Experimental Study of the $2I+1$ Rule Using the (d,α) Reaction on F^{19}, Na^{23}, Al^{27}, and P^{31}†*

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The angular distributions corresponding to the (d,α) production cross sections ($E_d\approx 9.3$ MeV) for the ground states and some of the low-lying states of O^{17}, Ne^{21}, Mg^{25}, and Si^{29} have been measured using silicon surface-barrier detectors. The discussion of the $2I+1$ rule within the context of these experimental observations is based on the premise that both the direct interaction (DI) and compound nucleus (CN) mechanisms contribute incoherently to the angular distributions, and the approximate validity of an *ad hoc* method used for the decomposition of these differential cross sections into DI and CN components. The integrated experimental differential cross sections, $\sigma_T(10°-170°)$ and $\sigma_B(90°-170°)$, and the integrated CN component, $\sigma_{CN}(10°-170°)$, are analyzed in terms of the $2I+1$ rule. The σ_{CN} values are found to be more nearly proportional to $2I+1$ than either the σ_T or σ_B values. This result is interpreted as indicating that the $2I+1$ dependence of the (d,α) cross sections is a characteristic associated with the CN mechanism rather than the DI mechanism. Furthermore, it is concluded that spin assignments based solely on the assumption of a $2I+1$ dependence of either the σ_T or σ_B values derived from (d,α) reactions would not as a rule be very reliable.

I. INTRODUCTION

THE original assertions concerning circumstances under which the cross section for a specific nuclear reaction leading to a definite final nuclear state of spin I might be expected to be proportional to $2I+1$ were made by Enge[1] and Sheline *et al.*[2] Although the importance and utility of such a rule is obvious, the experimental investigation of its existence and/or range of validity has been limited in scope and fragmentary in nature. The suggestion of Enge[1] concerning the so called $2I+1$ rule was made in connection with a method proposed for assigning spins to the members of certain closely spaced $(j\text{-}j)$ multiplets in Al^{28}, P^{32}, and K^{40}. It was argued that under the experimental conditions which prevailed, both the stripping and compound-nucleus mechanisms would yield (d,p) cross sections which are proportional to $2I+1$, and that the production cross section for each member of a multiplet should have approximately the same proportionality constant. The spin assignments made on this basis have since been well corroborated by other methods.[3] Sheline and co-workers[2] also proposed that a $2I+1$ rule be used as a criterion in the identification of the members of $(j\text{-}j)$ doublets and concluded that the ground state doublets of Al^{28}, and P^{32} are true $(j\text{-}j)$ doublets whereas the first excited state doublet of Al^{28} is not.

Ericson[4] and MacDonald[5] have within the context of the compound-nucleus (CN) theory listed and/or discussed certain of the conditions under which the statistical factor $2I+1$ would be expected to dominate the behavior of the cross section. Even if rather extensive experimental evidence were found for the existence of a $2I+1$ rule, its origin could not in every instance be unambiguously ascribed to the presence of a dominating CN reaction mechanism, since particular descriptions of certain direct-interaction (DI) processes, e.g., Newns' description[6] of two-nucleon pickup, yield expressions for the cross section involving a $2I+1$ factor. In view of the complexity of these latter expressions, it can be reasonably argued that the simple $2I+1$ dependence of the DI mechanism would in most cases be obscured by fluctuations in the other energy and angular-momentum-dependent factors.

Even though insufficient relevant experimental information is presently available for the empirical establishment of a useful $2I+1$ rule, the data do suggest the possible existence of such a rule under somewhat less restrictive circumstances than were originally assumed necessary. All but one of the reported experimental investigations which are specifically concerned with the $2I+1$ rule have used the (d,α) reaction. The $Al^{27}(d,\alpha)Mg^{25}$ reaction has been by far the most thoroughly studied in this connection. Measurements have been made at a large number of incident deuteron energies

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‡ National Defense Education Act Fellow, September 1960 to September 1963.
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[2] R. K. Sheline, N. R. Johnson, P. R. Bell, R. C. Davis, and F. K. McGowan, Phys. Rev. 94, 1642 (1954).
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[4] T. Ericson, Advan. Phys. 9, 425 (1960), and Nucl. Phys. 17, 250 (1960).
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in the range 1.5 to 10.5 MeV.[7–11] The strongest summarizing conclusion which characterizes the results of these studies over the entire range of incident energies is that some evidence for a $2I+1$ rule exists among the production cross sections associated with those states of Mg^{25} below 4.5 MeV which have a spin less than $\frac{7}{2}$. The studies of Abuzeid *et al.*[11] ($1.5<E_d<2.6$ MeV) and Hinds *et al.*[7] ($E_d=10.10$ MeV), aside from being among the most current and complete, have yielded results which are more or less representative composites of those reported by others for the higher and lower portion of the deuteron-energy interval. Abuzeid *et al.*[11] measured $\sigma(E,\theta)$ for ten of the lower energy states of $Mg^{25}(E_x<4.5$ MeV) at a number of deuteron energies in the interval 1.5 to 2.6 MeV, and the results may be summarized as follows:

(1) The differential cross sections vary slowly and smoothly with angle, and the magnitude in the most extreme case changes by less than a factor of 3 over the angular range covered.

(2) The angular distributions as a rule are approximately symmetric about 90°.

(3) The energy dependence of the cross sections at a constant angle is irregular.

(4) The energy-averaged, integrated (90° to 150°) cross sections follow the $2I+1$ rule when a correction for the barrier penetrability of the outgoing alpha particle is made.

Hinds *et al.*[7] measured $\sigma(\theta)$ for 48 states of Mg^{25} ($E_x\leq 7.4$ MeV) at a deuteron energy of 10.10 MeV, and their results as summarized by MacDonald[5] are as follows:

(1) The angular distributions do not have the structure characteristic of a simple direct interaction.

(2) The angular distributions are not symmetric about 90°, so that the reaction cannot be completely described by statistical compound-nucleus theory.

(3) The cross sections integrated over all angles, for final states of spin $I=\frac{1}{2}$ to $\frac{9}{2}$, are roughly proportional to $2I+1$.

(4) The same cross sections if integrated over angles greater than 90° are more nearly proportional to $2I+1$.

In addition, MacDonald[5] has discussed these results within the context of the statistical compound-nucleus theory with particular reference to the $2I+1$ proportionality of the cross sections and its dependence on the value of the level density spin cut-off parameter. The $Mg^{25}(d,\alpha)Na^{23}$ reaction has been studied by Hansen *et al.*[10] at a large number of incident deuteron energies in the ranges 3.35 to 3.70 MeV and 7.1 to 7.7 MeV. The energy-averaged (7.1 to 7.7 MeV), integrated (25° to 160°) cross sections for the low-lying states ($E_x<5$ MeV) of Na^{23} that have well established spins were found to satisfy the $2I+1$ rule. The general validity of the rule was then assumed and spin assignments for the remaining states were made. These investigators assert that these latter assignments did not disagree with implications of other existing data. It was also found that the energy-averaged (3.35 to 3.70 MeV), integrated (20° to 160°) cross sections for the same low-lying states of Na^{23} did not show the $2I+1$ proportionality nearly as well as the higher energy cross sections. The cross sections of the $Si^{28}(n,\alpha)Mg^{25}$ reaction for transitions to the ground and first nine excited states have been measured by Colli *et al.*[12] using neutron energies in the range 12.15 to 18.5 MeV. The corresponding energy-averaged cross sections are reported to show a linear $2I+1$ dependence which is interpreted by these investigators as arising from the CN reaction mechanism.

It is clear from the review and discussion presented above that the experimental clarification of the $2I+1$ rule will require much more empirical information than is presently available. The present paper reports the results and conclusions of an experimental study of the $2I+1$ rule as related to the (d,α) production cross sections for low-spin ($I\leq\frac{7}{2}$), low-lying ($E_x\leq 4.551$ MeV) states in a series of light nuclei.

II. EXPERIMENTAL

A detailed description of the Purdue University cyclotron facility has been presented elsewhere.[13] The charged particle spectra were measured using silicon surface-barrier detectors and a conventional electronic spectrometer configuration incorporating a 256-channel pulse-height analyzer.

The incident deuteron energy was 9.200 ± 0.030 MeV for the F^{19}, Na^{23}, and Al^{27} measurements; and 9.510 ±0.030 MeV for the P^{31} measurements. The normal beam cross section at the target position was circular with a 0.078-in. diameter. The azimuthal acceptance angle of the detector with respect to the center of the target was 2.3°, and the nominal solid angle subtended by the detector was 0.001 sr.

A summary of experimental information concerning the targets and measurements is presented in Table I. The general experimental procedures and the measures used in data validation for these investigations were similar to those previously described.[14] The assignment of a 15% probable systematic error in absolute cross

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[13] B. T. Lucas, S. W. Cosper, and O. E. Johnson, Phys. Rev. **133**, B963 (1964).

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TABLE I. Summary of some experimental information concerning the targets and measurements.

Target nucleus	Target material	Number and thickness of targets	Angular range (deg)	Number of angles
F^{19}	Stretched $\frac{1}{4}$-mil Teflon film	32 targets; 370–720 $\mu g/cm^2$	10–172.5	46
Na^{23}	Thermal vacuum evaporated metallic sodium on thin ($<20\ \mu g/cm^2$) Formvar-film backings	2 targets; 70 and 130 $\mu g/cm^2$	10–172.5	45
Al^{27}	Thermal vacuum evaporated metallic aluminum on thin ($<20\ \mu g/cm^2$) Formvar-film backings	2 targets; 90 and 120 $\mu g/cm^2$	10–170	40
P^{31}	Thermal vacuum evaporated red phosphorous powder on thin ($<10\ \mu g/cm^2$) Zaponite-lacquer backings	3 targets; 362, 566, and 642 $\mu g/cm^2$	10–172.5	35

sections is based on an appraisal of uncertainties in target thickness, beam integration, and experimental geometry. The nominal probable errors in the relative cross sections are estimated to be 5%.

III. RESULTS AND DISCUSSION

In Table II are given the energies, spins, and parities of the final nuclear states whose (d,α) production cross sections were measured along with the corresponding integrated cross sections: σ_T, 10° to 170°; and σ_B, 90° to 170°. The values of $\sigma_T/(2I+1)$ and $\sigma_B/(2I+1)$ which are associated with the various final states in Table II are plotted in Fig. 1. The values for states with nonunique spin assignments are indicated by connected solid circles plotted at the appropriate spin values. The dashed lines are required to be horizontal and are least-squares fitted to all the solid circles. (For this purpose, a spin of $\frac{7}{2}$ was assumed for the states of Si^{29} with nonunique assignments.)[15,16] The rms percentage deviation from the fitted line is indicated in each section of Fig. 1. Except for the $Al^{27}(d,\alpha)Mg^{25}$ case, the over-all linearity of the $\sigma_B/(2I+1)$ plots is improved over that of the $\sigma_T/(2I+1)$ plots. The crosses in the $Al^{27}(d,\alpha)Mg^{25}$ portion of Fig. 1 represent the results of Hinds *et al.*[7] These data were normalized at the point corresponding to the first excited state of Mg^{25}. Although different deuteron energies were used in the two investigations, the apparent spin dependence of the σ_B cross sections is about the same. On the basis of the five $Al^{27}(d,\alpha)Mg^{25}$ cross sections measured in the present study, there appears to be little difference between σ_T and σ_B in so far as the $2I+1$ proportionality is concerned, contrary to the conclusion of Hinds *et al.*[7]

Our discussion of the $2I+1$ rule and the interpretation of the experimental observations outlined above will be based on the premise that the DI and CN mechanisms contribute incoherently to the (d,α) angular distributions, and the approximate validity of an *ad hoc* method used for the decomposition of the angular distributions into their DI and CN components. Even though irrefutable experimental and/or theoretical support for the above premise and decomposition procedure cannot be given, some indirect evidence for their plausibility and approximate validity will be cited.

The angular distributions observed in the present investigation all display the following general characteristics: (1) an over-all oscillatory structure with pronounced forward and backward peaking, and asymmetry with respect to 90°; and (2) minima whose magnitudes in all cases differ significantly from zero. The first characteristic is usually taken as an indication that a DI mechanism is operative; however, the presence of the second is not so easily interpretable. In fact, it is not at all clear that this separation of characteristics should be taken to imply that they arise from separate and distinct physical mechanisms. There are no *a priori* reasons to believe that for the nuclei, reaction, and experimental circumstances of the present investigation that both the CN and DI mechanisms will not make contributions to the reaction cross sections. Furthermore, even if both processes are

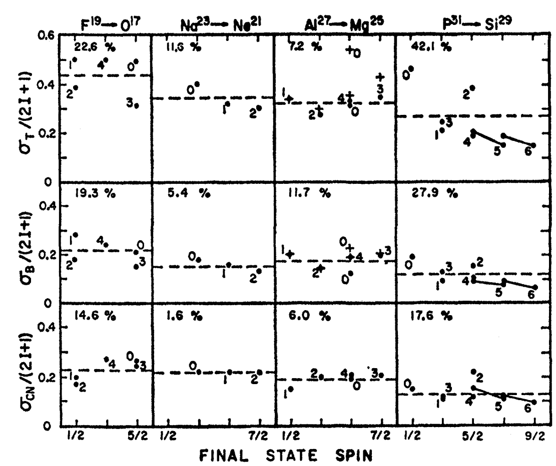

FIG. 1. A graphic representation of the $\sigma_T/(2I+1)$, $\sigma_B/(2I+1)$, and $\sigma_{CN}/(2I+1)$ ratios derived from the data presented in Table II. The values for final states with nonunique spin assignments are indicated by the connected solid circles which are plotted at the appropriate spin values. The dashed lines are required to be horizontal and are least-squares fitted to all the solid circles. (For this purpose, a spin of $\frac{7}{2}$ was assumed for the states of Si^{29} with nonunique assignments.) The percentage rms deviation from the fitted line is indicated in each section of the figure. The crosses in the $Al^{27} \rightarrow Mg^{25}$ section of the figure represent the results of Hinds *et al.*[7] These data were normalized at the point corresponding to the first excited state of Mg^{25}.

[15] A theoretical argument has been given for a $\frac{7}{2}^-$ spin assignment for the fifth excited state of Si^{29}. See V. A. Chepurnov and P. E. Nemirovsky, Nucl. Phys. **49**, 90 (1963).

[16] A theoretical energy-level analysis has predicted a $\frac{7}{2}^+$ level in the energy region of the sixth excited state of Si^{29}. See B. E. Chi and J. P. Davidson, Phys. Rev. **131**, 366 (1963).

TABLE II. Spins, parities, and energies of some states in O^{17}, Ne^{21}, Mg^{25}, and Si^{29}; and information related to the decomposition and interpretation of their production cross sections in the (d,α) reaction on F^{19}, Na^{23}, Al^{27}, and P^{31}.

Nuclear state	energy (MeV)	I^π	σ_T[a,b] (mb)	σ_B[a,b] (mb)	$d\sigma_{CN}$[c] (μb/sr)	σ_{CN}/σ_T	$\sigma_{CN}/2\sigma_B$
O^{17}-0	0	$5/2^+$	2.93±0.03	1.28±0.02	130	0.55	0.63
O^{17}-1	0.871	$1/2^+$	1.00±0.02	0.57±0.01	30	0.37	0.32
O^{17}-2	3.058	$1/2^-$	0.77±0.02	0.36±0.01	25	0.42	0.44
O^{17}-3	3.846	$5/2^{-}$[d]	1.87±0.03	0.90±0.02	115	0.76	0.79
O^{17}-4	4.551	$3/2^-$	1.99±0.03	0.96±0.02	90	0.55	0.57
Ne^{21}-0	0	$3/2^+$	1.59±0.02	0.69±0.01	71	0.55	0.64
Ne^{21}-1	0.350	$5/2^{+}$[e]	1.90±0.03	0.89±0.02	105	0.68	0.73
Ne^{21}-2	1.75	$7/2^{+}$[e]	2.40±0.03	1.04±0.02	137	0.71	0.82
Ne^{21}-3	2.79	$1/2^+$ }	4.58±0.04	1.97±0.03	...	...	...
-4	2.87	? }					
Mg^{25}-0	0	$5/2^+$	1.87±0.02	0.73±0.01	94	0.62	0.79
Mg^{25}-1	0.584	$1/2^+$	0.68±0.01	0.40±0.01	23	0.43	0.36
Mg^{25}-2	0.976	$3/2^+$	1.10±0.02	0.58±0.01	61	0.69	0.66
Mg^{25}-3	1.611	$7/2^+$	2.75±0.03	1.56±0.02	132	0.59	0.52
Mg^{25}-4	1.962	$5/2^+$	1.98±0.03	1.14±0.02	97	0.61	0.53
Si^{29}-0	0	$1/2^+$	0.92±0.02	0.38±0.01	26	0.34	0.41
Si^{29}-1	1.277	$3/2^+$	0.83±0.02	0.36±0.01	33	0.49	0.57
Si^{29}-2	2.027	$5/2^+$	2.27±0.03	0.90±0.02	106	0.58	0.73
Si^{29}-3	2.425	$3/2^+$	0.95±0.03	0.49±0.02	36	0.46	0.45
Si^{29}-4	3.067	$5/2^+$	1.14±0.03	0.59±0.02	55	0.60	0.58
Si^{29}-5	3.621	$(5/2,7/2)^-$	1.17±0.03	0.57±0.02	69	0.73	0.75
Si^{29}-6	4.078	$(7/2^+,9/2^+)$[f]	1.49±0.03	0.60±0.02	75	0.62	0.77
Si^{29}-11	5.249	? }	2.96±0.03	1.40±0.02	...	...	...
-12	5.279	? }					

[a] The experimental differential cross sections are integrated over the angular range from 10° to 170° c.m. to yield σ_T and from 90° to 170° c.m. to yield σ_B.
[b] The probable errors for σ_T and σ_B are derived from the errors associated with the individual data points of the experimental angular distributions which are in turn based on counting statistics and spectral-decomposition uncertainties.
[c] The determination of the magnitude of $d\sigma_{CN}$, the compound-nucleus differential cross section, is described in detail in the text. Its integral from 10° to 170° c.m. is designated σ_{CN}.
[d] See: C. Broude, T. K. Alexander, and A. E. Litherland, Bull. Am. Phys. Soc. **8**, 26 (1963); R. E. Segel, P. P. Singh, R. G. Allas, and S. S. Hanna, Phys. Rev. Letters **10**, 345 (1963); E. A. Silverstein, L. D. Opplinger, and R. A. Blue, Bull. Am. Phys. Soc. **9**, 68 (1964); and T. K. Alexander, C. Broude, and A. E. Litherland, Nucl. Phys. **53**, 593 (1964).
[e] See: P. Kienle and K. Wien, Nucl. Phys. **41**, 608 (1963); and D. Pelte, B. Povh, and W. Scholz, Nucl. Phys. **55**, 322 (1964).
[f] R. E. White, *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach, Science Publishers, Inc., New York, 1963), p. 549.

operative, there are no *a priori* bases for reliably estimating the extent to which one might dominate the other.

For certain isotopic-spin forbidden (d,α) transitions in light nuclei the differential cross sections have been found to be measurable, but small, and approximately isotropic.[17–19] The accepted interpretation of these isotopic-spin violations is that the reaction proceeds through intermediate compound-nucleus states whose isotopic-spin impurity arises from the Coulomb interaction.[20] Since the physical and experimental circumstances in those investigations are similar to the ones prevailing in the present studies, it seems reasonable to assume that the CN mechanism, though not necessarily dominant, is generally operative in the reactions of interest.

Within the framework of compound-nucleus theory the total differential cross section for a given transition may be written,

$$d\sigma/d\Omega=(d\sigma_R/d\Omega)+(d\sigma_P/d\Omega)+(d\sigma_I/d\Omega),$$

where the constituent terms are associated with the resonant interaction (CN mechanism), the nonresonant interaction (DI mechanism), and the interference between these two interactions.[21] It is reasonable to expect that the theoretical predictions based on the assumption of a statistical continuum of compound-nuclear levels should be physically approximated for situations in which many levels of the compound nucleus overlap. In particular, there should be no interference between the CN and DI processes.[21] In the present investigation, the nominal CN excitation energy is 25 MeV, and the corresponding level densities calculated from the expressions of Cameron[22] are $[(2I+1)/0.2]$ and $[(2I+1)/0.08]$ levels/keV for Ne^{21} and S^{33}, respectively. The rms spread in the incident beam energy (30 keV) and the energy spread due to finite target thickness (20-50 keV) would further ensure the excitation of a large number of these compound states.

It follows immediately from the incoherence of the DI and CN processes that the decomposition of an experimental angular distribution requires a detailed knowledge of only one of the components. Unfortunately, neither CN nor DI theory is completely free of ambiguities in so far as predicting angular distributions for specific reactions is concerned. For the purpose of decomposing these experimental angular distributions

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[20] W. M. MacDonald, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 956.
[21] H. Feshbach, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 625.
[22] A. G. W. Cameron, Can. J. Phys. **36**, 1040 (1958).

two assumptions about the CN differential cross sections will be made: (1) They are isotropic; and (2) the magnitude for a given transition will be well approximated by the value of the corresponding experimental angular distribution at its lowest observed minimum. The approximate isotropy observed in connection with the previously mentioned isotopic-spin forbidden (d,α) reactions,[17–19] because of the similarity in experimental and physical circumstances, suggests that the CN component of the present angular distributions may be of the same character. Furthermore, the complete statistical model which assumes a continuum of both the compound and final nucleus states predicts isotropic angular distributions if the level densities both have a $2I+1$ spin dependence.[21,23] Ericson[4] has investigated, in the classical limit, the decay from a continuum of compound-nuclear states to discrete final states and found that a level density with a $2I+1$ spin dependence yields isotropic angular distributions. Although the spin dependence of the level densities appears crucial in guaranteeing isotropy, present purposes do not warrant a fuller discussion of this complicated subject here.[24] Better over-all agreement between theoretical plane-wave DI predictions and some experimental (d,α),[14,25,26] (d,p),[27–29] and (p,d)[30] angular distributions results if an isotropic component with a magnitude approximately equal to that of the lowest minimum in the experimental distribution is subtracted from the experimental data before fitting is attempted. Furthermore, a general survey of the open literature has led to the conclusion that there are numerous instances for certain classes of reactions in which improved plane-wave DI fits could have been achieved if the decomposition indicated above had been performed. The following studies may be cited as examples; (d,p) by Rietjens *et al.*,[31] (p,d) by Kavaloski *et al.*,[32] (t,p) by Middleton and Pullen,[33] and (He^3,d) by Holmgren *et al.*[34] It is upon these considerations that the second assumption is based.

The quantitative results of this decomposition are included in Table II. The magnitudes of the CN differential cross sections were determined solely on the basis of the criterion detailed above and are given in the column labeled $d\sigma_{CN}$. The nominal probable errors in these $d\sigma_{CN}$ values range from 7 to 9% and are estimated on the basis of both subjective and statistical factors. The derived ratios σ_{CN}/σ_T and $\sigma_{CN}/2\sigma_B$ are also presented. Subject to the validity of this decomposition procedure, these ratios provide a measure of the relative contributions of the CN and DI mechanisms to the forward and backward angle cross sections. In Fig. 1, the plots of the $\sigma_{CN}/(2I+1)$ values for each reaction are shown. The improved constancy of the $\sigma_{CN}/(2I+1)$ values over the $\sigma_B/(2I+1)$ and $\sigma_T/(2I+1)$ values is clearly evident. Moreover, in a given reaction the $\sigma_{CN}/(2I+1)$ values for final states of the same spin, excluding the second and fourth excited states of Si^{29}, tend to be more nearly equal than the corresponding $\sigma_B/(2I+1)$ and $\sigma_T(2I+1)$ values. From an inspection of the $\sigma_{CN}/(2I+1)$ plots of Fig. 1, it appears that the σ_{CN} value of the second excited state in Si^{29} is anomalously large. The angular distribution corresponding to this level showed nothing abnormal. It can only be concluded that this cross section is either a marked exception to the $2I+1$ rule or is associated with a closely spaced doublet. The latter alternative appears unlikely since studies of this level by other means have yielded no evidence which would suggest a doublet structure.

From the experimental results and the considerations presented above, it may be concluded that the $2I+1$ rule is not generally valid for (d,α) cross sections integrated from 0° to 180°, but in some cases the cross sections integrated from 90° to 180° may more closely approximate a $2I+1$ dependence. The constancy of the $\sigma_{CN}/(2I+1)$ values suggests that the $2I+1$ dependence of the (d,α) cross sections is a characteristic associated with the CN process. This would explain the improvement in linearity which occurs in the $\sigma_B/(2I+1)$ values when the percentage of the CN contribution to the cross section is greater in the backward than in the forward direction. However, the present data do not support the general assumption that the CN process is more dominant in the backward direction, e.g., the $Al^{27}(d,\alpha)Mg^{25}$ results of this study. Consequently, spin assignments made from (d,α) reaction data and based solely on the assumption of a $2I+1$ dependence for either σ_T or σ_B would not as a rule be very reliable.

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