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Neutron-Neutron Scattering at Low Energies

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This paper investigates the information contained in a neutron-neutron scattering experiment at low energies which could be performed by colliding beams coming from an underground nuclear explosion. The significance of such an experiment is discussed from the point of view of a check on charge symmetry and charge independence, and it is found that because of the electromagnetic complications in proton-proton scattering, and because of the proton-neutron mass difference, the knowledge of neutron-neutron scattering would be of considerable value. The functional form of the experimental data which is most convenient for analysis and the approximate relative magnitude of the terms is investigated, and it is concluded that for the kind of experiment which is envisaged (measuring cross sections to 10% from 20 keV to 2 MeV) only two parameters should be kept in the effective-range expansion. The connection between the number and distribution of energies at which the cross section is measured and the error on the individual measurements, on the one hand, and the accuracy of the effective-range parameters deduced from the experiments, on the other, is given explicitly and is found also to depend on the absolute magnitude of the scattering length. The results show that ten 10% measurements, suitably distributed between 20 keV and 2 MeV, can determine the sign of the scattering length to four standard deviations, the magnitude of the effective range to 50–70%, and the magnitude of the scattering length to about 3%. Finally, the relationship between the variation of the effective-range parameters and the corresponding variation in the parameters of the scattering potential is studied, and it is found that, while this relationship is strongly shape-dependent, a small change in the potential parameters, in any case, results in a large change in the scattering length, but a small one in the effective range. Numerical relationships show that, even in the worst case, the variation in the scattering length is about eight times the variation in the potential parameter. It is concluded that a 10% experiment at 20 energies between 20 keV and 2 MeV would be able to get information on the potential parameters sufficiently accurately so that charge-dependent or charge-symmetry violating effects could be detected.

I. INTRODUCTION

THIS paper was prompted by considerations of the possibility of measuring neutron-neutron scattering at low energies in a colliding beam experiment utilizing a single underground nuclear explosion.[1] Experimental aspects of this problem will not be discussed here, but it appears that it may be possible to measure this scattering cross section to an accuracy as high as 10%, from about 20 keV to about 2 MeV. The questions under investigation are: (a) why the knowledge of low-energy neutron-neutron scattering would be of interest (Sec. I); (b) the relationship of experimental data to the effective-range parameters to be determined (Sec. II); (c) the dependence of the uncertainty in the effective-range parameters on the number, distribution, and error of the experimental data points (Sec. III); and (d) the relationship between the errors on the effective-range parameters and the uncertainty in the parameters describing the scattering potential (Sec. IV). The conclusions are stated in Sec. V.

A precision knowledge of the neutron-neutron scattering parameters at low energy would be of interest for several reasons. Firstly, it would furnish a test of charge symmetry. Although charge symmetry is rather firmly believed, the substantial evidence for it comes exclusively from nuclear structure. Since our knowledge of the relationship of nuclear structure to the nuclear two-body problem is far from complete, there is much to be said for a direct check of charge symmetry using the two-nucleon interaction itself. Furthermore, there are several phenomena which will cause an apparent

[1] Charles D. Bowman and William C. Dickinson, University of California, Lawrence Radiation Laboratory Report No. UCRL-7859, 1964 (unpublished).

deviation from charge symmetry, even after the Coulomb interaction is separated out in the proton-proton scattering, such as nucleon electromagnetic structure, magnetic moment interaction, and the neutron-proton mass difference.[2–4] One presumes that all these effects are electromagnetic in origin, and that the "pure" strong interaction between nucleons is charge symmetric. Yet, it is not even clear how such separation of these electromagnetic effects could be carried out. For instance, in comparing the width and depth of the "pure nuclear" scattering potentials of neutrons and protons, should one measure these parameters in the same absolute units, or in units of their respective masses? The investigation of these and similar questions would receive a large boost if charge symmetry could be checked directly to sufficiently high accuracy (i.e., to about 0.1%). It will be demonstrated in this note that such a high-precision check would be quite feasible with the experiments referred to above.

A second reason why low-energy neutron-neutron scattering would be of great interest is that it would facilitate tests of charge independence. At the present, direct tests of charge independence for nucleons involve a comparison of proton-proton scattering with the $T=1$ part of neutron-proton scattering. The former, however, is greatly hampered by the large electrostatic scattering which has to be separated out of the data and which completely overwhelms the scattering below, say, 100 keV. Furthermore, vacuum polarization corrections also have to be taken into account in the analysis of high-precision proton-proton scattering experiments. Furthermore, since the scattering experiments are restricted to above 100 keV, to get any energy range at all, experiments at relatively high energies (e.g., 5 MeV) must also be included in the analysis, which, in turn, necessitates the inclusion of higher partial waves as well as more terms in the effective-range expansion, and this complicates the analysis.[5] Thus, although proton-proton scattering experiments are, in general, much easier to carry out than neutron-neutron experiments, their interpretation is considerably more complicated.

A joint knowledge of neutron-neutron and proton-proton scattering would also make the test of charge independence easier for another reason. In previous comparison of proton-proton and neutron-proton scattering at low energies, a rather large difference was found in the scattering lengths. To this very day it is not clear, however, whether this discrepancy can be explained by electromagnetic effects or not. In addition to the magnetic moment interaction, nuclear form factors, and the neutron-proton mass difference already discussed above in connection with charge symmetry, there are here also effects due to the mass difference between neutral and charged pions. In previous calculations of these corrections,[2–4] there was enough of an uncertainty so that one could not tell whether the discrepancy disappears if these corrections are applied. With the neutron-neutron scattering also available for comparison, it would be easier to check these corrections against experiment, since some of them affect only charge independence, while others affect both charge symmetry and charge independence.

Finally, if there is a slightly unbound dineutron resonance state, it would have a marked effect on the low-energy neutron-neutron scattering cross section. In particular, there would be a dip superimposed on the straight-line behavior of $Q(T)$, to be defined by Eq. (2.4). The position and size of this dip would be valuable and unprecedented information concerning such a resonance.

In the low-energy region below, say, 2 MeV, only S waves will contribute substantially to the scattering. The veracity of this statement depends, of course, on the precision by which experiments are carried out. In this report we will consider experiments in which the error on the individual cross sections at various energies is of the order of 10%. With such experiments, up to 2 MeV, any waves other than $^1S_0$ can be neglected. A "proof" for this can be obtained by comparing it with p-p scattering, where at 2 MeV the S-wave phase shift is around 45°, while the largest of the three P phase shifts is only 0.5°, so that even the S-P interference terms in the differential cross section will be negligible compared to the experimental error. Furthermore, if one measures total cross sections, even this interference term is absent and the P-wave effects are truly infinitesimal.

With only the S waves present, the differential cross section will be isotropic in the center-of-mass system and the interaction can be characterized by only one parameter, the $^1S_0$-wave phase shift. The subject of investigation is, therefore, the energy dependence of this phase shift. This is usually expressed in terms of the effective-range theory which gives this energy dependence in terms of a power series in k, the center-of-mass momentum of one of the two colliding neutrons.[6] The number of terms one has to keep in this power series depends on the energy range under consideration and the precision of the experiments. In our case, since we expect that the effective-range parameters for neutron-neutron scattering will be of the same order of magnitude as those for proton-proton scattering, we can make a fairly reliable *a priori* estimate on the number of parameters to be kept. These and other quantitative considerations will be discussed in the next section.

[2] J. Schwinger, Phys. Rev. **78**, 135 (1950).
[3] E. E. Salpeter, Phys. Rev. **91**, 994 (1953).
[4] Riazuddin, Nucl. Phys. **7**, 217 and 223 (1958).
[5] H. P. Noyes, Phys. Rev. Letters **12**, 171 (1964).

[6] See e.g. M. J. Moravcsik, *The Two-Nucleon Interaction* (Clarendon Press, Oxford, England, 1963). The scattering length for p-p scattering in Table 8 on p. 45 should be negative.

In the absence of a direct measurement of neutron-neutron scattering, there have been proposals in the past to determine the low-energy scattering parameters from the final-state interaction in a reaction resulting in two neutrons and a photon.[7] The disadvantages of this method are as follows: (a) It depends on certain assumptions about the mechanics of the reaction and hence is not as firm as the direct measurement; (b) in its proposed form it measures only the absolute value of the scattering length, and not its sign or the corresponding effective range; (c) even the magnitude of the scattering length can be obtained only to about 25%, while the experiment discussed in this note can hope to do an order of magnitude better.

II. EXPERIMENTAL QUANTITIES AND EFFECTIVE-RANGE PARAMETERS

The effective-range formula for S waves is usually written in the following form:

$$k\cot\delta=-(1/a)+\tfrac{1}{2}r_0k^2-Pr_0^3k^4, \tag{2.1}$$

where k is the center-of-mass momentum of one of the neutrons, a is the scattering length, r_0 is the effective range, P the shape parameter, and δ the phase shift. On the other hand, the S-wave cross section can be written as

$$\sigma=4\pi\frac{d\sigma}{d\Omega}=4\pi\frac{\sin^2\delta}{k^2}=\frac{4\pi}{k^2+(k\cot\delta)^2}, \tag{2.2}$$

where σ is the total cross section, and $d\sigma/d\Omega$ the differential cross section. Furthermore, we have the kinematic relationship

$$k^2=mT/2, \tag{2.3}$$

where m is the neutron mass and T is the kinetic energy of the incoming neutron in the laboratory system.

Combining these equations we can write

$$Q(T)\equiv\left(\frac{d\sigma}{d\Omega}\right)^{-1}=\frac{1}{a^2}+\frac{m}{2}\left(1-\frac{r_0}{a}\right)T+\frac{r_0^2m^2}{4}\left(\frac{1}{4}+2P\frac{r_0}{a}\right)T^2. \tag{2.4}$$

We can rewrite this equation in practical units as follows:

$$Q(T)=\frac{1}{a^2}+\left(1-\frac{r_0}{a}\right)\epsilon+r_0^2\left(\frac{1}{4}+2P\frac{r_0}{a}\right)\epsilon^2, \tag{2.5}$$

where $Q(T)$ is given in $(\mathrm{F}^2/\mathrm{sr})^{-1}$, a and r_0 in F, and ϵ is defined as (T in units of MeV)/82.88. The shape parameter P is dimensionless.

We will now try to estimate the relative magnitude of the terms in Eq. (2.5). For proton-proton scattering the approximate values of the effective-range parameters are[5,6] $a=-7.8$, $r_0=2.8$, $P=0.02$, while for the $T=1$ neutron-proton amplitude $a=-24$, $r_0=2.7$, $P=0.02$. Hence, one can roughly estimate that for neutron-neutron scattering, we would have an r_0/a with a magnitude of the order of 0.3–0.1. With P of the order of 0.02, this means that in the term quadratic in ϵ, the terms $2Pr_0/a$ is only about 4% of the $\frac{1}{4}$ term. Furthermore, the whole quadratic term is very small compared to the linear term, since the coefficient of the linear term is of the order of 1, and the coefficient of the quadratic term is of the order of 2. Since, however, ϵ is only 0.02 even at 2 MeV, the upper limit of the range we are considering, the quadratic term as a whole is at most only 4% of the linear term. With 10% errors on the individual experimental points, therefore, we can ignore the quadratic term. If needed, however, one could include it as $r_0^2\epsilon^2/4$ (which, as we have just seen, is a very good approximation to it) and such a quadratic term would not add an extra parameter to be determined from the experiments since r_0 already appeared in the linear term. In other words, one could then fit the data against

$$Q(T)=\epsilon+(a^{-1}-\tfrac{1}{2}r_0\epsilon)^2 \tag{2.6}$$

which contains only two parameters.

It is evident from the above considerations that with a 10% experiment, it would not be possible to measure the shape parameter at all, and that a linear approximation for $Q(T)$ is very likely to be sufficient.

The value of this straight line at $\epsilon=0$ would give us directly a^{-2}. Since the square of a is involved, the sign of the scattering length is not determined by this term. On the other hand, for the same reason, an x% determination of this term gives us a $\frac{1}{2}x$% determination of the scattering length itself. The slope of the straight line gives the coefficient of the linear term and determines r_0/a. It also determines the sign of a.

III. EXPERIMENTAL ACCURACY AND UNCERTAINTY IN THE EFFECTIVE-RANGE PARAMETERS

As we saw in the previous section, a and r_0 are determined from the experimental data by a two-parameter fit. In planning a significant experiment one has to determine, therefore, the relationship between the particulars of the experiment and the uncertainty in the effective-range parameters obtained therefrom. The latter will depend on the number and distribution of energies at which the cross section is measured, on the size of the error attached to the individual measurements, and, as it turns out, on the values of the effective-range parameters themselves. In this section we will be given some quantitative information on these dependences, whose qualitative behavior is quite plausible.

As it is evident from Eq. (2.5), the quantities actually determined from the fit to the experiment are

[7] K. W. McVoy, Phys. Rev. 121, 1401 (1961).

the intercept I of $Q(t)$ with the ordinate and the coefficient C of the ϵ term. [The analysis is analogous but slightly different if Eq. (2.6) is used instead. Since the quantitative difference between Eqs. (2.6) and (2.5) will be very small in our case, all quantitative conclusions in this section hold also for Eq. (2.6).] As mentioned before, a given percentage error in I will result in half that percentage error in a. A given percentage error in C, however, will result in a percentage error in r_0 which will depend on the magnitude and sign of r_0/a. In particular, using $r_0=2.7$ (which is the effective range for proton-proton scattering, and which, not being sensitive to small differences in the potential depth and width, will likely be about the same for neutron-neutron scattering), we can say that a $p\%$ error in C will become a $(1+0.38|a|)p\%$ error in r_0/a. The uncertainty in r_0/a can be directly related to the uncertainty in r_0 since in practice a is always much better known than r_0. Having made these remarks we can now restrict ourselves to the uncertainty in I and C as determined from the experiments.

Perhaps the simplest relationship of the effects listed above is between the error on the individual data points and the errors on I and C: If, other things being equal, all experimental errors are doubled, the uncertainty in I and C will also double.[8]

Almost as simple a relationship exists between the number of data points and the uncertainties in I and C: Again, other things being equal, if the number of data points are increased by a factor of 2, the uncertainty in I and C decreases by $2^{1/2}$. (In this case, strictly speaking, other things cannot be held the same, since the increased number of data points will, by necessity, mean a slightly different distribution of energies.) One can see the above relationship by realizing that, given a certain number of "counts" N taken in the over-all energy range, it should make essentially no difference whether they are classified into n energy subintervals, in which case, each point will have a percentage error of $(N/n)^{-1/2}$, or into $2N$ energy subintervals, in which case, the individual percentage errors will be $(N/2n)^{-1/2}$.

The rest of the relationships cannot be predicted on general grounds but has to be investigated for the circumstances of each particular situation. Such a study for the problem under consideration yielded the following results.

1. Distribution of Energies and Errors

Since I is determined mostly by measurements at the lowest energies, its accuracy hinges on many pieces of data at low energies. C, on the other hand, is determined mainly by data at the ends of the over-all energy range. Hence, for an optimal determination of both quantities, it is most advantageous to perform most measurements near the low and high ends of the over-all energy range. Furthermore, the uncertainty in I depends fairly strongly on the value of the lowest energy at which measurements exist, while C is essentially independent of this as long as the size of the over-all energy range remains approximately the same.

2. Dependence on the Magnitude of the Effective-Range Parameters

The uncertainty in I and C depends also on I and C itself or, to be precise, on their ratio. For a given percentage error in the data points, I is determined more accurately if $R\equiv[Q(E_{\max})-Q(E_{\min})]/Q(E_{\min})$ is small than if it is large. For C, however, the situation is just the reverse.

The above qualitative statements can best be summarized quantitatively by giving a number of examples. This is done in Table I. The three variables explored are the above defined R, the distribution of energies D (including dependence on the lowest energy measured), and the distribution of relative errors E. Although only a few combinations in the three-dimensional space are given, the table suffices for a quantitative estimate to find out which features of a proposed experiment are the most critical. For a fixed r_0, the value of R can be directly related to the value of a. Since we do not expect much deviation from the value of $r_0=2.7$, the three blocks in Table I, showing the R dependence, can thus be labeled by the a value which corresponds to this value of r_0.

Table I shows that while C can be measured best for large negative a's, the corresponding precision in the determination of r_0 is best for intermediate a's. We also see that a $D(4)$, $E(1)$ type measurement would give a precision of 7–10% on I and at the same time would yield r_0/a to 50–70%, regardless of the magnitude of a. This would be sufficient to give a very reliable result for the sign of a. At the same time, the magnitude of a would be known to 3.5–5%.

IV. RELATIONSHIP BETWEEN THE EFFECTIVE-RANGE PARAMETERS AND THE DEPTH AND WIDTH OF THE POTENTIAL

Although the scattering length and effective range are the parameters immediately connected with the experimental data, the theoretically more significant quantities are those describing the scattering potential. At these low energies, and with the available experimental precision which can measure only two effective-range parameters, one can determine only the depth and the width of an "equivalent" central potential. The shape of this central potential cannot be determined, although the actual values of depth and width belonging to a given value of scattering length and effective range will depend on the assumed shape.

[8] P. Cziffra and M. J. Moravcsik, University of California, Lawrence Radiation Laboratory Report No. UCRL-8523 Rev., 1959 (unpublished).

TABLE I. Percentage errors in the intercept I and slope C of $Q(T)$ of Eq. (2.4), as functions of the distribution D of energies of the data points and of their errors E, as well as of the scattering length a, assuming that $r_0=2.7$. The percentage error in a is simply half the percentage error in I. The percentage error in r_0, given by $(1+0.38|a|)$ times the percentage error in C, is given in parentheses following the percentage error in C. The notation for the distributions is as follows:

$D(1)$: 20, 40, 70, 100, 200, 300, 500, 700, 1000, 2000 keV;
$D(2)$: 20, 30, 40, 70, 100, 150, 200, 500, 1000, 2000 keV;
$D(3)$: 20, 100, 300, 600, 800, 1000, 1200, 1400, 1700, 2000 keV;
$D(4)$: 20, 30, 40, 100, 500, 1200, 1400, 1600, 1800, 2000 keV;
$D(5)$: 100, 120, 140, 170, 200, 300, 500, 700, 1000, 2000 keV;
$D(6)$: 100, 120, 130, 140, 170, 200, 250, 500, 1000, 2000 keV.

The notation for the errors is as follows:

$E(1)$: 10% for all points;
$E(2)$: 10% for the five lowest energy points, 20% for the rest;
$E(3)$: 20% for the five lowest energy points, 10% for the rest;
$E(4)$: 10% for the two lowest and three highest energy points, 20% for the rest.

$a=-8.2$ F

	$E(1)$	$E(2)$	$E(3)$	$E(4)$
$D(1)$	6.5	7.1	10.5	8.5
	24 (102)	44 (189)	29.2 (124)	26.2 (111)
$D(2)$	5.8	6.5	8.7	8.0
	24 (102)	44 (189)	27.8 (118)	26.4 (112)
$D(3)$	8.9	9.4	16.6	10.0
	20.4 (86)	31.8 (135)	28.2 (120)	21.8 (93)
$D(4)$	7.1	7.3	14.0	9.1
	17.2 (73)	29.8 (127)	22.8 (97)	19.8 (84)
$D(5)$	7.1	8.3	11.1	9.3
	26.2 (111)	48.4 (206)	30.4 (130)	27.8 (118)
$D(6)$	6.6	7.9	9.5	8.8
	26.8 (114)	30.8 (132)	29.2 (124)	28.0 (119)

$a=-15.8$ F

	$E(1)$	$E(2)$	$E(3)$	$E(4)$
$D(1)$	8.3	8.6	14.3	10.0
	10.0 (70)	18.0 (126)	12.0 (84)	11.1 (77)
$D(2)$	7.0	8.1	11.7	9.3
	12.0 (84)	21.5 (150)	12.6 (88)	11.8 (83)
$D(3)$	10.5	11.7	21.5	11.8
	8.2 (58)	11.8 (83)	9.8 (69)	9.4 (66)
$D(4)$	8.0	8.4	16.2	10.1
	7.8 (55)	13.6 (95)	8.7 (61)	10.4 (73)
$D(5)$	11.0	13.3	18.0	13.2
	12.2 (85)	22.5 (156)	13.4 (94)	12.5 (87)
$D(6)$	10.3	13.3	15.1	12.7
	13.0 (91)	25.0 (175)	13.7 (96)	13.0 (91)

$a=-28.3$ F

	$E(1)$	$E(2)$	$E(3)$	$E(4)$
$D(1)$	11.0	12.0	21.5	12.7
	8.0 (93)	13.0 (151)	8.5 (99)	8.9 (104)
$D(2)$	10.0	11.6	17.8	11.8
	9.0 (105)	16.2 (187)	9.5 (110)	9.3 (108)
$D(3)$	15.0	15.2	30.0	15.1
	6.0 (70)	8.7 (102)	7.2 (83)	7.7 (90)
$D(4)$	10.4	10.8	20.2	12.0
	6.4 (74)	10.9 (127)	7.0 (81)	9.0 (105)
$D(5)$	22.6	30.6	27.0	25.2
	9.1 (106)	17.0 (198)	9.9 (115)	9.9 (115)
$D(6)$	22.2	31.5	30.5	25.0
	9.9 (115)	19.3 (224)	10.9 (127)	10.5 (122)

In this section we will investigate the sensitivity of the effective-range parameters to small variations in the potential depth and width. This is crucial in determining whether a given experiment can serve to detect certain noncharge-symmetric effects (such as neutron-proton mass-difference effects, whose magnitude is likely to be 0.1% in the potential parameters).

Let us denote the potential depth as V_0, and the potential width as b. One can then define[9] a parameter s as

$$s=KV_0b^2\,, \tag{4.1}$$

where the K has such dimensions as to make s dimensionless, and its numerical value [but not the functional form of Eq. (4.1)] is shape-dependent.

Further relationships of use to us are given in Ref. 10, Tables I and V. We will denote these by

$$r_0/s=1/t(s) \tag{4.2}$$

and

$$s=u(w)\,,\quad w\equiv r_0/a\,, \tag{4.3}$$

respectively. Again, the precise form of these functions is shape-dependent. Combining these two relations, we get

$$t[u(w)]\equiv f(w) \tag{4.4}$$

and therefore

$$b=r_0f(w) \tag{4.5}$$

and

$$V_0=C\frac{s}{b^2}=\frac{C}{{r_0}^2}\frac{s}{f^2(w)}\equiv\frac{C}{{r_0}^2}g(w)\,, \tag{4.6}$$

where $f(w)$ and $g(w)$ are shape-dependent, but the r_0 dependence of Eqs. (4.5) and (4.6) is not. In Eq. (4.6) we used $C=K^{-1}$ which is a shape-dependent constant.

From the above equations, one can obtain the percentage variations in b and V_0 as functions of the percentage variations in r_0 and a. The relations are

$$\frac{db}{b}=\left[1+wf^{-1}(w)\frac{\partial f(w)}{\partial w}\right]\frac{dr_0}{r_0}-wf^{-1}(w)\frac{\partial f(w)}{\partial w}\frac{da}{a} \tag{4.7}$$

and

$$\frac{dV_0}{V_0}=\left[wg^{-1}(w)\frac{\partial g(w)}{\partial w}-2\right]\frac{dr_0}{r_0}-wg^{-1}(w)\frac{\partial g(w)}{\partial w}\frac{da}{a}\,. \tag{4.8}$$

It should be noted that Eqs. (4.7) and (4.8) do not depend separately on r_0 and a, but only on their ratio w.

One can also see from the above equations that at $w=0$ (which, for instance, can be obtained with $a=\pm\infty$), if we assume that $g(0)\neq0$, and $\partial g(w)/\partial w|_0\neq\infty$, then an infinitely small change in V_0 or b results in an infinitely large change in a. This fact is well known and can be seen easily from the study of the wave function for this case. A similar infinite sensitivity can also occur for the r_0, but at different values of w, namely,

[9] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York 1952), pp. 55–56.
[10] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

TABLE II. Functions $F(w)$ and $G(w)$, defined by Eq. (4.11), as a function of w, defined by Eq. (4.3) for square well, exponential well, and Yukawa potential shapes.

	Square well		Exponential well		Yukawa well	
w	$F(w)$	$G(w)$	$F(w)$	$G(w)$	$F(w)$	$G(w)$
−0.200	0.323	−0.0363	0.456	−0.388	0.535	−0.651
−0.188	0.327	−0.0351	0.465	−0.398	0.549	−0.668
−0.176	0.330	−0.0337	0.475	−0.407	0.562	−0.687
−0.164	0.335	−0.0323	0.486	−0.418	0.577	−0.706
−0.152	0.339	−0.0308	0.496	−0.428	0.592	−0.726
−0.140	0.344	−0.0292	0.507	−0.439	0.607	−0.747
−0.128	0.348	−0.0274	0.519	−0.450	0.623	−0.768
−0.116	0.353	−0.0256	0.530	−0.461	0.641	−0.791
−0.104	0.358	−0.0236	0.543	−0.473	0.659	−0.815
−0.092	0.363	−0.0215	0.555	−0.486	0.677	−0.841
−0.080	0.368	−0.0192	0.568	−0.498	0.697	−0.867
−0.068	0.373	−0.0168	0.581	−0.512	0.717	−0.895
−0.056	0.378	−0.0142	0.596	−0.526	0.739	−0.925
−0.044	0.384	−0.0115	0.610	−0.540	0.762	−0.956
−0.032	0.389	−0.0086	0.625	−0.555	0.786	−0.989
−0.020	0.395	−0.0055	0.641	−0.570	0.811	−1.024
−0.008	0.401	−0.0023	0.657	−0.586	0.838	−1.061

for

$$wg^{-1}(w)\frac{\partial g(w)}{\partial w}\approx 2 \quad (\text{for } V_0) \tag{4.9}$$

and

$$wf^{-1}(w)\frac{\partial f(w)}{\partial w}\approx -1 \quad (\text{for } b). \tag{4.10}$$

Whether such a condition is physically realizable or not is not immediately clear.

One can also see from Eqs. (4.7) and (4.8) that the sensitivity coefficients depend only on the logarithmic derivatives of $f(w)$ and $g(w)$, defined as

$$F(w)\equiv f^{-1}(w)\partial f(w)/\partial w\,, \qquad G(w)\equiv g^{-1}(w)\partial g(w)/\partial w\,. \tag{4.11}$$

These functions are tabulated in Table II, for the values $-0.2<w<0$, which is the range of validity for negative w of the formulas in Ref. 10 from which they were derived. The table gives these functions for a square well, for the exponential well, and for the Yukawa well, thus covering the usual range of shapes under consideration. It is evident from Table II that there are very large variations among the various shapes in the sensitivity coefficients. In particular, $G(w)$ for a square well is not only an order of magnitude smaller than for the other two shapes, but its absolute value decreases with increasing w, reaching a very small value indeed around $w=0$. If we could calculate the small deviations from charge symmetry, this large variation in the sensitivity coefficients would give us a means to obtain information on the shape of the potential. Under the present circumstances, however, one can only assume a conservative approach and say that in the range of w which is most likely to be of interest to us ($-0.2<w<-0.1$), $F(w)$ is not larger than of the order of 0.6 and $G(w)$ is not larger than of the order of 0.75.

V. CONCLUSIONS

One can conclude from the above considerations that if the neutron-neutron scattering cross section could be measured with a 10% error at 20 energies between 20 keV and 2 MeV, with a concentration of energies near the low and high ends of the energy range, one could obtain the magnitude of the scattering length a to an accuracy of 2.8–3.7% and the sign of a with a quite high degree of confidence (the points corresponding to the two different signs of a being four standard deviations apart). This, in turn, would permit us to determine the parameters of the corresponding scattering potential to an accuracy of at least 0.3%, and perhaps 0.1%. With this precision charge-dependent and charge-symmetry violating effects could be detected. The experiment would also yield a 30–50% determination of the effective range r_0, but since r_0 is relatively insensitive to changes in the interaction potential, and since the error on this determination is quite large, the value thus obtained for r_0 would not be of much interest.

It is hoped that the foregoing considerations will serve as positive encouragement for the actual carrying out of a neutron-neutron scattering experiment in an underground nuclear explosion using colliding beams.

Note added in proof. After submitting the manuscript of this paper, I encountered an article [M. Bander, Phys. Rev. **134**, B1052 (1964)], in which the method of Ref. 7 is discussed further and some of the approximations used therein improved. In view of the fact, however, that some of the uncertainties of Ref. 7 remain unresolved, the estimate of the accuracy for the determination of the scattering length on the basis of such a calculation (i.e., an error of 1 F) appears somewhat optimistic, and the sign of the scattering length and value of the effective range remain undetermined.

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