Electromagnetic Mass Differences of Baryons and Mesons*

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The four independent baryonic and two mesonic electromagnetic (E.M.) mass differences are evaluated on the basis of the unitary symmetry model. An approximation scheme is devised and its relation to a dispersion-theoretic treatment by Cottingham is pointed out. Four contributions to the E.M. mass differences are taken into account: the elastic form-factor part, processes which involve the two isovector transitions (π^0-η) and (Σ^0-Λ), and what we call the self-induced mass differences. Unitary symmetry is used to provide values for quantities not given by experiments, e.g., form factors for strange particles and coupling constants. An earlier correct result for the pion mass difference is confirmed because it is shown that the chief contribution is due to the elastic form-factor part. All the observed baryonic mass differences can be approximately reproduced. The sign of the kaon mass difference can be explained, but the quantitative predictions for the kaon are still unreliable.

1. INTRODUCTION

ACCORDING to present conceptions, the masses of particles of specified spin and parity within a given SU_3 representation should be identical in the limit of exact unitary symmetry. Actually, no two members of the same unitary baryon or meson multiplet possess the same mass (if cognizance is taken of the fact that a unitary meson multiplet contains both particles and antiparticles). The observed mass differences within a unitary multiplet range in order of magnitude from several hundred MeV, between the various subgroups with the same T and Y (T is the isospin and Y the hypercharge), through several MeV, between the members of the same isotopic multiplet, to the very small value of 10^{-5} eV between the two members of the neutral kaon mixture. This clustering of mass differences around three distinct sets of values makes it natural to ascribe the observed mass spectrum to three types of splitting:

(1) the moderately strong (M.S.) splitting which is associated with an A_3^3 symmetry-breaking[1] term that conserves T and Y and yields mass differences of the order of $g^2 m_\pi$ (g^2 is the M.S. coupling constant);

(2) the electromagnetic (E.M.) splitting which is associated with an A_1^1 symmetry-breaking term that conserves Y (but not T) and yields mass differences of the order of αm_π (α is the fine-structure constant); and

(3) the weak (nonleptonic) splitting which is associated with an $(A_2^3+A_3^2)$ symmetry-breaking term that conserves neither Y nor T (but requires $\Delta Y=1$, $\Delta T=\frac{1}{2}$) and yields mass differences of the order $(G^2 m_\pi^{-4})m_\pi$ (G is the weak coupling constant).

The success of the Gell-Mann–Okubo[1,2] (GMO) mass formula for the M.S. mass differences within the better-known baryon and meson unitary multiplets confirms the hypothesis that the M.S. splitting is predominantly A_3^3 in character. However, no serious dynamical calculation has been made of the separate M.S. mass differences within any of the unitary multiplets (the GMO formula represents *one* relation among *all* the M.S. mass differences within a given unitary multiplet) because of our lack of knowledge of the M.S. interactions. Similarly, the Coleman-Glashow formula[3] for the E.M. mass differences within the $J=\frac{1}{2}^+$ baryon octet is consistent with the hypothesis that the E.M. splitting is predominantly A_1^1, but here again dynamical calculations of the separate E.M. mass differences within each isotopic multiplet of the $J=\frac{1}{2}^+$ baryon octet are in a rudimentary stage. Moreover, the Coleman-Glashow relation is essentially empty of content for the $J=0^-$ meson octet. Finally, it should be remarked that the $(K_1^0-K_2^0)$ mass difference is the only known "weak" mass difference and hence cannot serve as a "test" of any symmetry-breaking hypothesis, although its sign and magnitude are of great interest for dynamical theories.

In this paper, we focus our attention on a dynamical calculation of the separate E.M. mass differences within the $J=\frac{1}{2}^+$ baryon octet and the $J=0^-$ meson octet where the numbers are fairly well known by now (although improved data for the Ξ's would be very welcome). In Sec. 2, the isospin and unitary aspects of the E.M. mass-difference problem for the baryon and meson octets are defined more sharply. In Sec. 3, our method of calculation is presented and its relation to Cottingham's[4] rigorous dispersion-theoretic approach is discussed. In Sec. 4, the elastic form-factor contributions to the E.M. mass differences are considered in some detail starting with the latest experimental data for the nucleon, and Sec. 5 treats the other processes we include in our calculation. Finally, Sec. 6 contains our results and conclusions.

* Work supported in part by the U. S. Atomic Energy Commission.
† Work begun while the author was a predoctoral National Science Foundation Fellow.

[1] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).
[2] M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
[3] S. Coleman and S. L. Glashow, Phys. Rev. Letters 6, 423 (1961).
[4] W. N. Cottingham, Ann. Phys. (N. Y.) 25, 424 (1963).

2. ISOSPIN AND SU_3 CONSIDERATIONS

The most recent experimental values for the E.M. mass differences of the baryon and pseudoscalar meson octets are given in the center column of Table I. As defined there they are positive, and we shall consistently refer to them in this fashion. It is to be noted that in three of six cases the neutral member of a pair is heavier than the charged one. From a simple-minded point of view, the self-energy of a charged particle is positive as is the field energy of a magnetic dipole; furthermore, a spinless particle cannot carry a magnetic moment. In such an approach, one can understand the signs of Δ_-, Δ_Ξ, and Δ_π. The signs of Δ_N and Δ_+ are difficult to explain because in order to obtain the correct signs, the magnetic moments of n and Σ^0 would have to be large compared to those for p and Σ^+, respectively; this is known not to be true for the nucleons. The sign of Δ_K is even more difficult to understand since the kaon is spinless. Moreover, we shall see that membership of N, Σ, and Ξ in the same baryon octet, and π and K in the same meson octet, initially aggravates the problem of understanding the signs of some of the E.M. mass differences (particularly Δ_K). Indeed, the subtleties involved in predicting the correct signs of all six E.M. mass differences are quite considerable.

From the Nishijima–Gell-Mann formula, $Q=T_3+Y/2$, we see that the charge Q and hence the electromagnetic current operator has the isotopic-spin character of the third component of an isovector plus an isoscalar. Thus, if the E.M. mass differences are of second order in the E.M. interaction, that is, quadratic in the current, the mass operator must have the form:

$$\Delta m_\alpha = a_\alpha{}^S + a_\alpha{}^V T_3 + a_\alpha{}^T (T_3)^2, \qquad (2.1)$$

where α is the index denoting a particular isotopic multiplet (S is scalar, V vector, T tensor). For the isospinors (K,N,Ξ), Eq. (2.1) is effectively isoscalar plus isovector, but for the isovector particles (π,Σ), all three terms in principle should contribute. Actually, since π^- is the charge conjugate of π^+, they must have the same mass and hence $a_\pi{}^V=0$ in (2.1). This argument does not apply to the Σ hyperons. Until recently it seemed that $\Delta_-=\Delta_+$ within experimental error, that is, that the masses of the Σ's appeared to obey an equal spacing rule. This would have implied that $a_\Sigma{}^T\approx 0$ in (2.1). However, the latest experimental data (cf. Table I) yield $(\Delta_- - \Delta_+)=1.9$ so that $a_\Sigma{}^T=0.95$.

TABLE I. Total mass differences (in MeV).

	Experimental value[a]	$f=0.30$	$f=0.35$	$f=0.39$
$\Delta_N=M(n)-M(p)$	1.29 ± 0.02	-0.18	1.12	2.20
$\Delta_+=M(\Sigma^0)-M(\Sigma^+)$	2.85 ± 0.30	3.17	3.24	3.24
$\Delta_-=M(\Sigma^-)-M(\Sigma^0)$	4.75 ± 0.10	4.81	5.47	5.98
$\Delta_\Xi=M(\Xi^-)-M(\Xi^0)$	$6.1\ \pm1.6$	9.83	9.34	8.84
$\Delta_\pi=m(\pi^+)-m(\pi^0)$	4.59 ± 0.07	4.23	4.23	4.23
$\Delta_K=m(K^0)-m(K^+)$	$3.9\ \pm0.6$	{ 15 / -35	15 / -35	15 / -35

[a] The experimental values for Δ_N, Δ_π, and Δ_K are taken from W. H. Barkas and A. Rosenfeld, University of California Radiation Laboratory Report UCRL-8030 (unpublished). The earlier measurements of W. H. Barkas, J. N. Dyer, and H. H. Heckman, Phys. Rev. Letters 11, 26 (1963) gave the values for Δ_+ and Δ_- of 3.85 ± 0.8 and 4.2 ± 0.8, respectively, and led to the speculative equal-spacing rule for the Σ masses. The recent values of R. A. Burnstein, T. B. Day, B. Kehoe, B. Sechi-Zorn, and G. A. Snow, University of Maryland Technical Report No. 382 (unpublished), show a rather large departure from this rule. Δ_Ξ is taken from D. D. Carmony, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 12, 482 (1964).

The smallness of the E.M. mass differences led to isospin invariance where the violation of the exact symmetry which produced the E.M. mass differences was taken to be proportional to the charge (or equivalently T_3). Within this framework, no relation could be obtained relating the E.M. mass differences of different isotopic multiplets. The observed grouping of particles with identical baryon number, spin, and parity into unitary multiplets leads to the charge (current) becoming: $Q=T_3+Y/2=A_1{}^1$ where $A_\nu{}^\mu$ are the set of traceless generators of unitary, unimodular transformations in three dimensions.[1] If the E.M. mass differences are due to the current, then the mass differences should be given by $\Delta m\sim(A_1{}^1)^n$, where $n=2$ would be expected. Okubo[5] has shown that any tensor $(A_1{}^1)^n$ can be written as $(A_1{}^1)^n=f(Q,\mathbf{U}^2)$ where Q is the charge operator and $\mathbf{U}$ is the U spin defined so that (p,Σ^+) and (Σ^-,Ξ^-) are U spin doublets, $[n,(-\Sigma^0+\sqrt{3}\Lambda)/2,\Xi^0]$ is a U triplet and $[(\sqrt{3}\Sigma^0+\Lambda)/2]$ is a U singlet. Therefore, the splittings of the masses from the "unperturbed" values satisfy

$$\begin{aligned} M(p)-\bar{M}(N)&=M(\Sigma^+)-\bar{M}(\Sigma) \quad (Q=1,\ U=\tfrac{1}{2}),\\ M(\Xi^0)-\bar{M}(\Xi)&=M(n)-\bar{M}(N) \quad (Q=0,\ U=1),\\ M(\Sigma^-)-\bar{M}(\Sigma)&=M(\Xi^-)-\bar{M}(\Xi) \quad (Q=-1,\ U=\tfrac{1}{2}). \end{aligned} \qquad (2.2)$$

Upon adding the above equations, we obtain

$$\Delta_\Xi+\Delta_N=\Delta_+ + \Delta_- . \qquad (2.3)$$

This is the well-known Coleman-Glashow relation,[3] which should hold to all orders in the E.M. interaction. If, for some unknown reason, the mass operator had the transformation properties of a traceless tensor $A_1{}^1$ (for example, first order in the current), we would have the

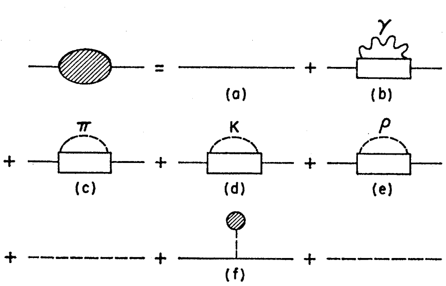

FIG. 1. Diagrammatic expansion of the self-energy of a particle.

[5] Okubo showed in Ref. 1 that $A_3{}^3=f(Y,\mathbf{T}^2)$. This equation for $A_1{}^1$ is similarly proved.

additional relations:

$$\Delta_+ = \Delta_- , \qquad (2.4)$$
$$M_{\Sigma\Lambda} = (1/\sqrt{3})(\Delta_+ - \Delta_N),$$

where $M_{\Sigma\Lambda}$ is the "Σ^0-Λ transition mass." This last equation actually has more general validity. Okubo[6] showed that it holds to first order in both the E.M. and M.S. interactions. Therefore, SU_3 does *not* predict equal Σ splitting unless the predominant interaction transforms as a tensor $A_1{}^1$.

For mesons, the general Coleman-Glashow relation is empty. If the meson mass operator is assumed to be proportional to $A_1{}^1$, we obtain:

$$\Delta_\pi = 0 , \qquad (2.5)$$
$$m_{\pi\eta}{}^2 = -(1/\sqrt{3})(m_{K^0}{}^2 - m_{K^+}{}^2),$$

where $m_{\pi\eta}{}^2$ is the "$(\pi^0$-$\eta)$ transition (mass)^2." The vanishing of Δ_π is to be expected because the operator $A_1{}^1$ only has isoscalar and isovector components and Δ_π is an isotensor. It follows that the observed finite value of Δ_π requires at least an $(A_1{}^1)^2$ contribution in the SU_3 language. The rather large value of Δ_π further implies that caution must be exercised in stating that the dominant contribution to the E.M. mass operator transforms as $A_1{}^1$ (cf. Sec. 1); this is also consistent with the new result for $(\Delta_- - \Delta_+)$.

3. METHOD OF CALCULATION

In order to calculate the self-mass of a particle, we must evaluate all contributions corresponding to diagrams which have only one line entering and leaving. Of course, we cannot calculate them all, and so to obtain an approximation scheme which justifies using only a few of the more manageable ones, we break them up into the series shown in Fig. 1. Any given diagram is included in one of the classes shown there if its lightest particle is the one shown drawn separately. In this way, no diagram is included more than once. We only want those subdiagrams which *distinguish* between different members of the same isotopic multiplet. Therefore, the part labeled (a), and any parts of (b), (c), (d), etc., in which the separated particle is coupled charge-symmetrically to the box, will not contribute to the E.M. mass differences.

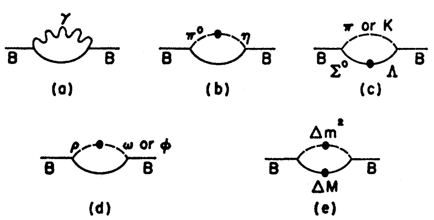

FIG. 2. Processes contributing to the baryonic E.M. mass differences.

[6] S. Okubo, J. Phys. Soc. Japan (to be published).

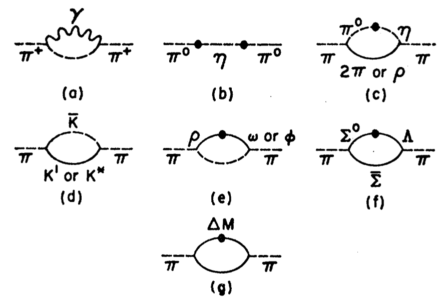

FIG. 3. Processes contributing to the pion mass difference.

In all parts of Fig. 1, the mass differences of the particles in the intermediate states will cause deviations from charge independence even when the coupling is charge-independent. For Fig. 1(b), which is already of order e^2, the mass differences will only give a second-order correction. However, Figs. 1(c) and 1(d) do receive contributions linear in the mass differences. We call these the self-induced mass differences, and denote them schematically in Figs. 2(e), 3(g), and 4(e) for the baryons, pion, and kaon, respectively.

Figure 1(a) represents the bare mass of the particle, and if it violates charge symmetry, that is, if the bare masses are unequal, then the electromagnetic mass differences are fundamental physical quantities and not calculable from other experimentally observed quantities such as masses and coupling constants. Coleman and Glashow[7] move partly in this direction by postulating that the tadpole diagrams of Fig. 1(f) contribute to the E.M. mass differences. Their procedure introduces one additional free parameter for all the particles, which parameter is adjusted to optimize the final results. We shall discuss this point more completely in Sec. 6.

We must now consider the baryons and mesons separately. Consider the baryons first. If we replace the box in Fig. 1(b) by the lowest mass state possible, which is the one-baryon state, we get the diagram shown in Fig. 2(a). We call this the elastic form-factor part. The next state contains a baryon and a meson and so is heavier. We hope that this and all higher states will give small enough contributions so that we can neglect them. Two photon states in any diagram give masses proportional to e^4, and we may certainly neglect them. Figure 1(c) can be treated similarly: if the box repre-

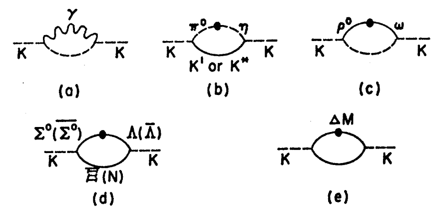

FIG. 4. Processes contributing to the kaon mass difference.

[7] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

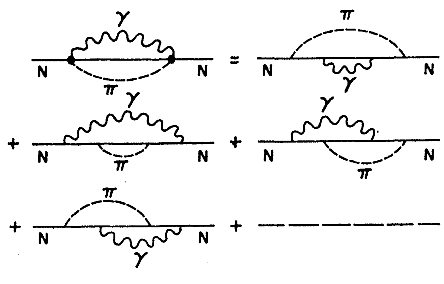

FIG. 5. Partial expansion of the contribution of the scattering process: $e+N \to_{\text{C.D.}} e'+N+\pi$ to the nucleon mass difference.

sents a baryon state, then only that coupling of the pions to the baryons which violates charge independence must be considered. The possibility of a transition between the π^0 and the η mesons[8] would produce such an effect. This results in the diagram of Fig. 2(b). Again, we neglect higher states for this box. The charge-dependent diagram extracted from Fig. 1(d) is caused by the transition between the Σ^0 and the Λ hyperons, and is shown in Fig. 2(c). For Fig. 1(e), the existence of a transition between the ρ^0 and the ω or ρ^0 and ϕ vector mesons would give the diagram shown in Fig. 2(d). However, for reasons which will be seen later, we shall not evaluate it. The self-induced contributions to the baryon mass differences are depicted by Fig. 2(e).

For the mesons, we distinguish between pion and kaon. For the pion, our approach results in Figs. 3(a)–(g). Fig. 3(a) is the elastic form-factor part; and Fig. 3(b), the contribution to the π^0 self-energy due to the (π^0-η) transition. Actually, Diagrams 3(c)–3(f) do not contribute to Δ_π. This can be seen on isospin grounds alone, since Δ_π is an isotensor and all these diagrams behave as isovectors. Alternatively, we can see that they vanish term by term. Figure 3(c) is of second order because the (π^0-η) transition is already of order e^2, and the coupling of η to (3π) or to ($\rho\pi$) is also of order e^2, because it violates G parity. Figure 3(d) can be seen to vanish to first order in Δ_K, $\Delta_{K'}$, and Δ_{K^*} because the diagrams with different charge complexions cancel. Figure 3(e) is of order e^4 because the (ρ^0-ω), (ρ^0-ϕ), (ω-2π), and (ϕ-2π) couplings all violate G parity. The different charge states of Fig. 3(f) all cancel, just as in the case of Fig. 3(d). The baryon mass differences in

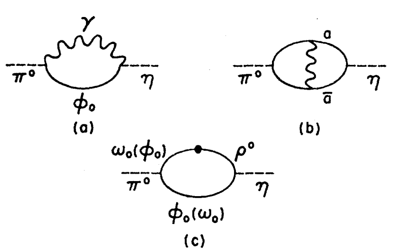

FIG. 6. Processes contributing to the ($\pi^0-\eta$) transition. In part (b) the particle denoted by a is either a K'^+, K^{*+}, p, or Ξ^-.

[8] Riazuddin and Fayazuddin, Phys. Rev. **129**, 2337 (1963).

the intermediate states of Fig. 3(g) give a self-induced contribution to Δ_π. A diagram similar to 3(f) but containing two (Σ^0-Λ) transitions does give a contribution to Δ_π, but this turns out to be very small. It will also turn out that the contributions of Figs. 3(b) and 3(g) are rather small, so that the only important contribution to the pion mass difference arises from Fig. 3(a). This explains the success of the calculation by Bose and Marshak,[9] who only considered the contribution of the elastic form factor.

Figures 4(a)–(e) are the corresponding diagrams for the kaons. Since Δ_K is an isovector, all processes contribute, including all the isovector baryonic mass differences in Fig. 4(e).

We end up with trying to compute the six E.M. baryon and meson mass differences by considering: (1) the elastic form-factor parts, (2) the three transitions (π^0-η), (Σ^0-Λ), and (ρ^0-ω or ϕ), and (3) the self-induced contributions. The elastic form factor and self-induced parts have mixed isospin character (scalar, vector, and tensor), while the three transitions are pure vector.

Cottingham,[4] in a dispersion-theoretic calculation, related the cross sections for elastic and inelastic elec-

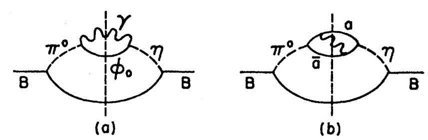

FIG. 7. (a) Contribution of the scattering process: $e+B \to_{\text{C.D.}} e' +B+\phi$ to the baryon mass difference. (b) Contribution of the scattering process: $e+B \to_{\text{C.D.}} e'+B+a+\bar{a}$ to the baryon mass difference [a same as in Fig. 6(b)].

tron-nucleon scattering to the n-p mass difference. This calculation includes all intermediate states which have at least one photon in the self-mass diagram. These are shown in Fig. 1(b). The elastic-scattering experiments determine the elastic form-factor part just as usual. However, he showed that measurement of the cross section for all energies and angles of the inelastically scattered electron completely determines the contributions to the mass difference of all the other states. Calculating the n-p mass difference in this way is very difficult. First, there is the large number of experiments needed to map out the cross section as a function of two variables. Second is the difficulty of separating out the neutron part of the deuteron scattering. Third is the fact that a large component of the cross section is symmetric in neutron and proton. This is the part which results in excitation of the ($\frac{3}{2}$,$\frac{3}{2}$) pion-nucleon resonance, and because *only* the isovector part of the current operator is effective in raising the nucleon isospin from $\frac{1}{2}$ to $\frac{3}{2}$, there will be no n-p mass splitting.

If one believes that the E.M. mass differences are due

[9] S. K. Bose and R. E. Marshak, Nuovo Cimento **25**, 529 (1962).

to processes which are second order in the electromagnetic interaction and that all such processes involve emission and reabsorption of photons, then Cottingham's approach should reproduce the mass differences correctly. The difficulty of extracting the necessary information for the nucleons and also the certainty that electron-hyperon and electron-boson experiments are in the very distant future has led us to our attempt to calculate the E.M. mass differences more indirectly.

A theoretical calculation of the E.M. mass differences which is in direct analogy with Cottingham's basically experimental approach would be to insert into the box of Fig. 1(b) one state after another and to evaluate as many of these diagrams as is necessary to reproduce the experimental values. One drawback is that convergence of such a series is probably not very good since the masses of succeeding states do not increase very fast (e.g., $N+2\pi$ is not much heavier than $N+\pi$). Also, the evaluation of even the first such term in which the intermediate particles are $(N+\pi)$ is not only difficult but also uncertain in that not much is known about the electropion production amplitude (especially on hyperons and mesons).

Although this approach is probably correct, it is impossible to carry through. Alternatively, our approximation method enables us to calculate numbers, but there is no guarantee that we have taken into account *all* the major contributions to the E.M. mass differences. In what follows we shall try to show a correspondence between the correct dispersion-theoretic method and our diagrammatic approach. Naturally, there are ambiguities in drawing the analogy because one cannot say to what dispersion-theory term particular diagrams contribute. Consider as an example Fig. 5, which represents the expansion of the second term in the Cottingham approach. In dispersion theory, we know that the intermediate state containing both a nucleon and a pion has the particles on their mass shells, and so each subdiagram shown on the right-hand side of Fig. 5 can only belong to the diagram on the left-hand side of Fig. 5. However, in a purely diagrammatic approach, with no mass-shell restrictions, we can cut each of the last three subdiagrams such that the intermediate state is either $(\gamma+N)$ or $(\gamma+N+\pi)$. Therefore, we cannot say if they contribute to the elastic form factor part or to the electropion production part. We shall avoid this difficulty by claiming that there is a correspondence between our method and the correct dispersion theory treatment if we can reduce our processes to diagrams which would appear in the dispersion approach, that is, diagrams with intermediate states which contain at least one photon.

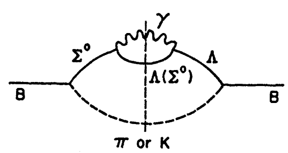

FIG. 8. Contribution of the scattering process: $e+B\rightarrow_{\text{C.D.}}e'+\Lambda(\Sigma^0)+K$ or π to the baryon mass difference.

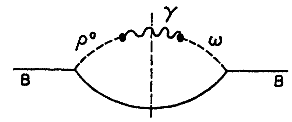

FIG. 9. Contribution of the scattering process: $e+B\rightarrow_{\text{C.D.}}e+B$ to the baryon mass difference.

When one recalls that the three transitions we consider have an electromagnetic origin and can be represented diagrammatically by graphs which have a virtual photon line, then we see that the diagrams we take are a special subclass of all those entering into the full calculation. This is represented in Figs. 6 and 7 for the $(\pi^0\text{-}\eta)$ transition. Rosen[10] showed that among the diagrams contributing to this transition are those of Fig. 6. If we insert these into Fig. 2(b), we get the diagrams of Fig. 7. Figure 8 is a diagram which arises from 2(c) if a particular graph which allows the $(\Sigma^0\text{-}\Lambda)$ transition process to occur is inserted. Figure 9 shows why it is probably incorrect to include the $(\rho^0\text{-}\omega)$ transition in this analysis. If this transition is dominated by the photon pole, it is already included in the elastic form-factor contribution. Figure 10 is a schematic way of viewing the self-induced mass differences. The particular diagram shown contributes to Δ_N.

Beneath each diagram of Figs. 7–10 are written the scattering processes to which they correspond. The letters C.D. underneath the arrows serve as a reminder that only the charge-dependent part of the diagram contributes to the mass differences. It is for this reason that we cannot argue backwards and predict important diagrams for inelastic electron scattering from processes which give large contributions to the E.M. mass differences. For example, from diagram 7(a) another graph for ϕ production can be obtained by replacing η by π^0, and if the $\phi\pi^0\gamma$ coupling is larger than the $\phi\eta\gamma$ coupling, it will give larger charge-symmetric scattering than the charge-dependent one. The charge-dependent diagrams for the E.M. mass differences correspond to the cross terms in the cross section, that is, the product of two partial amplitudes which lead to the same final state but have different internal lines and different isospin dependence. There is no reason to expect that these

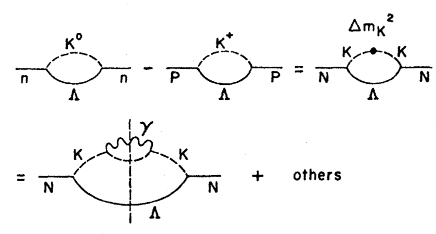

FIG. 10. Contribution to Δ_N induced by the kaon mass difference. This partially takes into account the contribution of the scattering process: $e+N\rightarrow_{\text{C.D.}}e'+\Lambda+K$ to the nucleon mass difference.

[10] S. P. Rosen, Phys. Rev. **132**, 1234 (1963).

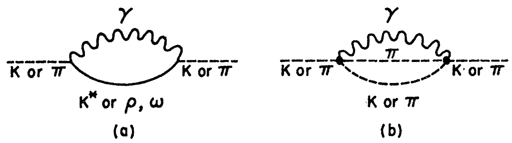

FIG. 11. (a) Contributions to the mesonic E.M. mass differences due to vector-meson intermediate states. (b) Contributions to the mesonic E.M. mass differences due to two-meson intermediate states.

contributions are larger or smaller than the direct ones.

We have done some approximate calculations which correspond to the first state heavier than the elastic form-factor part in the Cottingham approach for the kaons and pions. First, the intermediate states were taken to be a photon plus a vector meson resonance as shown in Fig. 11(a). The relative couplings were taken from unitary symmetry,[11] and are:

$$M(K^{*+}\to K^{+}+\gamma)=M(\rho^{+}\to\pi^{+}+\gamma)$$
$$=M(\rho^{0}\to\pi^{0}+\gamma)=-\tfrac{1}{2}M(K^{*0}\to K^{0}+\gamma)$$
$$=(1/\sqrt{3})M(\omega\to\pi^{0}+\gamma)\,,\quad(3.1)$$

where a Hamiltonian

$$\mathcal{H}_{I}=g\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}\phi^{\alpha}\partial^{\beta}\phi\quad(3.2)$$

was used. g is the effective coupling constant, and $F^{\mu\nu}$, ϕ^{α}, and ϕ are the photon, vector meson, and boson field operators, respectively. The width of the decay of K^{*+} into $(K^{+}+\gamma)$ was taken as 1 MeV. This calculation is quadratically divergent and was cut off at one nucleon mass. The results are $\Delta_K=-7$ MeV and $\Delta_\pi=2.3$ MeV. If the radiative decay width of K^{*+} is larger, as may be the case, then the mass differences become proportionately larger.

In view of this poor result, an estimate was made of the same diagram treating the $(K-\pi)$ system as unbound and putting the resonant effect into the amplitude for pion photoproduction on kaons. This is shown in Fig. 11(b), and the result is $\Delta_K=-0.17$ MeV if the electromagnetic decay width is again taken as 1 MeV. The $(\gamma+2\pi)$ state does not contribute to Δ_π because the octet model predicts equal photon coupling to $(\rho^{+}\text{-}\pi^{-})$ and to $(\rho^{0}\text{-}\pi^{0})$. Not much more than the sign of the $(K-\pi)$ contribution to Δ_K can be inferred from the calculation, because of the divergence in the resonant approach and because of the approximations made in the second. The large difference, a factor of 50 between the two calculations, casts doubt on naive calculations which treat the resonances as stable particles.

4. ELASTIC FORM-FACTOR CONTRIBUTIONS

If the electromagnetic form factors of a pair of particles are known, then a simple weighted integration over the momentum-transfer variable gives the contribution of diagrams 2(a), 3(a), or 4(a) to the E.M. mass difference. Feynman and Speisman[12] derived the proper relation and used it to show that the nucleon mass difference could be understood if suitable cutoff functions were introduced for the form factors. Marshak and Sudarshan[13] did the same thing for the Σ hyperons and showed that for certain ranges of values of the magnetic moments of the Σ's, the experimental masses could be reproduced. This early work must now be reconsidered as will be shown below.

Riazuddin[14] derived the relation between the boson form factors and the E.M. mass differences using dispersion theory. Cini, Ferrari, and Gatto[15] extended this to fermions and obtained a result identical to that of Feynman and Speisman. In both cases, the usual question of subtractions in the dispersion relations creates an uncertainty whether the derived expressions correctly give the mass differences. These subtraction terms are determined by requiring that the final result becomes equal to the perturbation-theory expression when the form factors are taken to be unity. If this method is accepted, then once the form factors are measured, their contributions to the E.M. mass differences can be evaluated. The only form factors determined so far and, practically speaking, the only ones likely to be measured even in the distant future, are the nucleon form factors. We shall not be able to proceed with our over-all program unless we can derive at least approximate expressions for the form factors of the hyperons and mesons. There are a number of ways to do this, and until a method can be developed to discriminate among them, they are all equally believable.

The first and simplest way to deduce the other form factors is to make use of unitary symmetry and impose the condition that the current operator transforms like the charge—that is, as the $A_1{}^1$ component of a traceless tensor. This yields the relations which were originally given for the magnetic moments[3]:

$$G_{\Sigma^+}=G_p;\quad G_{\Sigma^0}=-\tfrac{1}{2}G_n=(1/\sqrt{3})G_{\Sigma\Lambda}=-\tfrac{1}{2}G_{\Xi^0};$$
$$G_{\Sigma^-}=G_{\Xi^-}=-G_p-G_n;\quad F_{K^+}=F_{\pi^+};\quad F_{K^0}=0.\quad(4.1)$$

Here the G's are the electric and magnetic form factors of the baryons and the F's are the electric form factors of the mesons. $G_{\Sigma\Lambda}$ is the form factor for radiative Σ^0 decay. If we remember to include the diagram for the Σ^0 in which the intermediate state is $\Lambda+\gamma$ and if we neglect rather small corrections due to M.S. mass differences, we obtain for the elastic form-factor contributions to the E.M. mass differences:

$$\Delta_N=\Delta_+;\quad\Delta_\Xi=\Delta_-;\quad m_\pi\Delta_\pi=-m_K\Delta_K\,,\quad(4.2)$$

[11] S. Okubo, Phys. Letters 4, 14 (1963).

[12] R. P. Feynman and G. Speisman, Phys. Rev. 94, 500 (1954).

[13] E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. 106, 599 (1957).

[14] Riazuddin, Phys. Rev. 114, 1184 (1959).

[15] M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters 2, 7 (1959).

independent of the values of the nucleon and meson form factors.

The second and third methods involve constructing the form factors by determining the coupling constants of the vector mesons ρ^0, ω, and ϕ to the particle in question and to the electromagnetic field. If we can determine the SU_3-invariant coupling strengths of the vector mesons to the baryons, and also the strength of the photon–vector-meson interaction from the nucleon form factors, then we can write down the hyperon form factors in terms of these parameters. This assumes that the experimental values for the nucleon form factors are well fitted by three resonance terms with masses at the ρ^0, ω, and ϕ masses. Unfortunately, this is not possible and one or two cutoff masses are also needed.[16] This introduces an arbitrariness in that the strengths of these cutoff terms are not known because their unitary properties are not known. This difficulty does not arise for the charge form factors, because the zero-momentum-transfer values—that is, the electric charges—are known. We can circumvent the difficulty for the magnetic form factors as well only if we know the magnetic moments of the various particles. It may be that departures from exact unitary symmetry are small for the magnetic form factors at zero-momentum transfer. In that case, we can use the usual relations for the magnetic moments[3] ($\mu_{\Sigma^+}=\mu_p$; $\mu_{\Sigma^0}=-\frac{1}{2}\mu_n$; etc.). If this were not true, then we might as well use the first method [i.e., (4.2)], and only consider it an order-of-magnitude estimate.

If we are willing to match the magnetic moments as given by unitary symmetry, then we can allow for cutoff masses in the form-factor expressions. Even under these restrictions we can construct at least two sets of form factors. The first uses particle mixing in the manner of Okubo[17] and Sakurai,[18] and the second used the vector mixing procedure of Coleman and Schnitzer.[19] In Okubo's method, the "bare" particles ω_0 and ϕ_0 are members of a nonet in unitary space described by a tensor $W_\nu{}^\mu = V_\nu{}^\mu + (1/\sqrt{3})\delta_\nu{}^\mu\phi_0$. Here $V_\nu{}^\mu$ is the traceless tensor containing the octet members K^*, $\bar{K}^*$, ρ, and ω_0; and ϕ_0 is a unitary singlet. These are coupled to the baryons by means of D- and F-type couplings and to the pseudoscalar mesons via F-type coupling only. The effective photon–vector-meson coupling is

$$\mathcal{H}_I = g[\rho_\mu{}^0 + (1/\sqrt{3})\omega_{0\mu}]A^\mu, \tag{4.3}$$

where g is the photon–vector-meson coupling constant. The "physical" or observed particles ω and ϕ are related to the bare ones by

$$\omega = (1/\sqrt{3})(\omega_0 + \sqrt{2}\phi_0); \quad \phi = (1/\sqrt{3})(\sqrt{2}\omega_0 - \phi_0). \tag{4.4}$$

We now construct a fit to the nucleon form factors in the following form:

$$G_{E,M}{}^S = \frac{\alpha_{E,M}{}^S}{m_\omega{}^2 - t} + \frac{\beta_{E,M}{}^S}{m_\phi{}^2 - t} + \frac{\gamma_{E,M}{}^S}{M^2 - t},$$
$$G_{E,M}{}^V = \frac{\alpha_{E,M}{}^V}{m_\rho{}^2 - t} + \frac{\beta_{E,M}{}^V}{M^2 - t}, \tag{4.5}$$

where M is a cutoff mass. The particle-mixing method imposes the following two conditions on the parameters: $\alpha_{E,M}{}^S = \frac{1}{3}\alpha_{E,M}{}^V$ and $\beta_{E,M}{}^S = -\frac{2}{3}(1-2f_{E,M})\alpha_{E,M}{}^V$; $f_{E,M}/(1-f_{E,M})$ are the ratios of F- to D-type coupling of the vector-meson nonet to the baryon octet for the electric and magnetic form factors. The resulting values for the parameters are:

$$\begin{aligned} &\alpha_E{}^S = 7.4\ \mathrm{F}^{-2}; && \beta_E{}^S = -132\ \mathrm{F}^{-2}; && \gamma_E{}^S = 124\ \mathrm{F}^{-2}; && M^2 = 25\ \mathrm{F}^{-2};\\ &\alpha_E{}^V = 22.2; && \beta_E{}^V = -25.7; && f_E = -3.95; &&\\ &\alpha_M{}^S = 26.1; && \beta_M{}^S = 62.6; && \gamma_M{}^S = -88.6\ \mathrm{F}^{-2}; &&\\ &\alpha_M{}^V = 78.3; && \beta_M{}^V = -76; && f_M = 1.1. && \end{aligned} \tag{4.6}$$

This fit matches the experimental values and slopes[16] at zero-momentum transfer. It also reproduces the experimental observations[20] up to $t=-100\ \mathrm{F}^{-2}$ except for $G_E{}^P$. At $t=-45\ \mathrm{F}^{-2}$, it yields $G_E{}^P=0.06$ against the experimental value of 0.124 ± 0.040. These discrepancies for large t are not very important for the E.M. mass differences, because the largest contributions to the latter come from the range $-2M_N{}^2<t\leq 0$.

Unfortunately, constructing the form factors on the basis of particle mixing has forced us into a serious difficulty. The large values for the f parameters—1.1 and -3.95—cause some of the hyperon coupling constants to be very large, and this in turn causes the contributions to the E.M. mass differences to be unbelievably large. The actual results using (4.6) and constructing the hyperon form factors via particle mixing are:

$$\Delta_N = -0.80; \quad \Delta_+ = -5.32; \quad \Delta_- = 13.2; \quad \Delta_\Xi = 20.8\ \text{(in MeV)}. \tag{4.7}$$

These poor results make us turn to the vector-mixing theory[19] to construct the hyperon form factors. The

[16] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963).
[17] S. Okubo, Phys. Letters **5**, 165 (1963).
[18] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962); Phys. Rev. **132**, 434 (1963).
[19] S. Coleman and H. Schnitzer, Phys. Rev. **134**, B863 (1964).
[20] K. W. Chen, A. A. Cone, J. R. Dunning, Jr., S. G. F. Frank, N. F. Ramsey *et al.*, Phys. Rev. Letters **11**, 561 (1963).

chief difference between the two methods is that with the vector-mixing method, the bare ϕ_0 meson is considered to have its own independent coupling constant to the baryonic current and all 10 of the parameters α, β, γ are independent. This produces the following fit for the nucleon form factors:

$$\begin{array}{llll} \alpha_E{}^S=15.2\ \mathrm{F}^{-2}; & \beta_E{}^S=-12.3\ \mathrm{F}^{-2}; & \gamma_E{}^S=0; & M^2=22.8\ \mathrm{F}^{-2}; \\ \alpha_E{}^V=24.6; & \beta_E{}^V=-27.4; & & \\ \alpha_M{}^S=20.4; & \beta_M{}^S=-22.7; & \gamma_M{}^S=0; & \\ \alpha_M{}^V=84.9; & \beta_M{}^V=-80. & & \end{array} \tag{4.8}$$

Experiments are not yet sufficiently detailed to fix the parameters $\gamma_{E,M}{}^S$, so that for simplicity we take them to be zero. If they were nonzero, then the values of $\alpha_{E,M}{}^S$ and $\beta_{E,M}{}^S$ would change, perhaps as much as by a factor of two. This would not alter Δ_N much, since the parameters are chosen so that the form-factor expressions match experimentally observed curves, and it is these curves which determine the contribution to Δ_N. The hyperon mass differences could change by one MeV or so because of this uncertainty.

The resulting expressions (4.8) match all the electron scattering experiments quite well; however, they do not satisfy the relations

$$G_M{}^{n,p}(t=+4M_N{}^2)=G_E{}^{n,p}(t=+4M_N{}^2),$$

relations which, as emphasized by Wilson,[21] are necessary in order that the annihilation cross sections for proton-antiproton and neutron-antineutron at rest be isotropic. This is not too serious a fault, because we do not expect the form-factor fits to be exact, but only to describe the experimental observations in a certain region, in this case, $t\leq 0$. Near $t=4M_N{}^2$, large-mass terms will be important, whereas for $t<0$ they are negligible. We do not write out the hyperon form factors here, but refer the reader to the paper of Coleman and Schnitzer[19] for a description of the vector-mixing method.

For the pseudoscalar mesons, the form factors constructed using the vector-mixing method are:

$$\begin{aligned} \tfrac{1}{2}F_\pi=F_K{}^V&=\frac{1}{2}\left(\frac{am_\rho{}^2}{m_\rho{}^2-t}+\frac{(1-a)M^2}{M^2-t}\right),\\ F_K{}^S&=\frac{1}{2}\left(\frac{0.271am_\rho{}^2}{m_\omega{}^2-t}+\frac{1.39am_\rho{}^2}{m_\phi{}^2-t}+\frac{(1-a)M^2}{M^2-t}\right), \end{aligned} \tag{4.9}$$

where M is a cutoff mass taken to be the same for the isoscalar and isovector parts, and a is an unknown parameter. For complete ρ^0 dominance of the vector form factor, $a=1$.

The results for the baryon and meson mass differences are presented in Table II. The first number is the value calculated by using the fit (4.8) for the nucleon form factors and constructing the hyperon form factors through vector mixing. For the mesons, we use (4.9). The number in the parenthesis is the value obtained by still taking (4.8) for Δ_N and the same expression (4.9) for F_π, but using the relations (4.1) to get the hyperon and kaon form factors. In all cases, the charged particles receive larger contributions than the neutral ones because the neutral ones have small charge form factors, and the magnetic moments of the baryons, according to (4.1), are all of the same order of magnitude. The pion mass difference can almost be correctly given if $a=1$, but the kaon mass difference has the wrong sign; indeed, Δ_π is greater than zero and Δ_K is less than zero for all values of a. The quantity $\Delta_- - \Delta_+$ receives a sizeable contribution, near 2 MeV in both cases, and this is essentially the observed value (cf. Table I).

[21] D. G. Wilson (private communication).

5. OTHER CONTRIBUTIONS

If we compute the contributions to the E.M. mass differences of diagrams 2(b) in perturbation theory and use an interaction Hamiltonian for the (π^0-η) transition: $\mathcal{H}_I=m_{\pi\eta}{}^2\phi_{\pi^0}\phi_\eta$, we obtain convergent results. This is because of the k^{-4} dependence of the effective propagator for the (π^0-η) system:

$$G(k^2)=im_{\pi\eta}{}^2/[(k^2-m_\pi{}^2)(k^2-m_\eta{}^2)]. \tag{5.1}$$

If we use the value for $m_{\pi\eta}{}^2$, the (π^0-η) transition (mass)^2, of $-0.16m_\pi{}^2$ derived from unitary symmetry,[22] we obtain undesirable results. All the baryon mass differences receive negative contributions if the meson-baryon couplings are given in the usual SU_3-invariant way with the f parameter at any value greater than 0.25. However, it was pointed out by Hori *et al.*,[23] that the 3π decay amplitude of η vanishes when calculated in this way if the four-boson interaction is assumed to be unitary symmetric. This is because the two diagrams

TABLE II. Elastic form-factor contribution (in MeV).

$\Delta_N=$	-0.85	(-0.85)
$\Delta_+=$	-0.61	(-0.88)
$\Delta_-=$	1.61	(0.93)
$\Delta_\Xi=$	2.30	(1.27)
$\Delta_\pi=$	$6.60-2.91a+0.42a^2$; $=4.11$ at $a=1$ (same)	
$\Delta_K=$	$-2.52+0.29a-0.64a^2$; $=-2.87$ at $a=1$	
	$(-2.52+0.78a-0.09a^2$; $=-1.83$ at $a=1)$	

[22] S. Okubo and B. Sakita, Phys. Rev. Letters **11**, 50 (1963).
[23] S. Hori, S. Oneda, S. Chiba, and A. Wakasa, Phys. Letters **5**, 339 (1963).

TABLE III. ($\pi^0-\eta$) transition contribution (in MeV).

		$f=0.30$	$f=0.35$	$f=0.39$
$\Delta_N=$	$-8.48(1-4f)$	1.70	3.40	4.75
$\Delta_+=$	$16.7f(1-f)$	3.50	3.80	3.96
$\Delta_-=$	$16.7f(1-f)$	3.50	3.80	3.96
$\Delta_\Xi=$	$8.25(1-4f^2)$	5.28	4.20	3.22
$\Delta_\pi=$	0.12	0.12	0.12	0.12
$\Delta_K=$	0.23 (from K'); $\left\{\begin{array}{l}18 \text{ (from } K^*) \\ -32\end{array}\right.$	18 -32	18 -32	18 -32

for η decay, one with an η pole and the other with a π^0 pole, cancel because they differ only by a relative minus sign.

Barrett and Barton[24] remedied the situation by effectively studying the k^2 dependence of the transition (mass)^2, $m_{\pi\eta}^2$. They evaluated the ($\pi^0-\eta$) propagator in the form:

$$G(k^2)=iR_\eta/(k^2-m_\eta^2)+iR_\pi/(k^2-m_\pi^2)+\sigma(k^2)\,, \quad (5.2)$$

where $\sigma(k^2)$ is the contribution of intermediate states other than π^0 and η, and $R_{\eta,\pi}$ are two parameters which they evaluated by relating them to the E.M. mass differences of the baryons and to the meson-baryon coupling constants. The values they find, namely $R_\eta=(2\text{ to }4)\times10^{-3}$ and $R_\pi=-(1\text{ to }2)\times10^{-2}$, can explain the partial width for $\eta\to3\pi$. If we ignore $\sigma(k^2)$, this is equivalent to taking the transition (mass)^2 to be given by:

$$m_{\pi\eta}^2=k^2(R_\eta+R_\pi)-R_\eta m_\pi^2-R_\pi m_\eta^2. \quad (5.3)$$

For most of the range of k^2 ($k^2<m_\eta^2$) this is a positive quantity, in contrast to the constant value $-0.16m_\pi^2$ used previously, and we can expect to obtain positive contributions to the E.M. mass differences. Now the calculation is logarithmically divergent, however, and requires a cutoff. We use a cutoff of one nucleon mass consistently throughout. These results are shown in Table III for three values of f: 0.30, 0.35, and 0.39,[25] [$f/(1-f)$ is the ratio of F- to D-type coupling of the pseudoscalar mesons to the baryons].

For the mass difference, Δ_π, we must evaluate Fig. 3(b). It contributes an amount $m_{\pi\eta}^4/[2m_\pi(m_\eta^2-m_\pi^2)]=0.12$ MeV; this same value is obtained if we use the value $-0.16m_\pi^2$ for $m_{\pi\eta}^2$ or evaluate it from Eq. (5.3) at $k^2=m_\pi^2$.

Figure 4(b) is the relevant diagram for Δ_K. K' is the presumed scalar (K-π) resonance at 725 MeV. Its width was taken as the maximum value allowed by experiment, namely 15 MeV. K^* is the vector resonance at 888 MeV, which has a width of 80 MeV. The couplings invariant under unitary symmetry are:

$$\mathcal{H}_I=g[\bar{K}'(\boldsymbol{\tau}\cdot\boldsymbol{\pi}-\eta/\sqrt{3})K]+\text{c.c.} +ig'[(\tau^{\mu\nu}\bar{K}_\mu^*)(\boldsymbol{\tau}\cdot\boldsymbol{\pi}+\sqrt{3}\eta)\overleftrightarrow{\partial}_\nu K]+\text{c.c.} \quad (5.4)$$

[24] B. Barrett and G. Barton, Phys. Rev. **133**, B466 (1964).
[25] A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964). These authors determined the value 0.39 for f.

The operator $\tau^{\mu\nu}$ is introduced into the ($K^*K\pi$) interaction to maintain invariance under the "gauge" transformation for the K^* field: $K_\mu^*\to K_\mu^*+\partial_\mu\Lambda$, where Λ is a strangeness-carrying gauge field. We impose "gauge invariance" even though the strangeness-changing current to which K^* is coupled is not conserved, mainly to remove a divergence of very high degree. Figure 4(b) is biquadratically divergent if we do not impose "gauge invariance"—that is, take $\tau^{\mu\nu}=g^{\mu\nu}$ in Eq. (5.4), and then the contribution to Δ_K is -32 MeV for a cutoff of one nucleon mass. However, if we make the interaction Hamiltonian "gauge-invariant" by taking the familiar form for $\tau^{\mu\nu}$:

$$\tau^{\mu\nu}=g^{\mu\nu}-\partial^\mu\partial^\nu/\partial^2\,, \quad (5.5)$$

the divergence is reduced to a more acceptable logarithmic one. Unitary symmetric coupling of particles is expected to describe physical reality only in the limit when all unitary multiplets have identical masses. In that case, the currents would be conserved and gauge invariance should be imposed. Since there are mass differences and in this case the K-π mass difference is the one we are ignoring, we can expect to make large errors, but perhaps the sign of the resulting contribution, $\Delta_K=18$ MeV, can be considered to be significant. These two results for Δ_K are also given in Table III.

In analogy with the way we handled the (π^0-η) transition, we should calculate the contributions of Figs. 2(c) and 4(d) using an effective propagator for the (Σ^0-Λ) system:

$$G(k)=iR_\Sigma/(\gamma\cdot k-M_\Sigma)+iR_\Lambda/(\gamma\cdot k-M_\Lambda)+\tau(k)\,, \quad (5.6)$$

where $\tau(k)$ is due to intermediate states in the propagator other than Σ^0 and Λ (e.g., ΞK and $N\bar{K}$), and R_Σ and R_Λ are the residues of the propagator at the Σ^0 and Λ masses, respectively. In the case of the (π^0-η) propagator, Barrett and Barton evaluated the constants $R_{\eta,\pi}$ and the remainder term $\sigma(k^2)$. In the limit of vanishing E.M. mass differences and vanishing π^0-η mass difference, $\sigma(k^2)\to0$. In this limit, if we use the theorem that the integral of the spectral function for the propagator for two different fields over all allowed values for the mass should vanish (see Ref. 24), we would have $R_\eta+R_\pi=0$. We would expect, therefore, R_η to approach $-R_\pi$ if we could decrease the π^0-η mass difference [of course, $G(k^2)$ would vanish in this limit—

TABLE IV. ($\Sigma^0-\Lambda$) transition contribution (in MeV).

		$f=0.30$	$f=0.35$	$f=0.39$
$\Delta_N=$	$0.432(1-4f^2)$	0.28	0.22	0.17
$\Delta_+=$	$1.12f(1-f)$	0.23	0.25	0.27
$\Delta_-=$	$1.12f(1-f)$	0.23	0.25	0.27
$\Delta_\Xi=$	$-2.46(1-4f)$	0.49	0.98	1.38
$\Delta_\pi=$	0	0	0	0
$\Delta_K=$	$6.48-10.74f-15.17f^2$	1.89	0.86	−0.01

cf. Eq. (5.2)]. In the case of the (Σ^0-Λ) propagator, in which the mass difference between Σ^0 and Λ is only about 8% of the Λ mass, the approximation $R_\Sigma=-R_\Lambda$ should not be bad. If we neglect the remainder term, then the propagator becomes:

$$G(k)=iR_\Sigma(M_\Sigma-M_\Lambda)/[(\gamma\cdot k-M_\Sigma)(\gamma\cdot k-M_\Lambda)]. \quad (5.7)$$

This is the same thing as would be obtained by using an effective Hamiltonian for the (Σ^0-Λ) transition:

$$\mathcal{H}_I=M_{\Sigma\Lambda}(\bar\psi_{\Sigma^0}\psi_\Lambda+\bar\psi_\Lambda\psi_{\Sigma^0}) \quad (5.8)$$

if the transition mass, $M_{\Sigma\Lambda}$, is taken to be equal to $R_\Sigma(M_\Sigma-M_\Lambda)$. Unitary symmetry gives a value[6] for $M_{\Sigma\Lambda}$ of $(1/\sqrt{3})(\Delta_+-\Delta_N)=0.90$ MeV; we use this value and take the usual meson-baryon coupling constants. The resulting logarithmically divergent integrals are cut off at one nucleon mass and the results are shown in Table IV; again for $f=0.30$, 0.35, 0.39. As was mentioned in Sec. 3, there is a contribution to Δ_π from baryon-antibaryon loops which have two (Σ^0-Λ) transitions, but this will be of the order of $M_{\Sigma\Lambda}^2/2m_\pi\approx3\times10^{-3}$ MeV and we shall neglect it.

The contributions to the E.M. mass differences from the (ρ^0-ω) or (ρ^0-ϕ) transitions are shown in Figs. 2(d) and 4(c); Fig. 3(e) is of higher order in the E.M. interaction. We do not evaluate the (ρ^0-ω) and (ρ^0-ϕ) transitions for three reasons: First, as was mentioned earlier, if these transitions are dominated by a photon pole, we have already included them in the elastic form factor part. In a recent work, Zimerman, Riazuddin, and Okubo[26] derived the value for the (ρ^0-ω_0) transition (mass)^2 (ω_0 is the bare ω and member of an octet) of $-(0.11$ to $0.14)m_\pi^2$. The contribution to this transition (mass)^2 from the photon pole is:

$$m_{\rho\omega_0}^2=-g_{\rho\gamma}^2/\sqrt{3}\bar{m}^2\approx-m_\rho^2/8\pi\sqrt{3}=-0.69m_\pi^2, \quad (5.9)$$

where $\bar{m}$ is the average of the ρ^0 and ω_0 masses and $g_{\rho\gamma}$ is the photon-ρ^0 coupling constant. We evaluate $g_{\rho\gamma}$ by assuming ρ^0 dominance of the pion form factor; its value is $-m_\rho^2/g_{\rho\pi\pi}$, where $g_{\rho\pi\pi}$ is the coupling strength of ρ^0 to $\pi^+-\pi^-(g_{\rho\pi\pi}^2/4\pi\approx2)$. From this, we see that even in the worst case, omitting the (ρ^0-ω_0) contribution should cause errors of the order of the elastic form-factor parts which, except for Δ_N, are already uncertain by an MeV or so. Another reason for omitting the (ρ^0-ω) and (ρ^0-ϕ) transitions is that the intermediate states in Fig. 2(d) are heavier than those from the (Σ^0-Λ) transition, and the results in that case were already relatively small; hence we can expect these to be smaller. Finally, the calculation would be highly divergent and little reliance could be placed on it.

With regard to the self-induced mass differences, Katsumori[27] has calculated these effects for both the baryons and mesons. We use his calculations, except that we insert the meson-baryon coupling constants predicted by unitary symmetry and also use more recent values of the E.M. mass differences. These results, as a function of the f parameter, are given in Table V.

TABLE V. Self-induced mass differences (in MeV).

		$f=0.30$	$f=0.35$	$f=0.39$
$\Delta_N=$	$1.49-11.7f+7.86f^2$	−1.31	−1.65	−1.87
$\Delta_+=$	$2.08-8.34f+5.19f^2$	0.65	−0.20	−0.38
$\Delta_-=$	$-1.22-1.74f+13.4f^2$	−0.53	−0.19	0.14
$\Delta_\Xi=$	$1.18+1.83f+0.36f^2$	1.76	1.86	1.94
$\Delta_\pi=$	$11.7-23.4f-21.2f^2$	2.77	0.91	−0.65
$\Delta_K=$	$-3.3+132f-148f^2$	23.0	24.8	25.6

6. RESULTS AND CONCLUSIONS

Table I shows the total contributions from the four processes that we have considered. We feel that in a consistent evaluation of the meson mass differences the major contributions should come from the lightest intermediate states. This is well borne out by the fact that the elastic form-factor part yields almost the entire observed pion mass difference. There are many states for the mesons lighter than the baryon-antibaryon loops appearing in the (Σ^0-Λ) transition and self-induced contributions, which we have not evaluated. Therefore, we feel that it would be inconsistent to include these contributions to Δ_π and Δ_K. Accordingly, in Table I, we only include the results of Tables II and III for the meson mass differences; the effects of Tables IV and V for Δ_π and Δ_K will be discussed below in any case.

Examination of Table I shows that the situation for Δ_π is very satisfactory; the same result for Δ_π arises as that calculated earlier[9] when ρ^0 dominance of the vector form factor was assumed. If the parameter a had the value 0.85 (nearly complete ρ^0 dominance), the pion mass difference would be matched even more exactly. If baryon-antibaryon heavy mass states are included for Δ_π, the results are nearly unchanged (cf. Table IV); even the self-induced contribution (cf. Table V) leaves this conclusion essentially unchanged provided that f is in the vicinity of 0.35.

On the other hand, the kaon mass difference is the

[26] A. H. Zimerman, Riazuddin, and S. Okubo, Nuovo Cimento (to be published).

[27] H. Katsumori, Progr. Theoret. Phys. (Kyoto) 24, 35 (1960).

most difficult to explain using our procedure. The contribution of the (π^0-η) transition (cf. Table III) is essentially indeterminate because of the range of values (between 18 and -32 MeV) permitted by the method of calculation. Even if a large negative value were the actual contribution from this process, there could still be a large positive contribution from the self-induced process (cf. Table V) which would make the value of Δ_K positive. While this is not a satisfactory state of affairs, the possibility has at least been opened up of explaining the unusual sign of the kaon mass difference.

With regard to the baryon mass differences, it is evident that the inelastic contributions [particularly from the (π^0-η) transition] play an important role. These inelastic contributions give a marked improvement over the elastic form-factor results (cf. Table II), and actually yield quite good predictions for the four baryon mass differences when $f=0.35$. The largest discrepancy is for Δ_Ξ. One solution which might be considered is that the (Ξ-K) couplings are not as large as those predicted by unitary symmetry (although there is no real evidence for this). For example, if for $f=0.35$ we uniformly reduce all K-baryon coupling constants by a factor $g_K^2 \rightarrow \frac{1}{3}g_K^2$, the resulting baryon mass differences would be:

$$\Delta_N=1.75;\quad \Delta_+=3.73;\quad \Delta_-=5.98;\quad \Delta_\Xi=7.38. \qquad (6.1)$$

From (6.1) it is seen that Δ_Ξ now comes within the measured range of values at the expense of Δ_N, Δ_+ and Δ_-. In any case it is encouraging that when inelastic contributions are taken into account, the signs and magnitudes of the four baryon mass differences can be understood.

Coleman and Glashow[7] and, more recently, Coleman and Schnitzer[28] have calculated the six E.M. mass differences assuming that there are two contributions. The first is the tadpole diagram of Fig. 1(f) where the particle exchanged is postulated to be a neutral isovector scalar meson called $\pi^{0\prime}$. The second, called the nontadpole contribution, is assumed to receive its major share from the elastic form-factor process. In this way, using the strength of the $\pi^{0\prime}$ tadpole as a free parameter, the authors attempt to match all the mass differences and do obtain reasonable agreement, especially since the 2-MeV isotensor contribution[29] to the Σ mass differences from the elastic form-factor part is now consistent with the latest measurements of the Σ masses.

One may inquire further whether the fit between theory and experiment can be improved by adding the tadpole diagram to the diagrams which we have considered. Since the tadpole diagram effectively adds amounts to the baryon mass differences of Table I in the ratios 1:1.5:1.5:2 (cf. Coleman and Schnitzer's paper[28]), it is possible to improve the fit by working with $f=0.39$; the total values obtained in this way would be:

$$\Delta_N=1.29;\quad \Delta_+=1.88;\quad \Delta_-=4.62;\quad \Delta_\Xi=7.02; \qquad (6.2)$$

where the strength of the tadpole contribution is determined to match Δ_N. Despite the availability of one additional parameter, these values of the E.M. mass differences for $f=0.39$ with the tadpole diagram are only slightly better than those for $f=0.35$ with the tadpole contribution not included.

From these results, we feel justified in saying that it is possible to dynamically evaluate the E.M. mass differences in a consistent way without invoking the tadpole mechanism. Our (π^0-η) transition diagram in particular plays an analogous role to that of the tadpole diagram in emphasizing the isovector contribution to the mass differences. But the point is that the (π^0-η) diagram must be present, whereas the tadpole mechanism is involed in a purely *ad hoc* way.[30] If one believes that scalar tadpoles give sizeable contributions to the mass differences, then our calculations show that other processes besides the elastic form-factor part contribute significantly to the nontadpole portion. Socolow[31] has reached a similar conclusion on the basis of other considerations. He has evaluated the contributions of the decuplet to the baryon mass differences [Fig. 1(b), where the box is represented by a spin-$\frac{3}{2}^+$ resonance]; adding the elastic form-factor contributions to his results, he was able to obtain a reasonable fit to the baryon mass differences (except for the nucleon mass difference). However, the agreement for the hyperon mass differences would be substantially destroyed if the tadpole mechanism is the one responsible for correcting the nucleon mass difference. While we assess differently the relative importance of the decuplet diagram compared to some of those which we have considered, Socolow's calculations, in conjunction with ours, underline the need to properly evaluate the nontadpole diagrams in order to decide whether the tadpole diagram enters the picture at all.

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[28] S. Coleman and H. Schnitzer (to be published).

[29] Until the latest experimental values for the Σ masses came along, the proponents of the tadpole point of view were faced with the conflict between the apparent equal Σ mass spacing and the nonnegligible isotensor contribution; this obstacle is now overcome.

[30] The extension of the idea of "tadpole dominance" to nonleptonic weak interactions runs into difficulty with the Cabibbo scheme [cf. Coleman and Glashow (Ref. 9), p. B679] and hence loses some of its attractiveness as a general symmetry-breaking mechanism for the medium strong, electromagnetic and weak interactions.

[31] R. Socolow, thesis, Harvard University, 1964 (unpublished).