Study of a Vector-Meson Bootstrap Model*

R. E. CUTKOSKY AND M. LEON
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received 30 December 1964)

Some self-consistent solutions of a simplified bootstrap model are studied. Self-consistency is imposed through properties of the Bethe-Salpeter equation. The method of solution and properties of the self-consistent sets of particles are described.

1. INTRODUCTION

As a preliminary toward consideration of more complicated bootstrap systems, we have studied a simplified model of self-coupled vector mesons. In keeping with the bootstrap philosophy, the vector mesons are interpreted as bound states for which the constituent particles as well as the exchanged particles are also these same vector mesons. The masses are not required to be degenerate or nearly degenerate.

Section 2 discusses the bootstrap equations and their dynamical factors, while Sec. 3 details the approximations used in computing these factors. In Sec. 4 application is made to sets of mesons, and the solutions obtained for self-consistent particle masses are described.

2. BOOTSTRAP EQUATIONS

The bootstrap equations here employed are very similar to those of other workers which are derived from an assortment of starting points.[1] Our particular equations follow readily from a ladder approximation to the Bethe-Salpeter (B.S.) equation,[2] and are

$$g_{abc}=\sum_{efi} g_{aif}g_{bei}g_{cfe}D_{abc}{}^{efi}, \tag{1a}$$

$$1=N_a\equiv\sum_{bcefi} g_{aif}g_{bei}g_{cfe}g_{abc}W_{bc,fi}{}^{a,e}, \tag{1b}$$

corresponding to Figs. 1(a) and 1(b). All of the dynamics, i.e., the dependence on the six masses, is isolated in the D and W factors.

The D factor contains an integration over the internal loop momentum, while the external particles are on their mass shells. For the form factors at vertices (bei) and (cef) we choose a self-consistently determined vertex function, which, since one particle is on the mass shell, is a B.S. wave function similar to the one that appears at vertex (afi). We insert a simple pole term for the exchanged particle e as well as for the constituent particles f and i. The result is obviously a highly symmetric D:

$$D_{abc}{}^{efi}=D_{bac}{}^{fei}=D_{bca}{}^{fie}=\cdots,$$

and "vertex symmetry" is built into the theory.

By vertex symmetry we mean the requirement that the same coupling constant g_{abc} should be obtained, whether a, b, or c is the particle which is being considered as the bound state. This symmetry is not automatically maintained in approximate calculations based on the N/D method.[3] Crossing symmetry is also incorporated exactly, so far as the single-particle poles are concerned, but only approximately for the two-particle and other continuum states.

By using the Bethe-Salpeter equation, W can be shown to satisfy

$$W_{bc,fi}{}^{a,e}=K_a{}^{bc}\partial D_{abc}{}^{efi}/\partial(m_a{}^2), \tag{2}$$

where K corresponds to Fig. 1(c) and might be computed similarly to D. As we shall discuss below, it appears preferable, instead, to calculate K from the identity

$$K_a{}^{bc}D_{abc}{}^{efi}=I_{bc,fi}{}^{a,e}=K_a{}^{fi}D_{afi}{}^{ebc}; \tag{3}$$

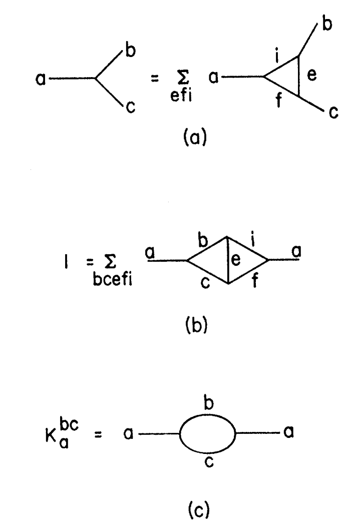

FIG. 1. The vertex (a) and normalization (b) equations. The closed loops are integrated over four momenta.

* Supported in part by the U. S. Atomic Energy Commission.

[1] H. M. Chan, P. C. DeCelles, and J. E. Paton, Phys. Rev. Letters **11**, 521 (1963); Nuovo Cimento **33**, 70 (1964); L. F. Cook and J. E. Paton (to be published); see also R. M. Rockmore, Phys. Rev. **132**, 878 (1963).

[2] R. E. Cutkosky and M. Leon, Phys. Rev. **135**, B1445 (1964). We follow the notation of this paper with minor changes. Since the appearance of this work, it has come to our attention that the main result, a general form for the normalization integral of a Bethe-Salpeter wave function, had been derived previously by a different method by G. R. Allcock, Phys. Rev. **104**, 1799 (1956). See also G. R. Allcock and D. J. Hooton, Nuovo Cimento **8**, 590 (1958), and K. Nishijima, Progr. Theoret. Phys. (Kyoto) **13**, 305 (1955).

[3] See, e.g., R. H. Capps, Phys. Rev. **134**, B1396 (1964); L. F. Cook and J. E. Paton, Ref. 1.

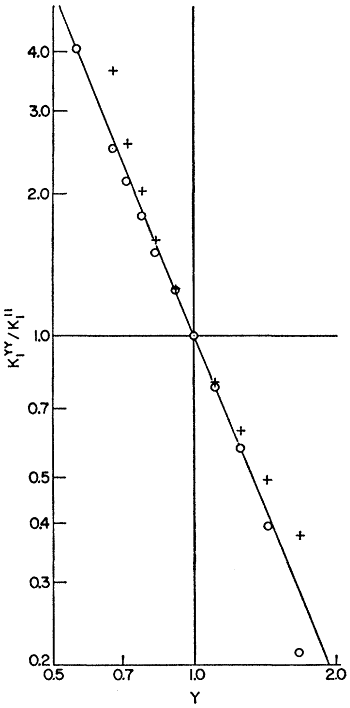

FIG. 2. The circles show calculated values of $D_{111}{}^{1yy}/D_{1yy}{}^{111}$, and the crosses show values of $D_{111}{}^{yyy}/D_{1yy}{}^{y11}$. The straight line shows a fit with $\beta=-2.4$. These values pertain to $x=\infty$.

I corresponds to Fig. 1(b) exactly as D corresponds to Fig. 1(a).[2] Thus,

$$K_a{}^{bc}/K_a{}^{fi}=D_{afi}{}^{ebc}/D_{abc}{}^{efi}. \tag{4}$$

By repeatedly using this identity and the D symmetry, we can derive three identities of the type

$$K_a{}^{fi}K_f{}^{ec}K_c{}^{ab}/(K_a{}^{bc}K_f{}^{ai}K_c{}^{ef})=1. \tag{5}$$

3. APPROXIMATIONS FOR D AND W

One of our aims is the study of the sensitivity of self-consistently determined masses to the details of the interaction. In this preliminary calculation we use scalar kinematics, i.e., scalar vertex functions V_{rst} and propagators

$$G_r=(p_r^2-m_r^2)^{-1}.$$

Instead of actually solving the Bethe-Salpeter equation, we use as an approximate amplitude one chosen to resemble qualitatively an actual wave function for the particle t:

$$V_{rst}=\left\{\frac{(x-1)m_r^2}{xm_r^2-p_r^2}\,\frac{(x-1)m_s^2}{xm_s^2-p_s^2}\right\}^{1/2}.$$

With this ansatz we have for D the familiar (regularized) triangle graph of perturbation theory. Numerical values were obtained by integration of the dispersion relation for the vertex.

Since without regularization, the integral for K (in contrast to D) is divergent, K, when computed directly, is strongly dependent on x. Furthermore we find that values of K so computed do not satisfy the identity, Eq. (5), at all well. We therefore evaluate K (to within a single over-all constant) from the D using Eq. (4), and find a satisfactory approximation to be

$$K_1{}^{1y}=K_1{}^{11}y^\alpha,\quad K_1{}^{yy}=K_1{}^{11}y^\beta \tag{6}$$

with α and β constant. (We do not need the general $K_1{}^{yz}$, and can always set at least one of the masses$=1$, since K and D are homogeneous functions.) Notice that the right-hand side of Eq. (4) depends on the mass of e; this dependence would cancel out if we were using the exact Bethe-Salpeter wave functions. We chose α and β so that Eqs. (6) would be a fair approximation for a range of e mass; an example of the fit is given in Fig. 2. In addition, the form of Eqs. (6) ensures that the identity, Eq. (5), is satisfied exactly.

The scale-invariance of Eqs. (1) implies that an adjustable parameter is needed in order that solutions be found.[4] It is convenient to use for this adjustable parameter the constant $K_1{}^{11}$, which is indeed especially sensitive to the high-momentum tail of the wave function.

4. SETS OF VECTOR MESONS

Instead of working with the full set of Eqs. (1), we simplify things by considering only those solutions with a certain amount of symmetry, including complete antisymmetry of the g's. In this case it is known that if the particle masses are equal, self-consistency is obtained if and only if the number of vector mesons is equal to the dimensionality of a simple Lie group.[5] We have considered the two simplest cases, $N=3$ and $N=8$, and

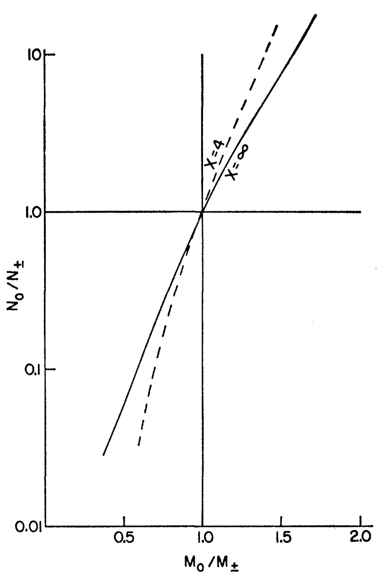

FIG. 3. A plot of $N_0/N_\pm$ versus $M_0/M_\pm$ showing the rapidity with which self-consistency is lost when the masses are made unequal.

[4] Thus we are not as strict with the bootstrap equations as, e.g., is P. D. B. Collins, Phys. Rev. **136**, B710 (1964).
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searched for additional self-consistent sets of particle masses. The search in particle masses was restricted so as to remain below the two-particle or anomalous thresholds.

For the case of three mesons, $\rho_\pm$ and ρ_0, there are only two masses and one coupling constant. Because of the adjustable parameter $K_1{}^{11}$, we can only require $N_\pm = N_0$ [cf. Eq. (1)]. We find only one solution, the completely symmetric one: $M_\pm = M_0$. This is shown graphically in Fig. 3.

For the case of eight mesons, we impose charge symmetry but not charge independence, giving the set

$$\{\phi, K^* = (K^{*+}, K^{*0}), \bar{K}^*, \rho_\pm, \rho_0\}$$

with four masses and four coupling constants. Equations (1) become as shown in Figs. 4 and 5. We have here introduced some Clebsch-Gordan factors[6] and have used the freedom of an adjustable parameter in the W factors to normalize the D's and thus make explicit the SU_3-symmetric solution. The actual coupling constants are then given by

$$2(6/D_0)^{1/2} g_{\rho\rho\rho}, \quad 3(2/D_0)^{1/2} g_{\phi K^* K^*}, \quad (6/D_0)^{1/2} g_{\rho K^* K^*},$$

with $D_0 \equiv D_{111}{}^{111}$.

A computer search yielded two solutions (for each value of x) in addition to the symmetrical one, as shown in Table I. One of these is a charge-independent solution in which the masses satisfy the Gell-Mann–Okubo formula quite well,[7] while the second is nearby with the $\rho_\pm$ and ρ_0 masses being separated.

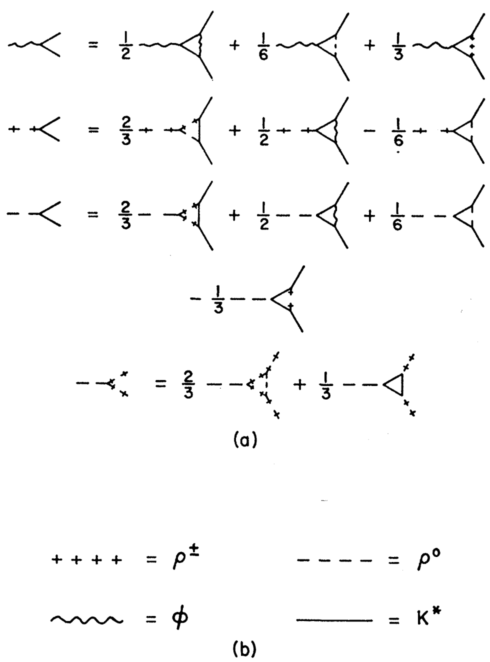

FIG. 4. The explicit form of the vertex equations for the case of eight mesons. Some factors have been extracted so that the symmetric solution has all g and $m=1$.

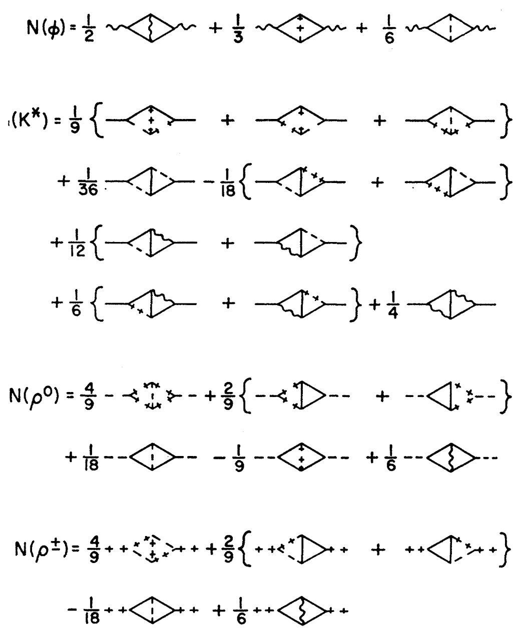

FIG. 5. Normalization equations, with the same notation as in Fig. 4.

In the computer search, coupling constants were determined from Eq. (1a) for a variety of trial masses, and the error in Eq. (1b) determined. For both values of x, it was observed that for mass values differing only slightly from equality, the error was smaller when the mass values satisfied the Gell-Mann–Okubo formula than when they disagreed with the formula or with charge independence. The charge-independent solutions with broken SU_3 symmetry lie on the prolongation of a common curve along which the error remains relatively small[7]; however, the actual magnitude of the mass splitting is seen to be large and to depend sensitively upon the form of the wave function. Note that the deviations from the Gell-Mann–Okubo rule are in the wrong direction to be encompassed by use of squares of masses.

Solutions with $g_{\rho\rho\rho} g_{\rho K^* K^*} < 0$ (with M_ρ large) were also found. The negative sign indicates that the ρ did not correspond to the most attractive eigenstate of the effective potential matrix

$$U_{bc,fi} = \sum_e D_{abc}{}^{efi} g_{bei} g_{cfe}.$$

These solutions were rejected as not being truly self-consistent.

We associate the existence of the charge-dependent solutions with the fact that the masses of the charge-independent solution satisfy the relation

$$M_\rho > M_{K^*} > M_\phi,$$

[6] E.g., J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).

[7] This is consistent with a qualitative discussion given by R. E. Cutkosky and P. Tarjanne, Phys. Rev. **132**, 1354 (1963).

TABLE I. Solutions of the bootstrap equations for $N=8$. M_{K^*} is the unit of mass. The coefficients which multiply the g's exhibited here to give the actual g's are given in the text.

	$M_{\rho_\pm}$	M_{ρ_0}	M_ϕ	$(g_{\phi K^*K^*})^2$	$(g_{\rho_\pm K^*K^*})^2$	$(g_{\rho_0 K^*K^*})^2$	$(g_{\rho\rho\rho})^2$
SU_3 symmetric:							
$x=2$ or $x=\infty$	1	1	1	1	1	1	1
Charge-independent							
$x=\infty$	1.55	1.55	0.91	1.49	0.66	0.66	0.16
$x=2$	1.33	1.33	0.93	1.37	0.83	0.83	0.26
Charge-dependent:							
$x=\infty$	1.63	1.41	0.90	1.46	0.47	1.01	0.13
$x=2$	1.47	1.19	0.93	1.37	0.47	1.49	0.24

so that the ρ is mainly a $(\bar{K}^*K^*)$ bound state. One can see from the second and third equations of Fig. 4(a) that a difference between $g_{\rho_\pm K^*K^*}$ and $g_{\rho_0 K^*K^*}$ can regenerate in such a case and the minus signs in these equations and in those for N_0 and $N_\pm$ allow a reciprocal amplification of mass and coupling-constant differences. This presumably would not happen if $M_\rho < M_{K^*} < M_\phi$, since then ρ would be mostly $(\rho\rho)$ and the first terms of the equations would dominate, leading to stability as for $N=3$.

PHYSICAL REVIEW VOLUME 138, NUMBER 3B 10 MAY 1965

Proton-Proton Scattering at 1.48 BeV*

A. M. EISNER,† E. L. HART, R. I. LOUTTIT, AND T. W. MORRIS
Brookhaven National Laboratory, Upton, New York
(Received 31 December 1964)

A sample of 2657 proton-proton scattering events at 1.48 BeV has been analyzed. The elastic cross section is 19.86 mb, and the elastic scattering is consistent with a simple opaque-disk optical model with $R=0.91$ F and $1\text{-}a=0.864$. The dominant feature of the inelastic scattering is the production of the (3/2, 3/2) isobar. The reaction $p+p \rightarrow p+n+\pi^+$ is interpreted satisfactorily in terms of the one-pion-exchange model.

INTRODUCTION

IN the last four or five years, a number of experimenters have investigated proton-proton interactions in the 1–3 BeV range utilizing electronic counters[1–3] or the liquid-hydrogen bubble chamber.[4–8] As a continuation of this investigation the BNL twenty-inch bubble chamber was exposed to a beam of 1.48-BeV protons from the Cosmotron. This paper reports the results from the analysis of the 2657 measured events. Detailed results are given only for elastic and $pn+$ events, because only in those cases were there sufficient events to make results statistically meaningful.

In previous experiments, the simple optical model of a purely absorbing disk[9] has been found to explain satisfactorily the angular distribution of elastic events over a wide range of scattering angles. This model is therefore used to analyze the elastic events and to correct for scanning biases due to the difficulty of observing small-angle scatters.

Momentum and effective-mass distributions for $pn+$ events are compared with the predictions of the Stern-

* Work performed under the auspices of the U. S. Atomic Energy Commission.

† Present address: Department of Physics, Harvard University, Cambridge, Massachusetts.

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