Alpha-Decay Properties of Some Thulium and Ytterbium Isotopes Near the 82-Neutron Closed Shell*

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The 84- and 85-neutron isotopes of Tm and Yb were produced by heavy-ion reactions on neutron-deficient rare-earth targets. These isotopes, which are 16 to 18 neutrons removed from the beta stability line, decay predominantly by alpha-particle emission. A search was made for proton emission in the decay of these nuclides and in Tm^{151} and Tm^{152}, but none was observed. The alpha-decay properties of these nuclides were measured and compared with the systematics of alpha decay of the 84- and 85-neutron isotopes. Evidence was obtained for the existence of a long-lived alpha-emitting isomeric state in Tm^{154}.

I. INTRODUCTION

IN previous papers, we have reported results on the alpha-decay properties of isotopes of the rare-earth elements between Tb and Er.[1–3] Most of the results were obtained for the 84- and 85-neutron isotopes which possess enhanced alpha-decay energies due to the effect of the 82-neutron closed shell.

The work reported here on Tm and Yb isotopes is a continuation of this study. The main objectives of this program are twofold: to provide a set of experimental alpha-decay data which can be used to study the effect of proton configuration on alpha-decay rates, and to obtain some information on the production and properties of ultra-neutron deficient nuclides.

II. EXPERIMENTAL DETAILS

The nuclides Tm^{153} and Tm^{154} were produced by $Pr^{141}(Ne^{20},xn)$ reactions and $Nd^{142}(F^{19},xn)$ reactions using 131–195-MeV Ne^{20} ions and 121–185-MeV F^{19} ions from the Berkeley heavy-ion linear accelerator (Hilac). Yb^{154} and Yb^{155} were produced by $Sm^{144}(O^{16},xn)$ reactions using 106–151-MeV O^{16} ions and by $Nd^{142}(Ne^{20},xn)$ reactions. Rare-earth oxide targets ($\sim$2 mg/cm^2) of enriched isotopes of Nd^{142} and Sm^{144} were used. The isotopic abundances of Nd^{142} and Sm^{144} were 93.93% and 94.6%, respectively. Relative excitation functions were obtained for each of the alpha activities studied by measuring the intensities of the alpha groups at different bombarding energies. Aluminum absorbers of varying thicknesses were used to degrade the energy of the heavy ion beam. The data of Northcliffe and Hubbard were used to determine energy losses in the aluminum absorbers, vacuum windows in the target assembly, and the targets.[4,5]

The activity produced during bombardment was collected by thermalizing the recoils ejected from the target in helium, and then pumping them through a small orifice onto a collection plate in vacuum. A gold-surface-barrier-alpha-particle detector was used to measure the alpha activity collected on the plate. Details of this system are reported in an earlier paper.[6]

Single-component decay curves were obtained by selecting the desired group in the alpha-particle spectrum using a single-channel pulse-height analyzer and following the decay of the group after bombardment.

Alpha-particle energy measurements were made using as standards the alpha particles from Dy^{150} (4.23 MeV),[1] Er^{152}(4.80 MeV),[3] and Po^{210} (5.30 MeV).[7]

III. RESULTS

A. $Pr^{141}+F^{19}$ and $Ce^{140}+Ne^{20}$

The reaction $Pr^{141}+F^{19}$ was used to study the energetics of the $(F^{19},8n)$ and $(F^{19},7n)$ reactions in the neutron-deficient rare-earth region. This information was used in assigning the mass numbers of the new Tm and Yb alpha activities. Using a Pr^{141} target, these reactions produce two known alpha emitters, Er^{152} and Er^{153}. Figure 1(a) shows the excitation functions for the production of these two nuclides. In addition to these activities, Ho^{151h} and Ho^{152h} [Fig. 1(b)], and Dy^{150} and Dy^{151} were also produced. The absolute cross sections for the production of Dy^{150} by the reaction $Pr^{141}+F^{19}$ have been measured by Alexander, and his results were used to convert our relative yield measurements to an absolute basis. The normalization point was taken at a bombarding energy of 166 MeV, where the cross section for Dy^{150} production was found by Alexander to be 260 mb.[8]

The information obtained from these measurements that was used in interpreting the Tm and Yb results was the position of the peaks of the excitation functions for the $(F^{19},8n)$ and $(F^{19},7n)$ reactions. As shown in Fig. 1(a), maxima were observed at excitation en-

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[5] E. L. Hubbard, Lawrence Radiation Laboratory Report UCRL-9053, 1960 (unpublished).
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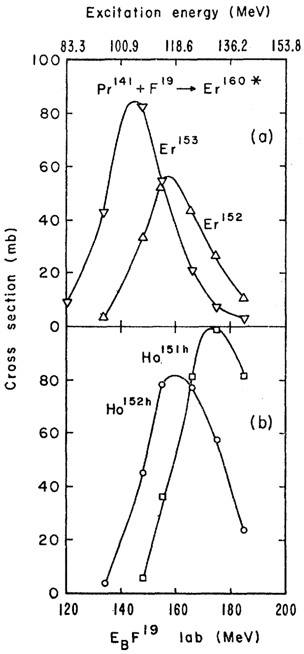

FIG. 1. Excitation functions for the reactions (a) $Pr^{141}(F^{19},8n)Er^{152}$(△), and $Pr^{141}(F^{19},7n)Er^{153}$(▽); and (b) $Pr^{141}(F^{19},9n+p8n)Ho^{151h}$ (□) and $Pr^{141}(F^{19},p7n)Ho^{152h}$ (○).

ergies of 117 and 105 MeV, respectively (the term "excitation energy" is defined as the difference between the center-of-mass bombarding energy and the Q value for compound nucleus formation calculated from the Seeger mass formula[9]).

Excitation functions were also obtained for the $(Ne^{20},8n)$ and $(Ne^{20},7n)$ reactions using a Ce^{140} target.

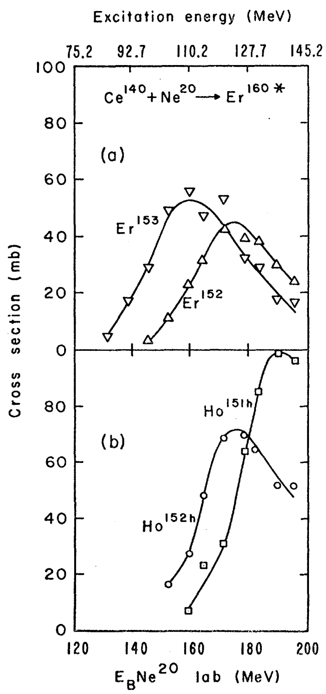

FIG. 2. Excitation functions for the reactions (a) $Ce^{140}(Ne^{20},8n)Er^{152}$(△), and $Ce^{140}(Ne^{20},7n)Er^{153}$(▽); and (b) $Ce^{140}(Ne^{20},9n+p8n)$-Ho^{151h}(□) and $Ce^{140}(Ne^{20},$-$p7n)Ho^{152h}$(○).

[9] P. A. Seeger, Nucl. Phys. 25, 1 (1961).

To convert to absolute cross sections, the cross-section data of Alexander for Dy^{150} production by $Ce^{140}+Ne^{20}$ were used.[8] The normalization point was 171-MeV bombarding energy where a Dy^{150} cross-section value of 262 mb was obtained by Alexander. The cross-section results for Er^{152} and Er^{153} are shown in Fig. 2(a), and for Ho^{151h} and Ho^{152h} in Fig. 2(b). The position of the maxima in the excitation functions for the $(Ne^{20},8n)$ and $(Ne^{20},7n)$ reactions correspond to excitation energies of 120 and 108 MeV, respectively.

The results obtained for the production of Ho^{151h} and Ho^{152h} by (H.I.,pxn) reactions (where H.I. means heavy ion) are also shown in order to point out the usefulness of these results in providing additional data for deducing mass assignments of new activities. Both Figs. 1 and 2 show that for (H.I.,xn) and (H.I.,pxn)

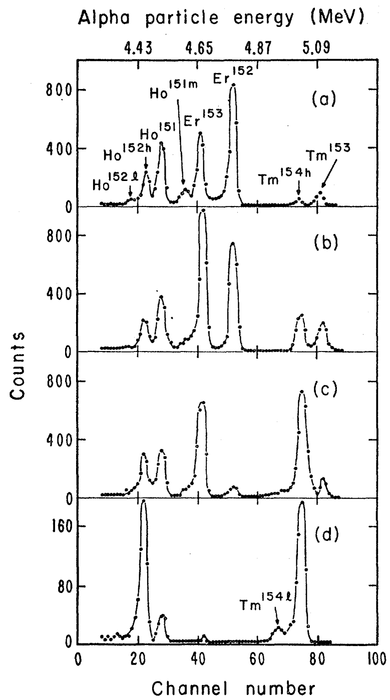

FIG. 3. Alpha-particle spectra taken from the $Pr^{141}+Ne^{20}$ runs showing the new Tm alpha emitters taken at bombarding energies of (a) 195 MeV, (b) 178 MeV, (c) 159 MeV, and (d) 138 MeV.

products, the independent excitation functions of isobaric pairs (e.g., Er^{152} and Ho^{152h}) peak at the same excitation energy in the neutron deficient rare-earth region (Er^{152} has a very small β^+/EC branch to Ho^{152h}).[3] The excitation function for Ho^{151h} is actually a sum of the Er^{151} and Ho^{151h} excitation functions, since Er^{151} is not an alpha emitter. For example, one would expect that the excitation function for the production of Tm^{153} by a (H.I.,xn) reaction will peak at approximately the same excitation energy as that for Er^{153} produced by a (H.I.,$p(x-1)n$) reaction from the same compound nucleus.

B. $Nd^{142}+F^{19}$ and $Pr^{141}+Ne^{20}$

The formation of Tm^{161} compound nuclei by $Nd^{142}+F^{19}$ and $Pr^{141}+Ne^{20}$ reactions produced three new alpha activities which were not seen in the $Pr^{141}+F^{19}$ or $Ce^{140}+Ne^{20}$ bombardments. On this basis, these activities were assigned to isotopes of thulium. Figure 3 shows alpha-particle spectra of these activities produced at different bombarding energies. In addition to these, the known alpha emitters of Dy, Ho, and Er were also observed.

$Tm^{153}(E_\alpha=5.11$ *MeV*)

The highest energy Tm alpha group observed has an alpha-particle energy of 5.11±0.02 MeV and decays with a half-life of 1.58 sec. This activity was tentatively assigned to the 84-neutron isotope Tm^{153} on the

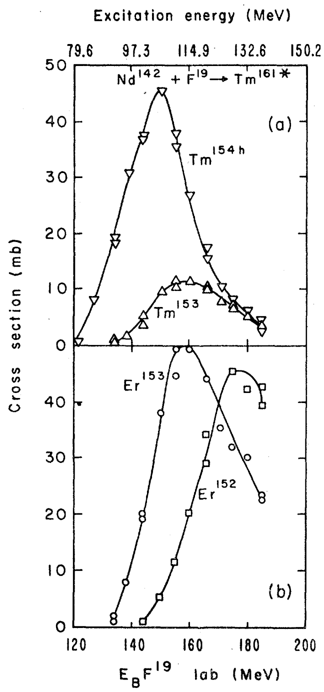

FIG. 4. Excitation functions for the reactions (a) $Nd^{142}(F^{19},8n)$-Tm^{153}(△) and $Nd^{142}(F^{19},7n)Tm^{154h}$(▽) where Tm^{154h} denotes the high-spin isomer. (b) The excitation functions for the reactions $Nd^{142}(F^{19},9n+p8n)Er^{152}$(□) and $Nd^{142}(F^{19},p7n)Er^{153}$(○).

basis of alpha-decay systematics. Conclusive evidence for this mass assignment was obtained from excitation function data. Figure 4 shows the excitation function for this activity when produced by $Nd^{142}+F^{19}$, and Fig. 5 shows the results obtained for $Pr^{141}+Ne^{20}$ which forms the same compound nucleus. In both cases, the excitation energies where the cross section was a maximum were very close to those observed in the Er work discussed above for (H.I.,$8n$) reactions. This new activity, therefore, must be due to Tm^{153}. Figures 4(b) and 5(b) show that the Er^{153} excitation functions peak at approximately the same energy as that for the activity we have assigned to Tm^{153}, a result which is consistent with our previous observa-

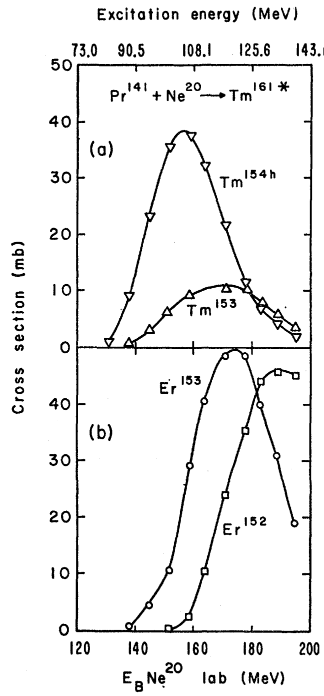

FIG. 5. Excitation functions for the reactions (a) $Pr^{141}(Ne^{20},8n)$-Tm^{153}(△) and $Pr^{141}(Ne^{20},7n)Tm^{154h}$(▽); and (b) $Pr^{141}(Ne^{20},9n+p8n)$-Er^{152}(□) and $Pr^{141}(Ne^{20},p7n)Er^{153}$(○).

tions of the similarity of the energetics of (H.I.,xn) and (H.I.,$p(x-1)n$) reactions in the rare-earth region.

$Tm^{154h}(E_\alpha=5.04$ *MeV*)

A second Tm alpha group decaying with a half-life of 2.98 sec was observed at an alpha-particle energy of 5.04 MeV. On the basis of alpha-decay systematics, the most likely mass assignment appeared to be Tm^{154}. The excitation functions for the production of this activity by $Nd^{142}+F^{19}$ and $Pr^{141}+Ne^{20}$ are shown in Figs. 4(a) and 5(a), respectively, and are labeled Tm^{154h}. These results, when compared with the Er data (Figs. 1 and 2), compare most favorably with the energetics of an (H.I.,$7n$) reaction which would produce Tm^{154} from a Tm^{161} compound nucleus.

$Tm^{154l}(E_\alpha=4.96$ *MeV*)

Alpha-particle spectra of activity collected from Nd^{142} $+F^{19}$ bombardments at energies below 134 MeV gave an indication of a weak alpha group just below the 5.04-MeV Tm^{154} peak. Experiments with $Pr^{141}+Ne^{20}$ at bombarding energies below 158 MeV also gave an indication of a weak alpha group at the same position. An alpha-particle spectrum showing this activity can be seen in Fig. 3(d). The alpha-particle energy of this activity is 4.96 MeV, and it decays with a half-life of 5 sec. The half-life was obtained from an analysis of alpha-particle spectra taken in a series of timed sequences after bombardment. This experiment clearly ruled out the possibility that the 4.96-MeV group was associated with the decay of the 2.98-sec Tm^{154} activity.

Because of the low intensity of this group, it was not possible to obtain a detailed excitation function. Qualitatively, it was determined that the excitation function peaks at a somewhat lower bombarding energy than the 5.04-MeV Tm^{154}, probably around 145 MeV when produced by $Pr^{141}+Ne^{20}$. Alpha-decay energy systematics do not favor the assignment of Tm^{155} to this activity because the difference between the alpha-decay energies of the 85- and 86-neutron isotopes of the elements below thulium have consistently been $\sim$400 keV. This, and the fact that the activity is apparently produced in low yield (a small alpha-decay branch would give the same result), suggests that the activity may be due to a low spin isomer of Tm^{154}. The energetics of the qualitative cross-section measurements also indicate that this is the most probable assignment. Similar results were obtained for the production of low spin isomers of holmium by high-energy-heavy-ion reactions.[2] Ho^{152}, which has the same number of neutrons as Tm^{154}, has a long-lived alpha-emitting isomeric state.[2] Here the difference in alpha-decay energies of the two states is 70 keV, which is close to the difference of 80 keV observed for the Tm^{154} case.

C. $Sm^{144}+O^{16}$ and $Nd^{142}+Ne^{20}$

The 84- and 85-neutron isotopes of ytterbium, Yb^{154} and Yb^{155}, were produced by $Sm^{144}(O^{16},xn)$ reactions and $Nd^{142}(Ne^{20},xn)$ reactions. The energetics of the (O^{16},xn) reactions have been studied in previous work on Dy, Ho, and Er alpha emitters,[2,3] and information on (Ne^{20},xn) reactions was obtained from results presented above on the Tm alpha emitters.

Alpha-particle spectra of activity collected from $Sm^{144}+O^{16}$ and $Nd^{142}+Ne^{20}$ bombardments, which form Yb compound nuclei, showed two new alpha groups which had not been seen in bombardments which produced isotopes up to thulium (Fig. 6). They were therefore assigned to isotopes of Yb. From excitation function data, they were shown to be due to Yb^{154} and Yb^{155}.

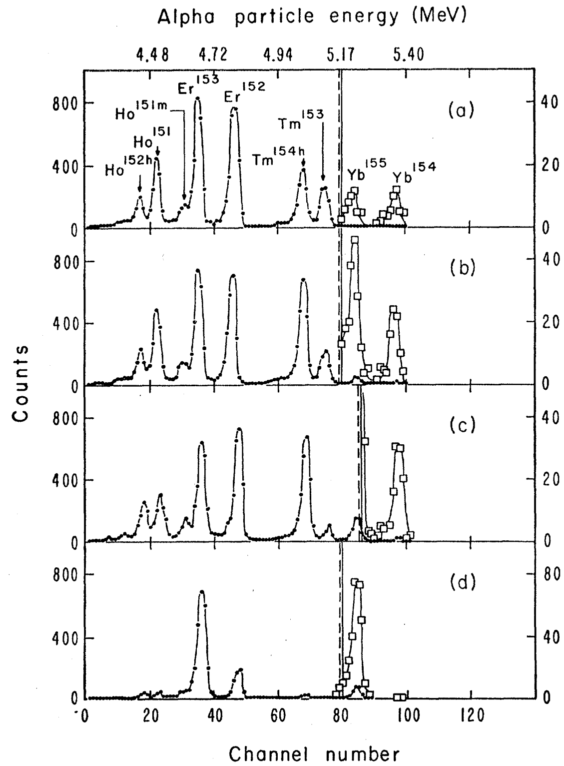

FIG. 6. Alpha-particle spectra taken from the $Nd^{142}+Ne^{20}$ runs showing the new Yb alpha emitters. The solid points refer to the scale on the left, and the squares to the scale on the right. Figures 6(a)–(d) were taken at bombarding energies of 195, 183, 171, and 145 MeV, respectively.

$Yb^{154}(E_\alpha=5.33\ MeV)$

One of the Yb alpha emitters has an alpha-particle energy of 5.33 MeV and decays with a half-life of 0.39 sec. As in the study of the Tm alpha emitters, the only means available for mass assignment was a comparison of the energetics of the excitation functions for this activity with those of known activities in this region. The activity was produced two different ways: the first was by a $Sm^{144}(O^{16},xn)$ reaction, and the second by a $Nd^{142}(Ne^{20},xn)$ reaction. The (O^{16},xn) results [Fig. 7(a)] for this activity showed that the peak cross section occurred at an excitation energy of 87 MeV. In our previous work with Ho and Er isotopes, we observed that the peak cross section for the $(O^{16},6n)$ reaction was at $\sim$85-MeV excitation energy, a value very close to that observed for this Yb activity. This result strongly suggests that this new nuclide is probably Yb^{154}.

Additional support for this mass assignment was obtained from the $Nd^{142}+Ne^{20}$ results. Here, the excitation function which is shown in Fig. 8(a) was compared with results previously obtained for $Ce^{140}+Ne^{20}$ and $Pr^{141}+Ne^{20}$. The position of the peak of the excitation function was found to correspond very closely to what had been previously observed for a $(Ne^{20},8n)$ reaction. This result is consistent with our mass assignment of Yb^{154}.

As a further check on our mass assignment, we looked at the excitation functions of the (H.I.,pxn) products [Figs. 7(b) and 8(b)], making use of our previous observation of the similarity of the energetics of isobars formed by (H.I.,xn) and (H.I.,$p(x-1)n$) reactions. For both the $Sm^{144}+O^{16}$ and $Nd^{142}+Ne^{20}$ results, the Tm^{154h} excitation functions peaked at approximately the same excitation energy as that for the activity we have assigned to Yb^{154}.

$Yb^{155}(E_\alpha=5.21\ MeV)$

The second Yb alpha activity that was observed has an alpha-particle energy of 5.21 MeV, and decays with a half-life of 1.65 sec. The assignment of this activity to Yb^{155} was made by a procedure analogous to that used for Yb^{154}, making use of excitation function data from previous work to identify the reaction producing the activity. It was first produced by the reaction $Sm^{144}(O^{16},xn)$, where the maximum cross section was observed at an excitation energy of 76 MeV. In the rare-earth region, this was found to be a characteristic value for $(O^{16},5n)$ reactions.

The $Nd^{142}+Ne^{20}$ bombardments also produced this activity, and the excitation function for this mode of production is shown in Fig. 8(a). The peak cross section was observed at an excitation energy of 106 MeV. From our $Ce^{140}+Ne^{20}$ and $Pr^{141}+Ne^{20}$ results, the $(Ne^{20},7n)$ reaction cross sections were found to peak at excitation energies of 108 and 106 MeV, respectively. This Yb activity must therefore have been produced by the reaction $Nd^{142}(Ne^{20},7n)Yb^{155}$.

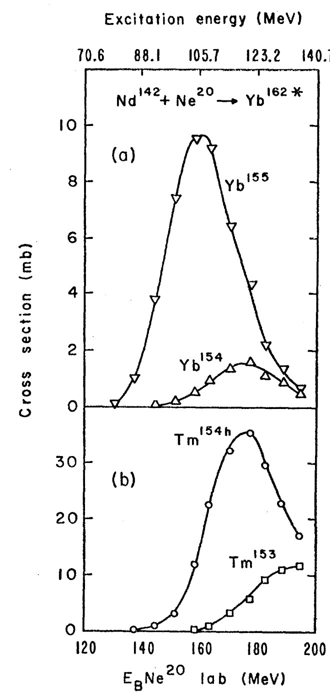

FIG. 8. Excitation functions for the production of Yb^{154} and Yb^{155} from Nd^{142}-(Ne^{20},xn) reactions, and for the production of Tm^{153} and Tm^{154} by $(Ne^{20},9n+p8n)$ and $(Ne^{20},p7n)$ reactions, respectively.

D. Search for Proton Radioactivity

The nuclides reported in this work are far on the neutron deficient side of the beta stability line. Proton binding energies of these nuclides are very small, and some of them may be proton unstable. Of the nuclides studied above, the Tm isotopes offered the best possibility of observing proton radioactivity, because odd-Z nuclei have lower proton binding energies. The Seeger semiempirical mass formula gives Tm^{153} a proton binding energy of $+0.727$ MeV, and Tm^{152} a value of $+0.423$ MeV.[9] The heaviest proton unstable Tm isotopes, according to this mass formula, is Tm^{150} ($Q_p=-0.228$ MeV). These, of course, represent large extrapolations from behaviors near beta stability, and are therefore subject to considerable uncertainty. However, the mass formula does reasonably well in predicting the alpha-decay energy of Tm^{153} (5.02 MeV compared to the experimental value of 5.25 MeV), so that there is some evidence for the validity of the mass formula in this region.

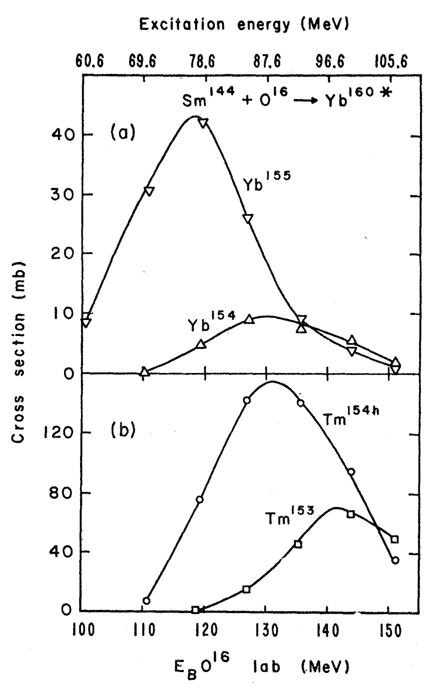

FIG. 7. Excitation functions for the production of Yb^{154} and Yb^{155} from $Sm^{144}(O^{16},xn)$ reactions, and for the production of Tm^{153} and Tm^{154} from the same compound nucleus.

The lighter Tm isotopes Tm^{151} and Tm^{152} appeared to be the most promising nuclides for the observation of proton radioactivity in this study. Because of the effect of the 82-neutron closed shell in suppressing the alpha-decay energies of the 82- and 83-neutron isotopes, there should not be any competition from alpha decay for these nuclides. The $Sm^{144}+O^{16}$ reaction using 152-MeV O^{16} ions was used to produce these nuclides because of the higher cross section for (O^{16},pxn) reactions in the neutron-deficient rare-earth region. This energy should give close to the maximum cross section for Tm^{152} production [see Fig. 7(b)], and probably a significant cross section for Tm^{151}. Particle spectra were obtained over the energy range of 0.75 to 3.40 MeV, and the results are shown in Fig. 9. No groups were observed in this region which could be attributed to proton radioactivity. The background in this region was essentially due to the low-energy tails of the intense rare-earth alpha groups. We estimate that our limit of detection of a proton group in this experiment is equivalent to a cross section of 0.2 mb, if the activity has a half-life greater than 0.04 sec and decays 100% by proton emission (activities with half-lives less than 0.02 sec cannot be detected in our experiments).

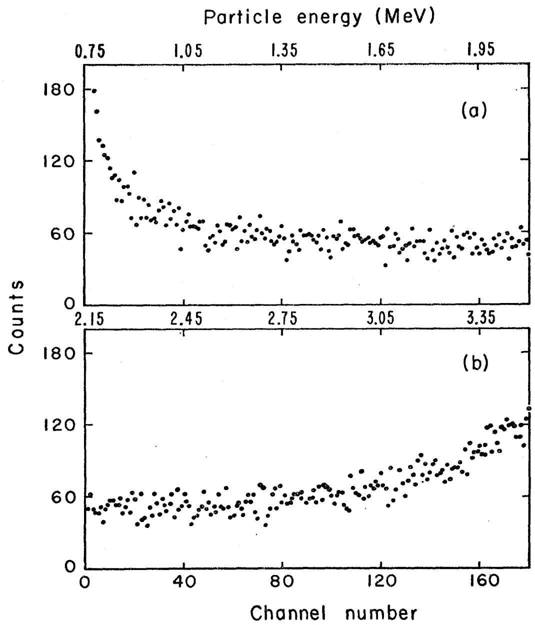

FIG. 9. Particle spectrum in the region of 0.75 to 3.4 MeV of activity collected from the reaction $Sm^{144}+O^{16}$ using 152-MeV O^{16} ions. This spectrum was taken in order to determine whether proton and delayed proton emitters could be detected in the neutron-deficient rare-earth region.

In order for a nuclide such as Tm^{151} to have a measurable proton-decay branch, the energy available for proton decay must be at least 0.6 MeV. This would give a proton-decay half-life of $\sim$10 sec (an approximate formula for calculating proton decay half-lives given by Goldansky was used to obtain this value.[10] Unfortunately, in our experiments, the background due to beta-decay pileup was very high below 0.75 MeV, so that the sensitivity for detecting proton groups in the critical region between 0.6 and 0.75 MeV was very poor.

IV. DISCUSSION

A. Alpha-Decay Energies

The alpha decay energies of the 84- and 85-neutron isotopes of Tm and Yb, when compared with the corresponding isotopes of Tb, Dy, Ho, and Er, show that the alpha-decay energies of these nuclides are a linear function of nuclear charge for constant neutron number. An expression for alpha-decay energies can be derived from the semiempirical mass formula which shows that alpha-decay energy and Z are linearly related if N is held constant. Only in the region of spherical nuclei, however, does this seem to be borne out experimentally.

The 86-neutron isotopes of Tm and Yb were undoubtedly produced in our experiments, but no alpha-decay branch was detected for these nuclides. No large alpha-decay hindrance factor is expected, so that the reason for the apparent small alpha branch of these nuclides can probably be attributed to competition from $E.C./\beta^+$ decay as a result of a significant lowering of alpha-decay energy between $N=85$ and $N=86$.

Comparisons of our experimental Q_α values with those calculated by the Seeger mass formula show that the Seeger formula gives very close values. The Cameron mass formula[11] is not particularly useful in this region because it predicts that the 85-neutron isotopes should have the maximum Q_α, whereas the experimental maximum occurs at $N=84$.

TABLE I. Summary of results.

Nuclide	Q_α(MeV)	Half-life (sec)	Estimated alpha branching ratio	δ^2 (MeV)[a]
Tm^{153}	5.25±0.02	1.58±0.15	0.90	0.065
Tm^{154} (high spin)	5.17±0.02	2.98±0.20	0.85	0.066
Tm^{154} (low spin)	5.09±0.03	5±1	···	···
Yb^{154}	5.47±0.02	0.39±0.04	0.98	0.091
Yb^{155}	5.35±0.02	1.65±0.15	0.90	0.080

[a] Calculated assuming $l=0$ alpha wave emission only.

B. Alpha Branching Ratios and Alpha Reduced Widths

It was not possible to obtain direct experimental values of alpha branching ratios for the Tm and Yb alpha emitters because of their low yield, and also because the beta-decay daughters are formed directly in greater yield than the parents. Some information obtained on the alpha branching ratio of Yb^{154} from a study of the alpha decay of Hf^{158} to Yb^{154} indicates that Yb^{154} has essentially a 100% alpha branch.[12]

In order to obtain some estimate of the alpha branching ratios of the Tm and Yb alpha emitters, beta-decay half-lives were estimated using previous data on the 84- and 85-neutron alpha-emitting isotopes of Dy, Ho, and Er. Using the Seeger semiempirical mass formula to estimate the energy available for beta decay and experimentally determined half-lives and alpha branching ratios of these nuclides, gross logft values were calculated. Logft values grouping around 5 were obtained for even-even, even-odd, odd-even, odd-odd, and high- and low-spin isomers.

A gross logft of 5 was taken as a reasonable lower limit for the alpha-emitting Tm and Yb isotopes, and beta half-lives were calculated on this basis. In all cases, it was found that the calculated beta half-life was much larger than the experimental value, indicating that the alpha branching ratios of these nuclides are close to unity. Estimated alpha branching ratios obtained from the calculated beta decay half-lives are given in Table I together with a summary of the Q_α and $t_{1/2}$ results.

[10] V. I. Goldansky, Nucl. Phys. **19**, 482 (1960).

[11] A. G. W. Cameron, Can. J. Phys. **35**, 1021 (1957).

[12] R. D. Macfarlane (unpublished results).

The alpha-reduced widths δ^2 given in Table I reflect the probability of alpha decay after the energy dependence has been removed. The exact definition of δ^2 and the method of calculation is that given by Rasmussen.[13] The δ^2 for the Tm isotopes are very close to those obtained for the Ho isotopes with the same neutron number, and the δ^2 for the Yb isotopes are in approximate agreement with those obtained for the corresponding Er isotopes. The approximate constancy of δ^2 for nuclides near the 82-neutron closed shell has been of particular interest. Theoretical calculations of relative reduced widths using pure single-particle wave functions show that large fluctuations in δ^2 can be expected, depending on the magnitude of the radial wave functions near the nuclear surface. Also, as a shell is being filled (in our case the $h_{11/2}$ proton shell), δ^2 should have a maximum value when the shell is half-filled ($Z=70$ for the $h_{11/2}$ proton shell). Experimentally, however, δ^2 for the 84- and 85-neutron isotopes has been reasonably constant for $Z=60$ to $Z=70$, except for a slight decrease at $Z=66$. If wave functions derived from residual pairing-force calculations are used, fluctuations in calculated δ^2 are essentially washed out.[14] The constancy of the experimental reduced widths for the 84- and 85-neutron isotopes clearly demonstrates the role of the residual pairing force in the alpha-decay process.

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Statistical Theory of Nuclear Collision Cross Sections. II. Distributions of the Poles and Residues of the Collision Matrix*

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The relationship between the statistical properties of the parameters defining the R matrix and the distributions and correlations of the poles and residues of the statistical collision matrix are explored by means of some limited numerical computations involving models for reactions in the presence of large numbers of competing strongly absorbed channels. The results shed light on the distributions of resonance energies and widths, and on the relationship between the partial width to spacing ratios and the channel transmission coefficients. The calculations also yield substantial channel-channel and resonance-resonance correlations in the complex amplitudes which define the collision-matrix pole residues. These are important for their effects on average cross section and fluctuation calculations. It is found that the investigated statistical relationships depend on the choice of R-matrix boundary conditions, and the implications of this for the choice of boundary conditions are discussed.

I. INTRODUCTION

THE statistical properties of the eigenvalue spectra and eigenvectors of complex and strongly interacting bound systems such as heavy nuclei have been extensively investigated.[1] In the continuum part of the spectrum, the results of this work are thought to be applicable to the artificial discrete states generated by the real constant boundary conditions of R-matrix theory.[2] The eigenvalues of this boundary value problem are then the poles of the R matrix and binary products of the corresponding eigenvectors form the matrix residues. It is of interest to translate such statistical models of the R matrix into statistical information regarding the poles and residues of the statistical collision matrix,[3] since it is the latter which directly affects the statistical properties of cross sections such as energy averages, mean square fluctuation, correlations, etc., as discussed in Ref. 3.

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[1] See, for example, the notes for lectures by N. Rosenzweig, in *Statistical Physics* (W. A. Benjamin, Inc., New York, 1963), Vol. 3, which also includes extensive references.

[2] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947). A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[3] P. A. Moldauer, Phys. Rev. **135**, B642 (1964). Extensive background references will be found there as well as more complete discussions of the concepts and symbols employed in this paper.