upon the neutrino mean free path as a function of the parameters describing the state of the stellar matter. For the densities ($\sim 10^{+10}-11^{+11}$ g/cc), temperatures (10–100 MeV), and neutrino energies ($\sim 10-100$ MeV) considered by Colgate and White, we find with the help of Eqs. (45) and (47) that energy deposition via neutrino-electron scattering is larger than energy deposition via neutrino absorption by nucleons. This result, which is contrary to what has been previously thought to be correct,[37] implies that neutrino scattering should be taken into account in future calculations of supernova explosions at high temperatures.[38]

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[37] Some previous workers have concluded that neutrino scattering is negligible compared to neutrino absorption for the conditions under consideration here. The basis for this erroneous conclusion was that the cross section for neutrino absorption contains an extra factor of ω, for large ω, compared to the cross section for neutrino scattering by electrons *at rest*. The results of Sec. IV show, however, that the cross sections for neutrino scattering by electrons in a gas can be much larger, due to an increased center of mass energy, than the scattering cross sections for electrons at rest. Moreover, for neutrino absorption by nucleons in a nucleus, a large fraction of the incident neutrino energy can be spent in overcoming the nucleon binding (Ref. 15).

[38] This is currently being done by S. A. Colgate (private communication).

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Backward Scattering in π-p Collisions at 3.15 and 4 BeV/c*

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On the basis of an analysis of the experimental data for forward scattering, both the upper limit and the lower limit of $d\sigma/dt$ for backward scattering are predicted. The experimental values of $d\sigma/dt$ in the backward direction lie within the region defined by two curves corresponding to the upper and lower limits of $d\sigma/dt$.

THE purpose of this paper is to predict the allowed region of $d\sigma/dt$ for backward scattering in π-p collisions at 3.15 and 4 BeV/c. The result can be derived from an analysis of the experimental data for $d\sigma/dt$ in the forward direction and from the unitarity condition of the S matrix.

In a previous paper[1] we adopted an empirical formula for the scattering amplitude $f(\theta)$ of π-p scattering,

$$f(\theta)=(ik/\sqrt{\pi})\{\exp\tfrac{1}{2}(A_0+A_1t)+C -\exp\tfrac{1}{2}[B_0+B_1(u-u_0)]\}(\mathrm{mb})^{1/2}, \quad (1)$$

and pointed out that if there exists a pronounced backward peak, there must be a scattering angle in the region 90–180° at which the imaginary part of scattering amplitude turns out to be zero [hereafter such a scattering angle is referred to as a zero point of $f(\theta)$ or simply a zero point], where $t=-2k^2(1-\cos\theta)$, $u=(m^2-\mu^2)^2/s-2k^2(1+\cos\theta)$, s is the square of the total energy in the center-of-mass system and u_0 is the value of u at 180°. Recent experimental data[2] for π^+-p scattering at 4 BeV/c have shown a pronounced backward peak and seem to support our prediction[1] for the zero point of $f(\theta)$. Using formula (1), we study in this paper backward scattering at 3.15 and 4 BeV/c from a phenomenological point of view.

Needless to say, the values of A_0 and A_1 can be estimated from the experimental data[2,3] for $d\sigma/dt$ in a very small $|t|$ region. In an intermediate $|t|$ region, for instance $|t|=1.5$–2.0 (BeV/c)^2, the first term $\exp\frac{1}{2}(A_0+A_1t)$ in Eq. (1) turns out to be so small that it may be neglected compared with the second term C. This makes it possible to estimate the value of C. The values of A_0, A_1, and C thus obtained are as follows[4]:

$$A_0=3.7,\ A_1=7.79\ (\mathrm{BeV}/c)^{-2},\quad \text{and}\quad C=0.3 \quad \text{for } \pi^-\text{-}p \text{ scattering at 3.15 BeV}/c, \quad (2)$$

$$A_0=3.7,\ A_1=7.34\ (\mathrm{BeV}/c)^{-2}\quad \text{and}\quad C=0.2 \quad \text{for } \pi^+\text{-}p \text{ scattering at 4 BeV}/c. \quad (2')$$

Note that the values of A_1 for π^--p and π^+-p scattering

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[1] S. Minami, Phys. Rev. 133, B1581 (1964).

[2] M. Aderholz *et al.*, Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Letters 10, 248 (1964).

[3] M. L. Perl, L. W. Jones, and C. C. Ting, Phys. Rev. 132, 1252 (1963).

[4] If $f(\theta)$ is expressed approximately by $f(\theta)\approx(ik/\sqrt{\pi})\times[\exp\frac{1}{2}(A_0+A_1t)+C]$, it follows from the optical theorem $\mathrm{Im}f(0^0)=(k/4\pi)\sigma_{\mathrm{tot}}$ that $\sigma_{\mathrm{tot}}(\pi^--p)$ at 3.15 BeV/$c\cong 29.4$ mb and $\sigma_{\mathrm{tot}}(\pi^+-p)$ at 4 BeV/$c\approx 28.9$ mb.

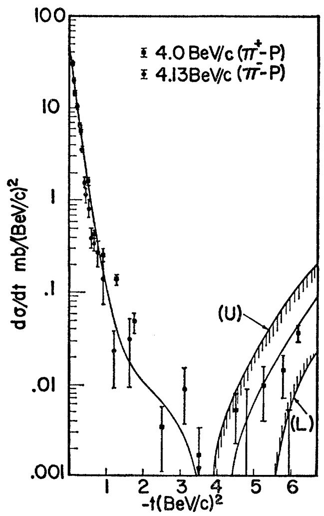

FIG. 1. Differential cross sections for elastic π^+-p scattering at 4 BeV/c. The curves (U) and (L) show, respectively, the upper and lower limit for the $d\sigma/dt$ in the backward direction. The solid curve shows our results obtained from the expression $|d\sigma/dt|=[\exp\frac{1}{2}(A_0+A_1t)+C-\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}]^2$ mb/(BeV/c)^2 with the following values of the parameters: $A_0=3.7$, $A_1=7.34$ (BeV/c)$^{-2}$, $C=0.2$, $B_0=-1.38$ and $B_1=0.68$ (BeV/c)$^{-2}$. The experimental data for π^+-p scattering at 4 (BeV/c) are from the work of Aachen-Berlin-Birmingham-Bonn-Hamburg-London(I.C.)-München Collaboration (Ref. 2). The experimental data for π^--p scattering at 4.13 BeV/c are from the work of Perl *et al.* (Ref. 3).

are equal to the values which have been estimated previously.[5]

As is well known, $f(\theta)$ can be expressed in the following form under the assumptions that the S matrix $[=\eta_l\exp(2i\delta_l)]$ for the lth partial wave has no spin dependence and that δ_l is equal to zero,

$$f(\theta)=(i/2k)\sum(2l+1)(1-\eta_l)P_l(\cos\theta), \qquad (3)$$

where $0\leq\eta_l\leq1$. When k and $[\exp\frac{1}{2}(A_0+A_1t)+C-\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}]$ are given in the units of (BeV/c) and (BeV/c)$^{-2}$ respectively,[6] the values of $(1-\eta_l)$ can be calculated by

$$(1-\eta_l)=(1-\eta_l)'-(1-\eta_l)''+(1-\eta_l)_c, \qquad (4)$$

$$(1-\eta_l)'=(k^2/\sqrt{\pi})\int_{-1}^{1}\exp(a_0+a_1x)P_l(x)dx, \qquad (5)$$

$$(1-\eta_l)''=(k^2/\sqrt{\pi})\int_{-1}^{1}\exp(b_0+b_1x)P_l(x)dx, \qquad (6)$$

$$(1-\eta_l)_c=(k^2/\sqrt{\pi})\int_{-1}^{1}CP_l(x)dx, \qquad (7)$$

where $a_0=A_0/2-A_1k^2$, $a_1=A_1k^2$, $b_0=B_0/2-B_1k^2$, and $b_1=-B_1k^2$.

Here it should be emphasized that $(1-\eta_l)$ must be smaller than unity owing to unitarity of the S matrix. If the value of $(1-\eta_0)'+(1-\eta_0)_c$ is larger than unity, then

$$|(1-\eta_0)''|\geq[(1-\eta_0)'+(1-\eta_0)_c-1]. \qquad (8)$$

Since the quantity $(1-\eta_l)''$ associated with odd l is negative,

$$|(1-\eta_1)''|\leq[1-(1-\eta_1)']. \qquad (9)$$

When both the conditions (8) and (9) are fulfilled, all the values of $(1-\eta_l)$ are smaller than unity, because both $(1-\eta_l)'$ and $|(1-\eta_l)''|$ are decreasing functions of l and tend to zero as l increases. The values of the parameters mentioned in Eqs. (2) and (2′) give us the following results:

$(1-\eta_0)'=0.740$, $(1-\eta_0)_c=0.696$, and $(1-\eta_1)'=0.666$ for π^--p scattering at 3.15 BeV/c,

$(1-\eta_0)'=0.785$, $(1-\eta_0)_c=0.606$, and $(1-\eta_1)'=0.721$ for π^+-p scattering at 4 BeV/c.

Thus the conditions (8) and (9), in our case, should be read as

$$\left.\begin{array}{l}|(1-\eta_0)''|\geq0.436\\|(1-\eta_1)''|\leq0.334\end{array}\right\}\ \text{for }\pi^--p\text{ scattering at 3.15 BeV}/c, \qquad (10)$$

$$\left.\begin{array}{l}|(1-\eta_0)''|\geq0.391\\|(1-\eta_1)''|\leq0.279\end{array}\right\}\ \text{for }\pi^+-p\text{ scattering at 4 BeV}/c. \qquad (10')$$

From Eq. (6), the quantities $(1-\eta_0)''$ and $(1-\eta_1)''$ can be expressed by

$$|(1-\eta_0)''|=(\xi/\sqrt{\pi})[\exp(b_0)/B_1]\times[\exp(-b_1)-\exp(b_1)], \qquad (11)$$

$$|(1-\eta_1)''|=(\xi/\sqrt{\pi})[\exp(b_0)/B_1]\times[\{\exp(-b_1)+\exp(b_1)\}+(1/b_1)\{\exp(-b_1)-\exp(b_1)\}], \qquad (12)$$

where the factor $\xi\approx1.606$ comes from a change of units from (mb)$^{1/2}$/(BeV/c) to (BeV/c)$^{-2}$ in the case where the expression

$$[\exp\tfrac{1}{2}(A_0+A_1t)+C-\exp\tfrac{1}{2}\{B_0+B_1(u-u_0)\}]$$

is given in the units of (mb)$^{1/2}$/(BeV/c). If B_1 (or $b_1=-B_1k^2$) for backward scattering is given, we can get both an upper limit and a lower limit of B_0 from Eqs. (11), (12), and (10) [or (10′)]. The value of B_1 is estimated here so that the curve, from which the upper limit of $d\sigma/dt$ for backward scattering is determined, may pass through the zero point t_0 of $f(\theta)$. That is,

$$\exp\tfrac{1}{2}[B_0+B_1(u-u_0)]_{t=t_0}=\exp\tfrac{1}{2}(A_0+A_1t_0)+C\cong C. \qquad (13)$$

[5] S. Minami, Phys. Rev. **135**, B1263 (1964).

[6] Note that $[\exp\frac{1}{2}(A_0+A_1t)+C-\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}]$ is usually given in the unit of (mb)$^{1/2}$/(BeV/c).

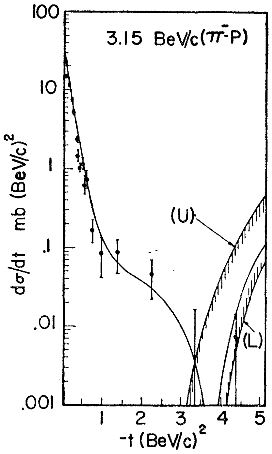

FIG. 2. Differential cross sections for elastic π^--p scattering at 3.15 BeV/c. The curves (U) and (L) show, respectively, the upper and lower limit for the $d\sigma/dt$ in the backward direction. The solid curve shows our results obtained from the expression $|d\sigma/dt|=[\exp\frac{1}{2}(A_0+A_1t)+C-\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}]^2$ mb/(BeV/c)^2 with the following values of the parameters: $A_0=3.7$, $A_1=7.79$ (BeV/c)$^{-2}$, $C=0.3$, $B_0=-0.84$ and $B_1=1.14$ (BeV/c)$^{-2}$. The experimental data are from the work of Perl *et al.* (Ref. 3).

So far as the description of the upper limit for $d\sigma/dt$ in the backward direction is concerned, this treatment is reasonable because the backward peak ought to appear in the region $|t|>|t_0|$. If the zero point of $f(\theta)$ is given, we can obtain from Eqs. (9) and (13) values of B_0 and B_1 corresponding to the upper limit of $d\sigma/dt$ in the backward direction. According to the experimental results[2] for π^+-p scattering at 4 BeV/c, the zero point t_0 seems to be in the neighborhood of $|t|=3.5$ (BeV/c)^2 (cf. Fig. 1). For π^--p scattering at 3.5 BeV/c, we introduce the assumption $|t_0|\approx 3$(BeV/c)^2, which seems to be consistent with the data[3] (cf. Fig. 2). The estimated upper limit of $d\sigma/dt$ for the backward scattering is shown in Figs. 1 and 2 by a curve (U) with a form

$|d\sigma/dt|=[C-\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}]^2$, where

$$B_0=-0.04 \quad \text{and} \quad B_1=1.11\ (\text{BeV}/c)^{-2} \quad \text{for } \pi^-\text{-}p \text{ scattering at 3.15 BeV}/c, \tag{14}$$

$$B_0=-0.83 \quad \text{and} \quad B_1=0.747\ (\text{BeV}/c)^{-2} \quad \text{for } \pi^+\text{-}p \text{ scattering at 4 BeV}/c. \tag{14'}$$

Next we study the lower limit of $d\sigma/dt$ for backward scattering with the assumption that the correct expressions for $d\sigma/dt$ at 3.15 and 4 BeV/c have the same B_1 values with those mentioned in Eqs. (14) and (14′), respectively.[7] Then, we obtain from Eq. (8) the following values of B_0 which give the lower limit of $d\sigma/dt$:

$$B_0=-1.14 \quad \text{and} \quad B_1=1.11\ (\text{BeV}/c)^{-2} \quad \text{for } \pi^-\text{-}p \text{ scattering at 3.15 BeV}/c, \tag{15}$$

$$B_0=-2.09 \quad \text{and} \quad B_1=0.747\ (\text{BeV}/c)^{-2} \quad \text{for } \pi^+\text{-}p \text{ scattering at 4 BeV}/c. \tag{15'}$$

The values of $|d\sigma/dt|=[C-\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}]^2$ with these B_0 and B_1 are shown by the curve labeled (L) in Figs. 1 and 2. As is seen from Fig. 1, the experimental values of $d\sigma/dt$ in the backward direction[2] lie within the allowable region surrounded by the two curves (U) and (L). Although we have no reliable data for π^--p backward scattering at 3.15 BeV/c, the predicted region for $d\sigma/dt$ in the backward direction seems to be consistent with the results obtained by Perl *et al.*[3]

Finally, we mention briefly a phenomenological analysis of the experimental data.[2,3] The solid curves in Figs. 1 and 2 show the values of $|d\sigma/dt|=[\exp\frac{1}{2}(A_0+A_1t)+C-\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}]^2$ with the following values of the parameters[8]:

For π^--p scattering at 3.15 BeV/c,

$$A_0=3.7,\ A_1=7.79\ (\text{BeV}/c)^{-2},\ C=0.3,\quad B_0=-0.84 \quad \text{and} \quad B_1=1.14\ (\text{BeV}/c)^{-2}, \tag{16}$$

For π^+-p scattering at 4 BeV/c,

$$A_0=3.7,\ A_1=7.34\ (\text{BeV}/c)^{-2},\ C=0.2,\quad B_0=-1.38 \quad \text{and} \quad B_1=0.68\ (\text{BeV}/c)^{-2}. \tag{16'}$$

It can be said that the experimental angular distributions[2,3] for π-p elastic scattering at 3.15 and 4 BeV/c can be well described by our empirical formula with the values of the parameters mentioned in Eqs. (16) and (16′), respectively. Note that the values of B_1 mentioned in Eqs. (16) and (16′) are nearly equal to those mentioned in Eqs. (14) and (14′), respectively. Since we have obtained the values of the parameters A_0, A_1, B_0, B_1, and C, the partial-wave analysis can easily be performed by making use of Eqs. (4), (5), (6), and (7). The detailed values of $(1-\eta_l)$ are omitted in the interest of brevity.

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[7] With regard to this assumption, there are grounds for controversy.

[8] Because the value of B_1 is not so large, particularly in π^+-p scattering at 4 BeV/c, the backward peak has a considerable effect on $d\sigma/dt$ in the forward direction. In other words, the effect of C term on $d\sigma/dt$ in the forward direction is reduced by the existence of $\exp\frac{1}{2}\{B_0+B_1(u-u_0)\}$ with the small B_1 value. The broad width of the backward peak may have something to do with the observed nonshrinking diffraction peak of $\pi-p$ scattering at high energy.