Group-Theoretical Consideration of Landau Level Broadening in Crystals

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A group-theoretical classification of the energy levels for a Bloch electron in a magnetic field is given. The fact is used that the magnetic translation group is a subgroup of the symmetry group for a free electron in a magnetic field, and it is shown that the broadening of the Landau levels in crystals is a general feature that follows from symmetry considerations. An explicit formula for the broadening in the case of cubic symmetry has been derived.

I. INTRODUCTION

THE broadening of the Landau levels in a periodic crystal potential has been treated by many authors.[1] In the present paper this problem is considered from a group-theoretical point of view. For a free electron in a constant magnetic field there exists a set of operators which commute with the Hamiltonian[2]:

$$\exp\left\{\frac{i}{\hbar}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)\cdot\mathbf{a}\right\}, \tag{1}$$

where $\mathbf{p}=-i\hbar(\partial/\partial\mathbf{r})$, $\mathbf{A}=\frac{1}{2}[\mathbf{H}\times\mathbf{r}]$, e is the magnitude of the electronic charge and $\mathbf{a}$ is an arbitrary vector. The infinitesimal operators in Eq. (1) were identified[2] with the coordinates for the center of the Landau orbits. Since the energy does not depend on the center of the orbit, there is an infinitely high degeneracy of the energy levels for a free electron in a constant magnetic field. In addition, when a periodic electric potential is introduced, operators again exist which commute with the Hamiltonian[3,4]

$$\exp\left\{\frac{i}{\hbar}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)\cdot\mathbf{R}_n\right\}, \tag{2}$$

where

$$\mathbf{R}_n=n_1\mathbf{a}_1+n_2\mathbf{a}_2+n_3\mathbf{a}_3 \tag{3}$$

is a Bravais lattice vector and $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are vectors of a unit cell. In previous papers[5] a magnetic translation group (M.T.G.) was defined which commutes with the Hamiltonian for a Bloch electron in a magnetic field. Although a detailed description of the energy levels and the eigenstates according to the irreducible representations of the M.T.G. was already given before,[4,5] we nevertheless repeat it in this paper by using a somewhat different construction which shows clearly how the Landau levels get broadened.

The relation between the operators defined in Eq. (1) and those defined in Eq. (2) is the same as between arbitrary translations for a free electron and discrete translations by $\mathbf{R}_n$ for a Bloch electron.

In order to find the Landau level broadening caused by the periodic potential of the lattice, we here use the fact that the operators [Eq. (2)] for a Bloch electron in a magnetic field form a subgroup of the operators [Eq. (1)] for a free electron in a magnetic field.[6] The usual procedure of comparing the irreducible representations of a group (the symmetry of the original problem) with the irreducible representations of its subgroup (the symmetry of the problem after the perturbing potential is introduced) allows one to find the splitting of the energy levels which is caused by the perturbation. (In our case the periodic potential of the lattice leads to the lowering of the symmetry.)

In Sec. II we present the classification of the energy levels for a Bloch electron in a magnetic field by using the irreducible representations of the magnetic translation group[5] (M.T.G.). In Sec. III we construct symmetry adapted functions for the irreducible representations of the M.T.G. from the eigenfunctions for a free electron in a magnetic field and calculate the broadening of the Landau levels.

II. CLASSIFICATION OF ENERGY LEVELS

Let us start[7] with a short description of the subgroups of the magnetic translation group.[5] We are interested here in commutative subgroups for the classification of the energy levels by means of the eigenvalues of the elements of these subgroups. The operators of the M.T.G. are defined as follows[5]:

$$\tau(\mathbf{R}_n|\mathbf{R}_1,\cdots,\mathbf{R}_i)=\exp\left\{\frac{i}{\hbar}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)\cdot\mathbf{R}_n\right\}\times\exp\left[2\pi i\frac{\Phi(\mathbf{R}_1,\cdots,\mathbf{R}_i)}{hc/e}\right], \tag{4}$$

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[6] The operators defined in Eqs. (1) and (2) do not form a group because a phase factor appears when products are formed. In Ref. 5 operators are defined which do form a closed set and therefore a group, called the magnetic translation group.

[7] Part of the results of this section were obtained elsewhere: See Refs. 3, 4, 5, and E. I. Blount, Phys. Rev. **126**, 1636 (1962).

where $\Phi(\mathbf{R}_1,\cdots,R_i)$ is the magnetic flux through the polygon enclosed by the vectors $\mathbf{R}_1, \cdots, \mathbf{R}_i, -\mathbf{R}_n$. We distinguish between the following cases:

(1) $\mathbf{H}$ does not lie in a lattice plane. The invariant commutative subgroups of the M.T.G. are given by the elements

$$\tau(n_i\mathbf{a}_i|\mathbf{R}_1,\cdots,\mathbf{R}_j), \quad \text{for} \quad i=1, 2, \text{or } 3, \tag{5}$$

where n_i takes all integer values.

(2) $\mathbf{H}$ lies in a lattice plane, say, in the plane given by the unit cell vectors $\mathbf{a}_1$ and $\mathbf{a}_3$. In addition to the invariant commutative subgroups defined by Eq. (5), the M.T.G. has the following invariant commutative subgroups:

$$\tau(n_1\mathbf{a}_1+n_3\mathbf{a}_3|\mathbf{R}_1,\cdots,\mathbf{R}_i), \tag{6}$$

where n_1 and n_3 assume all integer values.

(3) $\mathbf{H}$ lies in a direction of a lattice vector, say, $\mathbf{a}_3$ (the unit cell vector $\mathbf{a}_3$ can always be chosen in a direction of any lattice vector). In this case there are two possibilities:

(a) $\dfrac{\mathbf{H}\cdot\mathbf{a}_1\times\mathbf{a}_2}{hc/e}\neq\dfrac{n}{N}$, where n and N are integers.

In addition to the subgroup defined by Eq. (6) there is another invariant commutative subgroup of the M.T.G.:

$$\tau(n_2\mathbf{a}_2+n_3\mathbf{a}_3|\mathbf{R}_1,\cdots,\mathbf{R}_j) \tag{7}$$

for all integer values of n_2 and n_3.

(b) $\dfrac{\mathbf{H}\cdot\mathbf{a}_1\times\mathbf{a}_2}{hc/e}=\dfrac{n}{N}$ for integer n and N.

The invariant commutative subgroup of the M.T.G. is given by the following elements

$$\tau(l_1s_1\mathbf{a}_1+l_2s_2\mathbf{a}_2+n_3\mathbf{a}_3|\mathbf{R}_1,\cdots,\mathbf{R}_i), \tag{8}$$

where $s_1s_2=N$ (or $N/2$ for even n) and l_1, l_2 and n_3 take all integer values. (We assume that n and N have no common factor.) In our consideration of the Landau level broadening, we use the case (3b) because, in this case, it is easy to construct the irreducible representations of the M.T.G. However, we shall see that the broadening is a general feature for a Bloch electron in a magnetic field.

Let us consider the case (3b) of the M.T.G. in more detail. The latter can be written by means of its subgroup F consisting of the elements [Eq. (8)] as follows:

$$\sum_{\substack{i_1=0,\,1,\,\cdots,\,s_1-1\\ i_2=0,\,1,\,\cdots,\,s_2-1}} \tau(i_1\mathbf{a}_1+i_2\mathbf{a}_2|i_1\mathbf{a}_1+i_2\mathbf{a}_2)F. \tag{9}$$

The summation in Eq. (9) is a direct one. Since F is a commutative group we can easily find its representations. They differ slightly for odd and even n, but in both cases they are described by wave vectors $\mathbf{k}$:

$$D\{\tau(l_1s_1\mathbf{a}_1+l_2s_2\mathbf{a}_2+n_3\mathbf{a}_3|\mathbf{R}_1,\cdots,\mathbf{R}_i)\}$$
$$=\exp\left\{2\pi i\frac{\Phi(\mathbf{R}_1,\cdots,\mathbf{R}_i)}{hc/e}\right\}$$
$$\times\exp\left\{\frac{2\pi i}{2}l_1l_2n\right\}\exp\{i\mathbf{k}\cdot\mathbf{R}_N\}\cdots, \tag{10}$$

where $R_N=l_1s_1\mathbf{a}_1+l_2s_2\mathbf{a}_2+n_3\mathbf{a}_3$, the first phase factor is given in Eq. (4) and

$$\mathbf{k}=m_1\frac{\mathbf{K}_1}{s_1}+m_2\frac{\mathbf{K}_2}{s_2}+m_3\mathbf{K}_3, \quad 0\leq m_1, m_2, m_3<1 \tag{10a}$$

where $\mathbf{K}_1$, $\mathbf{K}_2$, $\mathbf{K}_3$ are unit cell vectors of the reciprocal lattice. All the representations of F are obtained when the wave vector $\mathbf{k}$ varies in the first magnetic Brillouin zone which is defined by the vectors $\mathbf{K}_1/s_1$, $\mathbf{K}_2/s_2$, and $\mathbf{K}_3$.[4] This Brillouin zone is smaller than the usual one for the same Bravais lattice by a factor of N because the commutative magnetic translation group F is constructed on the translations $s_1\mathbf{a}_1$, $s_2\mathbf{a}_2$, and $\mathbf{a}_3$ as on unit vectors. Since the group F commutes with the Hamiltonian, the energy levels and the states for a Bloch electron in a magnetic field can be labeled by the vectors $\mathbf{k}$ [Eq. (10a)]. The description is thus similar to the one for an electron in a periodic potential alone. The only difference is that the unit cell vectors for the commutative translations are larger and the Brillouin zone is therefore smaller. As a consequence of the definition of $\mathbf{k}$ the energy will be periodic in $\mathbf{k}$ with periods $\mathbf{K}_1/s_1$, $\mathbf{K}_2/s_2$, $\mathbf{K}_3$. The vector $\mathbf{k}$ varies continuously and the energy as a function of $\mathbf{k}$, $E(\mathbf{k})$, will exhibit a band structure. For a full description of the energy spectrum, we have to use the whole M.T.G. In order to see the effect of the other symmetry elements, let us take a similarity transformation:

$$\tau^{-1}(i_1\mathbf{a}_1+i_2\mathbf{a}_2|i_1\mathbf{a}_1+i_2\mathbf{a}_2)$$
$$\times\tau(l_1s_1\mathbf{a}_1+l_2s_2\mathbf{a}_2+n_3\mathbf{a}_3|l_1s_1\mathbf{a}_1+l_2s_2\mathbf{a}_2+n_3\mathbf{a}_3)$$
$$\times\tau(i_1\mathbf{a}_1+i_2\mathbf{a}_2|i_1\mathbf{a}_1+i_2\mathbf{a}_2)$$
$$=\tau(l_1s_1\mathbf{a}_1+l_2s_2\mathbf{a}_2+n_3\mathbf{a}_3|l_1s_1\mathbf{a}_1+l_2s_2\mathbf{a}_2+n_3\mathbf{a}_3)$$
$$\times\exp\{2\pi i(-l_1s_1i_2+l_2s_2i_1)n/N\}. \tag{11}$$

By applying both sides of Eq. (11) to an eigenvector $|m_1,m_2,m_3\rangle$ of the elements of F, we find that[5]

$$\tau(i_1\mathbf{a}_1+i_2\mathbf{a}_2|i_1\mathbf{a}_1+i_2\mathbf{a}_2)|m_1,m_2,m_3\rangle\rightarrow$$
$$\left|m_1-\frac{i_2}{s_2}, m_2+\frac{i_1}{s_1}, m_3\right\rangle \tag{12}$$

is again an eigenvector of the elements of F but corresponding to the eigenvalues $m_1-(i_2/s_2)$, $m_2+(i_1/s_1)$, m_3. The element in Eq. (12) will thus transform a wave function ψ_{m_1,m_2,m_3} into $\psi_{m_1-(i_2/s_2),m_2+(i_1/s_1),m_3}$. This again

is similar to the case of an electron in a periodic potential. In the latter case the rotations or reflections (denoted by α) of the space group transform the $\mathbf{k}$ vector from one point in the Brillouin zone into another: $\mathbf{k}\rightarrow\alpha\mathbf{k}$. For the description of the representations of space groups it suffices to take vectors only in a part of the Brillouin zone. For example, in a cubic symmetry it suffices to consider only 1/48 of the Brillouin zone. For the M.T.G. we have to take, according to relation (12), only $1/N$ of the magnetic Brillouin zone and by means of the $\mathbf{k}$ vectors in this region we can define all its irreducible representations[5]:

$$D_{j'}^{(m_1,m_2,m_3)}\{\tau(\mathbf{R}_n|\mathbf{R}_n)\}=\delta_{j',j-n_2}\exp\{-\pi i(n_2+2j')n_1(n/N)\}\times\exp\{i\mathbf{k}\cdot\mathbf{R}_n\}, \quad (13)$$

where j and j' take values from 0 to $N-1$ and

$$\mathbf{k}=m_1(\mathbf{K}_1/N)+m_2(\mathbf{K}_2/N)+m_3\mathbf{K}_3;\qquad 0\leq m_1, m_2, m_3<1. \quad (14)$$

[In the representation of $\tau(\mathbf{R}_n|\mathbf{R}_1,\cdots,\mathbf{R}_i)$ and additional phase factor

$$\exp\left\{2\pi i\frac{\Phi(\mathbf{R}_1,\cdots,\mathbf{R}_i)}{hc/e}\right\},$$

will appear in Eq. (13).] As a result of the symmetry elements that do not belong to F the energy as a function of $\mathbf{k}$, $E(\mathbf{k})$, will be defined in a $1/N^2$ part of the usual Brillouin zone. Denoting by $\mathbf{K}$ any vector that is obtained in relation (14) by assuming integers for m_1, m_2, and m_3, we get

$$E(\mathbf{k}+\mathbf{K})=E(\mathbf{k}). \quad (15)$$

Let us compare the result in Eq. (15) with the symmetry of the energy for an electron in a periodic potential. In the latter case the energy obeys the condition [Eq. (15)] for vectors of the usual Brillouin zone and also an additional relation which follows from the point symmetry of the space group:

$$E(\alpha\mathbf{k})=E(\mathbf{k}), \quad (16)$$

where α is a rotation or a reflection. The relation [Eq. (16)] for the magnetic case is expressed by the fact that the magnetic Brillouin zone is split into N equal parts and hence the energy variation need be considered only in one of these parts. From the representations [Eq. (13)] it follows that the degeneracy of the energy levels is N-fold.

III. LANDAU LEVEL BROADENING

Having described the energy levels for a Bloch electron in a magnetic field and the representations of the magnetic translation group, we can now solve the following problem: What happens to the Landau levels for an electron in a constant magnetic field when a weak periodic potential is turned on? We know the exact solutions for an electron in a constant magnetic field and we choose these in one of the representations given in Ref. 2. Let us denote the operators $\mathbf{p}-(e/c)\mathbf{A}$ by $\boldsymbol{\pi}$ and require the eigenfunctions of the Hamiltonian

$$(\mathbf{p}+(e/c)\mathbf{A})^2/2m \quad (17)$$

to be eigenfunctions of π_x too. By choosing a gauge $\mathbf{A}=\frac{1}{2}[\mathbf{H}\times\mathbf{r}]$ and by directing the z axis of the coordinate system in the direction of the magnetic field, we have[2]

$$\psi_{n,b}(xyz)=A_n\exp\left\{-\frac{ixy}{2\lambda^2}+ibx+\frac{i}{\hbar}p_z z\right\}\times\exp\left\{-\frac{1}{2}\left(\frac{y-\lambda^2 b}{\lambda}\right)^2\right\}H_n\left(\frac{y-\lambda^2 b}{\lambda}\right). \quad (18)$$

Here $\hbar b$ are the eigenvalues of π_x, $\lambda=(\hbar c/eH)^{1/2}$ is the radius of the cyclotron orbit and A_n is a normalization constant. The functions (18) obey the following equations:

$$\exp\left\{\frac{i}{\hbar}\pi_x a_x\right\}\psi_{n,b}=\exp\{iba_x\}\psi_{n,b},\qquad \exp\left\{\frac{i}{\hbar}\pi_y a_y\right\}\psi_{n,b}=\psi_{n,b-a_y/\lambda^2}, \quad (19)$$

where a_x and a_y are any translations in x and y directions. According to the interpretation given in Ref. 2 the operators π_x and π_y are connected with the y and x coordinates of the center of the cyclotron orbit. Equations (19) show that the energy does not depend on the continuous variable b, because a state with any eigenvalue of the operator π_x can be obtained from the state $\psi_{n,b}$ by applying operators $\exp\{(i/\hbar)\pi_y a_y\}$ that commute with the Hamiltonian (17). The energy is thus infinitely degenerate. By turning on a periodic potential the Hamiltonian of the problem will be

$$(\mathbf{p}+(e/c)\mathbf{A})^2/2m+V(\mathbf{r}). \quad (20)$$

The symmetry of the Hamiltonian (20) is lower than for the free electron in a magnetic field and we have to expect that the degeneracy of the energy levels will be partly removed. The operators that commute with (20) are given by Eqs. (19), but now a_x and a_y are components of lattice vectors. The magnetic translation group is thus a subgroup of the symmetry group for a free electron in a magnetic field. The operators $\exp\{(i/\hbar)\pi_y(R_n)_y\}$ will no longer give us a state with any eigenvalue of π_x from the state $\psi_{n,b}$; we obtain, instead, a discrete set of states that belong to the same energy. The infinite continuous set of functions (18) will split into an infinite number of discrete sets, each one leading, in general, to a different energy. Sets that are created from states with very close eigenvalues b will presumably have close energies. The Landau levels will thus be broadened.

A more detailed description of the broadening can be given for the case (3b) of the M.T.G. For simplicity we take a cubic crystal with the z axis along one of the principal axes of the crystal. In order to get the splitting, in the lowest order of perturbation theory, of the Landau levels which are described by the eigenfunctions (18), we have to use these eigenfunctions to construct symmetry adapted functions for the representations (13) of the M.T.G. Let us treat the simple case when $n=1$, and ignore the z dependence of the wave function which is of no interest in this treatment. The usual construction of symmetry adapted functions[4,5] leads to the following result (apart from a normalization constant):

$$\Psi_{n,j}{}^{\mathbf{k}}(xy)=\exp\left\{-\frac{2\pi}{N}im_2j\right\}\sum_m \tau(mN\mathbf{a}_2|mN\mathbf{a}_2)\times\exp\{-2\pi im_2m\}\psi_{n,(2\pi/a)(m_1-j)/N}, \quad (21)$$

where $\psi_{n,b}$ is the function defined in (18) for $z=0$, j takes values from 0 to $N-1$, $\mathbf{r}$ is a vector with components (x,y), and

$$\mathbf{k}=m_1(\mathbf{K}_1/N)+m_2(\mathbf{K}_2/N), \quad 0\leq m_1, m_2<1. \quad (22)$$

The functions (21) correspond to the classification of the energy levels for a Bloch electron in a magnetic field that is given in Sec. II. Two consequences follow from the above construction of the symmetry adapted functions (21): First, the Landau levels become N-fold degenerate instead of being of infinite degeneracy. Secondly, the Landau levels are broadened because of the dependence of the energy on the vector k. This dependence may be calculated by taking matrix elements of the periodic potential $V(\mathbf{r})$ in (20) between states (21), which are the correct functions for the lowest order perturbation theory. The matrix of the secular equation in the lowest order of perturbation theory will be automatically diagonal and we are left with the calculation of matrix elements of the potential $V(\mathbf{r})$ only between states (21) with the same n, j, and k. For a given n, the perturbation energy will depend on k only (we take therefore $j=0$):

$$V_n(\mathbf{k})=\int \Psi_n{}^{*\mathbf{k}}(xy)V(xy)\Psi_n{}^{\mathbf{k}}(xy)dxdy. \quad (23)$$

By expanding the potential $V(xy)$ in a Fourier series and by taking a proper normalization for the functions $\Psi_n{}^{\mathbf{k}}$, we get

$$V_n(\mathbf{k})=\sum_{p,q}\exp\{-2\pi i(m_1p+m_2q)\}H_{qp}(n), \quad (24)$$

where p and q take all integral values from $-\infty$ to $+\infty$; m_1 and m_2 define the $\mathbf{k}$ vector according to relation (22) and $H_{qp}(n)$ are the overlap integrals:

$$H_{qp}(n)=\frac{1}{2^n/\pi^{1/2}\lambda n!}V_{q,-p}\int \exp\left\{-\frac{1}{2}\left(\frac{y}{\lambda}\right)^2\right\}H_n\left(\frac{y}{\lambda}\right)\times\exp\left\{-\frac{1}{2}\left(\frac{y-qNa}{\lambda}\right)^2\right\}H_n\left(\frac{y-qNa}{\lambda}\right)\times\exp\left\{\frac{-2\pi}{a}ipy\right\}dy. \quad (25)$$

Here $V_{q,-p}$ is the Fourier transform of the potential $V(xy)$:

$$V_{q,-p}=\frac{1}{a^2}\int \exp\left\{-\frac{2\pi}{a}i(qx-py)\right\}V(xy)dxdy. \quad (26)$$

The integration in (26) is over an area of a unit cell in the x-y plane.

From the definition of the overlap integrals (25) and from the reality of the potential $V(xy)$, it follows:

$$H_{pq}{}^*(n)=H_{-p,-q}(n). \quad (27)$$

It can also be shown (see Appendix)

$$H_{pq}(n)=H_{q,-p}(n)=H_{-p,-q}(n). \quad (28)$$

Combining (27) and (28) we find that the overlap integrals $H_{pq}(n)$ are real.

The final expression for the perturbation energy (24) will be

$$V_n(\mathbf{k})=\sum_{pq}\cos(2\pi m_1p)\cos(2\pi m_2q)H_{pq}(n)=H_{oo}+2\sum_{p=1}^{\infty}[\cos(2\pi m_1p)+\cos(2\pi m_2p)]H_{op}(n)+2\sum_{p,l=1}^{\infty}\cos(2\pi m_1p)\cos(2\pi m_2q)H_{pq}(n). \quad (29)$$

As expected the perturbation energy $V_n(\mathbf{k})$ satisfies relation (15) and has the symmetry of the potential. The first term in (29) is a constant (it equals V_{oo}) and leads to a total shift of all energy levels. The other terms in the perturbation energy (29) will lead to both broadening and unequal spacing of the Landau levels. In order to estimate these two effects let us take the lowest order term of $V_n(\mathbf{k})$. We have (omitting the constant term H_{oo}):

$$V_n(\mathbf{k})=2V_{01}\exp\left\{-\left(\frac{\pi\lambda}{a}\right)^2\right\}L_n\left\{\left(\frac{\sqrt{2}\pi\lambda}{a}\right)^2\right\}\times(\cos2\pi m_1+\cos2\pi m_2), \quad (30)$$

where $L_n(x)$ is a Laguerre polynomial. The argument $(\pi\lambda/a)^2$ in both the exponential and the Laguerre polynomial is very large for all practically achievable

magnetic fields:

$$(\pi\lambda/a)^2\sim 10^{10}\times 1/H. \tag{31}$$

For fields $H\sim 10^5$ G, $(\pi\lambda/a)^2\sim 10^5$. For small quantum numbers n, the perturbation $V_n(\mathbf{k})$ will therefore be negligible. For high quantum numbers ($n>100$) we use the asymptotic expression for the Laguerre polynomials

$$L_n(x)\sim\frac{1}{\sqrt{\pi}}\exp\{\tfrac{1}{2}x\}x^{-1/4}n^{-1/4}\times\cos\left(2/n^{1/2}x-\frac{\pi}{4}\right)+O(n^{-3/4}). \tag{32}$$

The perturbation energy $V_n(\mathbf{k})$ will be

$$V_n(\mathbf{k})\sim V_{01}\left(\frac{2\pi^2\lambda^2}{a^2}n\right)^{-1/4}\cos\left(\frac{\lambda}{a}2\pi\sqrt{2}n-\frac{\pi}{4}\right)\times(\cos 2\pi m_1+\cos 2\pi m_2). \tag{33}$$

For $H\sim 10^5$ G and $n\sim 10^3$, we get

$$V_n(\mathbf{k})\sim 0.01V_{01}(\cos 2\pi m_1+\cos 2\pi m_2). \tag{34}$$

This is already a strong perturbation, and we have to expect that, for high quantum numbers n, the shift of Landau levels and the broadening will become considerable. However, when $n\to\infty$, $V_n(\mathbf{k})\to 0$. This is in agreement with the fact that very high-energy levels are not influenced by the periodic potential.

Finally, it is interesting to note that the formula (29) for the perturbation energy has the same form as in the case of the tight binding approximation. This fact is not surprising because the procedure used here for constructing symmetry adapted functions in the magnetic field is exactly the same as one uses for constructing Bloch-type functions from atomic orbitals.

IV. CONCLUSION

The Landau level broadening considered in this paper is an example of symmetry lowering which is caused by the introduction of a perturbation. In the case under consideration, a periodic electric potential is introduced as a perturbation into the problem of a free electron in a magnetic field. As a result of the perturbation, the degeneracy is partially removed and we get a broadening of the energy levels. It is a usual thing in quantum mechanics to expect that when an additional field is applied to a system the symmetry of the latter is lowered. This is, however, not the case when the additional applied field is a constant magnetic field. To see this let us compare the translation operators

$$\exp\{(i/\hbar)\mathbf{p}\cdot\mathbf{a}\}, \tag{35}$$

which commute with the Hamiltonian for a free electron ($\mathbf{a}$ is an arbitrary translation) with the operators (1) which commute with the Hamiltonian for an electron in a constant magnetic field. The latter do not form a subgroup of the operators (35). The same can be said about the case when a magnetic field is applied to a Bloch electron. The translation operators that commute with the Hamiltonian for a Bloch electron are

$$\exp\{(i/\hbar)\mathbf{p}\cdot\mathbf{R}_n\}. \tag{36}$$

When a magnetic field is also present, the operators that commute with the Hamiltonian are defined by Eq. (2). Again, the latter do not form a subgroup of the usual translation group (36). The fact that we get a *different* symmetry group by introducing a magnetic field probably explains the difficulties in solving the problem for a Bloch electron in a magnetic field. The only case when symmetry is lowered in the presence of a magnetic field is the example treated in this paper. This example is in fact very easy to treat.

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APPENDIX

We derive here the symmetry properties (28) of the overlap integrals $H_{pq}(n)$:

$$H_{pq}(n)=\frac{1}{2^n/\pi^{1/2}\lambda n!}V_{p,-q}\int\varphi_n\left(\frac{y}{\lambda}\right)\varphi_n\left(\frac{y-pNa}{\lambda}\right)\times\exp\left\{-\frac{2\pi}{a}iqy\right\}dy, \tag{A1}$$

where

$$\varphi_n\left(\frac{y}{\lambda}\right)=\exp\left\{-\frac{1}{2}\left(\frac{y}{\lambda}\right)^2\right\}H_n\left(\frac{y}{\lambda}\right). \tag{A2}$$

The integrals that appear in (A1),

$$I_{pq}=\int\varphi_n\left(\frac{y}{\lambda}\right)\varphi_n\left(\frac{y-pNa}{\lambda}\right)\exp\left\{-\frac{2\pi}{a}iqy\right\}dy \tag{A3}$$

are the Fourier transforms of the products $\varphi_n(y/\lambda)\times\varphi_n((y-pNa)/\lambda)$. Using the formula for a Fourier transform of a product of two functions, we get

$$I_{pq}=I_{q,-p}. \tag{A4}$$

Since we assume cubic symmetry for the potential energy relation (28) follows at once from the equality (A4).