sible to excite a phonon state in an even nucleus through (d,p) stripping (see, e.g., Ref. 32), but it is normally expected that the particle strength is distributed over many shell model states,[33] thus making a strong transition through one l_n value improbable. If we want to describe $Ti^{48}(1)$ as a vibrational state, it is therefore necessary to assume that most of the phonon strength is already present in $Ti^{47}(0)$. Under this assumption the $Ti^{48}(1)$ can be reached by "pairing off" the odd particle in Ti^{47}. This picture, however, is hardly consistent; first the configuration of $Ti^{47}(0)$ is almost pure $f_{7/2}$ (see Sec. 3.2); second, the coupling between particles and phonons demanded to give an appreciable phonon strength in $Ti^{47}(0)$ is so large that the concept of a definite phonon number no longer is valid, i.e., we approach the deformed coupling scheme; third, this picture does not explain why the $J=\frac{5}{2}$ state is the ground state in Ti^{47}.

[32] S. Yoshida, Nucl. Phys. **38**, 380 (1962).
[33] B. Mottelson, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960* (University of Toronto Press, Toronto, 1960), p. 525.

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Phenomenological Potentials and the $D(n,p)2n$ Reaction. II

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Recent direct-interaction calculations of the $D(n,p)2n$ reaction cross section at 14.4 MeV have been extended to include the nucleon-nucleon potentials of Hamada and Johnston and Lassila, Hull *et al.* The calculations have been modified to incorporate the tensor potentials in the description of the final-state two-body "particles." Proton energy spectra are presented, and a comparison with previous calculations which employed the Gammel-Thaler phenomenological potentials are made. Within the present calculation framework, better agreement with the experimental results is afforded in the Gammel-Thaler and Hamada-Johnston potential descriptions than in that of Lassila, Hull *et al.*

IN a previous paper[1] the $D(n,p)2n$ reaction cross section was calculated in a direct-reaction framework utilizing the phenomenological potentials of Gammel and Thaler.[2] Although the approximations involved in these earlier calculations may be viewed somewhat questioningly, the results were in good agreement with the experimental data. In a recent paper by Signell and Yoder,[3] several of the latest phenomenological nucleon-nucleon models were compared, and it was concluded that a "better" description was afforded, for example, by the Hamada-Johnston[4] potentials and the Yale[5] potentials. It was decided therefore to utilize these potentials in the calculation of the $D(n,p)2n$ reaction cross section and to note the effect on the results.

The cross-section computations were carried out in the same procedure as that of Ref. 1. By way of review, the reaction was viewed as progressing through an (n,p) mode and an (n,n') mode. The final state therefore was pictured as a continuum dineutron plus a free proton or as a continuum deuteron plus free neutron. For the interaction potential V_{int} we used $V_{\text{int}}=V_{23}+V_{13}$, where the subscripts 1 and 2 refer to the neutron and proton, respectively, in the deuteron and 3 refers to the incident neutron. The interaction potentials are, therefore, nucleon-nucleon potentials, and it is here that we employ the phenomenological potentials. In calculating the two-body continuum wave functions, however, we also use the phenomenological models.

The interaction between the two-body particle (deuteron and dineutron) and the free particle was neglected, and, furthermore, an $l=0$ approximation was used in describing the two-body particles as well as in describing the relative motion of the two-body particle and the free particle. The present calculations depart from the pre-

[1] D. R. Koehler and R. A. Mann, Phys. Rev. **135**, B91 (1964).
[2] J. L. Gammel and R. M. Thaler, Progr. Elem. Particle Cosmic Ray Phys. **5**, 99 (1960).
[3] P. Signell and N. R. Yoder, Phys. Rev. **132**, 1707 (1963).
[4] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).
[5] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).

vious calculations in that the wave function describing the triplet n-p continuum state is a solution of the coupled differential equations resulting from inclusion of the tensor force in the Schrödinger equation.

The singlet phase shifts for all three potential models are in good agreement with the experimental nucleon-nucleon scattering cross section. Better agreement in the case of the triplet phase shifts was afforded, however, by the Hamada-Johnston and the Gammel-Thaler potential than by the Yale potentials.

For the initial-state description of the ground-state deuteron, a Hulthén wave function which vanished at the hard core was used. With the approximation $l=0$, one gets a cross-section contribution only from the central even-parity potential components. The results are presented in Figs. 1–6.

In Fig. 1 we have compared the proton energy spectra

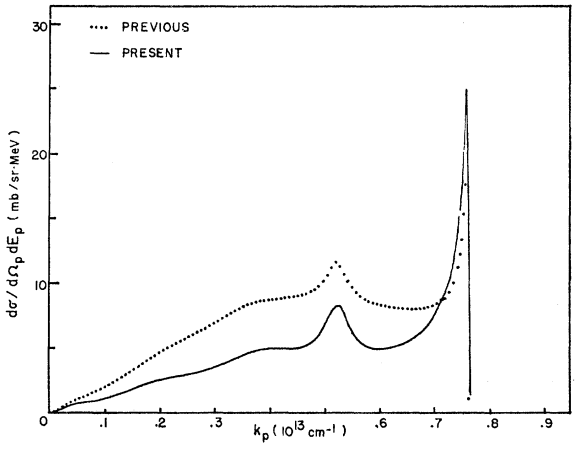

FIG. 1. Proton energy spectrum at 0° lab resulting from use of Gammel-Thaler potentials.

(at 0°), calculated in the present work, with the results of the previous calculations. These calculations have been performed with the Gammel-Thaler triplet potential parameters ${}^3V_c{}^+=87.724$ MeV, ${}^3\mu_c{}^+=1.2183(10)^{13}$ cm^{-1} for the central component, and ${}^3V_T{}^+=272.87$ MeV, ${}^3\mu_T{}^+=1.2183(10)^{13}$ cm^{-1} for the tensor component. The abscissa in all these figures is the wave number of the emitted proton in the laboratory system.

Figure 2 displays the dependence of these calculations on the particular Gammel-Thaler potential set. This figure is to be contrasted with a similar comparison in the previous work where a very marked quantitative dependence was observed as a function of the triplet potential strength. The present results seem to suggest the same sort of cross-section independence on triplet potential strength as do the nucleon-nucleon scattering cross sections. The previous strength-dependent results are explained then by the fact that the nucleon-nucleon scattering data were fitted with phase-shift calculations employing both the central and tensor phenomenological components, and, consequently, the two-body wave

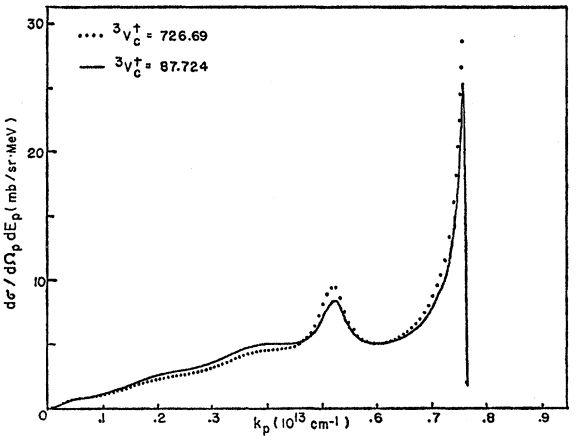

FIG. 2. Proton energy spectrum at 0° lab as a function of Gammel-Thaler triplet potential strength.

functions calculated with only a central potential component are incorrect.

Displayed in Figs. 3–5 are the cross-section results for the three potential descriptions and the primary contributing elements of the cross sections. The curves labeled B and C represent those transitions leading to a singlet-dineutron final state and a singlet-deuteron final state, respectively, while the curve A shows the contribution resulting from the interference of these two terms. Those transitions occurring from $I=1$ to $I'=1$ (I and I' are the spins of the two-body particles) in the total-angular-momentum state $J=\frac{1}{2}$ are shown in component D, and the curve E represents the total cross section.

Finally, in Fig. 6, a comparison is made of the three calculated proton energy spectra versus the experimental results of Ilakovac *et al.*[6] The Yale results have been reduced by a factor of $\frac{1}{4}$ in the figure. As mentioned

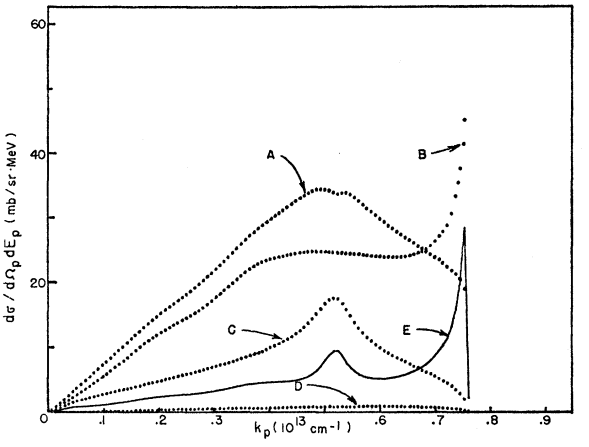

FIG. 3. The differential cross section and its components at 0° lab: Gammel-Thaler potentials.

[6] K. Ilakovac, L. G. Kuo, M. Petravić, I. Šlaus, and P. Tomaš, Phys. Rev. Letters **6**, 356 (1961).

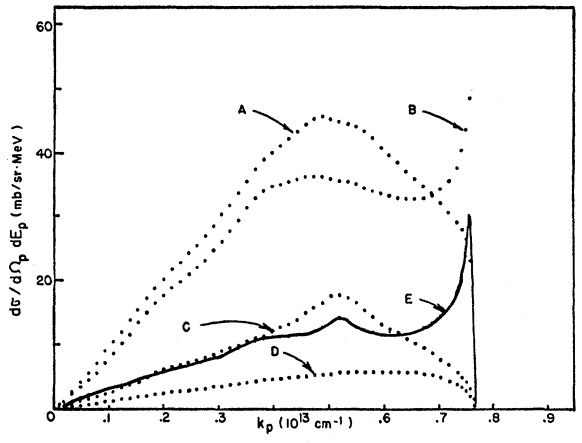

FIG. 4. The differential cross section and its components at 0° lab: Hamada-Johnston potentials.

above, the two-body wave functions and the associated asymptotic phase shifts, in the case of the singlet state, calculated from the three potential descriptions discussed here are in good agreement with one another. However, those integrals involving both the singlet potential and the singlet two-body wave functions are not consistently in good agreement and, furthermore, seem to be potential-shape-dependent.

A further disparity appears in those integrals that involve either the triplet-state two-body wave function or the triplet-state nucleon-nucleon potential. In these cases the nonagreement of the computed integrals can be attributed to the discrepancies in the triplet wave functions or the differences in potential shape or both.

An analysis of the agreement between experimental data and the theoretical calculations has been made, and, in this respect, a measure of the "goodness of fit" in the quantity

$$H=\frac{1}{n}\sum_{i=1}^{n}[(Y_i-y_i)^2]^{1/2}$$

has been defined. The quantity Y_i is the calculated cross-section value, and y_i is the corresponding experimental quantity; n is the number of datum points compared. This comparison yields $H=2.41$ mb (Gammel-Thaler), $H=5.50$ mb (Hamada-Johnston), and $H=38.0$ mb (Yale). A similar comparison with the later experimental data of Cerineo *et al.*[7] yields $H=2.97$ mb (Gammel-Thaler), $H=3.68$ mb (Hamada-Johnston), and $H=37.4$ mb (Yale). It is seen therefore that, within the direct-interaction framework and the model of the reaction process used here, the Gammel-Thaler and Hamada-Johnston potentials yield a better over-all fit to the experimental data.

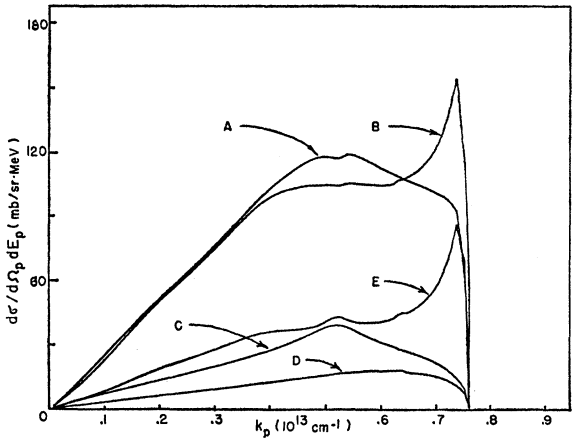

FIG. 5. The differential cross section and its components at 0° lab: Yale potentials.

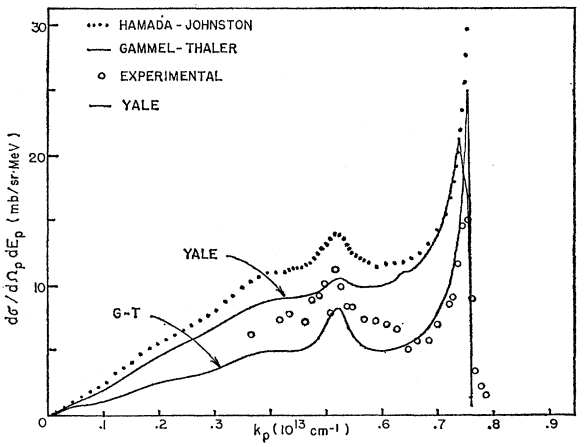

FIG. 6. Comparison of the theoretical cross-section computations versus the experimental cross-section results of Ilahovac *et al.* at 4° lab: the Yale results have been reduced by a factor of $\frac{1}{4}$ in this figure.

[7] M. Cerineo, K. Ilakovac, I. Šlaus, P. Tomaš, and V. Valković, Phys. Rev. **133**, B948 (1964).