As was shown in Sec. 2B, the equality of $M_{f\mu\nu}$ and $M_{y\mu\nu}$ requires that the condition $Z_3=0$ on the Yukawa theory be satisfied.[47]

[47] We note that the identification of G_0' and G_0 is only justified when G_0 is infinite and $Z_3=0$. Moreover, identification of μ_0' with μ_0 is not justified at all since Birula has used $\mu_0'=0$ whereas μ_0 is strictly undetermined, as mentioned above after Eq. (4.43).

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Higher Symmetries for the Vector Mesons*

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To explain the existence of nine vector mesons and the degeneracy of the unmixed masses of the ϕ and ω, we postulate that the vector mesons possess a higher symmetry than SU_3. We discuss various properties of the adjoint representation of SU_n and propose three simple models for the symmetry-breaking interaction. We specialize the results to SU_4, and find good agreement between mass formulas and experiment. We introduce a new quantum number, "hyperstrangeness," and six hyperstrange vector mesons with fractional hypercharge but integral electric charge. These particles, which form an isotopic doublet and an isotopic singlet (together with their antiparticles) are approximately degenerate with the ρ and K^*, respectively, and are stable with respect to the strong and electromagnetic interactions. We also discuss the production and decay of these particles.

I. INTRODUCTION

ONE of the interesting problems in the assignment of strongly interacting particles to representations of SU_3 is the existence of at least nine vector mesons ρ, K^*, $\bar{K}^*$, ω, and ϕ, which seem to belong to an octet and a singlet in the limit of exact unitary symmetry. The symmetry-breaking interaction complicates the situation by allowing mixing between the singlet (which we shall call ϕ_0) and the neutral[1] member of the octet (ω_0). The mass operator (or more generally the inverse propagator) is then no longer diagonal in the (ω_0,ϕ_0) representation and the physically observed particles (ω,ϕ) are those linear combinations of ϕ_0 and ω_0 which diagonalize the mass operator. In this scheme there is no *a priori* relationship between the masses of the singlet and the octet. These masses can be calculated from the experimental data in a given model, however, and the remarkable result is obtained that they are approximately equal.[2,3]

It is of course possible that this equality is merely coincidental, or that it has some deep significance that can only be understood by a detailed dynamical calculation (for example one of the many triplet models[4]). The purpose of this paper is to explore the possibility that this may be explained on a purely group theoretical level by postulating some higher symmetry for the vector mesons.

As a first attempt we might look for a group possessing a nine-dimensional self-conjugate representation with the correct isotopic spin and hypercharge content. There is no simple Lie group with these properties, and it appears that it is not even possible to use a nonsimple group without some specific dynamical hypotheses. We are therefore led to consider the possibility that there are in fact more than nine vector mesons and that the observed nine form part of a representation of some larger group.

It has been shown that the usual interpretation of vector mesons as gauge particles requires us to assign them to the adjoint representation of the underlying symmetry group.[5] A dynamical bootstrap model leads to the same result.[6] We will restrict ourselves to simple groups and it follows that the rank (r) of the group must be equal to the number of neutral vector mesons,[7]

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[1] We use "neutral" in the extended sense of having all additive quantum numbers zero.

[2] S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963); S. Okubo, Phys. Letters **5**, 165 (1963); J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

[3] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).

[4] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN (to be published); F. Gursey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964); J. Schwinger, Phys. Rev. Letters **12**, 237 (1964).

[5] S. L. Glashow and M. Gell-Mann, Ann. Phys. (N. Y.) **15**, 437 (1961); V. I. Ogievetskij and I. V. Polubarinov, *ibid.* **25**, 358 (1963).

[6] R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963).

[7] A. Salam, *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), p. 173.

and hence is at least three. Correspondingly, there are r additive quantum numbers—the third component of isotopic spin I_0, the hypercharge, which we will denote by $Y^{(2)}$, and a series of new quantum numbers $Y^{(3)}\cdots Y^{(r)}$ which we shall call hyperstrangeness. The simple groups of rank three are A_3 (or SU_4) with 15 parameters, B_3 (or O_7) with 21 parameters, and C_3 (or Sp_6) with 21 parameters. All of these contain SU_3 as a subgroup and all require the introduction of several hyperstrange vector mesons. The most economical choice, requiring only six additional particles, is SU_4.

This group has been considered by Tarjanne and Teplitz,[8] who assume that hyperstrangeness is conserved in reactions involving vector mesons only and not in reactions involving other particles. They then identify the 730-MeV $K\pi$ resonances as vector mesons, and include them with the other nine. We prefer to take a different point of view—to assume that hyperstrangeness is rigorously conserved in all strong interactions, and to use this conservation law to explain why the extra hypothesized vector mesons have not been observed.

In view of the rapidly changing experimental situation, we shall consider the possibility of the existence of further neutral vector mesons and hence of groups of higher rank. Such a possibility has also been considered by Hagen and MacFarlane[9] from a different point of view. It is therefore worthwhile to consider the problem in somewhat greater generality, although we shall find that SU_4 provides an adequate solution to the $\omega_0-\phi_0$ degeneracy. It turns out, as well, that the adjoint representation of SU_n may be handled quite simply[10] and the results are perhaps clearer than those obtained from a numerical computation for given n.

Our main concern in this paper will be with mass formulas. The most general mass formula derived under similar assumptions to those used by Gell-Mann and Okubo[11] involves $n-1$ arbitrary parameters for SU_n, and we shall propose various models to reduce this number.

II. THE VECTOR MESON SYSTEM

Coleman and Schnitzer[3] have pointed out several alternative approximations for the inverse propagator for a problem such as $\omega-\phi$ mixing. In particular, they distinguish between "particle mixing" where the inverse propagator is written

$$\mathbf{D}^{-1}(k^2)=k^2-\mathbf{M}_0-\boldsymbol{\delta}\mathbf{M} \tag{1}$$

and "vector mixing," where

$$\mathbf{D}^{-1}(k^2)=(1+\boldsymbol{\delta})k^2-\mathbf{M}_0. \tag{2}$$

The matrices $\boldsymbol{\delta}\mathbf{M}$ and $\boldsymbol{\delta}$ are real and symmetric, and $\mathbf{M}_0$ is real and diagonal. These authors point out that (2) is a more suitable approximation when vector particles are involved, and we shall use it in the following.

In exact analogy with the scheme usually used for SU_3[11] we impose the following conditions on the mass splitting operator:

(i) It commutes with hypercharge and hyperstrangeness.

(ii) It respects isotopic spin symmetry.

(iii) It transforms like a member of the adjoint representation of SU_n.

The most general form of the mass splitting matrix ($\boldsymbol{\delta}\mathbf{M}$ or $\boldsymbol{\delta}$) then is[12]

$$\sum_{l=2}^{n-1}(a_lY^{(l)}+b_lZ^{(l)}), \tag{3}$$

where the operators $Y^{(l)}$ and $Z^{(l)}$ are defined in Appendix I and the a's and b's are arbitrary constants. Since we are dealing with a system which is its own charge conjugate the a's are zero. In our model, where all the vector mesons are assigned to a single representation, $\mathbf{M}_0$ (the unbroken mass operator) is a multiple of the unit matrix and hence the mass formula involves $n-1$ parameters.

The physical particle masses are the zeroes of $D^{-1}(k^2)$. Some simple manipulations with matrices then show that (with $\mathbf{M}_0$ as above) if one uses the same form for the symmetry breaking in the two cases, any linear relationship between the masses[13] obtained from particle mixing is also true in the case of vector mixing if we merely replace the masses by their reciprocals, and that the same unitary transformation diagonalizes Eqs. (1) and (2).

We should now like to suggest various possible extra restrictions on the mass operator which will reduce the number of arbitrary parameters in (3). Though all these models will be discussed for all SU_n we shall be most interested in the application to SU_4. The content of the adjoint representation of SU_4 is given in Table I, and the weight diagram is shown in Fig. 1. For this

TABLE I. Content of the adjoint representation of SU_4.

Particle	I	$Y^{(2)}$	$Y^{(3)}$	Q
ρ	1	0	0	$0, \pm1$
K^*	$\frac{1}{2}$	1	0	0, 1
ω	0	0	0	0
ϕ	0	0	0	0
L	$\frac{1}{2}$	$\frac{1}{3}$	1	0, 1
l	0	$-\frac{2}{3}$	1	0

[8] P. Tarjanne and V. Teplitz, Phys. Rev. Letters **11**, 447 (1963).
[9] C. R. Hagen and A. J. MacFarlane, Phys. Rev. **135**, B432 (1964).
[10] D. Neville, Phys. Rev. **132**, 844 (1963) has noted this in a different connection.
[11] M. Gell-Mann, CSL Report 20, 1961 (unpublished); S. Okubo, Progr. of Theor. Phys. (Kyoto) **27**, 949 (1962).
[12] J. Ginibre, J. Math. Phys. **4**, 720 (1963).
[13] Since this is a calculation involving bosons we use the mass squared, as has already been anticipated by our choice of propagator.

group, the only particles which mix are the ω_0 and ϕ_0, and the linear combinations which diagonalize the propagator are

$$\begin{aligned}\phi&=(\sin\theta)\phi_0-(\cos\theta)\omega_0\,,\\ \omega&=(\cos\theta)\phi_0+(\sin\theta)\omega_0\,.\end{aligned}\tag{4}$$

Kim, Oneda, and Pati[14] have used the observed branching ratios for ϕ and ω decays to estimate the mixing angle, and find two solutions $\theta\approx 23°$ or $17°$. This disagrees with Sakurai's estimate based on the application of SU_3 to Eq. (1).

III. THE "$Z^{(l-1)}$ MODEL"

In the octet model, the symmetry breaking interaction is taken to transform like the operator $Z^{(2)}$ of Appendix I. A possible assumption is that this remains the symmetry-breaking interaction in SU_n. In fact, we can consider a model where the symmetry-breaking interaction transforms like $Z^{(l-1)}$ leaving invariance under a group isomorphic to $U_{n-l}\otimes U_{l-1}$. It is easy to construct model Lagrangians that have this property. In this case the mass operator (for particle mixing) may be written

$$\mathbf{M}=\mathbf{M}_0+b\mathbf{Z}^{(l-1)}\,.\tag{5}$$

The matrix elements of $\mathbf{Z}^{(l-1)}$ are given by Eq. (I.15) in Appendix I, and on diagonalizing we find $(l-2)$ neutral mesons with mass M_0+2bn/l which belong to the adjoint representation of the SU_{l-1} subgroup, $(n-l-1)$ neutral mesons with mass M_0, and two neutral mesons with masses $M_0+b\chi_\pm$, where

$$\chi_\pm=\frac{n(2-l)}{l}\pm\left[\frac{n(4-4l+nl)}{l}\right]^{1/2}.\tag{6}$$

The masses of the charged particles follow at once from Eq. (I.15b).

Specializing to $n=4$ and $l=3$ we find the mass formulas

$$\begin{aligned}2K^*&=\omega+\phi\,,\\ \sqrt{3}\phi+2\rho&=\sqrt{3}\omega+2K^*\,,\\ 3L&=2\rho+K^*,\\ 4K^*&=3l+\rho.\end{aligned}\tag{7}$$

As explained above, these results apply to vector mixing if the particle symbol is interpreted as standing for the inverse mass squared. Using the observed masses of ρ and K^* to predict the others, we obtain the results given in Table II. We see that the experimental masses of the ω and ϕ are predicted to within 3%. The mixing angle is given by

$$\theta=\cos^{-1}\left[\left(\frac{\chi_-}{\chi_--\chi_+}\right)^{1/2}\right]=\cos^{-1}\left[\frac{1}{(3-\sqrt{3})^{1/2}}\right]=27°$$

in agreement with one of the solutions of Kim, Oneda, and Pati.[14]

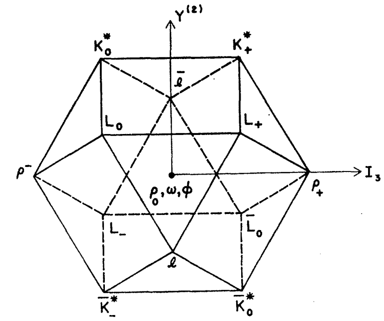

FIG. 1. The weight diagram for the regular representation of SU_4.

TABLE II. Particle masses in the $Z^{(2)}$ model. We use the masses of the ρ and K^* to predict the others.

Particle	Observed mass (MeV)	Mass predicted by Z^2 model (MeV)
ρ	755	$\cdots$
K^*	890	$\cdots$
ω	783	806
ϕ	1020	1015
L	$\cdots$	793
l	$\cdots$	953

IV. HIERARCHICAL SYMMETRIES

The conventional treatment of strong and electromagnet interactions postulates exact SU_3 symmetry, which is broken by an interaction transforming like the hypercharge $Y^{(2)}$ to give exact U_2 symmetry.[15] This is then broken by an interaction transforming like $Y^{(1)}$ (or I_3) to give the electromagnetic splittings.

If we adopt the Hagen-MacFarlane[9] scheme of a hierarchy of approximate SU_n symmetries, we may generalize the above scheme to a hierarchy of symmetry-breaking interactions. At each stage exact SU_n

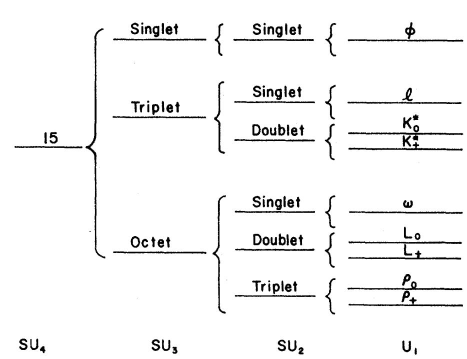

FIG. 2. Schematic indication of the mass splittings in the hierarchical symmetry scheme.

[14] Y. S. Kim, S. Oneda, and J. C. Pati, University of Maryland (to be published).

[15] That the symmetry is U_2 rather than $SU_2\otimes U_1$ has been emphasized by Michel, Proceedings of the Instanbul Summer School, 1962 (unpublished) and represents the fact that there is a relationship between the possible values of $Y^{(2)}$ and I that can occur.

symmetry is broken by an interaction transforming like $Y^{(n-1)}$ to give exact U_{n-1} symmetry. The representations of U_{n-1} furnish representations of SU_{n-1}, with a relationship between the quantum numbers specifying the representation of SU_{n-1} and the eigenvalues of $Y^{(n-1)}$. (It is these relations which ultimately ensure that the charge is integral.) The SU_{n-1} symmetry is then split to U_{n-2}, and so on. We assume that at each stage the magnitude of the splitting is such that each can be treated as a small perturbation on the one before.

There are several ways of embedding a U_{n-1} subgroup in SU_n and we may utilize this freedom to specify distinct hierarchies. Specifically, consider the generators $B_\alpha{}^\beta$ of SU_n defined in Appendix I. The scheme described above considers a symmetry breaking transforming like $B_n{}^n$, leaving the U_{n-1} subgroup generated by $B_\alpha{}^\beta$ ($\alpha, \beta = 1 \cdots n-1$). We might equally well choose any $B_r{}^r$ for the symmetry breaking,[16] and consider the U_{n-1} subgroup generated by $B_\alpha{}^\beta$ ($\alpha, \beta \neq r$).

For example, starting with SU_4 there are two possibilities for the U_3 subgroup—that generated by $B_\alpha{}^\beta$ ($\alpha, \beta = 1, 2, 3$) under which ρ, K^*, and ω_0 form an octet, L and l a triplet, and ϕ_0 a singlet (this is the conventional group of the eightfold way); and that generated by $B_\alpha{}^\beta$ ($\alpha, \beta = 1, 2, 4$) under which ρ, L, and ω_1 form an octet, K^* and l a triplet, and ϕ_1 a singlet. In terms of the $\psi_\alpha{}^\beta$ introduced in Appendix I

$$\begin{aligned} \phi_0 &= (3\psi_4{}^4 - \psi_3{}^3 - \psi_2{}^2 - \psi_1{}^1)/\sqrt{12}\,, \\ \omega_0 &= (2\psi_3{}^3 - \psi_2{}^2 - \psi_1{}^1)/\sqrt{6}\,, \\ \phi_1 &= -(3\psi_3{}^3 - \psi_4{}^4 - \psi_2{}^2 - \psi_1{}^1)/\sqrt{12}\,, \\ \omega_1 &= (2\psi_4{}^4 - \psi_2{}^2 - \psi_1{}^1)/\sqrt{6}\,. \end{aligned} \tag{8}$$

In a theory involving only vector mesons the distinction between these two possibilities is purely conventional, since we could always relabel the particles. Interactions with other particles do enable us to specify which particle is K^* and which L, however, and the large $K^*-\rho$ mass difference suggests we choose the second possibility in accordance with our postulate about the relative strengths of the splittings. Then the splitting from SU_4 to SU_3 is of the order of the $K^*-\rho$ mass difference (or 100 MeV), that from SU_3 to SU_2 is of the order of the $\rho-\omega$ mass difference[17] ($\sim$25 MeV), and that from SU_2 to U_1 is of the order of the $\rho^0-\rho^+$ mass difference ($\sim$5 MeV). The physical ϕ and ω would then be mixtures of ϕ_1 and ω_1, but in this scheme we would expect the mixing angle to be small, and to lowest order we may regard ϕ_1 and ω_1 as the physical particles. It is important to note that this is only a choice of the symmetry-breaking interaction and does not change the qualitative properties of the particles. In particular the assignment of quantum numbers in Table I remains unchanged. When we consider the SU_3 invariant couplings of the vector mesons to other strongly interacting particles we shall continue to assume that the octet generated by $B_\alpha{}^\beta$ ($\alpha, \beta = 1, 2, 3$) is the proper octet to couple. This way we do not have extra restrictions on the allowed couplings due to $Y^{(3)}$ conservation (since all members of this octet have $Y^{(3)} = 0$), and we can apply the standard phenomenological calculations found in the literature.[14] The mass formulas for hierarchical symmetry have been given by one of us,[18] and for SU_4 splittings to SU_3 we find

$$(2\phi_1 + \omega_1)/3 = l = K^*\,, \tag{9a}$$

$$\rho = \omega_1 = L\,. \tag{9b}$$

At this stage it is, of course, immaterial whether we write ρ or ω_1 on the left-hand side of Eq. (9a). In the subsequent split from SU_3 to SU_2 all of the particle masses are, in general, perturbed. Let us make the further assumption that this splitting transforms exactly as a member of an octet (that is, any SU_3 invariant part has been taken into account in the splitting from SU_4 to SU_3). Some assumption of this type is necessary to give content to the notion that the second splitting is weaker than the first. Then the ω_1 and ϕ_1 masses will be unperturbed by this splitting and, under the assumption that ω_1, ϕ_1 mixing is small we have the same statement for the physical particles ω and ϕ which we identify with them. Also the K^* will be perturbed much more than the l so we can make the approximation that (9a) applies to the ω and l even after the second splitting.[19] We also get the conventional Gell-Mann–Okubo formula for the octet

$$(3\omega + \rho)/4 = L\,, \tag{10}$$

giving $L = 776$ MeV (using inverse squared masses as before). Equation (9a) yields $l = 919$ MeV and indeed we see that the $\rho-\omega$ mass difference is essentially equal to the K^*-l mass difference.

Using Eq. (8) which can be solved for (ω_1, ϕ_1) in terms of (ω_0, ϕ_0) yielding

$$\begin{aligned} \phi_1 &= \tfrac{1}{3}\phi_0 - \tfrac{2}{3}\sqrt{2}\,\omega_0\,, \\ \omega_1 &= \tfrac{2}{3}\sqrt{2}\,\phi_0 + \tfrac{1}{3}\omega_0\,, \end{aligned} \tag{11}$$

and the assumption that (ω_1, ϕ_1) mixing is small so that they can be treated as the physical particles, we have $\theta = \sin^{-1}(\frac{1}{3}) \approx 19°$ again in agreement with one of the values of Kim, Oneda, and Pati.[14]

At each stage we are taking a mass splitting which transforms like one of the physical mesons. A possible generalization of this scheme is given in Appendix II.

[16] We may not choose $B_1{}^1$ or $B_2{}^2$ since $B_\alpha{}^\beta$ for $\alpha, \beta = 1, 2$ have been identified as the generators of the isotopic spin group.

[17] P. Singer, Phys. Rev. Letters **12**, 524 (1964) has recently suggested that the full $\rho-\omega$ mass difference is of electromagnetic origin.

[18] M. L. Whippman, University of Pennsylvania (to be published).

[19] Strictly speaking, with the above assumptions we may use the conventional Okubo formula to obtain $(2\phi+\omega)/3 = (13l + 2K^*)/15$, but to the accuracy to which we are working, the right-hand side may be replaced by l.

V. DISCUSSION

We have shown how the unexpected degeneracy between the unmixed ϕ_0 and ω_0 masses can be accounted for by postulating some higher symmetry scheme for the vector mesons. We have in mind a model such as that proposed by Cutkosky[6] in which the vector mesons bootstrap themselves, so at this stage we may confine our attention to the vector mesons alone. As a first attempt, SU_4 seems a promising candidate for the higher symmetry group, but we should emphasize that, though the predictions of a given symmetry scheme may be fairly straightforward, we are faced with an almost embarrassing profusion of possibilities when we discuss the symmetry-breaking interaction. In the case of SU_n, the most general form for the symmetry-breaking subject to our requirements gives a mass formula with $n-1$ arbitrary parameters. The purpose of this paper has been to point out how simple and perhaps physically reasonable choices can be made to reduce the number of parameters.

The most notable feature of the higher symmetry scheme is the prediction of six new vector mesons with fractional hypercharge but integral electric charge, and with nonzero values of the new quantum number hyperstrangeness. The crucial test of the theory is, of course, the existence of these particles but any suggested experimental search for these particles involves some assumption about their interactions with nonhyperstrange particles which will be highly conjectural.

So far, our discussion has been confined solely to the vector mesons and we now try to extend it to the baryons and pseudoscalar mesons. The first possibility is to assume that SU_4 symmetry holds for the vector mesons and that the other particles possess only SU_3 symmetry. Though, at first sight, such a scheme is unappealing, the bootstrap model for the vector mesons alone gives them a privileged position in the theory, and this scheme might arise if some dynamical mechanism suppressed the contributions of the other particles to the bootstrap. In such a scheme, one possible way of coupling the vector mesons to the other particles would be to form the SU_3 invariant couplings of the baryon and pseudoscalar meson octets to the SU_3 octet formed by ρ, K^*, and ω^0; the triplets L, l; and $\bar{L}$, $\bar{l}$ and the singlet ϕ_0. [Note that irrespective of our choice of octet for the symmetry breaking in the hierarchical model we choose the ρ, K^*, and ω_0 octet for coupling to other particles. As previously mentioned this guarantees hyperstrangeness conservation, and it also enables us to compare the mixing angle derived from Eq. (11) with that of Kim, Oneda, and Pati.[14]] In such a scheme, there would be no *a priori* relation between the $\rho\rho\pi$ and $L\bar{L}\pi$ coupling constants and we could increase the symmetry by assuming some such relation. In any event, since we assume that the strong and electromagnetic couplings conserve hyperstrangeness, the L and l would both be stable against decay by these interactions and could only decay weakly. Hence we would expect their lifetimes to be of the order of 10^{-9}-10^{-8} sec.

To consider the possible leptonic decays we first note that the well known $\Delta Y^{(2)}=0$, $\Delta T=1$, and $\Delta Y^{(2)}=1$, $\Delta T=\frac{1}{2}$ weak currents both have $\Delta Y^{(3)}=0$ since they have nonzero matrix elements between states of zero hyperstrangeness. Thus these currents contribute only to decays which conserve hyperstrangeness and we might expect to see a decay such as

$$l \rightarrow L+e+\nu. \tag{12}$$

(We use the symbol e to stand for any charged lepton and do not distinguish between neutrinos.)

For decays in which hyperstrangeness is changed, we might postulate a current transforming like the L^+ with $\Delta T=\frac{1}{2}$, $\Delta Y^{(2)}=\frac{1}{3}$, $\Delta Y^{(3)}=1$ which would induce decays such as

$$L \rightarrow \pi+e+\nu, \tag{13a}$$

$$l \rightarrow \bar{K}+e+\nu. \tag{13b}$$

The approximate degeneracy of L with ρ and l with K^* would severely inhibit the decays,

$$L \rightarrow \rho+e+\nu, \tag{14a}$$

$$l \rightarrow \bar{K}^*+e+\nu. \tag{14b}$$

If the rate for decays induced by the $\Delta Y^{(3)}=1$ current are significantly less than those induced by the $\Delta Y^{(3)}=0$ currents (just as the rate for $\Delta Y^{(2)}=1$ is less than that of $\Delta Y^{(2)}=0$), then one would expect most of the hyperstrangeness changing leptonic decays to be L decays since the transition (12) would presumably dominate (13b). The nonleptonic decays could be induced in a similar way by considering a weak Hamiltonian which transforms like a linear combination of K^{0*}, L^0, and l. An interesting possibility that follows from the existence of hyperstrangeness changing terms in the weak Hamiltonian is the hyperstrange decays of ordinary particles, though most of these are kinematically forbidden. We also have the possibility that associated with the divergence of the $\Delta Y^{(3)}=1$ axial vector current is a hyperstrange pseudoscalar meson which would give rise to Goldberger-Treiman relations in the usual way. Of course the possibility of more complicated transformation properties for the Hamiltonian cannot be excluded.

We also have the possibility that all strongly interacting particles have SU_4 symmetry. If we wished to assign the baryons to the adjoint representation, we would then predict a ninth neutral baryon and six hyperstrange baryons. We could also assign the baryon to some new representation, such as the 20-dimensional one which would contain only the observed octet of nonhyperstrange baryons and twelve hyperstrange baryons. We could then write down all possible SU_4 invariant couplings and would get a whole family of strong interactions involving hyperstrange particles and a large number of possibilities for weak decays.

In either case the hyperstrange particles could only be produced in pairs in reactions such as

$$p+p \to p+p+L+\bar{L}, \tag{15a}$$

$$p+\bar{p} \to L+\bar{L}, \tag{15b}$$

$$\pi+p \to \pi+p+L+\bar{L}. \tag{15c}$$

However, if the cross sections for these reactions were of the same order of magnitude for the corresponding reactions involving ρ's instead of L's the experiments would be difficult. The long L lifetime means that it would leave a visible track in a bubble chamber, but it is difficult to suggest a characteristic reaction that would serve as an unambiguous "signature" for the hyperstrange particles, and identification would probably have to be made by an accurate mass measurement either by counting bubble densities or by using Čerenkov counters. In either case fairly good resolution would be needed to distinguish L's from protons. It appears that these particles should be detectable in a suitably designed experiment but the difficulties are sufficiently great that it is not disturbing that they have not yet been noticed.

We have suggested three rather general models for constructing mass operators with simple properties. In the case of SU_4, the hierarchical symmetry model and the model of Appendix II gave the same results, while the $Z^{(2)}$ model gave a different but equally well-satisfied prediction for the masses. This indicates that a mass formula is not a particularly sensitive test of the theory. These models did lead to different mixing angles, but experiments are not yet accurate enough to provide a realistic value for this parameter. Unfortunately each of our models agrees with one of the estimates of Ref. 14 and we cannot distinguish between them on this basis at the present. It is apparent that a unique determination of the mixing angle will be of great interest.

The crucial test of the theory is, of course, the existence of hyperstrange particles. We are not able to suggest even an order of magnitude for their production cross section; presumably they should be formed in pairs in sufficiently high energy collisions, though the large particles mass ($\sim$900 MeV) will give a small pair production cross section at presently available energies.

Note added in proof. The possibility of applying SU_4 to elementary particles has recently been considered by several other authors, including Bjorken and Glashow [Phys. Letters **11**, 255 (1964)], Amati, Bacry, Nuyts, and Prentki (to be published) and Tarjanne (private communication).

ACKNOWLEDGMENT

One of us (M.L.W.) would like to thank Professor A. Ewart for a discussion.

APPENDIX I

We shall work throughout with the realization of U_n given by the n^2 operators $A_\alpha{}^\beta(\alpha,\ \beta=1\cdots n)$ with the commutation rules

$$[A_\alpha{}^\beta, A_\gamma{}^\delta]=\delta_\alpha{}^\delta A_\gamma{}^\beta-\delta_\gamma{}^\beta A_\alpha{}^\delta. \tag{I.1}$$

The corresponding generators of SU_n are defined by[20]

$$B_\alpha{}^\beta=A_\alpha{}^\beta-(1/n)\delta_\alpha{}^\beta A_\mu{}^\mu \tag{I.2}$$

and satisfy the same commutation relations as the A's. It is also convenient to introduce another set of operators of SU_l given by

$$Y^{(l)}=\frac{1}{l+1}\sum_{r=1}^{l+1} A_r{}^r - A_{l+1}{}^{l+1}. \tag{I.3}$$

A further set of tensor operators is defined by

$$T_\alpha{}^\beta=\{A_\alpha{}^\mu, A_\mu{}^\beta\}, \tag{I.4}$$

and correspondingly

$$S_\alpha{}^\beta=\{B_\alpha{}^\mu, B_\mu{}^\beta\}-(2/n)\delta_\alpha{}^\beta\Lambda_n, \tag{I.5}$$

$$Z^{(l)}=\frac{1}{l+1}\sum_{r=1}^{l+1} S_r{}^r - S_{l+1}{}^{l+1}, \tag{I.6}$$

where

$$\Lambda_n=B_\alpha{}^\beta B_\beta{}^\alpha \tag{I.7}$$

is the Casimir operator of SU_n. To make contact with physics we identify the isotopic spin operators as

$$I_+=B_1{}^2,\quad I_-=B_2{}^1,\quad I_3=Y^{(1)}. \tag{I.8}$$

The usual hypercharge operator is $Y^{(2)}$ in this scheme, and the charge, defined as

$$Q=\sum_{r=1}^{n-1} Y^{(r)}/r, \tag{I.9}$$

is an integer for all particles in the adjoint representation.[9]

The operators $\mathfrak{D}(A_\alpha{}^\beta)$ defined by

$$\mathfrak{D}(A_\alpha{}^\beta)A_\gamma{}^\delta=[A_\alpha{}^\beta, A_\gamma{}^\delta] \tag{I.10}$$

form a realization of the algebra, which becomes the adjoint representation when the A's are regarded as elements of a column vector. To avoid notational confusion it is convenient to denote the elements of basis vectors of the adjoint representation of U_n by $\psi_\mu{}^\nu$ and the operators by $A_\mu{}^\nu$. Then we have

$$A_\alpha{}^\beta\psi_\mu{}^\nu=\delta_\alpha{}^\nu\psi_\mu{}^\beta-\delta_\mu{}^\beta\psi_\alpha{}^n. \tag{I.11}$$

The corresponding basis for SU_n is

$$\phi_\alpha{}^\beta=\psi_\alpha{}^\beta-(1/n)\delta_\alpha{}^\beta\psi_\mu{}^\mu. \tag{I.12}$$

A convenient choice for the wave functions of the neutral particles (appropriately normalized) is

$$\lambda_r=\left(\frac{r}{r-1}\right)^{1/2}\left[\psi_r{}^r-\frac{1}{r}\sum_{i=1}^{r}\psi_i{}^i\right]. \tag{I.13}$$

In particular, $\rho_0=\lambda_2$, $\omega_0=\lambda_3$, $\phi_0=\lambda_4$.

[20] The summation convention applies to Greek but not to Latin indices.

We may now calculate the matrix elements of the various operators in this representation, and we find

$$\langle\lambda_r|S_l{}^l|\lambda_s\rangle=\begin{cases} 2n(l-1)/l & (r=s=l) \\ -2n[(l-1)/lr(r-1)]^{1/2} & (r=l,\ s>l) \\ -2n[(l-1)/ls(s-1)]^{1/2} & (r>l,\ s=l) \\ 2n/[rs(r-1)(s-1)]^{1/2} & (r,\ s>l) \\ 0 & \text{otherwise.} \end{cases} \tag{I.14a}$$

For $\alpha\neq\beta$

$$\langle\phi_\alpha{}^\beta|S_l{}^l|\phi_\gamma{}^\delta\rangle=\begin{cases} n & (\beta=\delta,\ \alpha=\gamma=l \quad\text{or}\quad \beta=\delta=l,\ \alpha=\gamma) \\ 0 & \text{otherwise.} \end{cases} \tag{I.14b}$$

$$\langle\lambda_r|Z^{(l-1)}|\lambda_s\rangle=\begin{cases} 2(2-l)n/l & (r=s=l) \\ 2n/l & (r=s<l) \\ 2n[(l-1)/lr(r-1)]^{1/2} & (r>l,\ s=l) \\ 2n[(l-1)/ls(s-1)]^{1/2} & (s>l,\ r=l) \\ 0 & \text{otherwise.} \end{cases} \tag{I.15a}$$

For $\alpha\neq\beta$

$$\langle\phi_\alpha{}^\beta|Z^{(l-1)}|\phi_\gamma{}^\delta\rangle=\delta_{\alpha\gamma}\delta^{\beta\delta}\times\begin{cases} 2n/l & (\alpha,\ \beta<l) \\ n(2-l)/l & (\alpha=l,\ \beta<l \quad\text{or}\quad \alpha<l,\ \beta=l) \\ n/l & (\alpha>l,\ \beta<l \quad\text{or}\quad \alpha<l,\ \beta>l) \\ -n(l-1)/l & (\alpha=l,\ \beta>l \quad\text{or}\quad \alpha>l,\ \beta=l) \\ 0 & \text{otherwise.} \end{cases} \tag{I.15b}$$

Finally we give the values of $Y^{(l-1)}$ acting on $\phi_\alpha{}^\beta$. These are

$$Y^{(l-1)}\phi_\alpha{}^\beta=\phi_\alpha{}^\beta\times\begin{cases} -1 & (\beta=l,\ \alpha<l) \\ 1 & (\beta<l,\ \alpha=l) \\ -1/l & (\beta<l,\ \alpha>l) \\ 1/l & (\beta>l,\ \alpha<l) \\ (l-1)/l & (\beta>l,\ \alpha=l) \\ (1-l)/l & (\beta=l,\ \alpha>l) \\ 0 & \text{otherwise.} \end{cases} \tag{I.16}$$

APPENDIX II

The $Z^{(l-1)}$ model assumes that the mass splitting transforms like one of the unmixed neutral mesons. In the hierarchical model, at each stage the mass splitting transforms like one of the physical mesons. Another possibility is to assume that the total mass splitting operator transforms like one of the physical mesons. The mass splitting operator (3) may equivalently be written

$$M=\sum_{r=3}^{n}\alpha_r S_r{}^r, \tag{II.1}$$

where $S_\alpha{}^\beta$ is defined in Eq. (I.5). Our assumption is that

$$\Phi=\sum_{r=3}^{n}\alpha_r\phi_r{}^r \tag{II.2}$$

is an eigenvector of M. Using Eqs. (I.14) of Appendix I, we find that p of the α's must be equal $(1\leq p\leq n-2)$ and the remainder must be zero. The symmetry group in the presence of such a symmetry-breaking interaction is then $\mathrm{SU}_{n-p}\otimes\mathrm{U}_p$. We then find for the masses of the neutral mesons:

$$\begin{aligned} &\text{one with mass} && M_0+(n-2p)a\,, \\ &(p-1)\text{ with mass} && M_0+(n-p)a\,, \\ &(n-p-1)\text{ with mass} && M_0-pa\,, \end{aligned}$$

where a is a constant. Again the masses of the charged particles may be determined from Appendix I.