potentials has been discussed, in particular for even $\Sigma\Lambda$ parity, by Dalitz,[14] by de Swart[8] and by Dalitz and Rajasekaran.[1] Our results for the meson-theory interaction III (which is appropriate to even $\Sigma\Lambda$ parity) show that the value of $f_{\Sigma\Lambda}$ is quite close to the comparable value $f_{\Sigma\Lambda}=0.276$ (for $f_{\Sigma\Sigma}=0$) which is obtained from the singlet Λ-N interaction.[9] (The attractive part of this, due to the exchange of two pions, has the same shape as for $V_{\Lambda\Lambda}$ but is proportional to $f_{\Sigma\Lambda}^2 f_{NN}^2$.) This is in agreement with the conclusions of Dalitz and Rajasekaran[1] which were based on the values of $a_{\Lambda\Lambda}$.

However, as has been emphasized especially by Dalitz,[1,14] any results deduced for $f_{\Sigma\Lambda}$ are very sensitive to the value used for the hard-core radius. (Thus for $r_c=0.35\mu_\pi^{-1}$ and $f_{\Sigma\Sigma}=0$, one has $f_{\Sigma\Lambda}=0.30$ for $a_{\Lambda\Lambda}=-1$ F.) This is because of the strong cancellation between the effects of the hard-core repulsion and the short-range attraction due to the exchange of two pions. One must therefore have some understanding of the relation between the hard-core radii for the Λ-N and Λ-Λ potentials if one is to reliably relate the attractive parts of these potentials. Furthermore, for $V_{\Lambda N}$, one can have, for example, exchange of single K mesons, which is not possible for $V_{\Lambda\Lambda}$.

Finally, if the event described in Ref. 3 is interpreted as ${}_{\Lambda\Lambda}Be^{11}$, then the conclusions about the Λ-Λ interaction will have to be modified accordingly. As pointed out by Dalitz,[14] the appropriate value $\Delta B_{\Lambda\Lambda}=4.5\pm1.0$ MeV for ${}_{\Lambda\Lambda}Be^{11}$ is quite similar to the value for the most probable interpretation ${}_{\Lambda\Lambda}Be^{10}$. With a rigid Be^9 core for ${}_{\Lambda\Lambda}Be^{11}$, a Λ-Λ-core model will yield very nearly the same results for $V_{\Lambda\Lambda}$ as are obtained for ${}_{\Lambda\Lambda}Be^{10}$ with a rigid core. Since the odd neutron in Be^9 has a separation energy of only 1.7 MeV, an α-α-n model might be expected to be quite good for Be^9 with a rms separation between the α particles which is rather larger than for ${}_{\Lambda}Be^9$. The contribution to $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}Be^{11})$ due to distortion may then be expected to be somewhat more than half of that for ${}_{\Lambda\Lambda}Be^{10}$, in view of our results for $E_{\alpha\alpha}(R_\Lambda)$ and for $E_{\Lambda\Lambda}$. The results for the Λ-Λ interaction will then be roughly intermediate between those obtained for ${}_{\Lambda\Lambda}Be^{10}$ with a rigid core, on the one hand, and with core distortion included, on the other.

ACKNOWLEDGMENTS

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[14] R. H. Dalitz, Phys. Letters 5, 53 (1963).

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Decays of the η Meson*

M. Foster, M. Peters, R. Hartung, R. Matsen, D. Reeder, and M. Good
Department of Physics, University of Wisconsin, Madison, Wisconsin

and

M. Meer, F. Loeffler, and R. McIlwain
Department of Physics, Purdue University, Lafayette, Indiana

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The decay modes of the η^0 meson have been investigated in the Lawrence Radiation Laboratory 72-in. hydrogen bubble chamber. The η's were produced in the reaction $\pi^+ + p \to \pi^+ + p + \eta$ at 1225 and 1275 MeV/c and were studied by analysis of all four-prong and two-prong+γ events. There appears to be a discrepancy in the measured branching ratio $R=\Gamma_\eta(000)/\Gamma_\eta(+-0)$ as compared with theoretical predictions based on various models relating R to the spectrum of T_{π^0} in $\eta^0\to\pi^+\pi^-\pi^0$. The theoretical predictions calculated from the observed spectrum are uniformly higher than the observed value $R=0.90\pm0.24$. For the Brown and Singer theory of a $T=0$, $S=0$ dipion resonance, we find $m_\sigma=407_{-12}^{+25}$ MeV, $\Gamma=117\pm15$ MeV, R(predicted)$=1.49\pm0.07$. The fit to the linear matrix-element expansion, $a=-0.41\pm0.06$, predicts $R=1.63\pm0.02$ (the amplitude $f=1+ay$ where $y=2\,T_{\pi^0}/T_{\pi^0\max}-1$). A fit may be obtained to both the spectrum and the branching ratio with an amplitude $f=(1-0.41y)e^{1.6iy}$, indicating that, although the magnitude of f is essentially smooth, a rapid variation in phase seems to be required to fit the branching ratio.

According to the accepted quantum numbers $0^{-+}=J^{PG}$ for the η meson, the final state reached in the decay $\eta\to\pi^+\pi^-\pi^0$ must have $T=1$, $J^P=0^-$. For this state, the most general decay amplitude is[1]

$$M=(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\boldsymbol{\tau}_3 f(\omega_1,\omega_2,\omega_3)+(\text{c.p.})\,, \qquad (1)$$

where (c.p.) means cyclic permutation of the indices 1, 2, 3; $\boldsymbol{\tau}_i$ is the isotopic spin vector of pion i; and f is

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[1] K. C. Wali, Phys. Rev. Letters 9, 120 (1962).

a scalar function of the pion energies, ω_i. Relative decay rates into $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ are given in terms of (1):

$$\Gamma_\eta(+-0)=2\int |f|^2 d\rho\,,$$
$$\Gamma_\eta(000)=\left(\int |f|^2 d\rho+2\int f^*(123)f(231)d\rho\right), \qquad (2)$$

where $d\rho$ means integration over the area of the Dalitz plot. $R=\Gamma_\eta(000)/\Gamma_\eta(+-0)$ depends on the size of the overlap integral appearing in (2), being $\frac{3}{2}P$ for complete interference and $\frac{1}{2}P$ for no interference. ($P=1.13$ is the correction factor for $\pi^0-\pi^+$ mass difference.)

If $f=$const, $R=(\frac{3}{2}P)=1.7$. Wali[1] has shown that if f is linear in ω_1, ω_2, ω_3, then $f=1+ay$ where

$$y=2T_{\pi^0}/T_{\pi^0_{\max}}-1\,. \qquad (3)$$

$$R=\frac{3}{2}\left(\frac{P}{1+\frac{1}{4}|a|^2}\right).$$

Brown and Singer[2] have proposed a Breit-Wigner form for f, based on the idea of a $T=0$ dipion resonance, postulated to explain the apparent enhancement of the decay modes $\eta\rightarrow\pi\pi\pi$ relative to $\eta\rightarrow\pi\pi\gamma$ and $\eta\rightarrow\gamma\gamma$. For the Brown and Singer form also, the branching ratio R is rigorously predictable, via Eq. (2), once the resonance parameters are determined.

This hypothesis is not inconsistent with previous data[3,4] on the $\pi^+\pi^-\pi^0$ spectrum and the branching ratio R.

We present here data on the decay modes (2) which fit all the above hypotheses rather poorly.

The η's were produced in the reaction $\pi^++p\rightarrow\pi^++p+\eta$ at 1225 and 1275 MeV/c in the LRL 72-in. hydrogen bubble chamber. The decays $\eta\rightarrow\pi^+\pi^-\pi^0$ and $\eta\rightarrow\pi^+\pi^-\gamma$ were studied by analysis of all four-prong events. Those fitting the four-constraint hypothesis

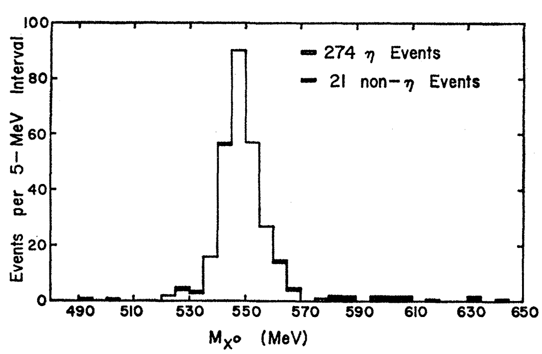

FIG. 1. Distribution of M_{x^0} in the reaction $\pi^++p\rightarrow\pi^+px^0$ for those events fitting $\pi^++p\rightarrow\pi^+p\pi^+\pi^-\pi^0$. For each event, the mass nearest the η mass is plotted.

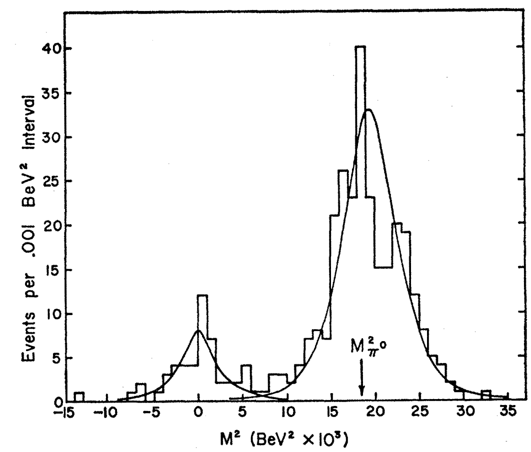

FIG. 2. Distribution of $M_{x^0}{}^2$ in the reaction $\pi^++p\rightarrow\pi^+p\pi^+\pi^-x^0$ for events fitting η production and decay. The curves are the resolution functions for $M_{\pi^0}{}^2$ and $M_\gamma{}^2$. Cutoffs to eliminate background in the $\eta\rightarrow\pi^+\pi^-\gamma$ events have not yet been made.

$\pi^++p\rightarrow\pi^+p\pi^+\pi^-$ were accepted as such. Events fitting the hypotheses $\pi^++p\rightarrow\pi^+p\pi^+\pi^-\pi^0$ and $\pi^++p\rightarrow\pi^+p\pi^+\pi^-\gamma$ were carefully checked on the scanning table for observable track scatters and were remeasured until two good measurements agreed. As shown in Fig. 1, almost all of the events accepted as fitting a missing π^0 fitted η production and decay (a two-vertex, two-constraint fit). Separation between the $\pi^+\pi^-\pi^0$ and $\pi^+\pi^-\gamma$ decay modes for the η is essentially complete, as may be seen from Fig. 2. The contamination of the sample of events fitting $\eta\rightarrow\pi^+\pi^-\pi^0$ from non-eta events is estimated to be 2%. In 14.6% of the events there was an ambiguity as to which π^+ came from η decay. The interpretation with the lowest χ^2 was chosen, and it is estimated that 30% of the ambiguous events were misinterpreted.

The decay $\eta\rightarrow 3\pi^0(\eta\rightarrow 2\gamma)$ was observed by detecting both the internal and external pair production initiated by one of the six (two) gamma rays eventually produced.

Most of the events with internal pair production were distinguished on the scanning table from other four-prong events by the bubble density of the Dalitz pair. Those four-prong events not fitting another hypothesis were also tested as possible Dalitz pairs. Events fitting the hypothesis $\pi^++p\rightarrow\pi^+p\pi^0$, $\pi^0\rightarrow e^+e^-\gamma$ were accepted as such. The remaining Dalitz pair events, which were candidates for multiple pion production and for η production, were checked on the scanning table for possible track scatters and were remeasured until two good measurements agreed. In most cases the proton could be distinguished by ionization.

Since high-energy electrons ($E\geqslant 100$ MeV) cannot be distinguished by their ionization from pions, the four-prong events fitting η production were examined further to estimate the probability of an $\eta\rightarrow 3\pi^0$, $\pi^0\rightarrow e^+e^-\gamma$, or $\eta\rightarrow 2\gamma$, $\gamma\rightarrow e^+e^-$ being misinterpreted as $\eta\rightarrow\pi^+\pi^-\pi^0$ or $\eta\rightarrow\pi^+\pi^-\gamma$. Under the assumption

2 L. M. Brown and P. Singer, Phys. Rev. 133, B812 (1964).
3 F. S. Crawford, Jr., R. A. Grossman, L. J. Lloyd, L. R. Price, and E. C. Fowler, Phys. Rev. Letters 11, 564 (1963); see also *ibid.* 13, 421 (1964).
4 A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 36, 977 (1964).

of a flat energy division $E_{e^+}/(E_{e^+}+E_{e^-})$,[5] it is estimated from the laboratory photon spectrum calculated by a Monte Carlo technique that in 2.8% (42.4%) of the $\eta\rightarrow 3\pi^0(\eta\rightarrow 2\gamma)$ events both the electron and the positron would have $E\geqslant 100$ MeV. Since the percentage of events with high-energy pairs in our sample of $\eta\rightarrow 3\pi^0(\eta\rightarrow 2\gamma)$ is consistent with the number calculated, it is assumed that the high-energy pairs have been correctly identified. It is important that no cutoff was necessary to eliminate Dalitz pairs from the sample of events fitting $\eta\rightarrow\pi^+\pi^-\pi^0$, for this would have biased the distribution.

For external pairs all film was scanned twice for all electron-positron pairs produced in the chamber. In each exposure containing a pair, the chamber was searched for possible two-prong origins for the gamma ray. After measurement, any pair that satisfied kinematic fit criteria to be a sample of single π^0 production and decay (a two-vertex, two-constraint fit) was removed from the sample. The remaining events were remeasured, and if the new measurement fit the π^0 hypothesis, the event was discarded. The remaining events were subjected to cutoffs for maximum and minimum distance from two prong to pair and for minimum total pair momentum. Only those events satisfying the requirement of colinearity of the neutral path and the vector sum of the electron momenta were accepted.

By Monte Carlo methods the average probability of converting a gamma ray from an $\eta^0\rightarrow 3\pi^0$ $(\eta^0\rightarrow 2\gamma)$ event is calculated. After corrections for scanning efficiency and cutoffs, the over-all detection efficiency is 0.0456 (0.0170).

The results of a maximum likelihood fit to all the data to determine the branching ratios,

$$T=\Gamma_\eta(\gamma\gamma)/\Gamma_\eta(+-0),\quad R=\Gamma_\eta(000)/\Gamma_\eta(+-0),$$

and the background contribution, are $T=1.61\pm0.39$; $R=0.83\pm0.20$. The curve used for the $2\pi^0$ spectrum is invariant phase space; it agrees with the spectra of $m^2(\pi^\pm\pi^-)$ in $\pi^\pm+p\rightarrow\pi^+p\pi^\pm\pi^-$ seen at the same energy.

The fitted background curve does, however, seem to be too high in the η region (Fig. 3). We have therefore investigated what different assumptions about background would do to the branching ratio R. (T is unaffected, since the background events, presumably mostly $2\pi^0$ events, do not fit $\eta\rightarrow\gamma\gamma$.) We find if we fit the background to the M_x^2 histogram near the η peak, we get $R=0.97\pm0.22$; and if we make the (incorrect) assumption of no background, $R=1.23$ ±0.21 results.

The best value for R is taken to be the average of the phase-space background and the fitted background values, with an error $\delta R=\pm0.1$ for background uncertainty folded in:

$$R_{\rm best}=0.90\pm0.24.$$

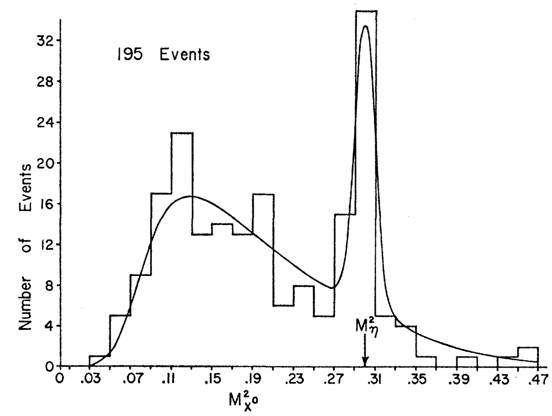

FIG. 3. Distribution of $M_{x^0}{}^2$ for $\pi^++p\rightarrow\pi^++p+x^0$, $x^0\rightarrow\gamma+Y^0$ or $X^0\rightarrow e^++e^-+Y^0$ for events not fitting single π^0 production and decay.

This value of R is in fair agreement with the world average of 1.16 ± 0.15 as determined from the report by Rosenfeld *et al.*[4] (Note that preliminary results of the present work are included in that report under the reference M. W. Peters, thesis.) Agreement with the similar experiment of Crawford *et al.*[6] ($R=0.83\pm0.32$) is good. The following discussion is based on the present result of $R=0.90\pm0.24$.

We note that R deviates markedly from the value 1.7 corresponding to a constant matrix element.

The Dalitz plot for the $\pi^+\pi^-\pi^0$ mode shows no dependence on $x=(T_{\pi^+}-T_{\pi^-})/\sqrt{3}Q$. The y dependence, as seen in Fig. 4, is well fitted by a linear matrix element. For this we have $a=-0.41\pm0.06$, R (predicted)$=1.63$ ±0.02, in disagreement with the measured value.

The Brown and Singer hypothesis fits the spectrum very well with

$$E_{\rm res}=(407_{-12}{}^{+25})\ \text{MeV},$$
$$\Gamma=(117\pm15)\ \text{MeV}.$$

For these parameters R(predicted)$=1.49\pm0.07$, again in disagreement with the measured branching ratio.[7]

The theory of Oneda[8] (P-wave dipion enhancement) fails to fit (R predicted$=1.63\pm0.02$), ($\beta=1.41\pm0.22$). Modifying the Brown and Singer hypothesis by allowing the width to vary as k is of no help (R predicted $=1.46\pm0.07$, $m=423_{-12}{}^{+35}$ MeV, $\Gamma=\gamma k_{\rm res}=117\pm13$ MeV).

We have tried to generalize the Brown and Singer hypothesis to include a general $T=0$ $\pi\pi$ scattering. We write for the three-body decay

$$f(123)=Ce^{i\delta}\sin\delta/k=C/(k\cot\delta-ik)$$

[5] This is a reasonable approximat ion tothe distribution calcalculated by W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, England, 1954), 3rd ed., p. 261, an d to the observed distribution for single π^0 production.

[6] F. S. Crawford, Jr., L. J. Lloyd, and E. C. Fowler, Phys. Rev. Letters **10**, 546 (1963).

[7] Our results agree with those of Crawford *et al.* (Ref. 3) for the linear matrix-element theory ($a=-0.45\pm0.07$, $R_{\rm pred}=1.63$ ±0.03), but for the dipion resonance they predict a slightly lower branching ratio ($m=392\pm9$MeV, $\Gamma=88\pm15$MeV, $R_{\rm pred}=1.28$ ±0.07).

[8] S. Oneda, Y. Kim, and L. Kaplan, Nuovo Cimento **34**, 655 (1964).

where $\delta = \delta_{12}$ is taken to be the $\pi\pi$ $T=0$ scattering phase shift at relative momentum $k = k_{12}$.

We then take as hypothesis $k \cot\delta = c_1 + c_2 k^2 + c_3 (k^2)^2 + \cdots$. We have attempted to fit the π^0 spectrum with various numbers of terms (up to four) in the expansion. It is easy to fit the spectrum; none of the choices made fit the branching ratio (see Fig. 4). The best fit with the first 3 terms in the expansion gives R(predicted) = 1.40. The best fit taking c_1 and c_3 as the parameters gives R(predicted) = 1.35.

Since the magnitude of f is essentially smooth, as shown by the spectrum, it seems that a rapid variation of the phase of f is required to bring down the overlap integral in (2) and fit the branching ratio. Accordingly we have tried an expansion of $f = \rho e^{i\phi}$ of the form

$$\rho = 1 + \epsilon y + \cdots .$$

$$\phi = \gamma y + \cdots \; (\epsilon, \gamma \text{real}) .$$

$\Gamma_\eta(+-0) \sim |1+\epsilon y|^2$ and thus the spectrum is fitted with $\epsilon = -0.41$ (as in linear matrix-element theory), independent of γ. The value $\gamma = 1.60 \pm 0.40$ then fits the experimental branching ratio. Such a value of γ implies a shift in phase of f by $183° \pm 46°$ in crossing the physical region.

We do not know what the meaning of such a form for f would be; in any case there is strong evidence for an interesting structure in the 3π final state in η decay.

Note added in proof. L. Brown and H. Faier[9] have recently obtained a prediction for the branching ratio R that agrees more closely with the experimental result.

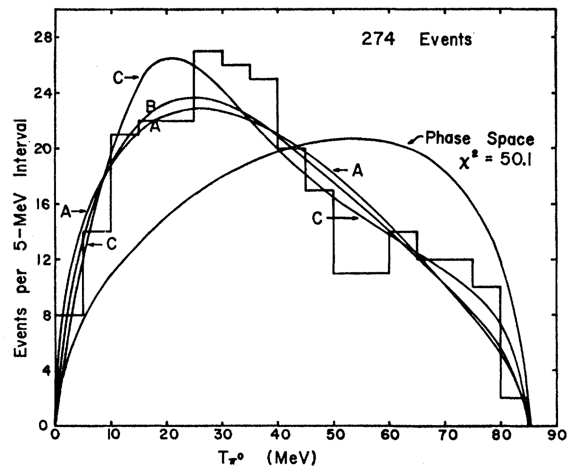

FIG. 4. Kinetic-energy distribution of π^0 in η decay, $\eta \to \pi^+\pi^-\pi^0$. Curve A: Linear matrix element; $a = -0.4$, $\chi^2 = 9.9$, R(predicted) = 1.63. Curve B: Brown and Singer; $m = 425$ MeV, $\Gamma = \gamma k = 118$ MeV, $\chi^2 = 8.4$, R (predicted) = 1.47. Curve C: $k \cot\delta = c_1 + c_3 (k^2)^2$, $c_1 = 1.26 f^{-1}$, $c_3 = -4.6 f^3$, $\chi^2 = 7.7$, R (predicted) = 1.35.

Assuming a modified Breit–Wigner form for the proposed $T=0$ dipion resonance and using a compilation of 708 events, including the 274 reported on here, they are able to predict a value as low as R(predicted) = 1.19. Thus, if the proposed resonance is found to exist, it may be the explanation for the rapid variation in phase of f.

We wish to thank Professor L. W. Alvarez and Professor Frank S. Crawford for making the exposure possible, and our scanning and measuring staffs for their tireless efforts. We are grateful to B. Sakita and C. Goebel for helpful discussions.

[9] L. Brown and H. Faier, paper presented at the Conference on Symmetry Principles at High Energy, Coral Gables, Florida, 1965 (unpublished).

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Possible Electromagnetically Induced Muonium-Antimuonium Conversion*

MICHAEL J. MORAVCSIK AND RICHARD SPITZER
Lawrence Radiation Laboratory, University of California, Livermore, California

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The possibility of the existence of an electromagnetically induced transition between muonium and antimuonium is examined. An experiment is suggested involving the formation of muonium by the injection of positive muons into a helium-filled resonant cavity, which is excited at a frequency corresponding to the difference in interaction energy between muonium and helium, on the one hand, and that between antimuonium and helium, on the other. The sign of antimuonium formation is the observation of the fast electrons from μ^- decay. The dependence of the number of these on which of the various cavity modes is excited gives information on the relative intrinsic parity of muonium and antimuonium. If this turns out to be odd, then this measurement, when combined with the usual relation for the product of the intrinsic parities of a Dirac particle and its antiparticle, would determine the relative intrinsic parity of the muon and electron to be imaginary. The conservation of parity in electromagnetic phenomena and the absence of electromagnetic μ-e transitions would then both find their natural explanation in the single assumption that the observation of electromagnetic phenomena must be compatible with invariance under space inversion.

I. INTRODUCTION

THE motivation for this paper is twofold. First, we hope to present a fairly detailed discussion of a type of experiment which could be used to investigate the possible existence of an electromagnetically induced transition between muonium and antimuonium. An interaction that results in such transitions would also lead to the process

$$e^- + e^- \to \mu^- + \mu^- . \qquad (1)$$

* Work done under the auspices of the U. S. Atomic Energy Commission.