Representation Mixing for SU_3 and G_2†

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A theory whose exact invariance with respect to SU_3 is broken by interactions transforming like the hypercharge generator of SU_3 is studied in perturbation theory. Physical particle states corresponding to unperturbed SU_3 multiplets are obtained by mixing into these multiplets possible impurities corresponding to other SU_3 multiplets, these impurities being as general as the perturbation allows. The effect of symmetry breaking on various physical quantities like masses and magnetic moments can then be obtained in terms of mixing parameters, and interesting formulas follow various restrictions of their values. Similar considerations applied to G_2 show that many of the conclusions which made exact G_2 invariance unattractive remain for approximate G_2 invariance.

RECENT events have given great support to the existence of SU_3 symmetry[1] as an approximate symmetry of the strong interactions. It therefore seems of interest to develop a theory of the approximate breakdown of the symmetry on the basis of particle (or representation) mixing. Here we discuss particle mixing correctly to lowest order in an effective symmetry-breaking interaction of the usual type, and briefly mention some results of similar work for G_2.[2]

We begin with some general remarks. Although violation of symmetry always causes representation mixing, it is to be noted that physical energies need not be the quantities most sensitive to this mixing. Consider, for example, the Stark effect on the degenerate $2s$ and $2p$ levels in hydrogen. The effect of the uniform electric field is to break space-inversion invariance so that the physical eigenstates are linear combinations of these levels. Further, the amount of mixing is independent of the applied-field strength and in fact maximal. This leads to the possibility of large electric-dipole moment even when the energy shift is very small. Low-energy nuclear physics gives other examples, wherein large admixtures (configuration mixing) may have little effect on energy levels, but significant effect on magnetic moments, radiative and β-decay rates, etc. Accordingly, we might expect in the particle physics context also, that quantities like magnetic moments and weak currents may provide more sensitive means of detecting mixing than quantities directly related to the symmetry-breaking interaction.

It is to be noted, also, that continuum states as well as other particle or resonant states can be mixed into physical particle states. We illustrate such a situation using the deuteron, regarding the tensor force as a perturbation which breaks orbital-angular-momentum conservation. With the tensor force absent there is only one bound state, an s state; with it present, the physical bound-state wave function exhibits mixing into the original pure s state of some continuum d state. Another example, which lacks the symmetry-breaking element, comes from the Lee model, where the physical V-particle state contains $N-\theta$ continuum states. It is possible, of course, that mixing of continuum states in physical particle states is relatively unimportant in particle physics.

We make the customary assumption that the symmetry-breaking interaction H_I for SU_3 has the tensor properties of the $I=0=Y$ member of the adjoint representation of SU_3, and apply perturbation theory. If a set of particles form a multiplet which is degenerate and belongs to the representation (λ,μ) of SU_3 in the absence of H_I, then the physical particle states involve the mixture into the unperturbed states of states of any (λ',μ') that occur in $(\lambda,\mu)\otimes(1,1)$, namely, (λ,μ), $(\lambda\pm1,\mu\pm1)$, $(\lambda\pm2,\mu\mp1)$, and $(\lambda\pm1,\mu\mp2)$. Matrix elements $\langle\lambda'\mu'II_ZY|H_I|\lambda\mu II_ZY\rangle$ occur. Their essential I and Y dependence can be evaluated[3] and the reduced matrix elements $\langle\lambda'\mu'\|H_I\|\lambda\mu\rangle$ can be viewed as parameters that determine the amount of the mixing of (λ',μ') into (λ,μ). We have obtained general formulas for the physical particle states corresponding to the original unperturbed multiplet, in terms of such mixing parameters, and shall describe their application to the derivation of mass formulas (accurate to 2nd order in H_I) and of magnetic-moment relationships (accurate to 1st order) for physical particles.[4]

Our most general parametric formula for the mass of a physical particle has the structure of the second-order Okubo mass formula.[5] But the parameters involved in it

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[1] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).
[2] R. E. Behrends and L. F. Landovitz, Phys. Rev. Letters **11**, 296 (1963); A. J. Macfarlane, N. Mukunda, and E. C. G. Sudarshan, Phys. Rev. **133**, B475 (1964).
[3] D. Lurié and A. J. Macfarlane, J. Math. Phys. **5**, 565 (1964).
[4] For all derivations and details, see N. Mukunda, Ph.D. thesis, University of Rochester, 1964 (unpublished).
[5] S. Okubo, Phys. Letters **4**, 14 (1963).

are the above-mentioned mixing parameters rather than parameters of unspecified nature. This allows systematic investigation of simplified situations with only a few nonvanishing mixing parameters.[6] Suppose only the representations 8, 10, and 27 mix. In addition to three relations for the 27 and one for the 10, which follows from the second-order Okubo mass formula, we find that there is one new relation for the 27 and one "mixing" relation involving multiplets from all three representations. Or, thinking of vector meson perhaps,[7] suppose only 8, 10, and $\overline{10}$ mix. We here get one relation

$$\begin{aligned}2[2M(8;\tfrac{1}{2}1)+2M(8;\tfrac{1}{2}-1)-3M(8;00)-M(8;10)]\\ =[M(10;0-2)-M(10;\tfrac{1}{2}-1)+M(10;\tfrac{3}{2}1)\\ -M(10;10)]+[M(\overline{10};02)-M(\overline{10};\tfrac{1}{2}1)\\ +M(\overline{10};\tfrac{3}{2}-1)-M(\overline{10};10)],\end{aligned}\quad(1)$$

with obvious notation, over and above the single relations that follow for the 10 and the $\overline{10}$ from the second-order Okubo mass formula.

We illustrate our work on electromagnetic form factors by writing down for the baryon magnetic moments the most general parametric expressions correct to first order in H_I:

$$\begin{aligned}\mu(p)&=-\beta-\tfrac{1}{2}\gamma-(2b+c)\\&\quad\times(\beta'+\tfrac{1}{2}\gamma')+(6/5)\mu(27),\\ \mu(n)&=\gamma+(2b+c)\gamma'+\mu(\overline{10})+\tfrac{3}{5}\mu(27),\\ \mu(\Sigma^+)&=-\beta-\tfrac{1}{2}\gamma+c(2\beta'+\gamma')\\&\quad+\mu(10)+\tfrac{3}{5}\mu(27),\\ \mu(\Sigma^0)&=-\tfrac{1}{2}\gamma+c\gamma'+\tfrac{1}{2}\mu(10)+\tfrac{1}{2}\mu(\overline{10})+\tfrac{3}{5}\mu(27),\\ \mu(\Sigma^-)&=\beta-\tfrac{1}{2}\gamma-c(2\beta'-\gamma')\\&\quad+\mu(\overline{10})+\tfrac{3}{5}\mu(27),\\ \mu(\Xi^0)&=\gamma-(2b-c)\gamma'+\mu(10)+\tfrac{3}{5}\mu(27),\\ \mu(\Xi^-)&=\beta-\tfrac{1}{2}\gamma-(2b-c)\\&\quad\times(\beta'-\tfrac{1}{2}\gamma')+(6/5)\mu(27),\\ \mu(\Lambda)&=\tfrac{1}{2}\gamma+c\gamma'+\tfrac{1}{4}\mu(1)+(27/20)\mu(27),\\ (2\sqrt{3})\mu_T(\Sigma^0-\Lambda)&=-3\gamma-\tfrac{3}{2}\mu(10)\\&\quad-\tfrac{3}{2}\mu(\overline{10})+\tfrac{3}{4}\mu(1)+(9/4)\mu(27).\end{aligned}\quad(2)$$

Here β and γ are the parameters that survive in limit of exact unitary symmetry; $\mu(1)$, $\mu(10)$, $\mu(\overline{10})$, and $\mu(27)$ are the contributions from mixing of all singlets, 10's, $\overline{10}$'s, and 27's, respectively, into the original octet[8]; the parameters β', γ', b', and c' describe the effect of mixing in another single octet into the original. The above expressions of course satisfy formulas first derived by Okubo[5] for magnetic moments treated to first order in H_I. If we mix into the original octet only another octet, we get an additional formula

$$\mu(\Sigma^0)+\mu(n)+\mu(\Xi^0)-3\mu(\Lambda)=0,\quad(3)$$

while if we mix in anything but another octet, the extra result

$$\mu(p)-\mu(\Sigma^+)+\mu(\Xi^0)-\mu(n)+\mu(\Sigma^-)-\mu(\Xi^-)=0\quad(4)$$

emerges. If we simplify the latter case further by mixing only 10 and $\overline{10}$, two more relations are found:

$$\begin{aligned}\mu(p)+\mu(\Xi^-)+2\mu(\Lambda)&=0,\\ \mu(p)+\mu(\Xi^0)-2\mu(\Lambda)-\mu(\Sigma^+)&=0.\end{aligned}\quad(5)$$

On the other hand, if we mix another octet and anything else into the original octet, we get nothing beyond the Okubo results.

We turn next to G_2, treating breakdown of exact invariance by the method outlined above. We adopt the usual assignments[2] and take the $I=Y=0$ member of the adjoint representation as the symmetry-breaking interaction H_I (so that H_I has K spin[2] of unity). We do not present the results of any mass calculations, but for the magnetic moments of the baryons, we have these results:

$$\begin{aligned}\mu(p)-\mu(\Sigma^+)=\mu(\Xi^-)-\mu(\Sigma^-)=3\mu(n)-3\mu(\Sigma^0),\\ \mu(n)=\mu(\Xi^0),\quad \mu_T(\Sigma^0-\Lambda)=0,\end{aligned}\quad(6)$$

[with $\mu(\Lambda)$ unrelated to the others but not necessarily zero], true to first order in H_I with the most general possible mixing. We conclude by considering nine processes[9]:

$$\begin{aligned}&(a)\ Y_1^*\to\Lambda\pi,\quad(b)\ \Sigma^0\to\Lambda\gamma,\quad(c)\ \phi\to\pi^0\gamma,\\ &(d)\ \omega\to\pi^0\gamma,\quad(e)\ Y_0^*\to\Sigma\pi,\quad(f)\ \phi\to K\bar{K},\\ &(g)\ \eta\to3\pi^0,\quad(h)\ \eta\to\pi^0\gamma\gamma,\quad(i)\ \pi^0\to2\gamma,\end{aligned}\quad(7)$$

which are forbidden by exact G_2 invariance. If we include the effect of H_I, we find by the representation mixing method that only processes (a) to (d) remain forbidden to first order in H_I. In the case of (a) and (b), this is an especially serious obstacle to the use of G_2 as an approximate symmetry of nature.

[6] S. Coleman, S. L. Glashow, and D. I. Kleitman, 1964 (unpublished), have considered the effect on masses of mixing specific pairs of representations of SU_3, obtaining results that follow also by simple specialization of our general formulas. They did not, however, actually construct the physical states, as we have done, and hence cannot discuss quantities like magnetic moments.

[7] B. W. Lee, S. Okubo, and J. Schechter, Phys. Rev. **135**, B219 (1964).

[8] Contributions from different representations of the type 1, 10, $\overline{10}$, or 27 combine linearly, but contributions from octets other than the original clearly do not.

[9] Y. Dothan and H. Harari, Nuovo Cimento (to be published).