(p\cdot xx_\alpha-x^2p_\alpha)\right]. \quad (8)$$

In these experessions, the four-momenta of the various particles are as indicated in Fig. 1, three-momenta are denoted by the same symbol in boldface type, and masses and energies by $m_{(\ )}$ or $E_{(\ )}$ with the appropriate subscript. We use the abbreviation $x\equiv(\omega,\mathbf{q})=q_1+q_2$, so that x^2 is the invariant mass squared of the Dalitz pair.

The expression for the decay rate, differential in the muon variables $d\mathbf{k}_\mu$ is obtained by a procedure outlined by Kroll and Wada,[2] which makes use of current conservation, $x_\alpha J_\alpha=x_\alpha J_\alpha{}^{(\text{pair})}=0$, to resolve J_α into transverse and longitudinal components. After a lengthy calculation, the differential decay rate, in the c.m. system (initial kaon at rest) can be written as

$$\begin{aligned}d\Gamma/d\mathbf{k}_\mu=&(ff'm_{\mathrm{W}}^{-2})^2e^4(2\pi)^{-7}\int dqdyd\varphi(2E_\mu)^{-1}\\&\times\{\tfrac{1}{2}A_0(1-y^2)+A_+[\tfrac{1}{4}(1+y^2)+m_e^2x^{-2}]\\&-A_-[\tfrac{1}{4}(1-y^2)-m_e^2x^{-2}]\cos2\varphi\\&-(m_KE_\nu)^{-1}qyx^{-2}[x^2(1-y^2)-4m_e^2]^{1/2}\\&\times\mathrm{Re}\sum_{\text{spins}}J_0J_x{}^*\cos\varphi\},\end{aligned} \quad (9)$$

where

$$A_-=m_K^{-1}c_7(c_4^2-c_3^2)|\mathbf{k}_\mu|^2\sin^2\theta, \quad (10)$$

$$A_+=A_-+m_K^{-1}c_7[\tfrac{1}{2}(c_4^2-c_3^2)(m_\mu^2-c_2)+2E_\nu c_4(E_\mu c_4-m_\mu c_3)], \quad (11)$$

$$\begin{aligned}A_0=&(m_KE_\nu)^{-1}[E_\mu E_\nu c_3^2+\tfrac{1}{4}(c_3^2-c_6^2)(m_\mu^2-c_2)\\&+m_\mu c_6(E_\nu c_3+c_5x\cdot k_\nu)\\&+c_5^2(x\cdot k_\mu x\cdot k_\nu-\tfrac{1}{2}x^2k_\mu\cdot k_\nu)\\&+c_3c_5(E_\mu x\cdot k_\nu+E_\nu x\cdot k_\mu-\omega k_\mu\cdot k_\nu)],\end{aligned} \quad (12)$$

$$\begin{aligned}\mathrm{Re}\sum_{\text{spins}}J_0J_x{}^*=&\tfrac{1}{2}|\mathbf{k}_\mu|\sin\theta[c_3^2(E_\nu-E_\mu)+c_4c_6(E_\nu+E_\mu)\\&+m_\mu c_3(c_4-c_6)+c_3c_5(x\cdot k_\nu-x\cdot k_\mu)\\&+m_\mu c_4c_5\omega],\end{aligned} \quad (13)$$

and

$$\begin{aligned}c_1=&2x\cdot k_\mu+x^2,\\c_2=&m_K^2-2m_K\omega+x^2,\\c_3=&m_\mu^2c_1^{-1}+[(2-\mu_W-\tfrac{1}{2}m_Km_W^{-2}Q_W)(m_K\omega-x^2)\\&+(1-\mu_W)x^2](m_W^2-c_2)^{-1},\\c_4=&m_\mu m_Kc_1^{-1},\\c_5=&-[(2-\mu_W-\tfrac{1}{2}m_Km_W^{-2}Q_W\omega)(m_K-\omega)\\&+(1-\mu_W)\omega](m_W^2-c_2)^{-1},\\c_6=&c_4+m_\mu m_Km_W^{-2}q^2(1-\mu_W)(m_W^2-c_2)^{-1}\\&-m_\mu(2m_K\omega-x^2)^{-1}(2m_K-\omega),\\c_7=&q^2E_\nu^{-1}x^{-2}.\end{aligned} \quad (14)$$

In Eq. (9), the variable y is a measure of the energy separation of the pair; $y=(E_1-E_2)q^{-1}$. The angle between the muon direction and $\mathbf{q}$ is denoted by θ and φ is the angle between the plane of the Dalitz pair and the decay plane (e.g., the plane of $\mathbf{q}$ and $\mathbf{k}_\mu$).

[7] See Ref. 1, where the notation is defined.

From the preceding equations, the distributions in the two independent variables of the Dalitz pair, x and y, as well as in φ can be obtained by integration. The kinematics of the decay, which determines the ranges of the variables, is discussed in the Appendix. As mentioned earlier, the orientation of the Dalitz pair relative to the reaction plane is of interest, since it is sensitive to the properties of the W boson. The y integration in Eq. (9) can easily be performed analytically. We then find that the distribution in φ may be written as

$$d\Gamma/d\mathbf{k}_\mu = (ff'm_W^{-2})^2 e^4 (2\pi)^{-7} \int_0^{2\pi} d\varphi \int_{q_{\min}}^{q_{\max}} \tfrac{1}{3} E_\mu^{-1} dq \times [\eta(1+2m_e^2 x^{-2})(A_0+A_+) - \tfrac{1}{2}\eta^3 A_- \cos 2\varphi], \quad (15)$$

where $\eta = (1-4m_e^2 x^{-2})^{1/2}$ and $q_{\min}$ and $q_{\max}$ are given by the appropriate expressions in the Appendix. In Eq. (15), the q integration can be performed numerically with the help of an IBM 7090. The resulting correlation in the angle φ can be conveniently expressed as

$$d\Gamma/d\mathbf{k}_\mu = (d\Gamma/d\mathbf{k}_\mu)(2\pi)^{-1} \int_0^{2\pi} d\varphi (1-\xi \cos 2\varphi). \quad (16)$$

The parameter ξ in Eq. (16) then determines the correlation between the plane of the Dalitz pair and the reaction plane. In Table II we give the results of our numerical integration for a variety of choices for the boson mass (expressed in units of m_K) and total magnetic moment μ_W (expressed in boson magnetons). (We have assumed $Q_W=0$ as mentioned earlier.[8]) We have chosen the muon energy and the angle θ to lie in the region where the W-boson effects in the radiative decay (1) are the largest. Specifically, we pick $E_\mu = 137$ MeV (kinetic energy) and $\theta = 166°$; [this choice corresponds to the maximum of curve (a) in Fig. 1 of Ref. 1]. We also recall[1] that the requirement of large muon energy is just what is needed to distinguish the decays (1) and (2) from the background due to

$$K \to \mu + \nu + \pi^0, \quad \pi^0 \searrow 2\gamma \text{ or } \gamma + e^+ + e^-. \quad (17)$$

TABLE II. Values of the coefficient of $\cos(2\varphi)$ in Eq. (16) for various m_W and μ_W.

m_W/m_K	$\mu_W=-2$	$\mu_W=-1$	$\mu_W=0$	$\mu_W=1$	$\mu_W=2$
1.0	−0.066	−0.066	−0.066	−0.053	0.133
1.9	−0.051	−0.031	0.012	0.091	0.139
2.5	−0.002	0.033	0.079	0.123	0.139
3.0	0.044	0.076	0.109	0.132	0.140
3.5	0.079	0.103	0.123	0.136	0.140
∞	0.140	0.140	0.140	0.140	0.140

[8] See also footnote 12 of Ref. 1. Of course, we are also neglecting all higher order corrections to the magnetic moment.

As seen from Table II, the value of ξ can be drastically reduced by the presence of the W, and can even be made to change sign.

The above numerical calculation also furnishes values for the differential decay rate integrated over the Dalitz pair variables x, y, and φ. This is expected to reflect the enhancement in the differential transition probability for the radiative process previously pointed out.[1] In Table III we give the ratio of $d\Gamma/d\mathbf{k}_\mu$, for various values of m_W and μ_W, to the differential rate assuming no boson to be present, i.e., $m_W=\infty$. The similarity between Table III and Table I, and Tables I and II in Ref. 1 is apparent.

TABLE III. Ratio for the differential decay rate $d\Gamma/d\mathbf{k}_\mu$ to that for $m_W=\infty$.

m_W/m_K	$\mu_W=-2$	$\mu_W=-1$	$\mu_W=0$	$\mu_W=1$	$\mu_W=2$
1.0	93.7	53.2	23.0	5.91	1.06
1.9	4.91	3.05	1.78	1.10	1.01
2.5	2.05	1.51	1.16	0.99	1.00
3.0	1.41	1.18	1.03	0.97	1.00
3.5	1.16	1.05	0.99	0.97	1.00

The shape of the distributions in q (which is essentially the same as the distribution in x, except for kinematic factors) and y are not especially sensitive to the presence of the W; it is the area under these distributions which is sensitive to the W, as can be seen from Table II. The distribution in x is sharply peaked for small values of x ($\gtrsim 2m_e$) corresponding to the predominance

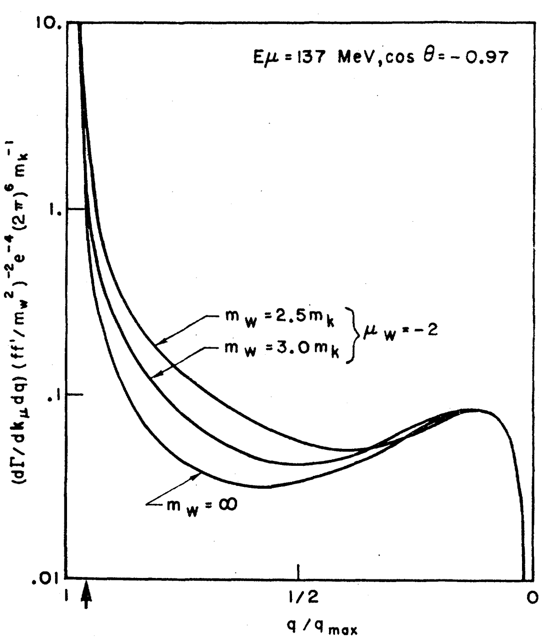

FIG. 2. Sample differential distribution in q for various values of the W parameters m_W and μ_W. Nearly half the area under the curves is between 1 and the arrow.

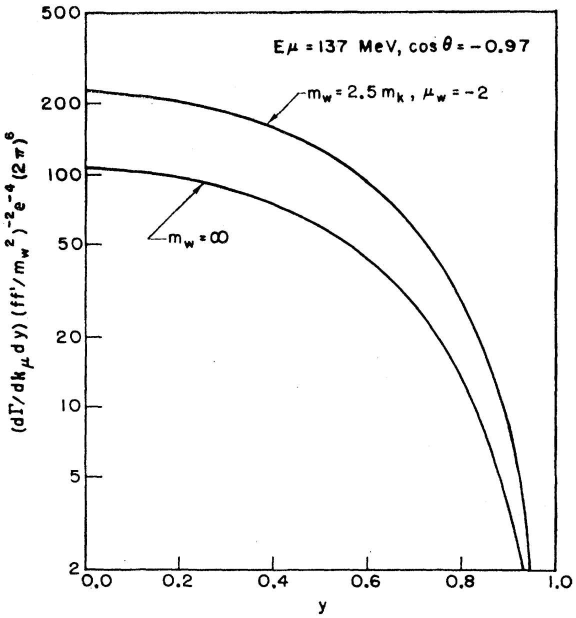

FIG. 3. Sample differential distribution in y for various values of the W parameters m_W and μ_W.

of "nearly real" photons in process (2).[9] For nearly all kinetic configurations of the muon, this corresponds to large values of q ($\lesssim q_{\max}$), i.e., just those that are enhanced by the W in the radiative process (1).[1] Therefore, the internal conversion coefficient ρ, i.e., the ratio of processes (2) and (1), is not expecially sensitive to the presence of the W either. Figures 2 and 3 give sample distributions in the variables q and y.

In summary, we repeat that the orientation of the plane of the Dalitz pair relative to the decay plane in the reaction $K\rightarrow\mu+\nu+e^++e^-$ can be affected in a striking manner by the presence of an intermediate vector boson. Our investigation indicates that this effect is greatest in a region where the Dalitz pair emerges at a large backward angle relative to the muon direction for energetic muons.

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APPENDIX

In this appendix we deal briefly with some of the kinematics for process (2). We choose to work in the c.m. system of the decay, where conservation of energy and momentum take the form

$$m_K=E_\mu+E_\nu+\omega$$
$$0=\mathbf{k}_\mu+\mathbf{k}_\nu+\mathbf{q} \tag{A1}$$

from which the neutrino energy is easily eliminated.

Our main interest is to find the restrictions on the variables q, x, and y, for fixed values of E_μ and $\cos\theta$. To this end, we first solve for ω and q as functions of x; [recall that $x=(\omega,\mathbf{q})$ and $x^2=\omega^2-q^2$]. The solutions are the roots of a quadratic equation,

$$q_\pm(x)=-|\mathbf{k}_\mu|t_1(x)\cos\theta\pm(m_K-E_\mu)t_2(x) \tag{A2}$$

$$\omega_\pm(x)=(m_K-E_\mu)t_1(x)\mp|\mathbf{k}_\mu|t_2(x)\cos\theta\,, \tag{A3}$$

where

$$t_1(x)=\tfrac{1}{2}(H^2+x^2)(H^2+|\mathbf{k}_\mu|^2\sin^2\theta)^{-1} \tag{A4}$$

$$t_2(x)=\tfrac{1}{2}[(H^2-x^2)^2-4x^2|\mathbf{k}_\mu|^2\sin^2\theta]^{1/2}\times(H^2+|\mathbf{k}_\mu|^2\sin^2\theta)^{-1} \tag{A5}$$

and $H^2\equiv(p-k_\mu)^2=m_K^2-2m_KE_\mu+m_\mu^2$. The region of validity for the solutions (A2) and (A3) depends on the values of E_μ and $\cos\theta$. The situation is as follows:

(i) For

$$m_\mu\leq E_\mu\leq E^*\equiv\tfrac{1}{2}[(m_K-2m_e)^2+m_\mu^2](m_K-2m_e)^{-1},$$

and $\cos\theta\geq0$, the proper solutions are $\omega_+(x)$ and $q_+(x)$. The range of x is $2m_e\leq x\leq m_K-E_\mu-|\mathbf{k}_\mu|$ and q is restricted to values between 0 and $q_{\max}\equiv q_+(x=2m_e)$.

(ii) For $m_\mu\leq E_\mu\leq E^*$ and $\cos\theta\leq0$ the same solutions as in (i) apply provided $2m_e\leq x\leq m_K-E_\mu-|\mathbf{k}_\mu|$. However, when $m_K-E_\mu-|\mathbf{k}_\mu|\leq x\leq x^*\equiv(H^2+|\mathbf{k}_\mu|^2\sin^2\theta)^{1/2}-|\mathbf{k}_\mu|\sin\theta$ both solutions $\omega_\pm(x)$ and $q_\pm(x)$ are possible. The range of q is still $0\leq q\leq q_{\max}$.

(iii) For $E^*\leq E_\mu\leq E_{\max}\equiv\tfrac{1}{2}m_K^{-1}(m_K^2+m_\mu^2-4m_e^2)$ only backward angles which satisfy the condition $\sin\theta\leq(H^2-4m_e^2)(4m_e|\mathbf{k}_\mu|)^{-1}$ are allowed. Again, both solutions are possible and the ranges of x and q are $2m_e\leq x\leq x^*$ and $q_-(x=2m_e)\equiv q_{\min}\leq q\leq q_{\max}$.

Since it is sometimes the case that q is a double valued function of x it is not convenient to make the substitution $dq\rightarrow dx$, since the derivative dx/dq can vanish. However, as mentioned earlier, the distribution in x is very sharply peaked for values near the minimum value of $2m_e$. In case (ii), q is still a single-valued function for $2m_e\leq x\leq m_K-E_\mu-|\mathbf{k}_\mu|$, which comprises most of the range of x provided $E^*-E_\mu\gg m_e$. Case (iii) applies for only the highest values of E_μ, since $E_{\max}-E^*\approx m_e(1-m_\mu^2m_K^{-2})\approx m_e$, and is of little practical interest.

Finally we note that the requirement that the transverse momentum of the pair be real yields the relation[9] $x^2(1-y^2)-4m_e^2\geq0$, from which one finds $|y|\leq\eta\equiv(1-4m_e^2x^{-2})^{1/2}$.

[9] See N. M. Kroll and W. Wada, Ref. 2.