The alpha-reduced widths δ^2 given in Table I reflect the probability of alpha decay after the energy dependence has been removed. The exact definition of δ^2 and the method of calculation is that given by Rasmussen.[13] The δ^2 for the Tm isotopes are very close to those obtained for the Ho isotopes with the same neutron number, and the δ^2 for the Yb isotopes are in approximate agreement with those obtained for the corresponding Er isotopes. The approximate constancy of δ^2 for nuclides near the 82-neutron closed shell has been of particular interest. Theoretical calculations of relative reduced widths using pure single-particle wave functions show that large fluctuations in δ^2 can be expected, depending on the magnitude of the radial wave functions near the nuclear surface. Also, as a shell is being filled (in our case the $h_{11/2}$ proton shell), δ^2 should have a maximum value when the shell is half-filled ($Z=70$ for the $h_{11/2}$ proton shell). Experimentally, however, δ^2 for the 84- and 85-neutron isotopes has been reasonably constant for $Z=60$ to $Z=70$, except for a slight decrease at $Z=66$. If wave functions derived from residual pairing-force calculations are used, fluctuations in calculated δ^2 are essentially washed out.[14] The constancy of the experimental reduced widths for the 84- and 85-neutron isotopes clearly demonstrates the role of the residual pairing force in the alpha-decay process.

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The author would like to thank Professor J. O. Rasmussen and Professor I. Perlman for the opportunity of carrying out this work at the Lawrence Radiation Laboratory. He would also like to thank Professor J. Alexander for making available unpublished cross-section data which were used in this work. The cooperation and assistance of A. Ghiorso and the Hilac personnel are gratefully acknowledged.

[13] J. O. Rasmussen, Phys. Rev. **113**, 1593 (1959).

[14] R. D. Macfarlane, J. O. Rasmussen, and M. Rho, Phys. Rev. **134**, B1196 (1964).

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Statistical Theory of Nuclear Collision Cross Sections. II. Distributions of the Poles and Residues of the Collision Matrix*

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The relationship between the statistical properties of the parameters defining the R matrix and the distributions and correlations of the poles and residues of the statistical collision matrix are explored by means of some limited numerical computations involving models for reactions in the presence of large numbers of competing strongly absorbed channels. The results shed light on the distributions of resonance energies and widths, and on the relationship between the partial width to spacing ratios and the channel transmission coefficients. The calculations also yield substantial channel-channel and resonance-resonance correlations in the complex amplitudes which define the collision-matrix pole residues. These are important for their effects on average cross section and fluctuation calculations. It is found that the investigated statistical relationships depend on the choice of R-matrix boundary conditions, and the implications of this for the choice of boundary conditions are discussed.

I. INTRODUCTION

THE statistical properties of the eigenvalue spectra and eigenvectors of complex and strongly interacting bound systems such as heavy nuclei have been extensively investigated.[1] In the continuum part of the spectrum, the results of this work are thought to be applicable to the artificial discrete states generated by the real constant boundary conditions of R-matrix theory.[2] The eigenvalues of this boundary value problem are then the poles of the R matrix and binary products of the corresponding eigenvectors form the matrix residues. It is of interest to translate such statistical models of the R matrix into statistical information regarding the poles and residues of the statistical collision matrix,[3] since it is the latter which directly affects the statistical properties of cross sections such as energy averages, mean square fluctuation, correlations, etc., as discussed in Ref. 3.

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[1] See, for example, the notes for lectures by N. Rosenzweig, in *Statistical Physics* (W. A. Benjamin, Inc., New York, 1963), Vol. 3, which also includes extensive references.

[2] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947). A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[3] P. A. Moldauer, Phys. Rev. **135**, B642 (1964). Extensive background references will be found there as well as more complete discussions of the concepts and symbols employed in this paper.

There are, of course, other formalisms which give more direct interpretations of the poles of the collision matrix.[4] These deal, however, always with "states" having complex energies and eigenvectors, requiring essentially twice as many statistical hypotheses, some of which may be physically redundant. In addition, these other formalisms involve parameters such as normalization constants or matrix elements whose evaluation requires additional dynamical assumptions. On the other hand, it must be recognized that assumptions regarding the R-matrix statistics are essentially models which are not simply related to specific dynamical models and which must be checked against experiment for verification. This fact will become particularly evident when we come to discuss the relationship between R-matrix boundary conditions and the statistical assumptions. Nevertheless, it appears that the statistical discussion proceeding through the R-matrix formalism is more firmly grounded at the present time since it is evidently connected, through a unique sequence of boundary conditions, to the true bound state problems and the almost bound states of slow neutron resonances, where its predictions have been amply confirmed.[5]

In the next section we review the theoretical relationship between the R matrix and the statistical collision matrix for the special conditions to be assumed here. Having so far failed in constructing a fully successful analytic method of relating the statistical properties of these two matrices, we then give the results of some numerical computations and discuss their implications.

II. THEORETICAL BACKGROUND

A detailed development of the relation between the R matrix and a locally valid pole expansion of the collision matrix has already been given.[3] We review here briefly the results for the special case of an energy region where distant poles do not contribute to the off-diagonal elements of the R matrix ($\mathbf{R}^{(0)}$ diagonal), and where there are no nearby thresholds. These conditions (which are in fact not nearly so restrictive as they may seem) imply that there are no direct reactions and no smooth effects due to distant resonances (except insofar as potential scattering may be regarded as such). Then, if the R matrix is

$$R_{cc'}(E)=\sum_\mu \frac{\gamma_{\mu c}\gamma_{\mu c'}}{E_\mu-E}, \tag{1}$$

it has been shown[3] that aside from constant phases the collision matrix may, in a certain energy interval, be approximated by

$$U_{cc'}(E)=\delta_{cc'}-i\sum \frac{g_{\mu c}g_{\mu c'}}{E-\mathcal{E}_\mu+\frac{1}{2}i\Gamma_\mu}, \tag{2}$$

where the $\mathcal{E}_\mu-\frac{1}{2}i\Gamma_\mu$ are the eigenvalues of the complex symmetric level matrix:

$$t_{\lambda\mu}[E_\mu\delta_{\mu\nu}-\textstyle\sum_c (S_c^0+iP_c)\gamma_{\mu c}\gamma_{\nu c}]=[\mathcal{E}_\lambda-\frac{1}{2}i\Gamma_\lambda]t_{\lambda\nu}, \tag{3}$$

where P_c is the penetrability function and S_c^0 the modified shift function of channel c, and

$$g_{\mu c}=(2P_c)^{1/2}\textstyle\sum_\nu t_{\mu\nu}\gamma_{\nu c}. \tag{4}$$

It can be shown, furthermore, that

$$\Gamma_\mu=\textstyle\sum_c \Gamma_{\mu c}, \tag{5}$$

where

$$\Gamma_{\mu c}=|g_{\mu c}|^2/N_\mu, \tag{6}$$

and

$$N_\mu=\textstyle\sum_\nu |t_{\mu\nu}|^2\geqslant 1. \tag{7}$$

In addition, we have defined[3]

$$\Theta_{\mu c}=(2\pi/D)N_\mu|g_{\mu c}|^2, \tag{8}$$

where D is the mean spacing of the $\mathcal{E}_\mu$. We have verified that in all cases of the calculations to be reported later,

$$\langle g_{\mu c}g_{\mu c'}\rangle_\mu=0 \quad \text{for} \quad c\neq c', \tag{9}$$

where $\langle\ \rangle_\mu$ is an average with respect to resonance index μ. When the relations (9) hold, the optical-model transmission coefficient for the partial wave c is given by[6]

$$T_c=\langle\Theta_{\mu c}\rangle_\mu(1-\tfrac{1}{4}Q_c\langle\Theta_{\mu c}\rangle_\mu), \tag{10}$$

where

$$Q_c=2B_c(1-\Phi_0)/\langle N_\mu^2\rangle_\mu, \tag{11}$$

and

$$B_c=|\langle g_{\mu c}^2\rangle_\mu|^2/\langle|g_{\mu c}|^2\rangle_\mu^2 \tag{12}$$

measures the anisotropy of the distribution of the $g_{\mu c}$ in the complex plane, going to zero for a completely isotropic distribution. The quantity Φ_0 depends on the distribution of the spacings of the $\mathcal{E}_\mu$ and goes to unity for large width to spacing ratios.[3] For most cases of $\Gamma/D>1$, Eq. (10) implies that $T_c\approx\langle\Theta_{\mu c}\rangle_\mu$. Under the same assumptions, the reaction cross section for transitions from partial wave c to a different partial wave c' is given by

$$\sigma_{cc'}=\pi\lambda\!\!\!^-_c{}^2\left\langle\frac{\Theta_{\mu c}\,\Theta_{\mu c'}}{\sum_{c''}\Theta_{\mu c''}}\right\rangle_\mu \quad c\neq c', \tag{13}$$

and angular distributions as well as fluctuations are expressible in terms of (13) and similar expressions.[3]

III. METHOD OF COMPUTATION

The solution of the problems posed in the Introduction were programmed for computation by means of the CDC 3600 digital computer. First, the R matrix

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[4] P. L. Kapur and R. Peierls, Proc. Roy. Soc. (London) **A166**, 277 (1938); H. Feshbach, Ann. Phys. (N. Y.) **5**, 357 (1958); J. Humblet and L. Rosenfeld, Nucl. Phys. **26**, 529 (1961).

[5] See, for example, J. B. Garg, J. Rainwater, J. S. Petersen, and W. W. Havens, Phys. Rev. **134**, B985 (1964) and earlier references cited there.

[6] P. A. Moldauer, Rev. Mod. Phys. **36**, 1074 (1964).

was generated by producing random uncorrelated and normally distributed real amplitudes $\gamma_{\mu c}$ with zero mean and unit standard deviation by averaging twelve pseudorandom numbers uniformly distributed in (0,1) and generated by the method of Rotenberg.[7] Up to 300 channel indices c could be accommodated. Four types of distributions of the E_μ were employed: (1) Equally spaced E_μ. (2) Uncorrelated E_μ. These were generated by drawing spacings $E_{\mu+1}-E_\mu$ at random from an exponentially distributed set of spacings. (3) Wigner repulsion, generated by drawing from a set of spacings distributed according to the Wigner distribution.[8] (4) Wigner repulsion anticorrelated, generated by drawing at random alternately from the upper and lower half of the above Wigner distribution of spacings. This was intended to test the effect of the anticorrelation of spacings actually present in the spacings of the random matrix model proposed by Wigner.[1,8]

The level matrix on the left-hand side of Eq. (3) was generated by specifying a set of channel penetration factors P_c and shift factors S_c^0. The procedure of Osborn[9] was employed to perform diagonalizations of

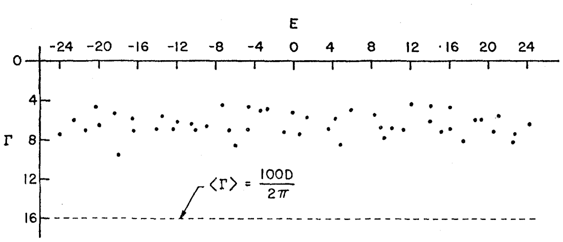

FIG. 1. Distribution of fifty poles of a collision matrix having 100 channels with random uncorrelated R-matrix amplitudes and R-matrix pole spacings selected from the Wigner distribution with anticorrelated neighboring spacings. The transmission coefficients of all channels are unity.

level matrices of dimension up to 50. At that maximum dimension, the complete program required about 15 min running time per matrix. The diagonalization method was checked for accuracy by means of a 3×3 complex matrix and by various internal consistency checks.

IV. CASE OF 100 BLACK CHANNELS

The case of 100 competing channels, each with transmission coefficient equal to unity and boundary conditions adjusted so that all S_c^0 vanished, was studied most thoroughly. Six independent 50×50 such matrices were diagonalized using six different independent distributions of the $\gamma_{\mu c}$, but the same Wigner anticorrelated distribution of the E_μ centered about $E=0$ and having unit average spacings. It was ascertained that the results were not significantly affected when

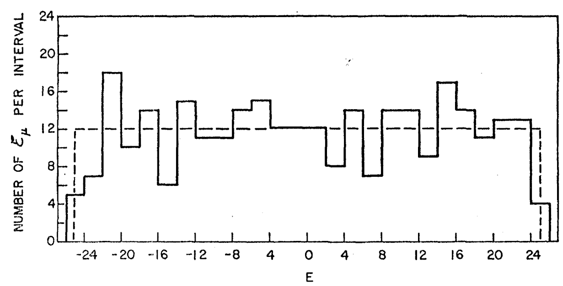

FIG. 2. Combined distribution in energy of the poles of six independent collision matrices as described in Fig. 1. The dashed line shows the underlying density of the R-matrix poles.

the dimensionality of the matrix was reduced in steps down to 10. It was found that the above conditions required the choice of $2\pi P_c=0.2$ for all c. Figure 1 shows the distribution of poles $\mathcal{E}_\mu-\frac{1}{2}i\Gamma_\mu$. (Note that, considered as a plot in the complex plane, the vertical scale in Fig. 1 is twice the horizontal scale.) We note first of all that the density of the $\mathcal{E}_\mu$ is uniform and equal to that of the E_μ and that the center of the distribution is unshifted. These facts are illustrated in Fig. 2 in which we compare the distribution of all 300 $\mathcal{E}_\mu$ in the six cases with the underlying density of the E_μ. This result, that the resonance level density is invariant under the transformation (3), should not be misinterpreted. In particular, it must not be assumed that the density of the $\mathcal{E}_\mu$ may be calculated directly by means of a statistical model of nuclear level densities.[10] Such models presumably apply to states arising from constant boundary conditions and these are not compatible with the assumption that the $S_c^0=0$ in all energy intervals. The shifts resulting from keeping the boundary conditions constant will tend to make the density of the $\mathcal{E}_\mu$ increase more slowly than that calculated by the statistical model.

The nearest-neighbor spacing distribution of the $\mathcal{E}_\mu$ is compared in Fig. 3 with the Wigner distribution which characterizes the spacings of the E_μ and with the exponential distribution which characterizes the spac-

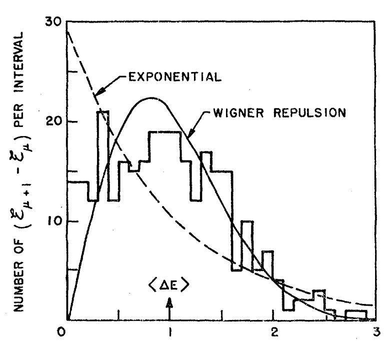

FIG. 3. Distribution of the energy spacings of the same poles as in Fig. 2 compared with the Wigner and exponential distributions.

[7] A. Rotenberg, J. Assoc. Comp. Mach. 7, 75 (1960).
[8] E. P. Wigner, *Fourth Canadian Mathematical Congress Proceedings* (University of Toronto Press, Toronto, Canada, 1957), p. 174.
[9] E. E. Osborn, SIAM 6, 279 (1958).

[10] For summaries see T. Ericson, Advan. Phys. 9, 425 (1960); and D. Bodansky, Ann. Rev. Nucl. Sci. 12, 79 (1962).

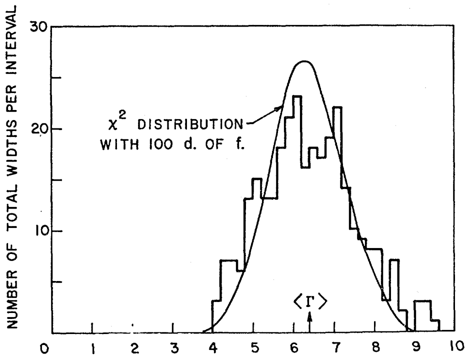

FIG. 4. Distribution of the total widths (twice the imaginary parts) of the poles of Fig. 2 compared with the chi-squared distribution with 100 degrees of freedom.

ings of random uncorrelated levels. It is clear that the level repulsion present in the E_μ is markedly reduced in the $\mathcal{E}_\mu$. However, small spacings are less common than in the exponential case. As a result, the function Φ_0 is actually closer to unity than in the case of full repulsion. Qualitatively, the same effect, namely, a partial reduction of the repulsion, resulted also when other E_μ spacing distributions were employed. Moreover, it appears that no other property of the collision matrix is very sensitive to the assumed spacing distribution of the E_μ.

In Fig. 4, the distribution of the total widths is shown to be slightly broader than the chi-squared distribution with 100 degrees of freedom. The latter characterizes the distribution of the quantities $2\Sigma_c P_c \gamma_{\mu c}{}^2$.[11] The average value $\langle \Gamma_\mu \rangle_\mu$ is 6.40 which does not differ significantly from the average value of $2\Sigma_c P_c \langle \gamma_{\mu c}{}^2 \rangle_\mu$ which is 6.37. It does differ very markedly from the average total width one would obtain from the relation

$$T_c = 2\pi \langle \Gamma_{\mu c} \rangle_\mu / D\,, \tag{14}$$

which would yield $\langle \Gamma_\mu \rangle_\mu = 15.9$. This discrepancy is easily explained as follows. From Eqs. (8) and (10),

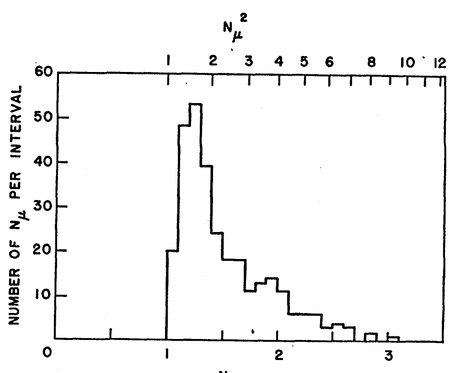

FIG. 5. Combined distribution of the normalization constants N_μ corresponding to the six collision matrices of Fig. 2.

[11] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).

we have

$$T_c \approx \langle \Theta_{\mu c} \rangle_\mu = (2\pi/D) \langle N_\mu | g_{\mu c} |^2 \rangle\,. \tag{15}$$

Comparing this with Eq. (6), we see that if the N_μ and the $|g_{\mu c}|^2$ were uncorrelated we would expect to obtain instead of (14)

$$T_c = \langle N_\mu{}^2 \rangle_\mu (2\pi \langle \Gamma_{\mu c} \rangle_\mu / D)\,. \tag{16}$$

In our case $\langle N_\mu{}^2 \rangle = 2.46$, for which Eq. (16) yields $\langle \Gamma_\mu \rangle_\mu = 15.6$. The deviation of this result from the value of 15.9 is not statistically significant.

The values of the normalization constants N_μ are thus of the greatest practical importance. Figure 5 shows the distribution of the N_μ obtained from six cases under consideration. This distribution has the average $\langle N_\mu \rangle_\mu = 1.52$ and the standard deviation $\sigma(N_\mu) = 0.39$. Of even greater interest is the apparent strong correlation of the values of N_μ belonging to neighboring $\mathcal{E}_\mu$. Numerical investigation of this correlation in the six cases produced positive correlation coefficients in all cases. In addition, the correlation was found to extend over a range of three consecutive $\mathcal{E}_\mu$ in some cases. These results are summarized in Table I,

TABLE I. Average resonance and channel correlations of the collision matrix parameters for 100 black channels.

Correlated quantities	$\rho^{(1)}$	$\rho^{(2)}$	$\rho^{(3)}$	$\rho^{(cc')}$
N_μ	+0.48	+0.13	+0.04	
$\Gamma_{\mu c}$	+0.19			+0.02
$\lvert g_{\mu c} \rvert^2$	+0.20			+0.12
$\Theta_{\mu c}$	+0.19			+0.20

where we employ the correlation coefficients defined by

$$\rho_N{}^{(n)} = \frac{\langle N_\mu N_{\mu+n} \rangle_\mu - \langle N_\mu \rangle^2}{\langle N_\mu{}^2 \rangle_\mu - \langle N_\mu \rangle^2}\,. \tag{17}$$

From Eq. (7), it is apparent that the correlations of the N_μ reflect correlations of the elements of the transformation matrix $\mathbf{t}$ which by Eqs. (4), (6), and (8) influence the statistical properties of the $g_{\mu c}$, $\Gamma_{\mu c}$, and $\Theta_{\mu c}$.

The channel parameters $|g_{\mu c}|^2$, $\Gamma_{\mu c}$, $\Theta_{\mu c}$ are closely related, differing statistically from one another only by factors of N_μ. Their distributions were found to be quite similar, following closely the Porter-Thomas distribution[11] of the $\gamma_{\mu c}{}^2$ but lacking somewhat in small values. This is illustrated in Figs. 6 and 7 where the distributions of the $\Gamma_{\mu c}$ and the $\Theta_{\mu c}$ for 25 channels in one of the cases are compared with the chi squared distributions with one and two degrees of freedom corresponding to the Porter-Thomas and exponential distribution laws, respectively. Examination of the normalized mean square deviations defined by

$$s(x_\mu) = \frac{\langle x_\mu{}^2 \rangle_\mu - \langle x_\mu \rangle_\mu{}^2}{\langle x_\mu \rangle_\mu{}^2} = \left(\frac{\sigma(x_\mu)}{\langle x_\mu \rangle_\mu} \right)^2, \tag{18}$$

shows that the distribution of the $\Theta_{\mu c}$ approaches most closely to the Porter-Thomas value of $s=2.0$, while the distribution of the $\Gamma_{\mu c}$ is distorted most toward the exponential value of $s=1.0$. (See Table II in the next section.) The phase angles of the $g_{\mu c}$ were found to cluster somewhat around the real axis yielding an average value of

$$\langle B_c \rangle_c = 0.39$$

which corresponds to a value of about $\frac{1}{2}$ for the ratio of the standard deviations of the imaginary and real parts of the $g_{\mu c}$.[12]

The N_μ produce two types of correlations of the channel parameters. They may be described as channel correlations and resonance correlations. The channel correlation coefficient

$$\rho_\Theta^{(c,c')} = \frac{\langle \Theta_{\mu c}\Theta_{\mu c'}\rangle_\mu - \langle \Theta_{\mu c}\rangle_\mu \langle \Theta_{\mu c'}\rangle_\mu}{\sigma(\Theta_{\mu c})\sigma(\Theta_{\mu c'})} \qquad (19)$$

arises from the fact that the same fluctuating N_μ enters into the $\Theta_{\mu c}$ for each channel. The resonance correlation coefficients

$$\rho_\Theta^{(n)} = \frac{\langle \Theta_{\mu c}\Theta_{\mu+n,c}\rangle_\mu - \langle \Theta_{\mu c}\rangle_\mu^2}{\sigma^2(\Theta_{\mu c})} \qquad (20)$$

are due to the resonance correlations of the N_μ or of the elements of **t**. Additional correlations between different channels *and* different resonances may be considered. The channel correlations of the $\Gamma_{\mu c}$ were found to be small and of varying signs with a slight bias toward positive correlations. Presumably this is due to the fact that the $t_{\mu\nu}$ occur bilinearly in both the numerator and denominator of the expression (6) for $\Gamma_{\mu c}$, and therefore, the correlation effects largely cancel. On the other hand, the $\Gamma_{\mu c}$ are of very little interest in the many channel case. Their only apparent significance lies in the fact that they add up to the total widths. Cross sections and their fluctuations are determined by the $\Theta_{\mu c}$ and the $|g_{\mu c}|^2$ which exhibit definite positive correlations as summarized in Table I. Such correlations are of importance in the theory of average cross sections where they tend to reduce the width fluctuation correction[3,13] and in the theory of cross-section fluctuations where they affect the interpretation of observed fluctuations.[3,14] [*Note added in proof.* It appears likely that the correlations observed here are in part consequences of the unitarity property which is automatically satisfied in the R matrix formalism, but is not necessarily satisfied by simple statistical assumptions regarding the parameters of a pole expansion of the collision matrix.]

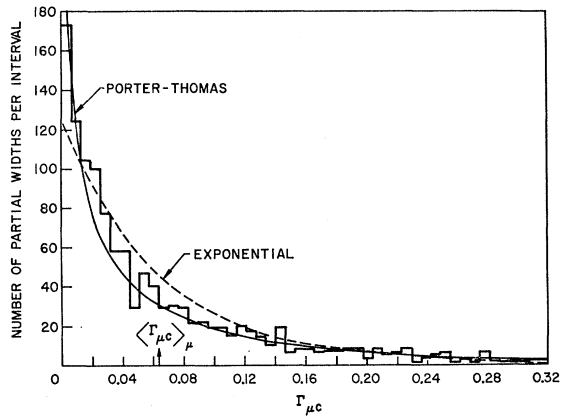

FIG. 6. Combined distribution of the partial widths of 25 of the channels of one of the collision matrices of Fig. 2 compared with the Porter-Thomas and exponential distributions.

V. EFFECTS OF VARIATION OF NUMBER OF CHANNELS AND BOUNDARY CONDITIONS

In order to see how the above results vary with changing numbers of strongly absorbed channels, we have diagonalized three 50×50 matrices corresponding to 300 competing channels with $T_c=0.92$ each, and also one 50×50 matrix corresponding to 20 competing channels with $T_c=0.83$. The latter computation yields insufficient data for a reliable statistical analysis and is cited here only to indicate the general trend. In all of these cases the R-matrix poles E_μ were uniformly spaced one energy unit apart. The results are summarized in Table II. It can be seen there that with increasing numbers of channels, both $\langle N_\mu\rangle_\mu$ and the $\sigma(N_\mu)$ increase, while B_c decreases, indicating a trend toward more isotropically distributed $g_{\mu c}$. The statistical samples provided by these calculations are too small to indicate any meaningful trend in the dispersions and

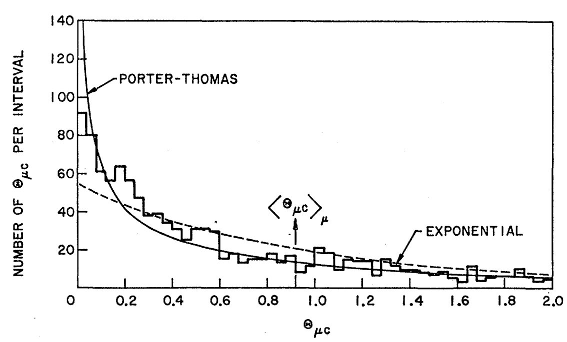

FIG. 7. Combined distributions of the $\Theta_{\mu c}$ of 25 of the channels of one of the collision matrices of Fig. 2 compared with the Porter-Thomas and exponential distributions.

[12] When the phases of the $g_{\mu c}$ are correctly defined, the major axis of the distribution of the $g_{\mu c}$ is found to be rotated away from the real axis by the potential scattering phase shift of channel c.

[13] P. A. Moldauer, in Proceedings of the Symposium on Statistical Properties of Atomic and Nuclear Spectra (unpublished). Obtainable from P. Kahn, Dept. of Physics, State University of New York at Stony Brook, Stony Brook, New York.

[14] T. Ericson, Ann. Phys. (N. Y.) **23**, 390 (1963); Phys. Letters **4**, 258 (1963); D. M. Brink and R. O. Stephen, Phys. Letters **5**, 77 (1963); **8**, 324 (1964); P. A. Moldauer, Phys. Letters **8**, 70 (1964).

TABLE II. Comparison of average parameters for the cases of 20, 100, and 300 strongly absorbed competing channels with 50 resonances spaced at unit energy intervals.

No. of channels	$\langle\Gamma_\mu\rangle$ / $s(\Gamma_\mu)$	$\langle N_\mu\rangle$ / $s(N_\mu)$	$\langle\Gamma_{\mu c}\rangle$ / $s(\Gamma_{\mu c})$	$\langle\lvert g_{\mu c}\rvert^2\rangle$ / $s(\lvert g_{\mu c}\rvert^2)$	$\langle\Theta_{\mu c}\rangle$ / $s(\Theta_{\mu c})$	B_c
20	1.94	1.18	0.097	0.144	0.83	0.53
	0.09	0.01	1.60	1.67	1.81	
100	6.40	1.52	0.064	0.108	1.0	0.39
	0.032	0.066	1.38	1.48	1.88	
300	14.34	1.69	0.047	0.081	0.92	0.33
	0.015	0.07	1.31	1.40	1.76	

correlations of $\Gamma_{\mu c}$ and $\Theta_{\mu c}$. In general, it is remarkable how little the statistical properties appear to change from the 100 to the 300 channel cases. The average total width of 14.3 units in the 300 channel cases is less than the value expected by Eq. (14) by more than a factor of three. This agrees roughly with the prediction of Eq. (16) and the value $\langle N_\mu{}^2\rangle_\mu = 3.06$.

In the R-matrix theory, the shift functions $S_c{}^0$ can be chosen to have arbitrary positive or negative real values by adjustment of the boundary conditions. It is therefore of interest to know whether and to what extent the relation between the statistics of R matrix and collision matrix parameters depends on the choice of boundary conditions. For this purpose the calculations described in Sec. IV were repeated, with all input parameters having the same statistics but now with nonvanishing shift factors given by $2\pi S_c{}^0 = -0.2$, -0.5, -1.0. The calculations were also repeated by adding closed channels with vanishing penetration factors and shift factors ranging up to a value of $-10/2\pi$. The results are summarized in Table III. Both the average values of the N_μ and their dispersions increase as the $S_c{}^0$ move away from zero. In addition, the centroid of the $\mathcal{E}_\mu$ is shifted by $-\Sigma_c(S_c{}^0/2P_c)\langle|g_{\mu c}|^2\rangle_\mu$ units with respect to the centroid of the E_μ and the density of the $\mathcal{E}_\mu$ is no longer uniform but decreases in the direction of the shift. Except for the direction of this shift, the

TABLE III. Dependence of average collision matrix parameters on R-matrix boundary conditions. Lines enclosed by brackets denote single calculation.

Number of channels	$2\pi(S_c{}^0+iP_c)$	$\langle\Gamma_c\rangle$	$\langle N_\mu\rangle$	$s(N_\mu)$	"T_c"	B_c
100	$0.2i$	6.40	1.52	0.066	1.0	0.39
100	$-0.2+0.2i$	6.51	2.12	0.10	2.1	0.33
100	$-0.5+0.2i$	6.52	3.07	0.13	4.5	0.49
100	$-1.0+0.2i$	6.23	5.18	0.11	12.4	0.49
{ 100	$0.2i$	6.29	5.57	0.14	14.2	0.52 }
{ 100	-1.0				0.0	}
{ 100	$0.2i$	6.29	7.86	0.08	27.5	0.96 }
{ 100	-10.0				0.0	}

statistical properties of the collision matrix do not appear to depend on the signs of the $S_c{}^0$.

The increased values of N_μ result in transmission coefficients greater than unity in Table III. These cases are then physically not meaningful, but serve to indicate the extent to which the dispersions of the $\gamma_{\mu c}$ must be reduced in order to maintain the physical values of the transmission coefficients in the face of boundary conditions with nonvanishing values of $S_c{}^0$. Of course, such reductions in the dispersions of the $\gamma_{\mu c}$ will result in corresponding reductions of the partial and total widths. The situation is summarized by observing that the relation (16) between the channel transmission coefficients and the average partial width to spacing ratios are dependent, through $\langle N_\mu{}^2\rangle$, upon the values of all the channel boundary conditions. However, to the extent that both the T_c and the $\langle\Gamma_{\mu c}\rangle/D$, or even the $\langle\Gamma_\mu\rangle/D$, are measurable quantities, the actual relationship between them cannot depend on the choice of R-matrix boundary conditions. It follows therefore, that for some sets of boundary conditions the distributions and correlations of the E_μ and the $\gamma_{\mu c}$ must be modified in such a way that either the relation (16) or the computed distributions of the N_μ are changed, or both. In other words, the bound-state random matrix model of the R-matrix parameters[1,8] cannot apply to all sets of boundary conditions.

Of course, the measurability of the channel transmission coefficients and the width to spacing ratios may be debatable, at least in practice. The T_c have at least upper bounds and the total widths are accessible by the analysis of cross-section fluctuations,[14] though the interpretation of such data is made more complicated by the above mentioned resonance correlations of the $|g_{\mu c}|^2$. But determinations of partial widths or resonance level spacings may be exceedingly difficult in the case of overlapping resonances and may even be impossible in principle. Nevertheless, it seems unlikely that agreement with all measurable quantities can be achieved in a reasonable way without modifications in the R-matrix statistics to suit the R-matrix boundary conditions.

The boundary conditions which set all $S_c{}^0$ locally equal to zero are unique in that they leave the pole distribution unshifted and the pole density invariant when passing from the R matrix to the collision matrix. They also minimize $\langle N_\mu\rangle_\mu$ and maximize $\langle\Gamma_\mu\rangle/D$ for a given set of channel transmission coefficients T_c. For these reasons, and because these boundary conditions relate the R-matrix poles to the physical processes in the same region of the energy spectrum where the poles are located, it may not be unreasonable to speculate that these boundary conditions are indeed the appropriate ones for the statistical application of R-matrix theory with the usual bound state model for the parameters.

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