

Further Analysis of the Decay $\pi \rightarrow e + \nu + \gamma^*$

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We have recalculated the structure-dependent part of the decay $\pi \rightarrow e + \nu + \gamma$, using the conserved vector current hypothesis, in order to derive information on the axial-vector-current form factor. Earlier calculations of Bludman and Young have been extended by including, as a phenomenological parameter, the axial-vector contribution relative to polar-vector, and by calculating integrals of the decay rate over various portions of phase space likely to be measured. The experiments can be analyzed in terms of the $\pi\rho$ coupling to the axial-vector current. We conclude that the recent experiments of Depommier *et al.* establish the presence of such axial-vector structure contributing to the decay, but that one cannot assign this component to a weak-interacting vector meson rather than to strong-interaction structure.

I. INTRODUCTION

THE radiative pion decay $\pi^- \rightarrow l^- + \nu + \gamma$ (where l is e or μ) has been studied as a source of information on weak-interaction structure by a number of authors.¹⁻⁵ The earlier work by one of us emphasized the possibility of distinguishing structure-dependent (SD) radiation from uninteresting inner bremsstrahlung, and of testing the conserved-vector-current hypothesis. With these aims in mind, the simplest assumptions were made about the axial-vector-current contribution, based upon the idea that nucleons made the principal contribution to this current. Since then, meson states of lower mass, which are more important, have been discovered. The conserved-vector-current (CVC) theory has been established, and experiments have been performed⁶ demonstrating the presence of an axial-vector contribution.

The present work is therefore devoted to an analysis of the axial-vector-current contribution and of the information on it that is obtainable in radiative decay experiments. Besides such experiments, the axial-vector current can also be studied through neutrino experiments.

II. AXIAL-VECTOR-CURRENT CONTRIBUTION TO RADIATIVE DECAY

The radiative decay amplitude can be written as a sum of three terms which are separately gauge-invariant:

$$\langle \gamma l \nu \text{ out} | \pi^- \rangle = \text{IB} + \text{SDA} + \text{SDV}, \quad (2.1)$$

where IB is the inner bremsstrahlung term, SDA is the structurally-dependent axial-vector term, and SDV

is the structurally-dependent vector term:

$$\begin{aligned} \text{IB} &= m f_\pi e \left(\frac{m_l m_\nu}{E_l E_\nu 4 P_0 k_0} \right)^{1/2} \\ &\times \bar{u}(l) \left[\frac{\not{p} \cdot \epsilon^*}{\not{p} \cdot k} - \frac{P \cdot \epsilon^*}{P \cdot k} + \frac{i \sigma_{\mu\nu} F_{\mu\nu}}{4 \not{p} \cdot k} \right] (1 + \gamma_5) v(\nu), \quad (2.2) \end{aligned}$$

$$\text{SDA} = (G/\sqrt{2}) \langle \gamma | A_\mu(0) | \pi \rangle l_\mu, \quad (2.3)$$

$$\text{SDV} = (G/\sqrt{2}) \langle \gamma | V_\mu(0) | \pi \rangle l_\mu. \quad (2.4)$$

[Our metric is one in which the fourth component of a four-vector is imaginary. P, k, \not{p}, P^ν and $\mu, 0, m, 0$ are the four-momenta and masses of π, γ, l, ν , respectively. ϵ_μ is the photon polarization vector. $F_{\mu\nu} = (2k_0)^{-1/2} \times (\epsilon_\mu^* k_\nu - \epsilon_\nu^* k_\mu)$, $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F_{\lambda\rho}$, $l_\mu = (m_l m_\nu / E_l E_\nu)^{1/2} \bar{u}(l) \times \gamma_\mu (1 + \gamma_5) v(\nu)$, $\sigma_{\mu\nu} = \frac{1}{2} i (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$. Our units are such that $e^2/4\pi = \alpha = 1/137$, $GM_p^2 = 1.02 \times 10^{-5}$.]

In the above expression, f_π is the amplitude for nonradiative π decay, related to the $\pi \rightarrow l + \nu$ partial-decay rate by

$$W_{l\nu} = (f_\pi^2/4\pi) \mu m^2 [1 - (m/\mu)^2]^2. \quad (2.5)$$

In (2.3), we exclude the contribution of the one-pion intermediate state because this is included in IB.

The polar- and axial-vector-current matrix elements

$$\langle \gamma | V_\mu(0) | \pi \rangle = i a (2P_0)^{-1/2} \tilde{F}_{\mu\nu} P_\nu, \quad (2.6)$$

$$\langle \gamma | A_\mu(0) | \pi \rangle = i b (2P_0)^{-1/2} F_{\mu\nu} P_\nu, \quad (2.7)$$

define form factors a, b , which are functions of the momentum transfer $s = -(P-k)^2$, and which are real, assuming time reversibility.

The CVC hypothesis implies^{1,4,5}

$$|a(0)| = 4(2\pi W_{\pi^0})^{1/2} e^{-1} \mu^{-3/2}, \quad (2.8)$$

where W_{π^0} is the rate of the decay $\pi^0 \rightarrow \gamma + \gamma$. Neglecting any momentum dependence of $a(s)$, SDV and IB have now been determined.

In the next section we will sketch what enters into a calculation of $b(s)$. First, however, we extend the results of Ref. 1 by computing various quantities in terms of a phenomenological $\tilde{b}(s)$.

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¹ S. A. Bludman and J. A. Young, Phys. Rev. **118**, 602 (1960).

² D. E. Neville, Phys. Rev. **124**, 2037 (1961).

³ V. F. Muller, Z. Physik **172**, 224 (1963).

⁴ V. F. Muller, Z. Physik **173**, 438 (1963).

⁵ V. G. Vaks and B. L. Ioffe, Nuovo Cimento **10**, 342 (1958).

⁶ P. Depommier, J. Heintze, C. Rubbia, and V. Soergel, Phys. Letters **7**, 285 (1963).

In place of $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$, we introduce $\mathfrak{F}_{\mu\nu}^+ = F_{\mu\nu} - \tilde{F}_{\mu\nu}$ and $\mathfrak{F}_{\mu\nu}^- = F_{\mu\nu} + \tilde{F}_{\mu\nu}$, which are amplitudes for positive and negative helicity photons, respectively. In terms of these noninterfering amplitudes and the parameter $\gamma(s) \equiv b(s)/a(s)$, Eq. (2.1) reads

$$\langle \gamma l \nu, \text{out} | \pi^- \rangle = \frac{iGa}{2P_0^{1/2}} \left[\left(\frac{1+\gamma}{2} \right) \mathfrak{F}_{\mu\nu}^- - \left(\frac{1-\gamma}{2} \right) \mathfrak{F}_{\mu\nu}^+ \right] P_\nu l_\mu \\ + \frac{ef_\pi m}{2(2P_0)^{1/2}} \left(\frac{m_\nu m_\nu}{E_l E_\nu} \right)^{1/2} (\mathfrak{F}_{\mu\nu}^+ + \mathfrak{F}_{\mu\nu}^-) \bar{u}(l) \left[\frac{\not{p}_\mu P_\nu}{(p \cdot k)(P \cdot k)} + \frac{i\sigma_{\mu\nu}}{4p \cdot k} \right] (1 + \gamma_5) v(\nu). \quad (2.9)$$

From Eqs. (2.8) and (2.9), the differential decay rate with respect to photon energy k and electron energy E is

$$\frac{d^2W}{dx dy} = A_{\text{IB}} \left[\frac{1-y+(m/\mu)^2}{x^2(x+y-1-m^2/\mu^2)} \right] \left[x^2 + 2(1-x)(1-m^2/\mu^2) + \frac{2x(m/\mu)^2(1-m^2/\mu^2)}{x+y-1-m^2/\mu^2} \right] \\ + A_{\text{SD}} \{ (1+\gamma)^2 [x+y-1-(m/\mu)^2] [(x+y-1)(1-x)-(m/\mu)^2] \\ + (1-\gamma)^2 [1-y+(m/\mu)^2] [(1-x)(1-y)+(m/\mu)^2] \} \\ + A_{\text{INT}} \left[\frac{1-y+(m/\mu)^2}{x(x+y-1-m^2/\mu^2)} \right] \{ (1+\gamma) [(1-x)(1-x-y)+(m/\mu)^2] \\ + (1-\gamma) [x^2 - (1-x)(1-x-y) - (m/\mu)^2] \}, \quad (2.10)$$

where $x \equiv 2k/\mu$, and $y \equiv 2E/\mu$ have the range

$$2m/\mu \leq y \leq 1 + (m/\mu)^2$$

$$1 - (y/2) - \frac{1}{2} [y^2 - 4(m/\mu)^2]^{1/2} \\ \leq x \leq 1 - (y/2) + \frac{1}{2} [y^2 - 4(m/\mu)^2]^{1/2}, \quad (2.11)$$

and

$$A_{\text{IB}} = \frac{W_{l\nu} \mu^2 \alpha}{2\pi m^2 [1 - (m/\mu)^2]} \left(\frac{m}{\mu} \right)^2, \\ A_{\text{SD}} = \frac{G^2 \mu^4 W_{\pi^0}}{4\alpha (2\pi)^3}, \\ A_{\text{INT}} = \frac{G\mu^3 (W_{\pi^0} W_{l\nu})^{1/2}}{(2\pi)^2 m [1 - (m/\mu)^2]} \frac{af_\pi}{|af_\pi|} \left(\frac{m}{\mu} \right)^2. \quad (2.12)$$

Using $W_{\mu\nu} = (1/2.55) \times 10^8 \text{ sec}^{-1}$ ⁷ and $W_{\pi^0} = 10^{16} \text{ sec}^{-1}$,⁸ we obtain

$$A_{\text{SD}} = 69 \text{ sec}^{-1}, \\ A_{\text{IB}} = 6300 (m/\mu)^2 A_{\text{SD}}, \\ A_{\text{INT}} = 160 (m/\mu)^2 A_{\text{SD}}. \quad (2.13)$$

For the electron mode, the interaction (INT) terms are small compared with SD, except in the region where the denominator is small (small x and y , or angle between \mathbf{k} and \mathbf{p} small); in this region, IB will also become large, and will dominate over both INT and SD. Thus, for the electron mode, the INT term may be

neglected. In what follows, only SD terms will be considered, and the approximation $(m/\mu)^2 \approx 0$ will be made.

Equation (2.10) then becomes symmetric between electron and neutrino energies y and $z \equiv 2E_\nu/\mu = 2 - x - y$:

$$d^2W_{\text{SD}}/dx dy = A_{\text{SD}} [(1+\gamma)^2 (1-z)^2 (1-x) \\ + (1-\gamma)^2 (1-y)^2 (1-x)]. \quad (2.14)$$

The terms proportional to $(1+\gamma)^2$ and $(1-\gamma)^2$ in d^2W are due to photons of left and right helicity, respectively. These contributions vanish respectively for neutrinos and for electrons of maximum energy. This is a consequence of the left helicity of the electron and the right helicity of the neutrino in π^- decay. If one of the leptons is of maximum energy, then both the other lepton and the photon must be emitted in the direction opposite to it. Angular-momentum conservation then requires that the photon spin be opposite to the total lepton spin. The photon helicity must always be the same in sign as that of the lepton oppositely emitted. Thus, only a left-helicity photon can be emitted when the *electron* is moving antiparallel to the photon direction; for the emission of a right-helicity photon to occur, the *neutrino* must be moving antiparallel to the photon. As a result of this, the $(1-\gamma)^2$ term cannot contribute substantially except in the region of small y , where IB dominates.

In any experiment, the partial decay rate d^2W_{SD} integrated over certain energy or angular intervals is measured. In obtaining the following formulas, the dependence of γ on s , or the photon energy k , is neglected. From Eq. (2.14), the SD decay rate for all

⁷ W. H. Barkas and A. H. Rosenfeld, University of California Radiation Laboratory Report UCRL-8030 Rev. (unpublished).

⁸ G. von Dardel, C. Dekkers, R. Mermod, J. D. Van Putten, M. Vivargent, G. Weber, and K. Winter, Phys. Letters 4, 51 (1963).

events such that $x \geq X$, $y \geq Y$ is

$$\int_X^1 dx \int_Y^1 dy (d^2 W_{SD}/dxdy) = A_{SD} \left\{ (1-\gamma)^2 \frac{1}{8} (1-X)^2 (1-Y)^3 \right. \\ \left. + (1+\gamma)^2 \frac{1}{12} (1-X)^2 (1-Y) [1-2X+3X^2+2(2X-1)Y+2Y^2] \right\} \quad (2.15)$$

for $X+Y \geq 1$.

The differential decay rate with respect to electron energy and the angle between electron and photon momenta $d^2 W/dxd\Omega$ has been previously given.⁹ The differential decay rate per unit solid angle for those events at a given angle for which $x \leq X$ is

$$\int_0^X dx \left(\frac{d^2 W_{SD}}{d\Omega} \right) = \frac{A_{SD}}{\pi} \left\{ \frac{(1+\gamma)^2 X^3}{2\lambda^3 (1-\lambda X)^3} \left[(35/6) + \frac{1}{8} \lambda (-10+35X) + \lambda^2 (15+20X - (7/4)X^2) \right. \right. \\ \left. \left. + \frac{1}{4} \lambda^3 (- (32/3) - 30X + 8X^2 - (7/6)X^3) + \frac{1}{2} \lambda^4 \left(\frac{1}{3} + 4X - 3X^2 + \frac{1}{3} X^3 - \frac{1}{4} X^4 \right) \right] \right. \\ \left. + \frac{(1+\gamma)^2}{8\lambda^4} (1-\lambda) (-35+45\lambda-7\lambda^2+\lambda^3) \left[\frac{X^2}{(1-\lambda X)^2} - \frac{2}{\lambda^2} \left(\ln(1-\lambda X) + \frac{\lambda X}{1-\lambda X} \right) \right] \right. \\ \left. + \frac{(1-\gamma)^2 (1-\lambda)^2 X^3}{2(1-\lambda X)^3 \lambda^3} \left[- (5/3) + \frac{1}{4} \lambda (- (16/3) - 5X) - \frac{1}{2} \lambda^2 (-\frac{1}{3} - 2X + \frac{1}{2} X^2) \right] \right. \\ \left. + \frac{(1-\gamma)^2}{8\lambda^4} (1-\lambda)^2 (-10+8\lambda-\lambda^2) \left[\frac{X^2}{(1-\lambda X)^2} - \frac{2}{\lambda^2} \left(\ln(1-\lambda X) + \frac{\lambda X}{1-\lambda X} \right) \right] \right\}, \quad (2.16)$$

where $\lambda = \sin^2(\theta/2)$, $\theta =$ angle between electron and photon momenta.

Integrated over all electron energies, we have

$$dW_{SD}/d\Omega = \int_0^1 dx (d^2 W_{SD}/dxd\Omega) = (A_{SD}/4\pi) \left\{ (1+\gamma)^2 \left[\frac{1}{12} \lambda^{-5} (420 - 750\lambda + 380\lambda^2 - 47\lambda^3) \right. \right. \\ \left. \left. + \lambda^{-6} (1-\lambda) (-\lambda^3 + 15\lambda^2 - 45\lambda + 35) \ln(1-\lambda) \right] + (1-\gamma)^2 (1-\lambda) \left[\frac{1}{3} \lambda^{-5} (30 - 39\lambda + 10\lambda^2) \right. \right. \\ \left. \left. + \lambda^{-6} (1-\lambda) (10 - 8\lambda + \lambda^2) \ln(1-\lambda) \right] \right\}. \quad (2.17)$$

The $(1-\gamma)^2$ and $(1+\gamma)^2$ terms vanish at $\lambda=1$ ($\theta=180^\circ$) and at $\lambda=0$ ($\theta=0^\circ$), respectively, because of angular momentum conservation, as previously discussed.

The total decay rate for all events with $\lambda > \Lambda$ ($\theta > \Theta$ where $2\Lambda = 1 - \cos\Theta$) is

$$\int_{\lambda > \Lambda} d\Omega (dW_{SD}/d\Omega) = 2\pi \int_{\Lambda}^1 2d\lambda (dW_{SD}/d\Omega) = (A_{SD}/2) \left\{ (1+\gamma)^2 \left[\frac{1}{6} \Lambda^{-4} (1-\Lambda) (84 - 114\Lambda + 34\Lambda^2 - \Lambda^3) \right. \right. \\ \left. \left. + 2\Lambda^{-5} (1-\Lambda)^2 (7 - 6\Lambda + \Lambda^2) \ln(1-\Lambda) \right] + (1-\gamma)^2 \left[(1-\Lambda)^2 \Lambda^{-4} (4 - 4\Lambda + \frac{1}{3} \Lambda^2) \right. \right. \\ \left. \left. + 2\Lambda^{-5} (1-\Lambda)^3 (2-\Lambda) \ln(1-\Lambda) \right] \right\}. \quad (2.18)$$

III. CONCLUSION

A. Phenomenological Analysis

Assuming the validity of the conserved vector-current hypothesis [Eq. (2.8)], an experiment on structure-dependent $\pi \rightarrow e + \nu + \gamma$ is essentially a measurement of the parameter γ , i.e., the amount of axial structure. Since W_{SD} depends on $(1+\gamma)^2$ and $(1-\gamma)^2$, a single measurement of the SD radiation leads to an equation quadratic in γ which has two solutions. One might hope to determine a single γ by looking at detailed energy spectra and angular distributions, or by measuring the circular polarization of the photon. (The various equations of the preceding section may be useful in this respect.) However, this is likely to be difficult since, as previously noted, the $(1-\gamma)^2$ contribution is large only in the region where IB is large and dominating the entire SD. This means that one is measuring mainly the magnitude of $(1+\gamma)^2$, which will yield two possible γ 's of different absolute values.

⁹ Equation (10) in Ref. 1. All the formulas in Ref. 1 from Eq. (9) on are in error and should be corrected by multiplication by a factor 2 (i.e., replace G_V^2 there by $2G_V^2$). If $W_{\pi^0} = 0.5 \times 10^{16} \text{ sec}^{-1}$, $(d^2 W_{SD}/dxd\Omega)_{180^\circ}/W_{\pi^0} = 7.0 \times 10^{-8} (1+\gamma)^2/4 \text{ sr}^{-1}$. Figure 2 of Ref. 1 is substantially correct with this value of W_{π^0} .

For the energy interval studied by Depommier *et al.*,⁶ for example, Eq. (2.15) gives

$$\iint d^2W_{SD} = [5.4 \times 10^{-9}(1+\gamma)^2 + 0.4 \times 10^{-9}(1-\gamma)^2] W_{\mu\nu} \quad \text{if } W_{\pi^0} = 0.5 \times 10^{16} \text{ sec}^{-1}, \quad (3.1)$$

$$\iint d^2W_{SD} = [10.9 \times 10^{-9}(1+\gamma)^2 + 0.8 \times 10^{-9}(1-\gamma)^2] W_{\mu\nu} \quad \text{if } W_{\pi^0} = 1 \times 10^{16} \text{ sec}^{-1}. \quad (3.2)$$

Depommier *et al.* quote

$$\begin{aligned} \gamma = 1.0 \quad \text{or} \quad -2.7 \quad \text{if } W_{\pi^0} = 0.5 \times 10^{16} \text{ sec}^{-1}, \\ \gamma = 0.4 \quad \text{or} \quad -2.1 \quad \text{if } W_{\pi^0} = 1 \times 10^{16} \text{ sec}^{-1}. \end{aligned} \quad (3.3)$$

As mentioned above, a single experiment is unable to choose between the two possible solutions for γ . The fact, emphasized by Depommier *et al.*, that one of the γ values (+0.4) obtained is compatible with a weakly interacting vector boson of mass ≈ 1400 MeV and no strong axial structure, may be suggestive, but must be regarded with great caution. Another γ value which fits the same experiment (-2.1) is compatible with no strong axial structure only if the boson mass is ≈ 600 MeV.¹⁰ More importantly, the abundance of meson resonances suggests the possibility of important strong interaction contributions to $\langle \gamma | A_\mu(0) | \pi \rangle$. We will now estimate how such mesons may contribute.

B. Strong Interaction Contributions to SDA

Any analysis of the matrix element (2.7) depends upon a knowledge of states n contributing to the weak axial-vector current and the electromagnetic coupling of these states. The following analysis is in the spirit of current pole-dominance weak-interaction calculations and is meant to be illustrative only.

We assume, as is reasonable for a vertex function, an unsubtracted dispersion relation for $\langle \gamma | A_\mu(0) | \pi \rangle$ or $\langle 0 | A_\mu(0) | \pi \gamma \rangle$. Then the absorptive part is given by

$$\text{Abs} \langle 0 | A_\mu(0) | \pi \gamma \rangle = - (2\pi)^4 i \sum_n \langle 0 | A_\mu(0) | n \rangle \times n | T | \pi \gamma \rangle \delta^4(P+k-Q_n), \quad (3.4)$$

where T is the electromagnetic transition operator. This weight function receives contributions from intermediate states n of $J=1^+$ and $T=1$, $G=-1$ (assuming normal G -conjugation behavior for the axial-current operator).

The recently discovered¹¹ A meson of mass $M_A = 1090$ MeV decays into $\pi + \rho$, and not into $K + \bar{K}$, so that it

¹⁰ The values of γ quoted in Ref. 6 if $W_{\pi^0} = 0.5 \times 10^{16} \text{ sec}^{-1}$ (1.0, -2.7) yield, with no strong axial structure, boson masses of 1000 and 610 MeV, respectively.

¹¹ G. Goldhaber, J. L. Brown, S. Goldhaber, J. A. Kadyk, B. C. Shen, and G. H. Trilling, Phys. Rev. Letters 12, 336 (1964). The spin-parity assignment 0^- is apparently preferred over 1^+ according to the Dalitz plot analysis by S. U. Chung, O. I. Dahl, L. M. Hardy, R. I. Hess *et al.*, Phys. Rev. Letters 12, 621 (1964).

has $G=-1$ and $T=1$ and spin-parity $0^-, 1^+, 2^-$. Suppose it is 1^+ , that its width ($\Gamma_A = 150$ MeV) can be neglected, and that its pole dominates the dispersion relation for $\langle 0 | A_\mu(0) | \pi \gamma \rangle$. Then if

$$\langle A | T | \pi \gamma \rangle = ie(4P_0Q_0)^{-1/2} M_A^{-1} \Lambda_{A\pi\gamma} F_{\mu\nu} \epsilon_\mu^{*\prime} Q_\nu, \quad (3.5)$$

$$\langle 0 | A_\mu(0) | A \rangle = (2Q_0)^{-1/2} M_A^2 \Lambda_{A\nu} \epsilon_\mu' \quad (3.6)$$

(where Q_ν and ϵ_μ' are the momenta and polarization of the A meson, and the Λ 's are appropriate coupling constants), we obtain

$$b(s) = e \Lambda_{A\nu} \Lambda_{A\pi\gamma} M_A / (M_A^2 - s) \approx e \Lambda_{A\nu} \Lambda_{A\pi\gamma} M_A^{-1}. \quad (3.7)$$

The coupling constant $\Lambda_{A\pi\gamma}$ can in turn be related to the coupling of the A to $\pi\rho$:

$$\begin{aligned} \langle A | T | \pi \rho \rangle = i(8Q_0P_0k_0'')^{-1/2} M_A^{-1} \\ \times \{ \Lambda_{A\pi\rho} [(k'' \cdot Q)(\epsilon'^* \cdot \epsilon'') - (k'' \cdot \epsilon'^*)(Q \cdot \epsilon'')] \\ + X_{A\pi\rho} [k''^{*\prime 2} (\epsilon'^* \cdot \epsilon'')] \}. \end{aligned} \quad (3.8)$$

Here k_μ'' and ϵ_μ'' are the momentum and polarization of the ρ . The form of (8) assumes conservation of the ρ current ($\partial_\mu j_\mu^\rho = 0$); it corresponds to:

$$\begin{aligned} \langle A | j_\mu^\rho | \pi \rangle \propto \Lambda_{A\pi\rho} [(k'' \cdot Q) \epsilon_\mu^{*\prime} - (k'' \cdot \epsilon'^*) Q_\mu] \\ + X_{A\pi\rho} [k''^{*\prime 2} \epsilon_\mu^{*\prime} - (k'' \cdot \epsilon'^*) k_\mu], \end{aligned} \quad (3.8a)$$

with $\epsilon'' \cdot k'' = 0$. Form factors $\Lambda_{A\pi\gamma}$, $X_{A\pi\gamma}$ can be defined for $\langle A | j_\mu^{em} | \pi \rangle$ analogously to (3.8a); these will be functions of $t \equiv -k^2 = -(Q-q)^2$; $\Lambda_{A\pi\gamma}(t=0)$ is the $\Lambda_{A\pi\gamma}$ of Eq. (3.5); at $t=0$, X does not contribute. If we further assume

$$\langle \rho | j_\mu^{em} | 0 \rangle = (2k_0'')^{-1/2} e m_\rho^2 (2\gamma_\rho)^{-1} \epsilon_\mu^{*\prime}, \quad (3.9)$$

with $\gamma_\rho^2/4\pi \approx \frac{1}{2}$,¹² and use unsubtracted dispersion relations for $\Lambda_{A\pi\gamma}$, $X_{A\pi\gamma}$, we obtain

$$\Lambda_{A\pi\gamma}(0) = \Lambda_{A\pi\rho} / 2\gamma_\rho, \quad (3.10)$$

$$b \approx e \Lambda_{A\pi\rho} \Lambda_{A\nu} / 2\gamma_\rho M_A. \quad (3.11)$$

This analysis is intended to suggest that the strongly interacting particle contribution to the axial-vector-current structure may be significant. The experimental data establishes the existence of some kind of axial-vector structure comparable in order of magnitude to the polar-vector structure.

¹² M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).