Relation among the $\Xi_{1/2}^*$, Y_1^*, $N_{3/2}^*$ Decay Widths in Broken Unitary Symmetry

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The decay amplitudes of the decuplet particles $\Xi_{1/2}^*$ (1530), Y_1^* (1385), and $N_{3/2}^*$ (1238) into a baryon and a pion have been calculated assuming that the interaction giving rise to such decays consists of a unitary symmetric part plus all the terms violating unitary symmetry which transform as the T_3^3 component of a unitary tensor. It is shown that contrary to what one would expect at first sight, only three undetermined coupling constants appear in the calculation of the four amplitudes G_1, G_2, G_3, G_4 for the decays (1) $\Xi_{1/2}^* \to \Xi+\pi$, (2) $Y_1^* \to \Sigma+\pi$, (3) $Y_1^* \to \Lambda+\pi$, (4) $N_{3/2}^* \to N+\pi$. One can therefore predict a relation among the above four amplitudes, which turns out to be:

$$G_4 = -(3/\sqrt{2})G_3 - (\sqrt{\tfrac{3}{2}})G_2 + (\sqrt{6})G_1.$$

With the present values of the widths this relation is satisfied remarkably well.

1. INTRODUCTION

THE decay amplitudes of the decuplet particles into the baryonic and pseudoscalar octets have been calculated by Lipkin[1] and by Glashow and Rosenfeld[2] using an SU_3 invariant interaction. Although the errors in the experimental decay widths are still large, it cannot be stated that the agreement is satisfactory.[3] In this paper we have calculated the same amplitudes, assuming that to the pure unitary symmetric interaction all the interaction terms violating unitary symmetry are added which transform as the T_3^3 component of a unitary tensor. The interesting point is that, contrary to what one would expect at first sight, only three undetermined coupling constants appear in the calculation of the four widths available experimentally:

$$\Xi^* \to \Xi+\pi, \quad Y_1^* \to \Sigma+\pi, \quad Y_1^* \to \Lambda+\pi, \quad N^* \to N+\pi;$$

one can therefore predict a relation among the above four widths, which appears to be satisfied remarkably well.

2. DERIVATION OF THE RELATION AMONG THE AMPLITUDES

We now show that the above four widths can be expressed in terms of only three coupling constants.

The SU_3 invariant interaction has the structure

$$D\otimes B\otimes P = 10\otimes 8\otimes 8,$$

where D, B, P are, respectively, the decuplet, the baryonic octet, and the pseudoscalar octet particles. Only one invariant term appears in the reduction of this product: call it s and call $g_s(p_1^2,p_2^2,p_3^2)$ the corresponding vertex function (here p_1, p_2, p_3 are the four momenta of the three legs of the vertex). The interaction terms transforming as T_3^3 can be obtained by forming the invariants from the reduction of the product

$$D\otimes B\otimes P\otimes T = 10\otimes 8\otimes 8\otimes 8,$$

where the T octet is finally specialized to the form T_3^3 and only the terms multiplying T_3^3 are taken.

Four independent invariants arise in the reduction of the product $D\otimes B\otimes P\otimes T$. In fact we have

$$P\otimes T = (P\otimes T)_{27} + (P\otimes T)_{10} + (P\otimes T)_{10}^* + (P\otimes T)_{8_1} + (P\otimes T)_{8_2} + (P\otimes T)_1,$$

where the suffix indicates the dimension of the representation. On further multiplication with $D\otimes B$, only the representations 27, 10^*, 8_1, and 8_2 can give rise to one (and only one) invariant so that we obtain four invariants:

$$a = D\otimes B\otimes (P\otimes T)_{27},$$
$$b = D\otimes B\otimes (P\otimes T)_{10^*},$$
$$c = D\otimes B\otimes (P\otimes T)_{8_1},$$
$$d = D\otimes B\otimes (P\otimes T)_{8_2}.$$

It follows that, in general, we have to do with five vertex functions, corresponding to the five interactions s, a, b, c, d.

The interesting point now is that the terms c and d cannot contribute to the pionic decay processes so that we remain with three vertex functions $g_s(p_1^2,p_2^2,p_3^2)$, $g_a(p_1^2,p_2^2,p_3^2)$, and $g_b(p_1^2,p_2^2,p_3^2)$. Indeed a typical element of the tensor $(P\otimes T)_8$ can be written as:

$$(P\otimes T)_{8\nu}{}^{\mu} = P_\lambda{}^\mu T_\nu{}^\lambda - \tfrac{1}{3}\delta_\nu{}^\mu P_\sigma{}^\lambda T_\lambda{}^\sigma.$$

Now only the P elements which multiply T_3^3 have to be considered; these are $P_3^\mu(\mu=1,2,3)$ and are all orthogonal to the pionic states.

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[1] H. J. Lipkin, Argonne National Laboratory 1963 (unpublished report).

[2] S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

[3] The ratios between the widths which one obtains with a purely unitary symmetric interaction are the following: $\Gamma(N^*)$:$\Gamma(Y_1^*\to\Lambda)$:$\Gamma(Y_1^*\to\Sigma)$:$\Gamma(\Xi^*)$=100:41.2:6.5:17.8. These numbers have been calculated with a value of the parameter X [compare the form of Eq. (10)] equal to that of Glashow and Rosenfeld (Ref. 2); they are, however, somewhat different from the values of Glashow and Rosenfeld because these authors have omitted the factor M_B which appears in Eq. (7) [compare also the Ref. 5]. In any case these values are *not* in agreement with the experimental data. In particular the ratio $Y_1^*(\Sigma)/Y_1^*(\Lambda)$ has been studied very accurately (see Ref. 7) and is certainly less than 1/25, a number very different from that obtained above (6.5/41.2).

Having remained only with the vertex part $g_s s + g_a a + g_b b$, we write below explicitly, s, a, and b omitting the space-time properties:

$$s = B_\alpha{}^\sigma D^{[\alpha\beta\gamma]} P_\beta{}^\lambda \epsilon_{\sigma\lambda\gamma}\,, \tag{1}$$

$$a = B_\lambda{}^\tau D^{[\alpha\beta\gamma]} \epsilon_{\alpha\tau\sigma} (P_\beta{}^\lambda T_\gamma{}^\sigma + P_\beta{}^\sigma T_\gamma{}^\lambda)\,, \tag{2}$$

$$b = B_\gamma{}^\sigma D^{[\alpha\beta\gamma]} \mathbf{S}_{\alpha\beta\sigma} P_\beta{}^\lambda T_\alpha{}^\nu \epsilon_{\lambda\nu\sigma}\,. \tag{3}$$

In the interaction a we have omitted a term which does not contribute to the pionic processes. In the above notation, $\mathbf{S}_{u,v,z}$ means symmetrization over the indices u,v,z in the ensuing expression. $D^{[\alpha\beta\gamma]}$ stands for $\frac{1}{6}\mathbf{S}_{\alpha\beta\gamma} D^{\alpha\beta\gamma}$.

We next extract the interactions giving rise to the decays: (1) $\Xi^{*-} \to \Xi^- + \pi^0$, (2) $Y_1^{*-} \to \Sigma^- + \pi^0$, (3) $Y_1^{*-} \to \Lambda^0 + \pi^-$, (4) $N^{*-} \to n + \pi^-$. The identification of the particles participating in such decays, with the symbols previously introduced, is the familiar one[4]:

D particles: $\Xi^{*-} = (1/2\sqrt{3}) D^{[233]}$,
$Y_1^{*-} = (1/2\sqrt{3}) D^{[223]}$, $N^{*-} = \frac{1}{6} D^{[222]}$;

B particles: $\bar{\Xi}^- = \bar{B}_3{}^1$, $\bar{\Sigma}^- = \bar{B}_2{}^1$, $\bar{\Lambda}^0 = -(\frac{2}{3})^{1/2} \bar{B}_3{}^3$;

$\bar{\Sigma}^0/\sqrt{2} + \bar{\Lambda}^0/\sqrt{6} = \bar{B}_1{}^1$, $-\bar{\Sigma}^0/\sqrt{2} + \bar{\Lambda}^0/\sqrt{6} = \bar{B}_2{}^2$, $\bar{n} = \bar{B}_2{}^3$;

P particles: $\pi^+ = P_2{}^1$, $\pi^0/\sqrt{2} + \eta_0/\sqrt{6} = P_1{}^1$,
$-\pi^0/\sqrt{2} + \eta_0/\sqrt{6} = P_2{}^2$.

Substituting these into Eqs. (1), (2), and (3) we get that the decays (1), (2), (3), and (4) above are determined by the following amplitudes G_1, G_2, G_3, G_4:

$$\begin{aligned}
&\Xi^{*-} \to \Xi^- + \pi^0\colon\ G_1 = -1/\sqrt{6}(g_{s1} + g_{a1} + 2g_{b1})\,,\\
&Y_1^{*-} \to \Sigma^- + \pi^0\colon\ G_2 = -1/\sqrt{6}(g_{s2} - g_{a2} + 2g_{b2})\,,\\
&Y_1^{*-} \to \Lambda^0 + \pi^-\colon\ G_3 = -1/\sqrt{2}(g_{s3} + g_{a3})\,,\\
&N^{*-} \to n + \pi^-\colon\ G_4 = g_{s4} - g_{b4}\,,
\end{aligned} \tag{4}$$

where g_{si}, g_{ai}, and g_{bi} are the values of the above vertex functions calculated for p_1^2, p_2^2, p_3^2 corresponding to the masses of the particles of the vertex.

To be as simple as possible, let us first neglect the dependence of g_{si}, g_{ai}, g_{bi} on the vertex index i; that is we put:

$$g_{si} = g_s,\ g_{ai} = g_a,\ g_{bi} = g_b\ (i = 1, \cdots, 4)\,. \tag{5}$$

We shall show in a moment that this simplification has practically no consequence on our results, this being due to the relatively small mass difference in the masses of the various vertices. The four amplitudes G_i are then expressed by the Eqs. (4) only in terms of the three parameters g_s, g_a, and g_b; by eliminating them we obtain

$$G_4 = -(3/\sqrt{2})G_3 - (\sqrt{\tfrac{3}{2}})G_2 + (\sqrt{6})G_1\,. \tag{6}$$

We now transform this relation in a relation among the widths. The ith decay width can be written:

$$\Gamma_i = G_i^2 q_i^3 (M_{Bi}/M_{Di}) \equiv G_i^2 \rho_i\,. \tag{7}$$

Here G_i is the amplitude defined above, q_i is the three momentum of one of the decay products in the rest system of the decaying particle, M_{D_i} is the mass of the decaying decuplet particle, and M_{B_i} is the mass of the produced baryon [in writing Eq. (7), the kinetic energy of B_i has been neglected in comparison to its mass[5]]; ρ_i is defined by the last equation. Also, in the following we write

$$\Gamma_i' = \Gamma_i/\rho_i\,. \tag{8}$$

Obviously the G_i's are determined from the Γ_i' only up to a phase factor, which, in view of the reality[6] of the G_i turns out to be just a sign. In practice only two signs are important because Γ_2' is practically zero; when these signs are chosen in the only way which fits the data, the Eq. (6) can be transcribed in terms of the widths explicitly as follows:

$$\begin{matrix}
[\Gamma'(N^* \to n+\pi^-)]^{1/2} & = & 3/\sqrt{2}[\Gamma'(Y_1^* \to \Lambda+\pi^-)]^{1/2} & \mp & \sqrt{\tfrac{3}{2}}[\Gamma'(Y_1^* \to \Sigma^-+\pi^0)]^{1/2} & - & \sqrt{6}[\Gamma'(\Xi^{*-} \to \Xi^-+\pi^0)]^{1/2} \\
10 & = & 17 & \mp & \text{`0'} & - & 6.95
\end{matrix}\,. \tag{9}$$

The experimental values, in arbitrary units, are given in the equation under (9); they have been calculated using the data of Table I.[7–9] The agreement is amazingly good; we cannot state, at this stage, whether it will persist when the large experimental errors which presently affect the widths will decrease.

TABLE I. Summary of the experimental data used in the present article. The symbols are defined in the text.

Decays	M_D[a] (MeV)	M_B[a] (MeV)	q[a] (MeV)	Γ^{total}: (MeV)	Γ_i (MeV)
1 $\Xi^{*-} \to \Xi^- + \pi^0$	1530	1321	148	7[b]	2.33
2 $Y_1^{*-} \to \Sigma^- + \pi^0$	1385	1197	119	$\leqslant 2$[c]	$\leqslant 1$
3 $Y_1^{*-} \to \Lambda^0 + \pi^-$	1385	1115	210	50[a]	50
4 $N^{*-} \to n + \pi^-$	1238	940	233	100[a]	100

[a] See Ref. 8.
[b] See Ref. 9.
[c] See Ref. 7.

[4] See, for instance, J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

[5] The correct formula would have $M_B + E_B$ instead of M_B [compare e.g., P. G. Federbush, M. T. Grisaru, and M. Tausner, Ann. Phys. (N. Y.) **18**, 23 (1962)]. The neglecting of the kinetic energy of the baryon implies a maximum error, in our case, of $\sim$1.5%. Note that the factor $(M_B + E_B)/M_D$ is often given in the literature as $[(M_D + M_B)^2 - m_\pi^2]/M_D^2$; they are the same thing.

[6] The fact that the G's are real—or better, pure imaginary—can be easily shown using unitarity and time reversal in the approximation in which it is possible to define asymptotic fields for the D particles; this can be seen either by standard methods or through the use of helicity amplitudes as in L. Durand III, P. C. De Celles, and R. B. Marr, Phys. Rev. **126**, 1882 (1962).

[7] P. L. Bastien, M. Ferro-Luzzi, and A. M. Rosenfeld, Phys. Rev. Letters **6**, 702 (1961).

[8] A. H. Rosenfeld, UCRL-10897, 1963 (unpublished).

[9] P. E. Schlein, D. D. Carmony, G. M. Pjerrou, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **11**, 167 (1963).

3. SOME CORRECTIONS TO EQ. (9)

We now discuss the statement made previously [Eq. (5)] concerning the quasi-independence of the $g_{\alpha i}$ from the vertex index i. To clarify this point let us assume for the vertex function a reasonable expression like:

$$g_\alpha(p_1^2,p_2^2,p_3^2)=g_\alpha X/(q^2+X^2)^{1/2} \quad \alpha=s,a,b\,, \qquad (10)$$

where the g_α's and X's are constants; q, the three momentum of the pion in the rest system of the resonance, should be thought expressed in terms of the masses of the legs of the vertex and therefore in terms of p_1^2, p_2^2, and p_3^2. We determine X to be:

$$X=2.5\,m_\pi\,, \qquad (11)$$

a value which results from fitting the shape of the N^* resonance.[10]

We are aware of the fact that X might be different for the s, a, and b interactions, but at this stage it looks unreasonable to enter in such details: even if the X's for the interactions s, a, and b were different, but with values of the same order or larger than that given in (11), our conclusions would not be appreciably changed.

From Eq. (10) we obtain for the coupling constants, which appear in the Eq. (4), the expression:

$$g_{\alpha i}=g_\alpha X/(q_i^2+X^2)^{1/2} \quad \begin{matrix} i=1,\cdots,4 \\ \alpha=s,a,b. \end{matrix} \qquad (12)$$

[10] For this, introduce the momentum-dependent width $\Gamma_i(q)$ for channel i, defined by $\Gamma_i(q)=\lambda q^3/(q^2+X^2)$. We now write the well-known Chew-Low equation

$$\tan\delta_{33}(q)=\tfrac{1}{2}\frac{\Gamma_4(q)}{\omega-\omega_R}\frac{\omega_R}{\omega}.$$

By fitting the experimental phases around the resonance, the value (11) for X is obtained; note that this is the same as that used in Ref. 2.

In the 10% approximation in which the changes of the denominators in (12) for the various vertices can be neglected, the $g_{\alpha i}$ are independent of the index i and our previous conclusion is justified.

A better approximation is obtained on using the Eqs. (4) with the Eq. (12) for the $g_{\alpha i}$. Because in each of the Eqs. (4) a term $X/(q_i^2+X^2)^{1/2}$ can be factorized, nothing changes in our final Eq. (9) except that Γ_i' is now defined as

$$\Gamma_i'=(\Gamma_i/\rho_i)[(q_i^2+X^2)/X^2] \qquad (13)$$

instead of by Eq. (8).

With the new values for Γ_i', our final Eq. (9) is satisfied practically in the same way as previously (right-hand side: 10; left-hand side: 9.8).

4. CONCLUSIONS

The Eqs. (4) and (12) allow us to determine the ratios of the coupling constants g_s, g_a, g_b. They are, in the approximation $X=\infty$,

$$g_s:g_a:g_b=1:0.45:-0.28$$

and they change by only 2% if $X=2.5M_\pi$. The unitary invariant interaction is 2 or 3 times larger than the interaction transforming as T_3^3, a situation in a sense similar to that which holds for the baryon-octet mass formula where the interaction transforming as T_3^3 and the invariant one have couplings in the ratio $\frac{1}{4}$. The reason why, in spite of this apparently large expansion parameter, things go as if a perturbation expansion limited to first order in T_3^3 were valid is not very clear for the masses and even less so for our case. But it is remarkable to see how well this prescription works; and it will be of course interesting to check whether the agreement persists when our experimental knowledge of the widths will be affected by errors smaller than the present ones.