Limits on Parity and Time-Reversal Noninvariance in *p-p* Scattering*

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A formalism for *p-p* elastic scattering is developed that includes the possibility of parity nonconservation and time-reversal noninvariance. Relevant experimentally measurable quantities are defined. An experiment measuring three of the quantities is described. These and other measurements relevant to parity nonconservation or time-reversal noninvariance are analyzed in terms of the formalism developed. No evidence is found for any violation of either invariance principle. The parity-conserving, time-reversal-noninvariant coupling of $^3P_2$ and $^3F_2$ states is at most 7% of its maximum possible value, in the energy range 140–210 MeV. The parity-nonconserving, time-reversal-invariant coupling of $^1S_0$ and $^3P_0$ states is at most 70% of its maximum possible value at 140 MeV. The parity-nonconserving, time-reversal-noninvariant coupling of $^1S_0$ and $^3P_0$ states is not bounded away from its maximum possible positive value, and is at most 60% of its extreme negative value at 140 MeV. The evidence against parity nonconservation is thus seen to be surprisingly weak. An experiment to sharpen the limits on parity nonconservation is suggested.

I. INTRODUCTION

SINCE the discovery of parity nonconservation in weak interactions, there have been several experiments performed to look for (or, alternatively, set upper limits on) parity-nonconserving terms in strong interactions. The majority of these have involved nuclear reactions at low energies: search for transitions forbidden by parity conservation, search for circular polarization of decay γ rays, etc. An extensive list of references is given by Michel.[1]

It is also of interest to perform experiments at higher energies, because parity nonconserving terms might become relatively larger as the energy increases. Such experiments performed to data are fewer in number and lower in accuracy.

This article is primarily concerned with evidence for parity conservation in *p-p* elastic scattering at energies below the threshold of π production. It is necessary also to consider evidence for time-reversal invariance. In Sec. II, a formalism that includes the possibility of parity nonconservation and time-reversal noninvariance is presented. Relevant experimentally measurable quantities are defined. An experiment measuring three of these quantities is described in Sec. III. An analysis of *p-p* scattering data relevant to parity conservation and time-reversal invariance is given in Sec. IV.

The conclusions are summarized in Sec. V. It is shown that Lee and Yang,[2] and Heer, Roberts, and Tinlot[3] estimated limits on parity nonconserving terms too small by a factor of 10, and while there is no evidence to suggest that parity is not conserved in *p-p* scattering, there is little evidence to require that it is conserved. The evidence for time-reversal invariance is stronger. Further analyses and experiments which could sharpen the limits on parity nonconservation are discussed.

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[1] F. C. Michel, Phys. Rev. **133**, B329 (1964).

[2] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[3] E. Heer, A. Roberts, and J. Tinlot, Phys. Rev. **111**, 645 (1958).

II. FORMULATION

A. General Form of the Scattering Matrix

We characterize the scattering of two spin-$\frac{1}{2}$ particles by the unit vectors **n**, **p**, and **q**, defined in terms of the initial ($\mathbf{P}_{1i}$) and final ($\mathbf{P}_{1f}$) momenta of particle one, in the center-of-mass system:

$$\mathbf{n}=(\mathbf{P}_{1i}\times\mathbf{P}_{1f})/|(\mathbf{P}_{1i}\times\mathbf{P}_{1f})|\,, \tag{1a}$$

$$\mathbf{p}=(\mathbf{P}_{1i}+\mathbf{P}_{1f})/|(\mathbf{P}_{1i}+\mathbf{P}_{1f})|\,, \tag{1b}$$

$$\mathbf{q}=(\mathbf{P}_{1f}-\mathbf{P}_{1i})/|(\mathbf{P}_{1f}-\mathbf{P}_{1i})|\,. \tag{1c}$$

The most general scattering matrix $\mathfrak{M}$ may then be written

$$\begin{aligned}\mathfrak{M}=&A+B\sigma_{1n}\sigma_{2n}+C(\sigma_{1n}+\sigma_{2n})+D(\sigma_{1n}-\sigma_{2n})+E\sigma_{1q}\sigma_{2q}\\&+F\sigma_{1p}\sigma_{2p}+J(\sigma_{1q}+\sigma_{2q})+K(\sigma_{1p}+\sigma_{2p})\\&+L(\sigma_{1p}\sigma_{2q}-\sigma_{1q}\sigma_{2p})+M(\sigma_{1n}\sigma_{2p}+\sigma_{1p}\sigma_{2n})\\&+N(\sigma_{1n}\sigma_{2q}+\sigma_{1q}\sigma_{2n})+P(\sigma_{1q}-\sigma_{2q})\\&+Q(\sigma_{1q}\sigma_{2n}-\sigma_{1n}\sigma_{2q})+R(\sigma_{1p}-\sigma_{2p})\\&+S(\sigma_{1n}\sigma_{2p}-\sigma_{1p}\sigma_{2n})+T(\sigma_{1p}\sigma_{2q}+\sigma_{1q}\sigma_{2p})\,.\end{aligned} \tag{2}$$

(We have used the standard notation $\sigma_{1n}=\boldsymbol{\sigma}_1\cdot\mathbf{n}$, where $\boldsymbol{\sigma}_1$ is the Pauli spin matrix. The coefficients A, B, C, etc., are complex functions of the energy and scattering angle.)

The effects of space reflection (P), time reversal (T), and exchange of the two particles (X) on the various terms in $\mathfrak{M}$ are shown in Table I.

If $\mathfrak{M}$ describes the scattering of two identical particles (as in *p-p* scattering), invariance under exchange of particles (X) is required. Further, if $\mathfrak{M}$ described *n-p* scattering, charge symmetry implies invariance under (X). Eliminating the six terms that change signs under (X), we are left with 5 invariant terms A, B, C, E, F, and five terms that imply parity nonconservation (R,S), time-reversal noninvariance (T), or both (P,Q).

B. Phase-Shift Parametrization

The conventional phase-shift parametrization of the scattering matrix assumes conservation of parity and time-reversal invariance. The case when these assumptions are *not* made is treated by Woodruff.[4] Parity nonconservation allows the coupling of a singlet state to two triplet states of the same J value. Woodruff described this by two additional mixing parameters η_j and ζ_j, which vanish if parity is conserved. Time-reversal invariance requires the matrix coupling two states of the same parity to be symmetric, and the matrix coupling two states of opposite parity to be antisymmetric. Time-reversal noninvariance allows the matrix coupling states to become unsymmetric. This is described by three additional parameters (for even J): $\lambda_{2,j}$ which involves the parity-conserving coupling, and $\lambda_{1,j}$ and $\lambda_{3,j}$ which involve the parity-violating couplings. Time-reversal invariance requires $\lambda_{2,j}=0$, $|\lambda_{1,j}|=|\lambda_{3,j}|=\frac{1}{2}\pi$.

Woodruff[4] has made an unfortunate choice in his parametrization, in that his mixing may be described as the three successive mixings

$$^1J_J \leftrightarrow {}^3(J-1)_J;\ {}^3(J-1)_J \leftrightarrow {}^3(J+1)_J;\ {}^1J_J \leftrightarrow {}^3(J-1)_J.$$

He relies for his mixing of the states $^1J_J \leftrightarrow {}^3(J+1)_J$ on the indirect path $^1J_J \rightarrow {}^3(J-1)_J \rightarrow {}^3(J+1)_J$. Thus, as the coupling ϵ between $^3(J-1)_J$ and $^3(J+1)_J$ approaches zero, his parameters ζ and η coalesce into one parameter, i.e., $\zeta+\eta$. Hence, for small ϵ, the quantity $\zeta-\eta$ is very poorly defined by the experimental data and has large errors. Further, $J=0$ is not properly treated by Woodruff's parametrization, and must be considered as a special case.

We parametrize by the following three successive mixings:

$$^1J_J \overset{\zeta_J}{\leftrightarrow} {}^3(J+1)_J;\ {}^3(J+1)_J \overset{\epsilon_J}{\leftrightarrow} {}^3(J-1)_J;\ {}^1J_J \overset{\eta_J}{\leftrightarrow} {}^3(J-1)_J.$$

TABLE I. Behavior of the various amplitudes in $\mathfrak{M}$ under exchange of two particles (X), space reflection (P), and time reversal (T). x indicates that the term changes sign under the indicated operation.

Term	Invariant	X	P	T	PT
A	Yes				
B	Yes				
C	Yes				
E	Yes				
F	Yes				
D	No	x			
J	No	x	x	x	
K	No	x	x		x
L	No	x		x	x
M	No	x	x		x
N	No	x	x	x	
P	No		x	x	
Q	No		x	x	
R	No		x		x
S	No		x		x
T	No			x	x

[4] A. E. Woodruff, Ann. Phys. (N. Y.) 7, 65 (1959).

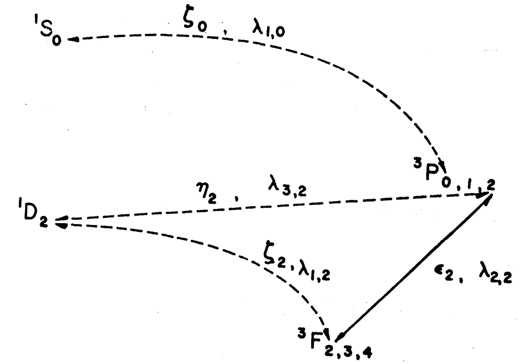

FIG. 1. The coupling of p-p states for $l \leqslant 3$. The solid arrow shows the parity-conserving transition between $^3P_2$ and $^3F_2$ states, described by the mixing parameter ϵ_2. Time-reversal noninvariance in this transition is characterized by a nonzero value of $\lambda_{2,2}$. The parity-nonconserving transitions indicated by the dashed arrows are described by the mixing parameters ζ_0, η_2, and ζ_2. Time-reversal noninvariance in these transitions is characterized by nonzero values of $\cos\lambda_{1,0}$, $\cos\lambda_{3,2}$, and $\cos\lambda_{1,2}$.

The difficulties present in Woodruff's choice, which occur for $\epsilon \approx 0$, show up in this choice for $\epsilon \approx 90°$. Since $\epsilon_0 \equiv 0$, and $\epsilon_2 \approx -12°$, at these energies, our parametrization seems preferable.

The situation for $l \leqslant 3$ is shown pictorially in Fig. 1, modified from Woodruff.[4] The solid arrow shows the parity conserving transition between $^3P_2$ and $^3F_2$ states, described by the mixing parameter ϵ_2. Time-reversal noninvariance in this transition is characterized by a nonzero value of $\lambda_{2,2}$. The dashed arrows show parity-nonconserving transitions between $^1S_0$ and $^3P_0$, described by ζ_0, between $^3P_2$ and $^1D_2$ described by η_2, and between $^1D_2$ and $^3F_2$ described by ζ_2. Time-reversal noninvariance in these transitions is characterized by nonzero values of $\cos\lambda_{1,0}$, $\cos\lambda_{3,2}$, and $\cos\lambda_{1,2}$, respectively.

Introducing only this one change into Woodruff's treatment, we write

$$\begin{pmatrix} \alpha_j & \alpha_+{}'^j & \alpha_+{}''^j \\ \alpha_-{}'^j & \alpha_{j-1,j} & \alpha_+{}^j \\ \alpha_-{}''^j & \alpha_-{}^j & \alpha_{j+1,j} \end{pmatrix} = U_J^{-1} \begin{pmatrix} e^{2i\delta_j} & 0 & 0 \\ 0 & e^{2i\delta_{j-1,j}} & 0 \\ 0 & 0 & e^{2i\delta_{j+1,j}} \end{pmatrix} U_J - \alpha_{\text{Coulomb}}, \quad (3)$$

$$U_J = B_J C_J D_J, \quad (4)$$

$$B_J = \begin{pmatrix} \cos\eta & \sin\eta e^{i\lambda_3} & 0 \\ -\sin\eta e^{-i\lambda_3} & \cos\eta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (5a)$$

$$C_J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\epsilon & \sin\epsilon e^{i\lambda_2} \\ 0 & -\sin\epsilon e^{-i\lambda_2} & \cos\epsilon \end{pmatrix}, \quad (5b)$$

$$D_J = \begin{pmatrix} \cos\zeta & 0 & \sin\zeta e^{i\lambda_1} \\ 0 & 1 & 0 \\ -\sin\zeta e^{-i\lambda_1} & 0 & \cos\zeta \end{pmatrix}. \quad (5c)$$

Keeping terms linear in the parity-violating param-

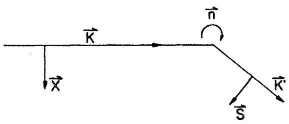

FIG. 2. The laboratory coordinate system used to define the experimental parameters. **k** is the direction of the incident particle; **k′**, the direction of the scattered particle.

eters ζ_j and η_j, and dropping all higher terms, we obtain:

$$\alpha_j = e^{2i\delta_j} - e^{2i\phi_j}, \quad (6a)$$

$$\alpha_{j-1,j} = \cos^2\epsilon e^{2i\delta_{j-1,j}} + \sin^2\epsilon e^{2i\delta_{j+1,j}} - e^{2i\phi_{j-1}}, \quad (6b)$$

$$\alpha_{j+1,j} = \sin^2\epsilon e^{2i\delta_{j-1,j}} + \cos^2\epsilon e^{2i\delta_{j+1,j}} - e^{2i\phi_{j+1}}, \quad (6c)$$

$$\alpha_+{}^j = (e^{2i\delta_{j-1,j}} - e^{2i\delta_{j+1,j}}) \cos\epsilon \sin\epsilon e^{i\lambda_2}, \quad (6d)$$

$$\alpha_-{}^j = \text{same as } \alpha_+{}^j, \text{ with sign of } \lambda_2 \text{ reversed}, \quad (6e)$$

$$\alpha_+{}'^j = e^{2i\delta_j}\eta e^{i\lambda_3}\cos\epsilon - e^{2i\delta_{j-1,j}}(\eta e^{i\lambda_3} + \zeta e^{i\lambda_1}\sin\epsilon e^{-i\lambda_2}) \times \cos\epsilon + e^{2i\delta_{j+1,j}}\zeta e^{i\lambda_1}\sin\epsilon\cos\epsilon e^{-i\lambda_2}, \quad (6f)$$

$$\alpha_-{}'^j = \text{same as } \alpha_+{}'^j, \text{ with signs of all } \lambda\text{'s reversed}. \quad (6g)$$

$$\alpha_+{}''^j = e^{2i\delta_j}(\zeta e^{i\lambda_1} + \eta e^{i\lambda_3}\sin\epsilon e^{i\lambda_2}) - \zeta e^{2i\delta_{j+1,j}} e^{i\lambda_1}\cos^2\epsilon - e^{2i\delta_{j-1,j}}(\eta e^{i\lambda_3} + \zeta e^{i\lambda_1}\sin\epsilon e^{-i\lambda_2})\sin\epsilon e^{i\lambda_2}, \quad (6h)$$

$$\alpha_-{}''^j = \text{same as } \alpha_+{}''^j, \text{ with signs of all } \lambda\text{'s reversed}. \quad (6i)$$

Phase shifts are Blatt–Biedenharn; ϕ_l is a Coulomb phase shift. Our expressions are identical to Woodruff's[4] except for the four terms $\alpha_\pm{}'^j$, $\alpha_\pm{}''^j$. We have omitted the subscript j from ϵ, η, ζ, λ_1, λ_2, λ_3.

The relation between the α's and the $\mathfrak{M}$ matrix is given by Woodruff.[4] The parity-conserving, time-reversal invariant terms are unchanged, to first order in the violations. The five violating terms are given by

$$T = \tfrac{1}{4}(ik)^{-1} \sum_{\text{odd}\, l} (\alpha_+{}^{l+1} - \alpha_-{}^{l+1}) \frac{(2l+3)}{[(l+1)(l+2)]^{1/2}} [(l+1)P_l(\theta)\sin\theta + P_l{}^1(\theta)\cos\theta], \quad (7)$$

$$P = -\tfrac{1}{4}(M_{0s}+M_{s0})\sin(\theta/2) - (1/2\sqrt{2})(M_{1s}+M_{s1})\cos(\theta/2), \quad (8a)$$

$$Q = \tfrac{1}{4}i(M_{0s}-M_{s0})\cos(\theta/2) - (i/2\sqrt{2})(M_{1s}-M_{s1})\sin(\theta/2), \quad (8b)$$

$$R = \tfrac{1}{4}(M_{0s}+M_{s0})\cos(\theta/2) - (1/2\sqrt{2})(M_{1s}+M_{s1})\sin(\theta/2), \quad (8c)$$

$$S = -\tfrac{1}{4}i(M_{0s}-M_{s0})\sin(\theta/2) - (i/2\sqrt{2})(M_{1s}-M_{s1})\cos(\theta/2), \quad (8d)$$

where

$$M_{s0} = -(k)^{-1} \sum_{\text{even}\, l} P_l(\theta)\{[(2l+1)(l+1)]^{1/2}\alpha_+{}''^l + [(2l+1)l]^{1/2}\alpha_+{}'^l\}, \quad (9a)$$

$$M_{s1} = (k)^{-1} \sum_{\text{even}\, l} P_l{}^1(\theta)\left[-\left(\frac{2l+1}{2l+2}\right)^{1/2}\alpha_+{}''^l + \left(\frac{2l+1}{2l}\right)^{1/2}\alpha_+{}'^l\right], \quad (9b)$$

$$M_{0s} = (k)^{-1} \sum_{\text{odd}\, l} P_l(\theta)\{[(l+1)(2l+3)]^{1/2}\alpha_-{}'^{l+1} + [l(2l-1)]^{1/2}\alpha_-{}''^{l-1}\}, \quad (9c)$$

$$M_{1s} = (k)^{-1} \sum_{\text{odd}\, l} P_l{}^1(\theta)\left[\left(\frac{2l+3}{2l+2}\right)^{1/2}\alpha_-{}'^{l+1} - \left(\frac{2l-1}{2l}\right)^{1/2}\alpha_-{}''^{l-1}\right]. \quad (9d)$$

C. Experimental Quantities

The experimental parameters are most conveniently defined with respect to a laboratory coordinate system. Our coordinate system is shown in Fig. 2. The five unit vectors are defined by $\mathbf{k}$, direction of the incident particle; $\mathbf{k}'$, direction of the scattered particle; $\mathbf{n} = \mathbf{k}\times\mathbf{k}'/|\mathbf{k}\times\mathbf{k}'|$, the sense of the scattering; $\mathbf{x} = \mathbf{n}\times\mathbf{k}$; $\mathbf{s} = \mathbf{n}\times\mathbf{k}'$. All but $\mathbf{n}$ are polar vectors; $\mathbf{n}$ is an axial vector.

The relevant experimental parameters are defined by the two equations

$$I = I_0[1 + \langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{n}\mathcal{A}_n + \langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{x}\mathcal{A}_x + \langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{k}\mathcal{A}_k], \quad (10)$$

$$I\langle\boldsymbol{\sigma}\rangle_f = I_0\{[P_n + D\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{n} + \rho\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{x} + \alpha\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{k}]\mathbf{n} + [P_s + \Delta\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{n} + R\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{x} + A\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{k}]\mathbf{s} + [P_{k'} + \Delta'\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{n} + R'\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{x} + A'\langle\boldsymbol{\sigma}\rangle_i\cdot\mathbf{k}]\mathbf{k}'\}. \quad (11)$$

I_0 is the differential cross section for an unpolarized beam. I is the differential cross section for a polarized beam. $\langle\boldsymbol{\sigma}\rangle_i$ is the polarization of the incident beam. $\langle\boldsymbol{\sigma}\rangle_f$ is the polarization of the scattered beam. (To fix the sign of the parity-nonconserving parameters, a right-hand rule is used to convert axial vectors to polar vectors.)

Equations (10) and (11) are natural extensions of the formalism of Wolfenstein[5]; similar definitions of parameters have been given by Gammel and Thaler,[6] and Woodruff.[4]

The $\mathcal{A}$ parameters are asymmetry parameters. For an incident beam polarized along a vertical axis, $\mathcal{A}_n$ is the left-right asymmetry for scattering in the horizontal plane, and $\mathcal{A}_x$ is the up-down asymmetry for scattering in a vertical plane. (Equally well, these parameters are

[5] L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).

[6] J. L. Gammel and R. M. Thaler, *Progress in Elementary Particle and Cosmic Ray Physics* (North-Holland Publishing Company, Amsterdam, 1960), Vol. 5, p. 99.

measured by keeping the sense of the scattering fixed and changing the sign of the incident polarization.) For the measurement of $\mathcal{A}_k$, the incident beam is polarized along its direction of motion, either parallel or antiparallel. $\mathcal{A}_k$ is the parallel-antiparallel asymmetry. (It can *only* be measured by changing the sign of the incident polarization, and not by changing the sense of the scattering.) If parity is conserved, then $\mathcal{A}_x=\mathcal{A}_k=0$.

The P parameters are polarization parameters. If the incident beam is unpolarized,

$$\langle\boldsymbol{\sigma}\rangle_f=P_n\mathbf{n}+P_s\mathbf{s}+P_{k'}\mathbf{k}'. \tag{12}$$

If parity is conserved, $P_s=P_{k'}=0$. If (time reversal) $\times$(parity) or if time reversal alone is conserved, then $P_n=\mathcal{A}_n$.

D, Δ, Δ', R, R', ρ, A, A', and α are triple scattering parameters, relating initial and final polarizations. D, R, A, R', and A' are the conventional parameters.[5] Δ, Δ', ρ, and α couple polarization in the scattering plane to polarization normal to the scattering plane, and all vanish if parity is conserved. Time-reversal invariance implies a relation among R, A, R', and A'.

The experimental quantities of particular concern to us are $P_n-\mathcal{A}_n$, $\mathcal{A}_x$, P_s, and Δ. To calculate these from the $\mathfrak{M}$ matrix, we must relate the unit vectors $\mathbf{n}$, $\mathbf{s}$, $\mathbf{k}'$, describing the scattering event in the lab to the unit vectors $\mathbf{n}$, $\mathbf{p}$, $\mathbf{q}$, describing the event in the center-of-mass system. The vector $\mathbf{n}$ is the same in the lab and the center-of-mass system. In a nonrelativistic approximation, $\mathbf{s}=\mathbf{q}$, $\mathbf{k}'=\mathbf{p}$, $\theta_{\text{c.m.}}=2\theta_L$. For evaluating the small quantities $\mathcal{A}_x$ and P_s, we shall use these approximations. Taking the appropriate traces, we obtain:

$$I_0(P_n-\mathcal{A}_n)=4\,\text{Im}[T^*(E-F)]+4\,\text{Im}(PR^*+SQ^*), \tag{13a}$$

$$I_0P_s=2\,\text{Re}(AP^*-EP^*+CQ^*-RT^*)+2\,\text{Im}(BS^*+RC^*+FS^*+QT^*), \tag{13b}$$

$$I_0\mathcal{A}_x=I_0\mathcal{A}_p\sin\theta_L+I_0\mathcal{A}_q\cos\theta_L, \tag{13c}$$

$$I_0\mathcal{A}_p=2\,\text{Re}(AR^*-FR^*-CS^*-PT^*)+2\,\text{Im}(QB^*+QE^*+PC^*+TS^*), \tag{13d}$$

$$I_0\mathcal{A}_q=2\,\text{Re}(AP^*-EP^*+CQ^*-RT^*)+2\,\text{Im}(SB^*+SF^*+CR^*+TQ^*), \tag{13e}$$

$$I_0\Delta=2\,\text{Re}(BQ^*+CP^*-EQ^*+TS^*)-2\,\text{Im}(AR^*+FR^*-CS^*-PT^*), \tag{13f}$$

$$I_0=|A|^2+|B|^2+2|C^2|+|E|^2+|F|^2+2(|P|^2+|Q|^2+|R|^2+|S|^2+|T|^2). \tag{13g}$$

D. Elastic Scattering from Spin-Zero Complex Nuclei

It is necessary to consider the effect of violations of parity and time-reversal invariance in elastic scattering from spin-zero nuclei, as such scattering is used both to produce the polarized beam, and to measure polarizations. The most general scattering matrix may be written

$$\mathfrak{M}_0=A_0+C_0\sigma_{1n}+P_0\sigma_{1s}+R_0\sigma_{1k'}. \tag{14}$$

In impulse approximation, these coefficients are proportional to

$$A_{pp}+A_{np},\quad C_{pp}+C_{np}+D_{np},\quad P_{pp}+P_{np}+J_{np},$$

and

$$R_{pp}+R_{np}+K_{np}.$$

The observables of interest to us are given by

$$I_0P_n=2\,\text{Re}C_0A_0^*-2\,\text{Im}R_0P_0^*, \tag{15a}$$

$$I_0\mathcal{A}_n=2\,\text{Re}C_0A_0^*+2\,\text{Im}R_0P_0^*, \tag{15b}$$

$$I_0P_s=2\,\text{Re}A_0P_0^*-2\,\text{Im}C_0R_0^*, \tag{15c}$$

$$I_0P_{k'}=2\,\text{Re}A_0R_0^*+2\,\text{Im}C_0P_0^*, \tag{15d}$$

$$I_0\mathcal{A}_x=(2\,\text{Re}A_0R_0^*-2\,\text{Im}C_0P_0^*)\sin\theta_L+(2\,\text{Re}A_0P_0^*+2\,\text{Im}C_0R_0^*)\cos\theta_L, \tag{15e}$$

$$I_0=|A_0|^2+|C_0|^2+|P_0|^2+|R_0|^2. \tag{15f}$$

III. THE EXPERIMENT

A. General Description

The experiment can most readily be described by a reference to Fig. 3. A 140-MeV, 65% polarized proton beam having its polarization vertical passes through a solenoid magnet (P). If the solenoid is turned on, the polarization precesses 90° about the direction of motion, so that on leaving the solenoid the beam has a polarization P_1 in the horizontal plane and perpendicular to the direction of motion. The beam strikes a liquid-hydrogen target (2), and particles scattered through an angle θ_2 in the horizontal plane, defined by counters A, B, then strike the analyzing scatterer (3). Particles scattered through an angle θ_3 in the vertical plane containing the line from the hydrogen target to the analyzing scatterer are detected by the counter telescopes CD or EF. The angle θ_3 of these telescopes can be reversed in sign; we

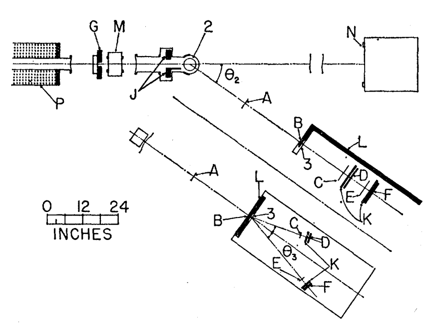

FIG. 3. Scale drawing of the experimental arrangement for the measurement of $\mathcal{A}_x$, P_s, and Δ showing: (2) hydrogen target, (3) analyzing scatterer (A–F) scintillation counters, (G) main defining slits, (J) antiscattering slits, (K) copper absorber, (L) iron shielding, (M) ion chamber, (N) Faraday cup, and (P) solenoid magnet.

denote by U and D, respectively, the up and down positions. The direction of the current through the solenoid, and hence the sign of the incident polarization P_1 can be reversed; we denote the possibilities by N (for normal), R (for reversed), and O (for off).

Let $I(k,m)$ be the rate of fourfold coincidences ($ABCD$ or $ABEF$) for counter telescope position k and solenoid current direction m, where k is either U or D, and m is either N, R, or O. Then we define

$$e_{3s}=\frac{I(D,N)+I(U,R)-I(U,N)-I(D,R)}{I(D,N)+I(U,R)+I(U,N)+I(D,R)}, \quad (16a)$$

$$e_{N-R}=\frac{I(D,N)+I(U,N)-I(D,R)-I(U,R)}{I(D,N)+I(U,N)+I(D,R)+I(U,R)}, \quad (16b)$$

$$e_{U-D}=\frac{I(U,N)+I(U,R)-I(D,N)-I(D,R)}{I(U,N)+I(U,R)+I(D,N)+I(D,R)}, \quad (16c)$$

$$e_{U-D}{}^{\mathrm{off}}=\frac{I(U,O)-I(D,O)}{I(U,O)+I(D,O)}. \quad (16d)$$

If P_1 is the polarization of the incident beam, and P_3 the spin-analyzing power of the third scattering, then

$$e_{3s}=RP_1P_3, \quad (17a)$$

$$e_{N-R}=\mp\mathcal{A}_xP_1, \quad (17b)$$

$$e_{U-D}=\pm P_sP_3, \quad (17c)$$

$$e_{U-D}{}^{\mathrm{off}}=P_3[P_1\Delta\pm P_s]. \quad (17d)$$

The upper sign is used for θ_2 scattering in the same sense as the first scattering (south), the lower sign for θ_2 being in the opposite sense (north). Note that the third scattering serves no purpose in the measurement of $\mathcal{A}_x$; the up and down positions are averaged over, and the entire third scattering system serves merely as a detector.

The primary purpose of the experiment was the measurement of R [Eq. (17a)]. That aspect of the experiment has already been described.[7,8] The reader is referred to Refs. 7 and 8 for further details on the apparatus and experimental procedure. A bonus of measuring R by means of a spin-precession solenoid is that one *simultaneously* obtains a measurement of $\mathcal{A}_x$ and P_s [Eqs. (17b) and (17c)]. Some additional data taken with the solenoid off provide a measurement of Δ. It is to these aspects of the experiment that the present article is devoted. Unfortunately, at the time when the data were obtained, it was not fully appreciated that they would yield P_s and $\mathcal{A}_x$, as well as R. Hence, there are certain errors to $\mathcal{A}_x$ present that could have been eliminated. The following subsections discuss treatment of systematic errors, errors to $\mathcal{A}_x$, errors to P_s, consequences to these measurements of parity nonconservation in p-carbon scattering, the results for $\mathcal{A}_x$ and P_s, and the Δ measurement.

[7] E. H. Thorndike, thesis, Harvard University, 1960 (unpublished).

[8] E. H. Thorndike, J. Lefrançois, and R. Wilson, Phys. Rev. **120**, 1819 (1960).

B. Treatment of Systematic Errors

This experiment contains several important systematic errors. This section will define the term "systematic error" as used here, and give a way of representing them. Errors (on measurements, at two or more different angles) that are *correlated* are called systematic. If an error on a measurement at one angle is *uncorrelated* with those at other angles, the error is called "random."

The data of this experiment were collected during four different experimental runs, two with the second scattering "north" and two "south." Hence the following four possibilities exist for the systematic nature of an error: (1) random; (2) systematic over a run, but independent from run to run; (3) systematic over the sense of the second scattering angle (N or S), but independent between the two senses; (4) systematic over the entire experiment.

In this article, an error systematic over several angles θ_i, will be represented as a *correction* to the measured quantity, of the form $\alpha A(\theta_i)$. $A(\theta_i)$ is a calculable function, while α is unknown, with probable value 0, and standard deviation ±1. α has the same value at all angles over which the error is systematic, thus allowing for the correlation from angle to angle.

Two errors systematic over the same range of angles can be combined if their angular dependence is the same (in practice, nearly the same). Thus, if $\alpha A(\theta_i)$ and $\beta B(\theta_i)$ represent two systematic errors, and $A(\theta_i)/B(\theta_i)\approx$constant, these errors are combined and represented by $\gamma C(\theta_i)$, where $\gamma=0\pm1$, and $C(\theta_i)=\{[A(\theta_i)]^2+[B(\theta_i)]^2\}^{1/2}$. For purposes of analysis, it does not matter whether the two errors that have been combined are in any way related as to their causes; what is important is that both are systematic over the same range, and both have the same angular behavior.

C. Errors to $\mathcal{A}_x$

In addition to errors to $\mathcal{A}_x$ from counting statistics, several other sources of error were considered, including those affecting the rate at which protons arrive at the B counter, and those affecting the detection efficiency of the third scattering system. The sources of error considered are listed in Table II. The quantity in parenthesis is the estimate of error to e_{N-R} (standard deviation) at the most unfavorable angle θ_2.

The most serious sources of error are (1), (3), (5), (8). [In an experiment specifically designed to measure $\mathcal{A}_x$, all of these but (1) could be greatly reduced.] We now discuss these in turn, and mention their systematic nature.

TABLE II. Sources of error to $\mathcal{A}_x$.

(1) Nonzero values for undesired components of polarization	$(0.04P_1\mathcal{A}_n \lesssim 0.004)$
(2) Nonlevelness of apparatus at θ_2 and θ_3 scattering	$(0.015P_1\mathcal{A}_n)$
(3) Monitoring changes with solenoid.	(0.005)
(4) θ_2 alignment changes with solenoid.	(0.0007)
(5) Beam-energy changes with solenoid, and with position across defining slit.	(0.007)
(6) Large asymmetry in background or random coincidences	(0.0004)
(7) Poor solenoid-current regulation.	(0.001)
(8) Volume of target-intersected changes with solenoid.	(0.005)
(9) θ_3 alignment shifts due to solenoid, sine-bar distortions, and polarization effects.	(0.0005)
(10) Changes in solid angle at third scattering.	(0.0002)
(11) Coupling of any two of above, which alone would cancel.	(0.001)

(1) If the incident beam emerging from the cyclotron has a component of polarization P_t, transverse to the beam direction and horizontal, then, on leaving the solenoid, there will be a vertical component of polarization, which will change sign as the solenoid is reversed, and give rise to an asymmetry[9] $P_t\mathcal{A}_n$, where $\mathcal{A}_n$ is the normal asymmetry parameter. A nonzero value for P_t would be caused by parity nonconservation in the p-carbon (first) scattering. This possibility will be elaborated upon in subsection III-E. It will also be caused by misaligning the beam defining slits relative to the median plane of the cyclotron. Out of the median plane there are horizontal components to the cyclotron magnetic field, which will precess the (large) vertical component of polarization into a horizontal component. Further, if the beam-defining slits are misaligned relative to the median plane, the first scattering will not be horizontal, again introducing a horizontal component of polarization.

Measurements have been made of the horizontal transverse and the longitudinal component of polarization, with the defining slits deliberately misaligned. (The beam was scattered from carbon; the longitudinal component was determined by interposing a 44° bending magnet between solenoid and carbon scatterer.) The results are

$$\Delta P_t/P_1 = 0.14 \pm 0.05/\text{in.},$$

$$\Delta P_{\text{longitudinal}}/P_1 = 0.16 \pm 0.08/\text{in.}$$

A calculation has been made of the longitudinal and horizontal transverse component of polarization, per inch misalignment of defining slits. The results are sensitive to the precise form of the fringing field of the cyclotron. Reasonable assumptions give the sign unambiguously but allow a leeway of a factor of 3 as to magnitude.

[9] The term "asymmetry" will mean e_{N-R} when $\mathcal{A}_x$ is under discussion, and e_{U-D} when P_s is under discussion.

In comparison with the experimental results, the calculation yields the correct signs, and is quite consistent with the observed magnitudes.

As a misalignment of the defining slits of much more than $\frac{1}{4}$ in. is unlikely, we take $P_t \lesssim 0.04P_1$ and thus arrive at the error quoted. As the slit was reset for each run, this error is systematic within a run but independent from run to run. Note that it does not allow for parity nonconservation in p-carbon scattering (see Sec. III-E).

(3) Either monitor, the ion chamber or the Faraday cup, could read slightly differently for the two solenoid current directions. The former is sensitive to low-energy particles from slit-edge scattering, which may be solenoid dependent. The latter has a cross-sectional area comparable to the beam, and hence all the beam may not be contained by the cup. Small motions of the beam due to the solenoid may cause the fraction of the beam missed to shift. It was observed that the ratio of the two monitors was solenoid-dependent. An average of the two monitors was used, a value of $\frac{2}{3}$ their difference was taken as the monitoring error, systematic within a run. A value of 0.002 was taken as the random monitoring error.

(5) The primary beam energy shifted a fraction of a MeV with solenoid reversal. Because the third scattering cross section was energy-dependent, this introduced a false asymmetry. As the energy shift was measured for 8 out of 12 angles, and at least once each run, a correction could be made. It was typically 0.003, but became as large as 0.01. The estimated error to this correction was typically ± 0.003, but became as large as ± 0.007, and was random in nature.

(8) Reversing the solenoid moves the beam horizontally across the cylindrical hydrogen target by a small amount. If the lines of traversal of the target are not symmetrically located about the center, the path lengths in the target will be different, introducing a false asymmetry. For a $\frac{1}{4}$-in. beam misalignment, a $\frac{1}{16}$-in. beam shift with solenoid causes a false asymmetry of 0.0047. This was taken as the error, systematic within a run but independent from run to run.

D. Errors to P_s

Sources of error to P_s (other than counting statistics) that were considered are listed in Table III. The

TABLE III. Sources of error to P_s.

(1) θ_3 misalignments	(0.025)
(2) Solid-angle variations	(0.025)
(3) Asymmetries from polarization effects	(0.011)
(4) Energy spread across the B counter	(negligible)
(5) Monitoring	(negligible)
(6) Coupling of any two of the above effects, which alone would cancel	(negligible)

quantity in parenthesis is the estimate of error to P_s, at the most unfavorable angle θ_2.

(1) and (2) The treatment of these sources of errors is discussed in Refs. 7 and 8. The θ_3 misalignment was measured, corrected for, and the error in the correction estimated. In addition to the treatment of Refs. 7 and 8, consideration was given to the mechanical distortions of the sine bar and misalignments of the θ_3 pivot, effects which cancel in an R measurement, because of the use of the solenoid, but introduce errors to P_s.

(3) The parameter P_n is introduced to a P_s measurement by a tilting of the θ_3 scattering plane or the θ_2 scattering plane. The parameter A is introduced by a nonzero component of initial longitudinal polarization. Such a component could be present because of defining slit misalignment, as discussed in Sec. III C, or because of parity nonconservation in the first (p-carbon) scattering, as discussed in Sec. III E.

E. Effect of Parity Nonconservation in p-Carbon Scattering

p-carbon elastic scattering occurs as the first scattering, used to polarize the beam, the third scattering, used as a polarization analyzer in the P_s measurement, and as the second scattering in subsidiary experiments to study the primary beam polarization. As parity nonconservation is about as probable in p-carbon scattering as p-p scattering, we must see what effect it will have on our measurements.

If the primary (once-scattered) beam, on entering the solenoid, has a vertical polarization P_{1n}, a horizontal transverse polarization P_{1t}, and a longitudinal polarization P_{1l}, then our measured asymmetries are

$$e_{N-R}=\pm(P_{1n}\mathcal{a}_{2x}-P_{1t}\mathcal{a}_{2n})\,, \tag{18a}$$

$$e_{U-D}=\pm(P_{2s}\mathcal{a}_{3n}-P_{2n}\mathcal{a}_{3x}+P_{1l}A\,\mathcal{a}_{3n})\,. \tag{18b}$$

(The upper signs refer to second scattering in the same sense as the first scattering; the lower signs, the opposite sense.) Hence our measured parameters are

$$\mathcal{a}_{2x}{}^{\text{meas}}=\mathcal{a}_{2x}-\mathcal{a}_{2n}P_{1t}/P_{1n}\,, \tag{19a}$$

$$P_{2s}{}^{\text{meas}}=P_{2s}-P_{2n}\mathcal{a}_{3x}/\mathcal{a}_{3n}+P_{1l}A\,. \tag{19b}$$

Owing to parity nonconservation in the first scattering, there will be horizontal components of polarization in the beam. These will precess about the vertical magnetic field of the cyclotron, through an angle $\alpha\approx110°$, so that

$$P_{1t}=P_{1s}\cos\alpha-P_{1k'}\sin\alpha\approx-\tfrac{1}{3}P_{1s}-P_{1k'}\,, \tag{20a}$$

$$P_{1l}=P_{1s}\sin\alpha+P_{1k'}\cos\alpha\approx P_{1s}-\tfrac{1}{3}P_{1k'}\,. \tag{20b}$$

If the beam defining slits are not deliberately aligned off the median plane, these expressions for P_{1t} and P_{1l} are the ones to use in Eqs. (19).

If, on the other hand, one replaces the hydrogen second scatterer with a carbon scatterer, and adjusts the slits to obtain a null result in the e_{N-R} measurement, then

$$\mathcal{a}_{2x}=P_{1t}\mathcal{a}_{2n}/P_{1n}\,. \tag{21}$$

Since the first and second scatterings are both p-carbon, at the same scattering angle and energy, one can replace 2's by 1's in the above equation. We also use $P_{1n}=\mathcal{a}_{1n}$, to first order in parity violating terms. This gives $P_{1t}=\mathcal{a}_{1x}$, as the expression to use in Eq. (19a), if the main slits were adjusted as described above.

This second method of slit adjusting has the following virtue. The difference $P_{2s}{}^{\text{meas}}-\mathcal{a}_{2x}{}^{\text{meas}}$ becomes

$$P_{2s}{}^{\text{meas}}-\mathcal{a}_{2x}{}^{\text{meas}}=P_{2s}-\mathcal{a}_{2x}+\mathcal{a}_{2n}(\mathcal{a}_{1x}/\mathcal{a}_{1n}-\mathcal{a}_{3x}/\mathcal{a}_{3n})+P_{1l}A\,. \tag{22}$$

(We have used $P_{2n}=\mathcal{a}_{2n}$ and $P_{1n}=\mathcal{a}_{1n}$ in the middle term, which is correct to first order in small amplitudes.) $\mathcal{a}_{1x}/\mathcal{a}_{1n}$ and $\mathcal{a}_{3x}/\mathcal{a}_{3n}$ both refer to p-carbon scattering. By setting the angle θ_3 equal to θ_1, and by checking that the slit adjustment remains unchanged if the energy of the primary beam is degraded, one can eliminate this middle term. P_{1l} in the last term is not simply related to the properties of the first scattering, however.

The procedure used in this experiment was to align the slits on the median plane (to $\pm\frac{1}{4}$ in.). However, with this method of adjustment, P_{1t} was found equal to $\mathcal{a}_{1x}$ to ±0.05. A similar measurement with a 44° spin-precession magnet inserted gave P_{1l} equal to $\mathcal{a}_{1x}$ to ±0.05.

It is apparent that the possibility of parity nonconservation in p-carbon scattering greatly complicates the interpretation of the p-p results. This is considered in Sec. IV, where the results are analyzed.

F. Results

The quantity $\mathcal{a}_x$, with its random and systematic errors, is given by

$$\mathcal{a}_x=\mathcal{a}_x{}^0\pm\Delta\mathcal{a}_x+(h+h_i)H+j_iJ+bB+cC\,. \tag{23}$$

Similarly

$$P_s=P_s{}^0\pm\Delta P_s+k_jK+l_iL+m_iM+nN\,. \tag{24}$$

The quantities $\mathcal{a}_x{}^0$, $\Delta\mathcal{a}_x$, H, J, B, and C appear in Table IV, while the quantities $P_s{}^0$, ΔP_s, K, L, M, and N appear in Table V. Note that there are two measurements at each angle, one for each sense of scattering (north and south). The quantities h, h_i, j_i, b, c, k_j, l_i, m_i, and n allow for systematic errors, and separately have a value of 0.0 ± 1.0. Those with no subscript (h,b,c,n) have the same value for all runs, and allow for errors systematic throughout the entire experiment. Those with subscript i (h_i,j_i,l_i,m_i) have the same value for all measurements within a run, but different values for different runs (i.e., $i=$I, II, III, IV), thus allowing for errors systematic within a run but independent from run to run. k_j takes on two values, one for $j=$north

TABLE IV. Results of the $\mathcal{A}_x$ measurements.

θ_0	Run	$\mathcal{A}_x^0$	$\Delta\mathcal{A}_x$	B	C	H	J
N 15°	II	−0.0198	0.0154	0.0006	0.0003	0.0095	0.0111
N 20°	II	+0.0169	0.0145	0.0003	0.0005	0.0097	0.0111
N 25°	I	−0.0103	0.0169	0.0003	0.0006	0.0094	0.0121
N 30°	II	+0.0265	0.0143	0.0003	0.0008	0.0068	0.0111
N 35°	II	+0.0080	0.0154	0.0003	0.0009	0.0052	0.0111
N 40°	II	−0.0135	0.0169	0.0003	0.0011	0.0022	0.0111
S 15°	IV	+0.0022	0.0129	0.0006	0.0003	0.0095	0.0081
S 20°	IV	−0.0066	0.0117	0.0003	0.0005	0.0097	0.0081
S 25°	IV	−0.0185	0.0098	0.0003	0.0006	0.0094	0.0081
S 30°	III	−0.0323	0.0095	0.0003	0.0008	0.0068	0.0076
S 35°	III	−0.0326	0.0089	0.0003	0.0009	0.0052	0.0076
S 40°	III	−0.0288	0.0089	0.0003	0.0011	0.0022	0.0076

scattering angles, and another for j=south scattering angles.

$\mathcal{A}_x^0$ and P_s^0 are the measured values of the parameters, while $\Delta\mathcal{A}_x$ and ΔP_s are the random errors to them. H includes errors of type (1), (2), and (10) (it is essentially $0.004P_2$). J includes errors of type (3), (8), (9), and (11); the errors (3) and (8) dominate. B is error (4), while C is error (6). K and L include errors of type (1) and (2); M, type (3); N, type (3), is $0.04AP_1$.

TABLE V. Results of the P_s measurements.

θ_2	Run	P_s^0	ΔP_s	K	L	M	N
N 15°	II	−0.0278	0.0242	0.0121	0.0048	0.0107	0.0097
N 20°	II	−0.0390	0.0253	0.0123	0.0056	0.0094	0.0086
N 25°	I	−0.0289	0.0368	0.0147	0.0075	0.0084	0.0079
N 30°	II	−0.0495	0.0376	0.0158	0.0099	0.0056	0.0053
N 35°	II	−0.0362	0.0600	0.0254	0.0154	0.0050	0.0048
N 40°	II	−0.0904	0.1000	0.0241	0.0241	0.0026	0.0026
S 15°	IV	+0.0046	0.0228	0.0121	0.0048	0.0107	0.0097
S 20°	IV	+0.0334	0.0209	0.0123	0.0056	0.0094	0.0086
S 25°	IV	+0.0105	0.0226	0.0147	0.0075	0.0084	0.0079
S 30°	III	−0.0376	0.0277	0.0158	0.0099	0.0056	0.0053
S 35°	III	−0.0023	0.0392	0.0254	0.0154	0.0050	0.0048
S 40°	III	+0.0265	0.0530	0.0241	0.0241	0.0026	0.0026

G. Measurement of Δ

One measurement of e_{U-D}^{off} was made at 25° north, which by Eq. (17d) yields a value of Δ. The data were analyzed in the same fashion as other asymmetry measurements. The errors peculiar to this measurement are the "mixing in" of R and A (due to nonzero values of undesired components of polarization of the incident beam). Further, parity nonconservation in the third scattering would introduce a contribution

$$\mathcal{A}_{3x}/\mathcal{A}_{3n}(D-P_n/P_1)\approx-\tfrac{1}{4}\mathcal{A}_{3x}/\mathcal{A}_{3n}.$$

None of these effects is apt to exceed 0.02 in magnitude. The measured value is

$$\Delta-P_s/P_1=e_{U-D}^{\text{off}}/P_1P_3=+0.08\pm0.06.$$

Taking

$$|P_s/P_1|<0.03,$$

and combining this and the above mentioned errors quadratically with the error from counting statistics, one obtains the result

$$\Delta=+0.08\pm0.07.$$

IV. PHASE-PARAMETER FITS

In this section, the data of Sec. III, and some other measurements are fitted by means of the phase-shift parametrization of Sec. II B. In this fitting, the parameters representing a noninvariance are taken to first order only, hence the invariant amplitudes ($ABCEF$) and the invariant phase parameters are unchanged. They are taken from a phase shift-set ($YLAM$ or $YRB1$) of Breit and collaborators.[10]

It is felt that noninvariances are more apt to occur at low angular momentum where the short-range part of the interaction (due to exchange of many pions, kaons, hyperons, etc.) plays a more important role, rather than at high angular momentum, where the long-range part of the interaction (due to exchange of one or two pions) is dominant. For this reason, the noninvariant parameters for the lowest relevant J value are adjusted to obtain the best fit to the data, while noninvariant parameters for all higher J values are set equal to zero. Hence, fits to $\mathcal{A}_x$, P_s, and Δ adjust $\zeta_0\sin\lambda_{1,0}$ and $\zeta_0\cos\lambda_{1,0}$ while fits to $P_n-\mathcal{A}_n$ adjust $\lambda_{2,2}$.

A. $P_n-\mathcal{A}_n$

Hwang *et al.*[11] have measured $P_n-\mathcal{A}_n$ at 142 MeV. (Their notation differs from ours: $P_2=\mathcal{A}_n$, $P_2'=P_n$.) In addition to the random error quoted in their Table VII, there is a systematic error of $\pm0.06\mathcal{A}_n$, due to the uncertainty in the polarization of the incident beam. In fitting, this is allowed for by a parameter d, whose measured value is 0 ± 1. The other adjustable parameter, $\sin\lambda_{2,2}$, allows for a time-reversal noninvariant coupling between ${}^3P_2$ and ${}^3F_2$ states.

The fitted value of $P_n-\mathcal{A}_n$ is obtained by inserting $\sin\lambda_{2,2}$ into Eqs. (6d), (6e), (7), and (13a); cross sections, invariant phase shifts, and invariant amplitudes needed in Eqs. (7) and (13a) are taken from[10] YLAM. The measured value of $P_n-\mathcal{A}_n$ is obtained by adding $0.06\mathcal{A}_n d$ to the values listed in Table VII of Ref. 11. The quantity

$$\chi^2=\sum_{\text{measurements}}\left[\frac{(\text{fitted value})-(\text{measured value})}{\text{random error}}\right]^2+d^2 \tag{25}$$

is minimized with respect to $\sin\lambda_{2,2}$ and d.

If $\sin\lambda_{2,2}$ is held fixed at 0, minimizing χ^2 yields

[10] G. Breit, M. H. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960); M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, *ibid.* **122**, 1606 (1961); and (private communications).

[11] C. F. Hwang, T. R. Ophel, E. H. Thorndike, and R. Wilson, Phys. Rev. **119**, 352 (1960).

$d=1.13$, $\chi^2=9.9$, χ^2(expected) $=8$. If $\sin\lambda_{2,2}$ is also adjusted to minimize χ^2, one obtains $\sin\lambda_{2,2}=-0.050$, $d=0.01$, $\chi^2=7.5$, χ^2(expected) $=7$. $\sin\lambda_{2,2}$ can vary by ± 0.033 from the above-mentioned value before χ^2 increases by 1, if d is simultaneously readjusted. If[10] YRB1 is used to supply the invariant quantities, these conclusions are not materially altered.

Abashian and Hafner[12] obtain a value of 0.029 ± 0.018 for $P_n-\mathcal{A}_n$ at 210 MeV, 30° c.m.; Hillman, Johansson, and Tibell[13] obtain 0.007 ± 0.023 at 176 MeV, 31° c.m., and 0.011 ± 0.022 at 179 MeV, 50° c.m.

Using YLAM at 210 MeV for invariant quantities, Abashian and Hafner's result requires $\sin\lambda_{2,2}=-0.10\pm 0.006$. Using YLAM at 210 and 140 MeV, and interpolating linearly, Hillman, Johansson, and Tibell's measurements require $\sin\lambda_{2,2}=0.022\pm 0.038$. (The errors indicate the variation in $\sin\lambda_{2,2}$ that increases χ^2 by 1.) If YRB1 is used instead of YLAM, these results are not significantly altered.

Assuming that $\sin\lambda_{2,2}$ has a smooth energy dependence, the three sets of measurements suggest that $\sin\lambda_{2,2}$ is not larger in magnitude than 0.07, at something near the 95% confidence level. Hence this time-reversal noninvariant parameter is not greater than 7% of its maximum possible value in the energy range 140 to 210 MeV.

B. $\mathcal{A}_x$, P_s, and Δ

$\mathcal{A}_x$ and P_s have been fitted by inserting $\zeta_0\cos\lambda_{1,0}$ and $\zeta_0\sin\lambda_{1,0}$ into Eqs. (6h), (6i), (8), (9), and (13). Cross sections, invariant phase shifts and invariant amplitudes were taken from[10] YLAM or YRB1. Measured values of $\mathcal{A}_x$ and P_s were obtained from Eqs. (23) and (24), Tables IV and V of Sec. III. The quantities h, h_i, j_i, b, etc., which allow for systematic errors were treated as parameters to be varied to minimize χ^2. Since each of these has a measured value of 0 ± 1, each contributed directly to χ^2, i.e.:

$$\chi^2=\sum_1^{24}\left[\frac{(\text{fitted value})-(\text{measured value})}{\text{random error}}\right]^2 +n^2+b^2+h_{\rm II}{}^2+h_{\rm III}{}^2+\text{etc.} \quad (26)$$

(During Run I, measurements were made at only one angle (N 25°); hence $h_{\rm I}$, $j_{\rm I}$, $l_{\rm I}$, and $m_{\rm I}$ which involve errors systematic within a run, were eliminated, by adding the errors associated with them in quadrature to the random error listed.)

For various conditions, χ^2 has been minimized, and the fitting parameters and the error matrix determined. (The criterion of an increase in χ^2 of 1 was used in defining the error matrix.) The results are given in Table VI. The uncorrelated errors (diagonal elements of the error matrix) were always larger than the correlated errors (nondiagonal elements) and hence, the former are quoted in the table.

TABLE VI. Fits to the 140-MeV $\mathcal{A}_x$ and P_s measurements. Listed are the values of the adjustable parameters $\zeta_0\cos\lambda_{1,0}$ and $\zeta_0\sin\lambda_{1,0}$, χ^2; the contributions to χ^2 from the $\mathcal{A}_x$ and P_s measurements separately; and the phase shifts used for the invariant quantities.

$\zeta_0\cos\lambda_{1,0}$	$\zeta_0\sin\lambda_{1,0}$	χ^2	$\chi^2\mathcal{A}$	$\chi^2{}_P$	Phase shifts
fixed at 0	fixed at 0	24.01	17.03	6.98	...
-0.041 ± 2.300	-0.066 ± 0.100	23.42	16.24	7.18	YLAM
fixed at 0	-0.065 ± 0.085	23.42	16.24	7.18	YLAM
0.803 ± 1.800	fixed at 0	23.82	16.75	7.06	YLAM
0.181 ± 0.600	-0.065 ± 0.150	23.21	16.26	6.94	YRB1
fixed at 0	-0.095 ± 0.110	23.30	16.27	7.04	YRB1
0.349 ± 0.450	fixed at 0	23.39	16.56	6.84	YRB1

If both noninvariant parameters are held fixed at zero, the first entry in the table results. The χ^2 of 24.01 is very reasonable for 24 degrees of freedom. The remaining entries in the table are obtained if either $\zeta_0\sin\lambda_{1,0}$, $\zeta_0\cos\lambda_{1,0}$, or both are adjusted to minimize χ^2; invariant quantities are taken from either YLAM or YRB1. In no case does χ^2 decrease by more than 0.8, hence the data are completely consistent with zero values for the noninvariant parameters.

While the results for $\zeta_0\cos\lambda_{1,0}$ are sensitive to variations in the invariant quantities (YLAM versus YRB1), $\zeta_0\cos\lambda_{1,0}$ is never bounded away from ± 0.5. Since an exact treatment (rather than a first-order one) replaces ζ_0 by $\sin\zeta_0\cos\zeta_0$, the range of this parameter is from $+0.5$ to -0.5, and hence these data place no limits whatsoever on $\zeta_0\cos\lambda_{1,0}$.

The results for $\zeta_0\sin\lambda_{1,0}$ are less sensitive to variations in the invariant quantities and can be quoted as $\zeta_0\sin\lambda_{1,0}=-0.065\pm 0.150$.

The effects of parity nonconservation in the first and third scatterings are allowed for by the systematic error parameters h and n. When either or both of these were held fixed at zero, there was no significant change in the results. Further, the correlated errors between these parameters and $\zeta_0\sin\lambda_{1,0}$ were at most $\frac{1}{2}$ of the error quoted above for $\zeta_0\sin\lambda_{1,0}$. It is thus felt that the amount of parity nonconservation in the first and third scatterings that is consistent with the results of Sec. IIIE, will *not* alter the value or error quoted for $\zeta_0\sin\lambda_{1,0}$ above.

The expressions for Δ at 140 MeV, 50° c.m. are

$$\Delta(\text{YRB1})=0.18\zeta_0\cos\lambda_{1,0}-0.16\zeta_0\sin\lambda_{1,0},$$

$$\Delta(\text{YLAM})=0.25\zeta_0\cos\lambda_{1,0}-0.17\zeta_0\sin\lambda_{1,0},$$

while the measured value (Sec. III) is $+0.08\pm 0.07$. This result coupled with the previous restriction on $\zeta_0\sin\lambda_{1,0}$, suggests $\zeta_0\cos\lambda_{1,0}=0.40\pm 0.35$. Thus $\zeta_0\cos\lambda_{1,0}$ is consistent with 0 or $+\frac{1}{2}$, but is bounded away from $-\frac{1}{2}$. Unlike the results for $\mathcal{A}_x$ and P_s, the relation

[12] A. Abashian and E. M. Hafner, Phys. Rev. Letters **1**, 255 (1958).

[13] P. Hillman, A. Johansson, and G. Tibell, Phys. Rev. **110**, 1218 (1958).

between Δ and $\zeta_0 \cos\lambda_{1,0}$ is not particularly sensitive to the choice of invariant quantities.

Oxley, Cartwright, and Rouvina[14] have measured $\mathcal{A}_x$ at 210 MeV and 57° c.m. They obtained $|P_1\mathcal{A}_x| = 0.0025\pm0.0100$. As this was the pioneering experiment in polarization phenomena they obtained only a crude estimate of P_1, their beam polarization from 19° scattering off carbon. Subsequent studies[15] suggest $P_1\approx0.3$, giving $|\mathcal{A}_x| = 0.0081\pm0.0333$. (The sign of $\mathcal{A}_x$ is not readily ascertainable from the article of Oxley *et al.*) The expressions for $\mathcal{A}_x$ at 210 MeV, 57° c.m. are

$$\mathcal{A}_x(\text{YRB1}) = 0.002\zeta_0 \cos\lambda_{1,0} + 0.013\zeta_0 \sin\lambda_{1,0},$$

$$\mathcal{A}_x(\text{YLAM}) = 0.023\zeta_0 \cos\lambda_{1,0} + 0.074\zeta_0 \sin\lambda_{1,0}.$$

The coefficients vary widely from YRB1 to YLAM. However, neither can limit either $|\zeta_0 \cos\lambda_{1,0}|$ or $|\zeta_0 \sin\lambda_{1,0}|$ to less than $\frac{1}{2}$.

C. High-Energy Reactions in Complex Nuclei

Chamberlain *et al.*[16] have measured $\mathcal{A}_x$ for scattering of 315-MeV protons by carbon at 15° lab. They obtain $|P_1\mathcal{A}_x| = 0.01\pm0.02$. Estimating P_1 at 0.5 leads to $|\mathcal{A}_x| = 0.02\pm0.04$.

An impulse-approximation calculation of $\mathcal{A}_x$ has been performed, taking $A_0 = A_{np} + A_{pp}$, $C_0 = C_{np} + C_{pp}$, $P_0 = P_{pp}$, and $R_0 = R_{pp}$. Invariant *p-p* quantities were taken from[10] YLAM; invariant *n-p* quantities from[10] YLAN-3M. These quantities were not at hand at 315 MeV, so the evaluations were performed at 140 and 210 MeV for 15° lab scattering angle.

$$\mathcal{A}_x(140\text{ MeV}) = 0.130\zeta_0 \sin\lambda_{1,0} + 0.031\zeta_0 \cos\lambda_{1,0},$$

$$\mathcal{A}_x(210\text{ MeV}) = 0.093\zeta_0 \sin\lambda_{1,0} + 0.031\zeta_0 \cos\lambda_{1,0}.$$

The coefficients obtained by using YRB1 instead of YLAM were smaller. In order to obtain a useful limit, it is necessary for the coefficient of either $\zeta_0 \sin\lambda_{1,0}$ or $\zeta_0 \cos\lambda_{1,0}$ to exceed 0.2 in magnitude at 315 MeV. By extrapolation, it appears that neither coefficient would be that large. Hence, the $\mathcal{A}_x$ measurement of Chamberlain *et al.* provides no limit on $\zeta_0 \cos\lambda_{1,0}$ or $\zeta_0 \sin\lambda_{1,0}$.

Jones, Murphy, and O'Neill[17] have measured the longitudinal polarization of neutrons produced at 0° by bombarding beryllium with 350-MeV protons. They find the polarization is less than 4×10^{-3}, at the 95% confidence level.

A crude impulse-approximation calculation can be performed by treating the reaction $p+\text{Be}\rightarrow n+x$ as an elastic charge-exchange scattering. Then Eq. (15d), which gives $P_{k'}$, applies. A_0 is taken as $A_{np}(180°\text{ c.m.})$; C_0 is taken as $C_{np}(180°\text{ c.m.})$; P_0 and R_0, similarly taken as the *n-p* parameters at 180°, are split up into the isotopic spin-0 and isotopic spin-1 parts. The former is independent of ζ_0, and is thus ignored; the latter is set equal to $1/\sqrt{2}$ times $P_{pp}(180°)$ or $R_{pp}(180°)$. Because $C_{np}(180°)\equiv0$, and because that part of $R_{pp}(180°)$ that is proportional to ζ_0 is also identically zero, $P_{k'}$ contains no terms proportional to ζ_0, and hence places no limit on it.

Three other experiments should be mentioned at this point. Heer, Roberts, and Tinlot[3] have measured the up-down asymmetry ($\mathcal{A}_x$ type parameter) in production of π^+ mesons by 210-MeV protons on aluminum. They obtained $\mathcal{A}_x = -0.016\pm0.042$. Davis *et al.*[18] have measured the up-down asymmetry in decay γ rays from production of π^0 mesons by 540-MeV polarized neutrons. They find asymmetry parameters consistent with zero with an uncertainty of ±0.02 to ±0.05. Garwin *et al.*[19] have measured the circular polarization of γ rays from π^0 decay. As the decay is believed to proceed through the channel $\pi^0\rightarrow\bar{p}+p$, this experiment bears on parity conservation of the Yukawa interaction. They find a polarization of $(2.0\pm9.0)\%$.

While these three experiments give evidence for parity conservation of the Yukawa interaction, and hence in *p-p* scattering, an involved meson-theoretic calculation is required to obtain upper limits on $\zeta_0 \sin\lambda_{1,0}$ or $\zeta_0 \cos\lambda_{1,0}$ from them.

V. DISCUSSION

To the best of the author's knowledge, the above-mentioned experiments exhaust the list bearing directly on time-reversal invariance or parity conservation in *p-p* scattering. The conclusions that can be drawn from them are these:

(1) There is no evidence for any violation of either invariance.

(2) The parity-conserving, time-reversal noninvariant coupling of $^3P_2$ and $^3F_2$ states ($\sin\lambda_{2,2}$) is at[20] most 7% of its maximum allowed value, in the energy range 140–210 MeV.

(3) The parity-nonconserving, time-reversal invariant coupling of $^1S_0$ and $^3P_0$ states ($\zeta_0 \sin\lambda_{1,0}$) is at[20] most 70% of its maximum value at 140 MeV.

(4) The parity-nonconserving, time-reversal noninvariant coupling of $^1S_0$ and $^3P_0$ states ($\zeta_0 \cos\lambda_{1,0}$) is not bounded away from its maximum positive value and is at [20] most 60% of its extreme negative value, at 140 MeV.

The evidence against parity nonconservation is seen

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[14] C. L. Oxley, W. F. Cartwright, and J. Rouvina, Phys. Rev. **93**, 806 (1954).

[15] Private communication from C. L. Oxley to Heer, Roberts, and Tinlot, mentioned in Ref. 3.

[16] O. Chamberlain, E. Segrè, R. D. Tripp, C. Wiegand, and T. J. Ypsilantis, Phys. Rev. **93**, 1430 (1954).

[17] D. P. Jones, P. G. Murphy, and P. L. O'Neill, Proc. Phys. Soc. (London) **72**, 429 (1958).

[18] D. G. Davis, R. C. Hanna, F. F. Heymann, and C. Whitehead, Nuovo Cimento **15**, 641 (1960).

[19] R. L. Garwin, G. Gidal, L. M. Lederman, and M. Weinrich, Phys. Rev. **108**, 1589 (1957).

[20] Limits are based on two standard deviations, and hence are at the 95% confidence level.

to be surprisingly weak. This conclusion is at variance with the conclusions drawn by other authors[2,3] about the *same data*. These authors were rather sketchy in describing the procedure for arriving at their limits on parity nonconservation. They appear to have done the following: Ignoring possible cancellations, they set

$$I_0\mathcal{A}_x \approx |\text{parity-conserving amplitude}| \times |\text{parity-nonconserving amplitude}| \quad (27)$$

$$I_0 \approx |\text{parity-conserving amplitude}|^2; \quad (28)$$

hence

$$\mathcal{A}_x \approx \left|\frac{\text{parity-nonconserving amplitude}}{\text{parity-conserving amplitude}}\right|. \quad (29)$$

They take

$$F \equiv \left|\frac{\text{parity-nonconserving amplitude}}{\text{parity-conserving amplitude}}\right|$$

and arrive at $F \gtrsim 10^{-1}$ to 3×10^{-2} as their limits on parity nonconservation.

In fact, cancellations are large, and Eq. (29) overestimates $\mathcal{A}_x$ frequently by a factor of 10, resulting in values of F low by the same factor.

The rest of the difference is due to a different definition of the amount of parity nonconservation. The values of the invariant phase shifts already imply certain limits on parity nonconservation. For example,

$$P = (1/2k)\sin(\theta/2)(e^{i\delta_0} - e^{i\delta_{1,0}})\zeta_0 \cos\lambda_{1,0}. \quad (30)$$

If $\zeta_0 \cos\lambda_{1,0}$ has its maximum value of $\frac{1}{2}$, P will be $0.05 \sin(\theta/2)$ F while a typical invariant amplitude is 0.3 F. Alternatively, if ζ_0 has its maximum value of $\frac{1}{2}$, the contribution to the cross section from parity-nonconserving terms is 2% of that from parity-conserving terms. (These statements, of course, depend on the values of δ_0 and $\delta_{1,0}$; they have been taken from[10] YLAM at 140 MeV. The statements would be modified only slightly for phase shifts at 210 MeV, or from YRB1.)

The measure of parity nonconservation used in this paper ($2\zeta_0 \cos\lambda_{1,0}$ or $2\zeta_0 \sin\lambda_{1,0}$) indicates by what factor the nonconserving amplitudes are smaller than the maximum values allowed them by the given invariant phase shifts. Lee and Yang,[2] and Heer, Roberts, and Tinlot[3] compare the nonconserving amplitude with a typical conserving amplitude, and should have lower estimates by a factor ≈ 7 on this account.

The error of a factor of ≈ 10 from neglecting cancellation, and the difference of a factor of ≈ 7 from the different choice of a measure of parity nonconservation, reconcile the results quoted by Lee and Yang,[2] and by Heer, Roberts, and Tinlot,[3] with the results of this paper. (Note further that the investigations of this paper were limited to the $J=0$ state.)

It is of interest to consider what further investigations might improve the limits on parity nonconservation. Two analyses suggest themselves:

(1) An extension of the analysis of Sec. IV to the $J=2$ noninvariant parameters ζ_2, η_2, $\lambda_{1,2}$, and $\lambda_{3,2}$. The weak limits on the $J=0$ parameters are due to the extensive cancellations in the expressions for $\mathcal{A}_x$ and P_s. This cancellation may not be present in the expressions involving $J=2$ parameters.

(2) The extension of a conventional phase shift analysis of quantities like σ, $\mathcal{A}_n$, R, A, D, to include parity-nonconserving phase parameters like ζ_0, $\lambda_{1,0}$ as parameters to be searched on. It is possible that the extensive measurements of σ, $\mathcal{A}_n$, R, etc., will have more to say about parity nonconservation than the sparser measurements of P_s, $\mathcal{A}_x$, and Δ. (Note that a first-order treatment is inadequate here; the treatment should be exact, or at least go to second order in the parity-violating parameters.)

In evaluating the usefulness of further experiments one must consider the limits placed on the accuracy of the measurement by systematic errors, the amount of running time required to obtain a random error comparable with the systematic errors, and the sensitivity of the quantity being measured to the parity nonconserving parameters. Of the four double-scattering parameters ($\mathcal{A}_x, \mathcal{A}_k, P_s, P_{k'}$), and the four triple-scattering parameters ($\Delta, \Delta', \rho, \alpha$), the most promising appears to be $\mathcal{A}_k$. A measurement to ± 0.003 should be possible, requiring a few hours running time per point. Such a measurement would determine $\zeta_0 \sin\lambda_{1,0}$ to ± 0.02, and $\zeta_0 \cos\lambda_{1,0}$ to ± 0.10, at 140 MeV.

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