boson mediates the μ-e decay, nonlocality is introduced, which gives a finite value for ρ. However, it is known from the recent high-energy neutrino experiment that the mass of the weak boson, if any, is not smaller than 1.3 BeV. The lower limit seems to tend to increase incessantly. After all, the weak boson will not be able to produce such large values of ρ as observed, although one cannot estimate it in any reliable way.

We have also shown that the μ-e puzzle cannot be solved, at least within the framework of quantum electrodynamics. If it were formally solved, we should encounter evident contradiction with the experiments on the longitudinal polarization of the electron in the μ-e decay. If one constructs a self-consistent theory with $m_e^0 \neq m_\mu^0$, the weak interactions are accommodated without any contradiction.

Outside of quantum electrodynamics, we have two alternative ways of avoiding similar difficulties. One of them is to formulate without a Lagrangian or Hamiltonian a self-consistent deviation theory which cannot be described by the Lagrangian theory in an equivalent way.

The origin of the contradictions pointed out here lies in the strict conservation of the weak currents. If one assumes the weak vertices to be nonvanishing on the light cone, one is led, as is well known, to the massless scalar bosons. The massless bosons are eliminated if the weak vertices are zero on the light cone. However, the present arguments lead to the contradictions independently of the behavior of the weak vertices near the light cone.

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Self-Consistent Calculation of the Scattering Amplitude and the Diffraction Peak

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We investigate the $\pi\pi$ scattering at low-momentum transfers, in order to understand diffraction scattering. A self-consistent calculation of the position $\alpha(s)$ and reduced residue $\gamma(s)$ of the Pomeranchuk-Regge trajectory is carried out using the Balázs method. The result of the calculation under certain simplifying approximations is that the s dependence of $\gamma(s)$ is responsible for the sharp forward peaking in the high-energy scattering and can roughly reproduce the experimental width of the diffraction peak derived from the factorization theorem.

I. INTRODUCTION

RECENT experiments at 5–20 BeV have shown a substantial shrinkage with increasing energy of the forward peak width of p-p and K^+-p elastic scattering, whereas only a slight shrinkage was observed for π-p and K^--p scattering.[1]

It was pointed out[2] that the three Regge pole approximation[3] may still explain the above features of the high-energy scattering if the following assumptions are made:

(i) The slope of the Pomeranchuk trajectory is assumed small in order to understand the absence of strong shrinkage in π-p scattering.

(ii) The s dependence of the residue function is important for the sharp forward peaking in high-energy scattering.

It is very interesting, therefore, to investigate whether one can get (i) and (ii) theoretically, starting from the Mandelstam representation and using unitarity and crossing symmetry. It is the purpose of this paper to investigate the asymptotic behavior of the model of pion-pion scattering to clarify the diffraction mechanism at high energy and low momentum transfer.

Attention is focused on small-momentum-transfer behavior of the position $\alpha(s)$ and residue $\gamma(s)$ of the top-level Pomeranchuk trajectory. This trajectory controls the high-energy scattering at low momentum transfers. There have been several "bootstrap" methods proposed for calculating the π-π amplitude from the requirement of analyticity, unitarity, and crossing symmetry.[4–10] We

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have set p ubootstrap equations for $\alpha(s)$ and $\gamma(s)$, using the Balázs version[10] of a Reggeized strip approximation for positive-signature partial waves to solve the N/D equation. The s dependence of the residue function $\gamma(s)$ is explicitly taken into account in our equations, in contrast to the assumption of a nearly constant behavior in Ref. 10.

To produce the bootstrap cycle, the s-channel Pomeranchuk trajectory is required to agree self-consistently with the t-channel Pomeranchuk trajectory. The input information in this calculation is (i) the value of σ_t, which we take from the factorization theorem[11] as $\sigma_t=15$ mb; (ii) the ρ-meson parameters; and (iii) the Chew-Frautschi saturation principle $\alpha(s=0)=1$.

In this calculation it is assumed that the only significant contributions to the partial-wave amplitude arise from the Pomeranchuk-Regge pole and the low-energy resonance poles. This assumes, of course, that there are no cuts to the right of the poles in the angular-momentum plane. In general, this is not likely to be the case, as has been argued by Mandelstam.[12] However, even if these cuts do exist, the fact that the high-energy total cross sections can be fitted quite well by a sum of simple poles indicates that, perhaps, the pole approximation is adequate in the forward direction. If so, then, at least for moderately large energies, it is reasonable to expect the poles to continue to dominate the cuts for a small range of momentum transfers near the forward angle.

In Sec. II we shall briefly describe the Balázs approximation[6,10] to solve the N/D equation, confining ourselves to the $I=0$ and positive signature partial wave.

In Sec. III, we shall show, after making some simplifying assumptions, how to complete a bootstrap cycle for $\alpha(s)$ and $\gamma(s)$ for the Pomeranchuk trajectory at small momentum transfer by making use of the N/D equation for the positive-signature partial wave.

The formula for the diffraction width is given in Sec. IV.

In Sec. V we give the results of the numerical computation of $\alpha(s)$ and $\gamma(s)$. We conclude that the s dependence of $\gamma(s)$ is responsible for the sharp forward peaking in the high-energy scattering and roughly reproduces the experimental width of the diffraction peak derived from the factorization theorem.

II. THE BALÁZS APPROXIMATION

We shall begin by summarizing the approximation method introduced by Balázs[6,10] to solve the Chew-Mandelstam N/D equation, confining ourselves to the isospin zero and positive-signature partial wave.[13]

From the Mandelstam representation the $I=0$ positive-signature partial wave can be defined in terms of the absorptive part, $\tilde{A}_t{}^0(t', 4(\nu+1))$, in the t channel with $I=0$ in the s channel as[14,15]

$$A_l{}^+(\nu)=\frac{1}{\pi\nu}\int_4^\infty dt'\tilde{A}_t{}^0(t', 4(\nu+1))Q_l\left(1+\frac{t'}{2\nu}\right). \quad (1)$$

Here $\nu=s/4-1$ and s is the square of the total energy in the barycentric system with pion mass 1. The amplitude defined in this way agrees with the physical partial-wave amplitudes at positive integral values of l and can be analytically continued to unphysical values. It satisfies the generalized unitarity condition[16]

$$\mathrm{Im}[A_l{}^+(\nu)]^{-1}=-(\nu/(\nu+1))^{1/2}R_l{}^+(\nu). \quad (2)$$

We shall define

$$B_l{}^+(\nu)=A_l{}^+(\nu)/\nu^l \quad (3)$$

to avoid a kinematical singularity.[17] Using the N/D decomposition,

$$B_l{}^+(\nu)=N_l{}^+(\nu)/D_l{}^+(\nu), \quad (4)$$

one can set up an effective-range approximation, in which the distant part of the left-hand cut is replaced by a few (in fact, two) poles. The position of the poles are fixed *a priori* so as to approximate the kernel in the equation for the numerator function sufficiently well in the region of interest.[18] Thus, this gives

$$N_l{}^+(\nu)=\sum_{i=1}^{2} f_i/(x_i{}^{-1}+\nu), \quad (5)$$

with $x_1{}^{-1}=6.25$ and $x_2{}^{-1}=50$, and it involves two unknown residues f_1 and f_2. Normalizing $D_l{}^+(\nu)$ to unity at $\nu=\nu_0$, one then obtains

$$D_l{}^+(\nu)=1-\frac{\nu-\nu_0}{\pi}\int_0^\infty d\nu'\left(\frac{\nu'^{2l+1}}{\nu'+1}\right)^{1/2}\frac{R_l{}^+(\nu')}{(\nu'-\nu_0)(\nu'-\nu)}\times\sum_{i=1}^{2}\frac{f_i}{x_i{}^{-1}+\nu'}. \quad (6)$$

For given $R_l{}^+(\nu')$, Eqs. (5) and (6) can be solved to give $B_l{}^+(\nu)$ through Eq. (4) in terms of the two residues f_1 and f_2. These residues can then be determined by matching the values of the amplitude in Eqs. (3) and (4), at two suitable chosen points (which lie between $\nu=0$, and $\nu=\nu_L$) with values calculable from Eq. (1).

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[18] See Ref. 6 for details regarding this method.

To evaluate Eq. (1), one can split the integral into two parts:

$$A_l^+(\nu)=A_l^{+(L)}(\nu)+A_l^{+(H)}(\nu), \tag{7}$$

where

$$A_l^{+(L)}(\nu)=\frac{1}{\pi\nu}\int_4^{t_0} dt' \tilde{A}_l^0(t', 4(\nu+1))Q_l\left(1+\frac{t'}{2\nu}\right), \tag{8}$$

$$A_l^{+(H)}(\nu)=\frac{1}{\pi\nu}\int_{t_0}^{\infty} dt' \tilde{A}_l^0(t', 4(\nu+1))Q_l\left(1+\frac{t'}{2\nu}\right). \tag{9}$$

Here t_0 is the width of the strip[19] or a separation point in the t channel between the low-energy region which is dominated by resonances and the high energy which is dominated by a s-channel Regge trajectory. If $A_l^{+(H)}(\nu)$ is approximated by a single Regge-pole term it has the form:

$$A_l^{+(H)}(\nu)=-\nu^l(2\alpha(s)+1)\frac{\beta(s)}{\nu^{\alpha(s)}}\frac{c_1(\alpha)c_2(l)}{a(s)-l}(\tfrac{1}{2}t_0)^{\alpha(s)-l} \tag{10}$$

where

$$c_1(\alpha)=\frac{2^\alpha\Gamma(\alpha+\frac{1}{2})}{\pi^{1/2}\Gamma(\alpha+1)}, \tag{11}$$

$$c_2(l)=\frac{\pi^{1/2}\Gamma(l+1)}{2^{l+1}\Gamma(l+\frac{3}{2})}. \tag{12}$$

The Regge-pole hypothesis also enables one to compute $R_l^+(\nu)$ for $\nu\gtrsim\frac{1}{4}t_0$, by calculating the contribution from Regge poles in the t and u channels to the partial-wave amplitude[10] $A_l^+(\nu)$

$$A_l^+(\nu)=\sum_I\frac{\beta_{0I}}{\nu}\left\{\frac{1}{2}\int_{-4\nu}^{0} dt' P_l\left(1+\frac{t'}{2\nu}\right)A^I(t', 4(\nu+1))\right.$$
$$\left.-\frac{\sin\pi l}{\pi}\int_{-\infty}^{-4\nu} dt' Q_l\left(-1-\frac{t'}{2\nu}\right)A^I(t', 4(\nu+1))\right\}, \tag{13}$$

and using the generalized unitarity condition Eq. (2). Here $\beta_{II'}$ is a crossing matrix,

$$\beta_{II'}=\begin{pmatrix}\frac{1}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{1}{2} & -\frac{5}{6}\\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6}\end{pmatrix}.$$

III. BOOTSTRAP OF THE TOP LEVEL TRAJECTORY

Let us confine our attention to the positive-signature partial wave in the neighborhood of $l=1$, since we are especially interested in the behavior of $\alpha(\nu)$ and $\beta(\nu)$ of the top-level trajectories at small momentum transfer.

The position of the Regge trajectory $\alpha(\nu)$ can be found by calculating a value $l=\alpha(\nu)$ for which

$$D_l^+(\nu)=0. \tag{14}$$

The residue function $\beta(\nu)$ of $A_l^+(\nu)$ at $l=\alpha(\nu)$ is then

$$\beta(\nu)=-\frac{N_{\alpha(\nu)}^+(\nu)}{\partial D_{\alpha(\nu)}^+(\nu)/\partial\nu}\nu^{\alpha(\nu)}\frac{d\alpha(\nu)}{d\nu}. \tag{15}$$

[19] G. F. Chew, Phys. Rev. **129**, 2363 (1963).

The reduced residue $\gamma(\nu)$, defined by

$$\gamma(\nu)=\frac{\beta(\nu)}{\nu^{\alpha(\nu)}}=-\frac{d\alpha(\nu)}{d\nu}\frac{N_{\alpha(\nu)}^+(\nu)}{\partial D_{\alpha(\nu)}^+(\nu)/\partial\nu}, \tag{16}$$

is related at $\nu=-1$ to the total cross section at infinity, while its first derivative

$$\frac{d\gamma(\nu)}{d\nu}=-\frac{d\alpha(\nu)}{d\nu}\frac{\partial}{\partial\nu}\left(\frac{N_{\alpha(\nu)}^+(\nu)}{\partial D_{\alpha(\nu)}^+(\nu)/\partial\nu}\right)_{l=\alpha(\nu)}$$
$$-\left(\frac{d\alpha(\nu)}{d\nu}\right)^2\frac{\partial}{\partial l}\left(\frac{N_l^+(\nu)}{\partial D_l^+(\nu)/\partial\nu}\right)_{l=\alpha(\nu)}$$
$$-\frac{d^2\alpha(\nu)}{d\nu^2}\frac{N_{\alpha(\nu)}^+(\nu)}{\partial D_{\alpha(\nu)}^+(\nu)/\partial\nu}, \tag{17}$$

at $\nu=-1$ is related to the diffraction width. We then impose the Chew-Frautschi saturation condition

$$\alpha(\nu=-1)=1, \tag{18}$$

and expand $\alpha(\nu)$ around $\nu=-1$ in a power series

$$\alpha(\nu)=1+\epsilon(\nu+1)+\epsilon'(\nu+1)^2+\cdots. \tag{19}$$

As far as the total cross section is concerned, a knowledge of ϵ, $N_1^+(\nu)$, and $\partial D_1^+(\nu)/\partial\nu$ is adequate. The latter two quantities are given in terms of $\gamma(\nu)$, $\alpha(\nu)$ and the ρ-meson parameters, and therefore provide a bootstrap cycle.

To obtain the value of $d\gamma(\nu)/d\nu$ at $\nu=-1$, a knowledge of

$$\frac{\partial}{\partial l}\left(\frac{N_l^+(\nu)}{\partial D_l^+(\nu)/\partial\nu}\right)_{l=\alpha(\nu)}$$

at $\nu=-1$ and ϵ' is required. Unfortunately, neither of these is calculable at present. Part of the difficulty lies in the fact that unlike $N_l^+(\nu)$ for a fixed l, $N_{\alpha(\nu)}^+(\nu)$ has no left-hand cut. Therefore, even though it is possible to calculate $N_1^+(\nu)$ by the usual approximation of replacing the left-hand cut by poles, it is not clear how to evaluate $N_l^+(\nu)$ along the trajectory $l=\alpha(\nu)$. We hope, however, that the second and third terms in Eq. (17) are not too large compared to the first term and that by retaining only the first term we can at least obtain the order of magnitude of $d\gamma(\nu)/d\nu$ at $\nu=-1$.

In what follows we shall neglect the second and third terms in Eq. (17) and replace $d\alpha(\nu)/d\nu$ at $\nu=-1$ by ϵ. The problem of investigating $\gamma(\nu)$ and $d\gamma(\nu)/d\nu$ then reduces to calculating $N_1^+(\nu)$ and $\partial D_1^+(\nu)/\partial\nu$. In order to calculate $N_1^+(\nu)$ and $\partial D_1^+(\nu)/\partial\nu$, we applied the Balázs approximation explained in Sec. II to the 1^+ unphysical state, by requiring the following two conditions:

Condition 1. The Chew-Frautschi saturation principle must hold, which is equivalent to putting

$$D_1^+(-1)=0. \tag{20}$$

Condition 2. The values of the amplitudes Eq. (4) should be the same as that given by Eq. (1) at the matching point ν_F. We shall take $\nu_0=\nu_F=-2$.[6] The sensitivity of the solution to the variation of ν_F will be discussed at the end of Sec. V. By requiring condition 1 together with Eq. (6) we obtain

$$\sum_{i=1}^{2} f_i I(\nu=-1, x_i^{-1})=1, \tag{21}$$

where

$$I(\nu,x_i^{-1})=I_E(t_0,\nu,x_i^{-1})+I_I(t_0,\nu,x_i^{-1}) \tag{22}$$

and

$$I_E(t_0,\nu,x_i^{-1}) = -\frac{1}{\pi}\int_0^{t_0/4} d\nu'\left(\frac{\nu'^3}{\nu'+1}\right)^{1/2}\frac{1}{(\nu'+2)(\nu'+x_i^{-1})(\nu'-\nu)}, \tag{23}$$

$$I_I(t_0,\nu,x_i^{-1}) = -\frac{1}{\pi}\int_{t_0/4}^{\infty} d\nu'\left(\frac{\nu'^3}{\nu'+1}\right)^{1/2}\frac{R_1^+(\nu')}{(\nu'+2)(\nu'+x_i^{-1})(\nu'-\nu)}. \tag{24}$$

Here the elastic approximation is made for $\nu<t_0/4$ and $R_1^+(\nu)$ for $\nu>\frac{1}{4}t_0$, is calculated by the method of Sec. II, to give

$$R_1^+(\nu)=\frac{4\pi}{\sigma_t}\frac{\nu(\epsilon\ln 2\nu)^2}{\nu\epsilon\ln 2\nu+\nu\epsilon\ln 2\nu\exp[-\nu\epsilon\ln 2\nu]+2\exp[-\nu\epsilon\ln 2\nu]-2}, \tag{25}$$

which is expressed in terms of the unknown slope ϵ of the t- and u-channel Pomeranchuk poles for given σ_t. Here the relations $\alpha(0)=1$ and

$$\beta(0)=-\sigma_t/8\pi^2, \tag{26}$$

which can be deduced with the help of the optical theorem, have been used.

Condition 2 gives us

$$\sum_{i=1}^{2} f_i\frac{1}{x_i^{-1}+\nu_F}=\frac{1}{\nu_F}[A_1^{+(L)}(\nu_F)+A_1^{+(H)}(\nu_F)], \tag{27}$$

where $A_1^{+(H)}(\nu_F)$ is expressed in terms of the Pomeranchuk-Regge trajectory in the s channel by Eqs. (10) and (16), and $A_1^{+(L)}(\nu_F)$ will be expressed in terms of the input parameters of the low-energy resonances. We then obtain

$$\sum_{i=1}^{2}\frac{f_i}{x_i^{-1}-2} = -\tfrac{1}{2}A_1^{+(L)}(-2)/\{1-(\tfrac{1}{2}t_0)^{-\epsilon}/\sum_{i=1}^{2} f_i[I(-2,x_i^{-1})]\}. \tag{28}$$

The unknown parameter $\epsilon=4(d\alpha/ds)(0)$ which is a slope of the Pomeranchuk trajectory in the s, t, and u channels can also be expressed by the following relation, obtained by Eq. (15) together with Eqs. (5) and (6) at $\nu=-1$,

$$\epsilon=\frac{\sigma_t}{8\pi^2}[1+\sum_{i=1}^{2} f_i J(-1,x_i^{-1})]\Big/\sum_{i=1}^{2}\frac{f_i}{x_i^{-1}-1}, \tag{29}$$

where

$$J(\nu,x_i^{-1})=(\partial/\partial\nu)I(\nu,x_i^{-1}). \tag{30}$$

Now $A_1^{+(L)}(\nu_F)$ can be obtained from Eq. (8) provided we know the low-energy resonance parameters. At $\nu=\nu_F$, we are within the t-channel Lehman ellipse, so it is appropriate to expand $\tilde{A}_t^0(t',4(\nu+1))$ into the partial waves of that channel. These partial waves are then approximated by one ρ and one f_0 exchange amplitude

$$A_1^{+(L)}(\nu_F)=A_{1\rho}^{+(L)}(\nu_F)+A_{1f_0}^{+(L)}(\nu_F).$$

$A_{1\rho}^{+(L)}(\nu_F)$ and $A_{1f_0}^{+(L)}(\nu_F)$ are then expressed in terms of the ρ and f_0 masses and widths.[6]

If σ_t and $A_1^{+(L)}(\nu_F)$ are given, we can calculate a set of three unknown parameters f_1, f_2, and ϵ for several values of the separation point by solving the three equations: Eq. (21) [with Eqs. (22), (23), (24), (25), Eq. (28), and Eq. (29) [with Eq. (30)]. We thus have a bootstrap cycle for $\alpha(s)$ and $\gamma(s)$ for small momentum transfer by calculating self-consistent values of f_1, f_2, and ϵ. Numerical solutions of these equations will be presented in Sec. V.

IV. WIDTH OF THE DIFFRACTION PEAK

In this section we shall express the width of the diffraction peak in terms of f_1 and f_2 which are calculable from the bootstrap mechanism using the N/D method as explained in Sec. III.

Using the amplitude[20] $A^I(s,t)$ the differential cross section is obtained as follows:

$$\frac{d\sigma}{ds}=16\pi\left|\frac{A^I(s,t)}{\frac{1}{2}t}\right|^2, \tag{31}$$

where at large t

$$A^I(s,t)=F(s)(\tfrac{1}{2}t)^{\alpha(s)}, \tag{32}$$

[20] See G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin and Company, Inc., New York, 1961).

$$F(s)=\pi[2\alpha(s)+1]c_1[\alpha(s)]\epsilon\frac{N_1^+(\nu)}{\partial D_1^+(\nu)/\partial\nu}\times\tfrac{1}{2}[-i+\cot\tfrac{1}{2}\pi\alpha(s)]. \quad (33)$$

We then obtain

$$d\sigma/ds=G(s)(\tfrac{1}{2}t)^{2(\alpha(s)-1)}, \quad (34)$$

where

$$G(s)=16\pi|F(s)|^2. \quad (35)$$

In order to evaluate the diffraction width $1/b$ one has only to take the logarithmic derivative of Eq. (34) at $s=0$

$$b=(d/ds)\ln(d\sigma/ds)|_{s=0}=a+\tfrac{1}{2}\epsilon\ln\tfrac{1}{2}t. \quad (36)$$

Here

$$a=(d/ds)\ln G(s)|_{s=0}\cong-\frac{1}{2}\left[\sum_{i=1}^{2}\frac{f_i}{(x_i^{-1}-1)^2}\right]\Big/\sum_{i=1}^{2}\frac{f_i}{x_i^{-1}-1}, \quad (37)$$

since main ν dependence comes from $N_1^+(\nu)$.

It should be noted from Eq. (36) that it is appropriate to define the asymptotic energy as those values of t for which

$$a\ll\tfrac{1}{2}\epsilon\ln\tfrac{1}{2}t. \quad (38)$$

V. NUMERICAL RESULTS AND DISCUSSION

In this section, we shall present the numerical solution of the bootstrap equations for $\alpha(s)$ and $\gamma(s)$ of the Pomeranchuk trajectory, as was explained in Sec. III. We shall also briefly compare these results with high-energy experiments.

Taking the experimental mass and half-width of the ρ meson as 750 and 50 MeV, respectively,[21] we obtain

$$A_{1\rho}^{+(L)}(\nu_F)=-0.0366. \quad (39)$$

We shall approximate $A_1^{+(L)}(\nu_F)\approx A_{1\rho}^{+(L)}(\nu_F)$, since when the experimental mass and half-width of the f_0 meson are taken as 1250 and 37.5 MeV, respectively,[21] $A_{1f_0}^{+(L)}(\nu_F)$[22] is less than $\frac{1}{10}A_{1\rho}^{+(L)}(\nu_F)$.

TABLE I. The solution f_1, f_2 and ϵ for the bootstrap equations of $\alpha(s)$ and $\gamma(s)$ at three separation points; $t_0=80$, 120, and 160.

	t_0	f_1	f_2	μ unit	ϵ CF unit $(=1/50\times4)$
Sol. 1	80	−1.510	20.49	0.073	0.91
Sol. 2	120	−2.017	26.46	0.063	0.79
Sol. 3	160	−2.593	33.35	0.053	0.67

[21] W. H. Barkas and A. H. Rosenfeld, Lawrence Radiation Laboratory Report, UCRL-8030 April 1963, revised ed. (unpublished).

[22] We see that the force due to the exchange of f^0 is small compared with that due to ρ since the crossing matrix element is small and the mass is large. Therefore it should be noted that the bootstrap mechanism to produce the f^0 particle only by the exchange of f^0 is not sufficient but ρ exchange must always be included.

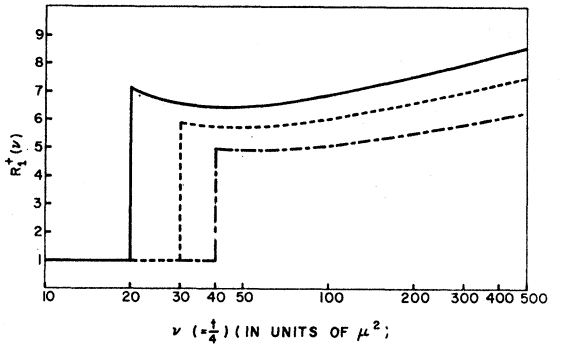

FIG. 1. The inelasticity factor $R_1^+(\nu)$ for each solution at $t_0=80$, 120, and 160. The function rises infinitely logarithmically. ——— line for $t_0=80$; – – – – line for $t_0=120$; —— – —— line for $t_0=160$.

Our procedure was to solve Eqs. (21), (28), and (29), given σ_t and $A_1^{+(L)}(\nu_F)$ as above. We iterated the solution until the input values approached to the output values within an accuracy 5%.

We have obtained solutions for f_1, f_2, and ϵ at several values of the separation point t_0 given in Table I. In choosing t_0, we assume that the Regge behavior sets in immediately above the resonance region. Since f_0 is the highest known resonance in the $\pi\pi$ system we thus take $t_0\approx80$. We also obtained solutions at $t_0\approx120$ and 160. The solutions are not sensitive to the variations of these separation points.

For each solution in Table I the inelasticity factor $R_1^+(\nu)$ is also plotted for $t_0=80$, 120, and 160 in Fig. 1. In order to compare the calculated diffraction width with the value which follows from experiment and factorization at high energy, we shall tabulate the width for each of the solutions 1, 2, and 3 in Table II. The experimental diffraction width of π-π scattering would be deduced with the help of the factorization theorem. Assuming the Pomeranchuk pole to be dominant, we have

$$\{F_{\pi N}(s)\}^2=F_{\pi\pi}(s)F_{pp}(s). \quad (40)$$

Defining a_i to be $2(d/ds)\ln F_i(s)|_{s=0}$, Eq. (40) leads to

$$a_{\pi\pi}=2a_{\pi p}-a_{pp}\approx10\ \text{BeV}^{-2}, \quad (41)$$

according to the Brookhaven data.[1] These values of $a_{\pi\pi}$ should be compared with the theoretical value of a in Table II. The residue function $\gamma(\nu)$ is also plotted for each of the solutions 1, 2, and 3 in Fig. 2.

Although the calculated slope tabulated in Table I is larger than the slope due to Foley *et al.*,[1] we have roughly reproduced the experimental width of the diffraction peak from self-consistent calculations of the Pomeranchuk trajectory. We must mention, however, that we have overestimated the inelasticity factor $R_1^+(\nu)$ in the intermediate energy region, since if the amplitude is approximated only the by Pomeranchuk

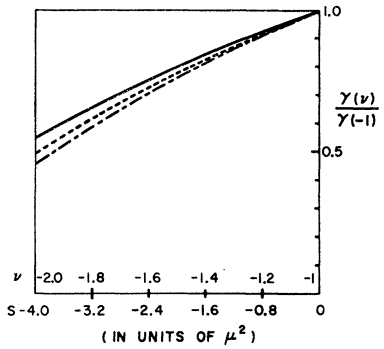

FIG. 2. The residue function $\gamma(\nu)/\gamma(-1)$ for each of the solutions 1, 2, and 3 in Table II. ——— line for $t_0=80$; - - - line for $t_0=120$; ——— line for $t_0=160$.

trajectory in the t and u channels it has only an imaginary part. The inclusion of the nearby singularities gives a fairly large real part which reduces the $R_1^+(\nu)$ in the intermediate energy region. Then, as is understood from the bootstrap equations (21) [with (22), (23), and (24)], the magnitude of f_1 and f_2 will become large, thus reducing the slope ϵ and enlarging the width a slowly. To give a rough idea, we shall show that the extreme case $R_1^+(\nu)=1$ for all $\nu>0$ leads to a solution $f_1=-7.31$, $f_2=86.0$, which gives us $\epsilon=0.0262=0.33$ CF unit [CF unit$=(4/50)(1/\mu^2)$] and $a=0.316$ $(=16.2$ BeV$^{-2})$. Therefore we have a lower limit for ϵ and an upper limit for a.

In the above discussion we fixed the positions of the effective poles on the left-hand cut *a priori* at $x_1^{-1}=6.25$ and $x_2^{-1}=50$, so as to make a good approximation to the kernel in the region of interest. The results are not expected to depend much upon this choice, as already emphasized by Balázs.[6] We have verified that this is indeed the case. To check this, we changed only the position of the nearby pole, since this is more important to the results. With $x_1^{-1}=10$ and $x_2^{-1}=50$ (in the case of $t_0=80$) we have $\epsilon=0.80$ CF and $a=0.143=7.31$ BeV^{-2}. Thus the results are not very sensitive to changes in the values of x_1^{-1} and x_2^{-1} even if the changes make the kernel approximation marginal.

In this calculation the matching point ν_F was also fixed *a priori* at $\nu_F=-2$, as discussed in Ref. 6. The parallel calculation has also been carried out in the case of $\nu_0=\nu_F=-3$ to see the sensitivity of the solution to the variation of ν_F. Taking the same high- and low-energy parameters and $t_0=120$, we iterated the solution until the input values approached the output values within 5%. We obtain $f_1=-1.567$, $f_2=20.88$, which gives us $\epsilon=0.0406=0.51$ CF unit and $a=0.214$ $(=10.9$ BeV$^{-2})$. These values should be compared with the solution at $t_0=120$ in the case $\nu_0=\nu_F=-2$, which gives us $\epsilon=0.063=0.79$ CF unit and $a=0.199$ $(=10.2$ BeV$^{-2})$ as tabulated in Tables I and II. The value of ϵ is slightly altered, but the value of a remained essentially the same.

TABLE II. Width of the calculated diffraction peak for each solution 1, 2, and 3 at squares of center-of-mass energy $t=80$ and 160. a^{-1}=energy independent part of the diffraction width. b^{-1}=total diffraction width.

		a		b	
	t	μ unit	BeV^{-2} unit	μ unit	BeV^{-2} unit
Sol. 1 ($t_0=80$)	80	0.171	8.75	0.305	15.6
	160	0.171	8.75	0.330	16.9
Sol. 2 ($t_0=120$)	80	0.199	10.2	0.315	16.1
	160	0.199	10.2	0.337	17.2
Sol. 3 ($t_0=160$)	80	0.215	11.0	0.314	16.0
	160	0.215	11.0	0.333	17.0

VI. CONCLUDING REMARKS

In order to understand the behavior of high-energy diffraction peak, it is not enough to consider the variation in s of $\alpha(s)$ alone. The self-consistent calculation of this paper makes it plausible that the dependence of the residue $\gamma(s)$ contributes significantly to this remarkable forward peaking of the diffraction scattering. The value of the diffraction width calculated by this method seems to be in qualitative agreement with experiment. It must be kept in mind, of course, that we have neglected the contribution of the second and third terms in Eq. (17). Our calculation is based on the hope that such an approximation will not change the order of magnitude of the width. Although the value for the slope was large compared with the Brookhaven data,[1] this fact could be traced to the especially simple approximations employed here. In particular the inelasticity for intermediate energies has been overestimated through the one-Pomeranchuk-pole approximation. More reliable values of the slope and width could therefore be obtained if other singularities near the Pomeranchuk trajectory were included.

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