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Lorentz Invariance and the Interpretation of $SU(6)$ Theory. II

M. A. B. BÉG AND A. PAIS
The Rockefeller Institute, New York, New York
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The relativistic completion of $SU(6)$ discussed in a foregoing paper is extended so as to take account of a more general form of the meson matrix. Low-frequency phenomena are discussed further. To order v/c all recoil effects are now included. The **70**$^-$ fermion representation is briefly treated.

I. INTRODUCTION

In a previous paper[1] we have begun a study of the following question: To what extent is it possible to impose simultaneously the requirements of Lorentz covariance and of $SU(6)$ invariance in a theory of strong interactions? For this purpose a procedure was described called "relativistic completion." Given the question, we gave the *minimal* requirements necessary for the interpretation of results obtained so far. In the present paper we discuss the completion process in some further detail. Where earlier we had considered its application only in the presence of axial-vector form factors (for pseudoscalar mesons) and vector form factors (for vector mesons), we now include also the presence of Pauli terms or tensor form factors (for vector mesons) and pseudoscalar form factors (for pseudoscalar mesons).[2] As a result, we are now able to give a more general discussion of fermion recoil. As in I, we find it convenient to discuss these questions first for interactions of the fundamental $SU(6)$ sextet of fermions. Thereafter we treat the interaction for the case of the **56** representation of baryons (Sec. II). In Sec. III we discuss recoil effects to order v/c. We find in particular that the relation[3] $g_A=5g/3$ still holds true in the presence of the tensor and pseudoscalar form factors. In Sec. III we also comment briefly on the relativistic completion for the $SU(6)$-covariant electromagnetic[4] and semileptonic[5] interactions. In Sec. IV we indicate how to construct the completed **70**$^-$ representation and the effective vertex for the corresponding baryon (**56**$^+$)–meson (**35**$^-$) decay.

Just as was done in I, we shall consistently use the following definition: The terms "$SU(6)$ representation" and "$SU(6)$ invariance" shall refer exclusively to the structure of one-particle states with zero three-momentum.

Nevertheless, it was shown in I that the $SU(6)$ theory can give unique predictions for the structure of certain *effective* matrix elements, vertices, etc., which involve one or more particles with nonzero three-momentum. Examples are the decay matrix element for $N^*\rightarrow N+\pi$, and its relation to the p-wave vertex $N\rightarrow N+\pi$ and to the s-wave vertex $N\rightarrow N+\rho$, even though all these quantities vanish if all the particles involved have zero three-momentum. Such predictions came about through a fully specified interplay of the $SU(6)$ group (as defined above) and the Lorentz group, leading to unique answers in the static limit. When in the following we use terms like "completed $SU(6)$ structure" we shall mean that particles are involved which obey the $SU(6)$-group requirements as defined above (in other words, they form supermultiplets) and to which the completion procedure (boosting) dictated by the Lorentz group has been applied.

In I it was found necessary to make a sharp distinction between the applicability of the completion of $SU(6)$ to effective matrix elements on the one hand and to local Lagrangian field theory on the other. In the latter case, completed $SU(6)$ can not apply. The origin of the breakdown of the completion could be pinpointed as the kinetic energy in the free-particle part of the Lagrangian.[6]

This raises the main dynamical problem now to be understood. If the effective matrix elements and vertices discussed above arise from an underlying local field theory, and if the latter cannot be $SU(6)$ completed, then why should at least some of these effective matrix elements, etc. give evidence at all of patterns prescribed by $SU(6)$, as they apparently do? A possible answer would appear to be that in strong interactions there must be a strong damping of high virtual frequencies. This means for approximate reconciliation of $SU(6)$ and local field theory was illustrated[7] by a naive field-theoretical comparison of the self-energy of a spin-1 and a spin-0 meson, both belonging to the same $SU(6)$ supermultiplet.

It will be shown in Sec. II that the inclusion of Pauli terms and of pseudoscalar terms necessitates a more refined treatment of the relativistic completion for the 12×12 meson matrix already introduced in I. The need for this refinement can perhaps be made clear by first considering the following simple problem in $SU(2)$. Take two **2** representations (spinors) of $SU(2)$. Then it

[1] M. A. B. Bég and A. Pais, Phys. Rev. **133**, B1514 (1965). This paper will be referred to as I.

[2] For the meson-baryon (spin $\frac{1}{2}$ and $\frac{3}{2}$) vertex, with baryons on the mass shell, one of course has only one form factor for the coupling of pseudoscalar particles and three for the coupling of vector particles, in the symmetry limit. The inclusion of the quadrupole form factor for vector mesons involves a "breakdown" of the completed $SU(6)$.

[3] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964) Eq. (8).

[4] M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964).

[5] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 51 (1965).

[6] This observation was also made by K. Bardakci, J. Cornwall, P. Freund, and B. W. Lee, Phys. Rev. Letters **13**, 698, 1964.

[7] See I, Sec. III.

is elementary to construct from these a bilinear form which behaves as the (adjoint) **3** representation (vector). This construction is unique. Likewise, take a **6** (sextet) and a **6*** (antisextet) representation of $SU(6)$. The process of constructing the (adjoint) **35** representation from these is still elementary and unique. At this point let us emphasize once more that $SU(6)$ here refers to zero 3-momentum. Now we wish to bring into play a second group, the Lorentz group. Thus we now take a relativistically completed **6** and a similar **6***. These boosted representations are now not only representations of $SU(6)$ but also of the Lorentz group.

Problem: Construct out of these boosted representations the corresponding boosted **35**. This is a somewhat novel problem in that we apply two noncommuting groups. We wish in particular (extending at this point the construction of I) to give the more general boosted form of **35**, where the **35** also carries a prescribed parity (parity itself being defined relative to the (**6***,**6**) system). This is the problem we discuss in the next section.

In the course of the derivations in Sec. II, we find it illuminating to perform an elementary unitary transformation which casts the 12×12 meson matrix derived in I in the form of a direct sum of two 6×6 matrices; see Eq. (2.37) below. Each of these 6×6 matrices *separately* represents actually a boosted **35** representation with the required covariance properties under the *proper* Lorentz group, but with indefinite properties under reflection. The joining of these two 6×6 matrices then ensures having definite (in our case, odd) parity of the meson representation.

Quite independently of any $SU(6)$ aspects such a doubling (from 2×2 to 4×4) is met in the characterization of the representation $\mathfrak{D}^{[1/2,1/2]}$ of the Lorentz group (4-vector) in the $SL(2,C)$ language,[8] if one assigns a definite parity to such a representation. Correspondingly, there is only one group $SU(6)$ in the present game, as is also evident from the fact that the relativistic completion of the **35** does not involve the introduction of new fields (but only of additional field *components* that can be eliminated for $\mathbf{q}=0$). In particular, the completed $SU(6)$ does not involve parity doubling.

We conclude the Introduction by summarizing the role of symmetries, exact and approximate, as they appear in the $SU(6)$ theory.

(1) $SU(6)$: refers to a zero–three-momentum property of one-particle states.

(2) Completed $SU(6)$: refers to the way, prescribed by the homogeneous Lorentz group, in which an $SU(6)$ representation behaves for nonzero three-momentum. In general, vertices, S-matrix elements, etc. are definable *only* with reference to the completed $SU(6)$.

While orbital angular momentum is alien to the definition of $SU(6)$, the dictates of Lorentz invariance have shown that $SU(6)$, by completion, leads nevertheless to certain *unique* P-wave predictions which are in reasonable agreement with experiment. Our completion procedures are *not* the same as saying that an arbitrary spurion of the type $(\boldsymbol{\sigma}\cdot\mathbf{q})$ has been introduced, as is readily seen in the following. $(\boldsymbol{\sigma}\cdot\mathbf{q})$ transforms like the (1,3) part of a **35**. Applied to the P-wave interaction of mesons with baryons, the spurion language would lead to the algebraic structure $\mathbf{1}\subset(\mathbf{56}^*\otimes\mathbf{56}\otimes\mathbf{35}\otimes\mathbf{35})$. As[9] $\mathbf{35}\times\mathbf{35}$ contains **35** twice and also **405**, it follows that the spurion device would generally not be unique. [See further the end of Sec. II(a).] This uniqueness is maintained, however, in the completed $SU(6)$ picture.

(3) Broken completed $SU(6)$: exemplified[1] by the precepts of local field theory. The completion does not work for the free kinetic energy. Thus a local Lagrangian for which the *interaction* is $SU(6)$-complete will in higher order give $SU(6)$-violating effects. For example it will generate mass splittings within supermultiplets. An example was given in I, Sec. III. These mass splittings are all of the "first stage" kind.[9] Recouplings which could lead to recurrences of $SU(6)$ supermultiplets[10] may perhaps also be generated.

(4) Broken $SU(3)$: the kinetic energy does not intrinsically violate $SU(3)$. Note that the requirement of invariance under[3] $SU(4)(T)\times SU(2)(X)\times W(Y)$ is perhaps a natural physical way to break $SU(3)$ ("second stage"[9]).

(5) Embeddings of $SU(6)$: $SU(6)$ can readily be embedded in[11–13] $U(6)\otimes U(6)$. For the low-frequency phenomena which led to the recognition of $SU(6)$ as a useful group, $U(6)\otimes U(6)$ does not play any role of practical importance. Indeed, $U(6)\otimes U(6)$ can have meaning only if one neglects all fermion masses. The procedures in I and in the present paper, on the other hand, do not at any stage require such a drastic neglect which, moreover, complicates[14] the assignment of massive physical particles to specific representations. However, this group could possibly be relevant for weak interactions.[14]

II. THE MESON MATRIX

(a) General Discussion

In this section we consider the general construction of the meson wave function at finite momentum $\mathbf{q}$. This wave function satisfies the following criteria:

(i) It transforms in a well-defined way under $L\otimes SU(3)$. For example, the meson matrix of I transforms according to

$$\mathfrak{D}_{(L)}{}^{(1/2,1/2)}\otimes\mathfrak{D}_{(3)}{}^{(2,1)}\oplus\mathfrak{D}_{(L)}{}^{(1/2,1/2)}\otimes\mathfrak{D}_{(3)}{}^{(0,0)}\oplus\mathfrak{D}_{(L)}{}^{(1/2,1/2)}\otimes\mathfrak{D}_{(3)}{}^{(2,1)}\,,\quad(2.1)$$

[8] See, e.g., R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics and All That* (W. A. Benjamin, Inc., New York, 1964).

[9] A. Pais, Phys. Rev. Letters **13**, 175 (1964).

[10] Reference 9, footnote 14.

[11] R. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters **13**, 678 (1964).

[12] K. Bardakci, J. Cornwall, P. Freund, and B. W. Lee, Phys. Rev. Letters **13**, 698 (1964).

[13] K. Bardakci, J. Cornwall, P. Freund, and B. Lee, Phys. Rev. Letters **14**, 48 (1965).

[14] See Ref. 5, footnote 18.

where the representations of the Lorentz group are labeled in the usual manner[8] and the notation $\mathfrak{D}_{(n)}{}^{(\lambda_1,\lambda_2\cdots\lambda_{n-1})}$ is used for representations of $SU(n)$, $(\lambda_1\geq\lambda_2\geq\cdots\geq\lambda_{n-1})$.

(ii) At $\mathbf{q}=0$ it transforms irreducibly under $SU(6)$ according to $\mathfrak{D}_{(6)}{}^{(2,1,1,1,1)}$.

(iii) It is characterized by a definite parity and this parity is to be taken as odd.

The construction proceeds most smoothly by starting with

$$\mathfrak{D}_{(L)}{}^{[1/2,0]}\otimes\mathfrak{D}_{(L)}{}^{[1/2,0]} =2\mathfrak{D}_{(L)}{}^{[0,0]}\oplus2\mathfrak{D}_{(L)}{}^{[1/2,1/2]}\oplus\mathfrak{D}_{(L)}{}^{[1,0]}. \quad (2.2)$$

In order to implement discrete operations (C, P, and T) we take $\mathfrak{D}_{(L)}{}^{[1/2,0]}$ to be an irreducible representation of the extended Lorentz group and thus a reducible representation of the proper part $L_+{}^\uparrow$. The sixteen Dirac matrices 1, γ_5, γ_μ, $\gamma_5\gamma_\mu$, $\sigma_{\mu\nu}$ furnish a complete basis for the representation matrices. Hence they also furnish a basis for tensors transforming according to $\mathfrak{D}_{(L)}{}^{[1/2,0]}\otimes\mathfrak{D}_{(L)}{}^{[1/2,0]}$. Suppressing $SU(3)$ indices, the meson wave function may therefore be written as

$$\mathfrak{M}(\mathbf{q})=a+b\gamma_5+c^\mu\gamma_\mu+d^\mu\gamma_5\gamma_\mu+e^{\mu\nu}\sigma_{\mu\nu}, \quad (2.3)$$

where, in the absence of any preferred direction in space-time, $a, \cdots, e$ depend only on the 4-vectors q and ϵ, ϵ being a polarization vector such that $q\cdot\epsilon=0$. Hence

$$a=a(q^2), \quad (2.4)$$

$$b=b(q^2), \quad (2.5)$$

$$c^\mu=c_1(q^2)q^\mu+c_2(q^2)\epsilon^\mu, \quad (2.6)$$

$$d^\mu=d_1(q^2)q^\mu+d_2(q^2)\epsilon^\mu, \quad (2.7)$$

$$e^{\mu\nu}=e_1(q^2)g^{\mu\nu}+\tfrac{1}{2}e_2(q^2)(\epsilon^\mu q^\nu+\epsilon^\nu q^\mu) +\tfrac{1}{2}e_3(q^2)(\epsilon^\mu q^\nu-\epsilon^\nu q^\mu)+e_4(q^2)q^\mu q^\nu. \quad (2.8)$$

Collecting terms we find

$$\mathfrak{M}(\mathbf{q})=\mathfrak{M}_0(\mathbf{q})+\mathfrak{M}_1(\mathbf{q};\epsilon_0,\boldsymbol{\varepsilon}), \quad (2.9)$$

where

$$\mathfrak{M}_0(\mathbf{q})=a+b\gamma_5+c_1(q\cdot\gamma)+d_1\gamma_5(q\cdot\gamma), \quad (2.10)$$

$$\mathfrak{M}_1(\mathbf{q};\epsilon_0,\boldsymbol{\varepsilon})=c_2(\epsilon\cdot\gamma)+d_2\gamma_5(\epsilon\cdot\gamma)+e_3\epsilon^\mu q^\nu\sigma_{\mu\nu}. \quad (2.11)$$

The parity of the mesons can be adjusted through

$$\gamma_4{}^\dagger\mathfrak{M}_0(\mathbf{q})\gamma_4=\mathfrak{M}_0(-\mathbf{q}), \qquad \text{Scalar} \quad (2.12)$$

$$\gamma_4{}^\dagger\mathfrak{M}_0(\mathbf{q})\gamma_4=-\mathfrak{M}_0(-\mathbf{q}), \qquad \text{Pseudoscalar} \quad (2.13)$$

$$\gamma_4{}^\dagger\mathfrak{M}_1(\mathbf{q};\epsilon_0,\boldsymbol{\varepsilon})\gamma_4=\mathfrak{M}_1(-\mathbf{q};\epsilon_0,-\boldsymbol{\varepsilon}), \text{ Vector} \quad (2.14)$$

$$\gamma_4{}^\dagger\mathfrak{M}_1(\mathbf{q};\epsilon_0,\boldsymbol{\varepsilon})\gamma_4=\mathfrak{M}_1(-\mathbf{q};-\epsilon_0,\boldsymbol{\varepsilon}), \text{ Axial-vector}. \quad (2.15)$$

For negative parity we obtain, with a redefinition of the coefficients,

$$\mathfrak{M}(\mathbf{q})=\left[if^P\gamma_5-f^A\gamma_5\frac{(q\cdot\gamma)}{\mu_{00}}\right]\otimes P +\left[if^V(\epsilon\cdot\gamma)+if^T\frac{q^\mu\epsilon^\nu}{\mu_{00}}\sigma_{\mu\nu}\right]\otimes V, \quad (2.16)$$

where P and V are tensors transforming according to $\mathfrak{D}_{(3)}{}^{(2,1)}$ and $\mathfrak{D}_{(3)}{}^{(0,0)}\oplus\mathfrak{D}_{(3)}{}^{(2,1)}$, respectively. The relative weight of singlet and octet in V is determined by constraint (ii). This constraint also implies

$$f^P(\mu_{00}{}^2)=f^T(\mu_{00}{}^2), \quad (2.17)$$

$$f^A(\mu_{00}{}^2)=f^V(\mu_{00}{}^2). \quad (2.18)$$

We postpone the proof of Eq. (2.17) and (2.18) until Sec. III(c) below. From now on the symbols f^P, f^T, f^A, f^V will be used only for the values on the mass shell.

We are thus led to a one-parameter family of 12×12 matrices as the requisite wave functions, the parameter being (f^T/f^V). The wave functions can be rendered unique only by imposing symmetries above and beyond $SU(6)$. Such a symmetry was indeed implicit in I. The meson matrix quoted in I is in fact odd under the γ_5 transformation. There is no reason to exclude parts even under γ_5 and the corresponding even-to-odd ratio is the free parameter (f^T/f^V). The significance of this parameter will be brought out more fully in Sec. III; at the moment it is sufficient to point out that all the $SU(6)$ predictions hitherto quoted in the literature are independent of the value of this parameter and thus are not affected by giving it an explicit value.

In the introduction we noted that our treatment is not equivalent to a spurion picture. We can now state more precisely what this inequivalence consists of. The following can be shown: (a) The double occurrence of **35** in **35**⊗**35** leads, and leads only to the arbitrariness in the ratio (f^T/f^V) mentioned above; (b) the completed $SU(6)$ picture corresponds furthermore to the absence of contributions due to **405**.

(b) Vertices in the Restricted Case

We first recapitulate some results obtained in I, referring to the treatment of the **35** given in I as the restricted case.

In order to describe the **6** representation, we introduced in I a 12-component wave function $w^\lambda(\mathbf{p})$, where

$$w^\lambda(\mathbf{p})=u_+{}^{iaA}(\mathbf{p}), \quad \lambda\equiv(i,a,A)=1,2,\cdots12. \qquad =u_-{}^{iaA}(\mathbf{p}). \quad (2.19)$$

$i=1$, 2 refers to spin up, down, respectively; $A=1$, 2, 3 is the contragredient $SU(3)$ index, and a is the index which doubles the number of components. The subscript $+(-)$ refers to particle (antiparticle) states. We have used the explicit representation[15]

$$u_+{}^{iaA}(\mathbf{p})=N(p)\begin{bmatrix}\chi^i\\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p_0+M}\chi^i\end{bmatrix}t^A, \quad (2.20)$$

$$u_-{}^{iaA}(\mathbf{p})=(\gamma_5)_b{}^a u_+{}^{ibA}(\mathbf{p}), \quad \gamma_5=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix}. \quad (2.21)$$

[15] See I Eqs. (22), (23). M is the common sextet mass, $p_0=(p^2+M^2)^{1/2}$.

We define the adjoint wave function $\bar{w}_\lambda(\mathbf{p})$ by

$$\bar{w}_\lambda(\mathbf{p}) = (w(\mathbf{p})^\dagger \gamma_4)_\lambda. \tag{2.22}$$

In this paper we work with four Hermitian Dirac matrices γ_μ, $\mu=1, \cdots, 4$. The following explicit representation is convenient:

$$\boldsymbol{\gamma} = \rho_2 \boldsymbol{\sigma}, \quad \gamma_4 = \rho_3, \quad \gamma_5 = -\gamma_1\gamma_2\gamma_3\gamma_4. \tag{2.23}$$

With the help of the bilinear product $\bar{w}_\lambda(p_2) w^\mu(p_1)$ we construct the 12×12 matrix $\mathfrak{M}_\mu{}^\lambda(\mathbf{q})$ for a meson supermultiplet with common mass μ_{00} by imposing the following conditions.

(i) The quantity

$$\bar{w}_\lambda(\mathbf{p}_2) w^\mu(\mathbf{p}_1) \mathfrak{M}_\mu{}^\lambda(\mathbf{q}), \quad \mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1, \tag{2.24}$$

shall have all the Lorentz-covariance properties of a meson-fermion vertex.

(ii) For $\mathbf{q}=0$, the only nonvanishing combination of PS and V fields which may occur in $\mathfrak{M}$ are given by $M_\alpha{}^\beta$, $\alpha=(i,A)$, $\beta=(j,B)$, where

$$M_\alpha{}^\beta = i\delta_i{}^j P_A{}^B + (\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})_i{}^j V_A{}^B. \tag{2.25}$$

Note that

$$\tfrac{1}{2} M^\dagger{}_\beta{}^\alpha M_\alpha{}^\beta = P_A{}^B P_B{}^A + (\boldsymbol{\varepsilon} V_A{}^B)(\boldsymbol{\varepsilon} V_A{}^B). \tag{2.26}$$

(iii) $\mathfrak{M}$ shall be such that Eq. (2.24) contains only $V(V)$ and $PS(PV)$ vertices. This condition confines us to the restricted case and implies that $\gamma_5^{-1}\mathfrak{M}\gamma_5 = -\mathfrak{M}$.

The conditions (i)–(iii) determine $\mathfrak{M}$ uniquely and one has[16]

$$\mathfrak{M}_\lambda{}^\mu(\mathbf{q}) = \begin{bmatrix} -N_\alpha{}^\beta(\mathbf{q}), & M_\alpha{}^\beta(\mathbf{q}) \\ -M_\alpha{}^\beta(\mathbf{q}), & N_\alpha{}^\beta(\mathbf{q}) \end{bmatrix}, \tag{2.27}$$

$$M_\alpha{}^\beta(\mathbf{q}) = \frac{iq_0}{\mu_{00}} \delta_i{}^j P_A{}^B + (\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})_i{}^j V_A{}^B, \tag{2.28}$$

$$N_\alpha{}^\beta(\mathbf{q}) = i\frac{(\boldsymbol{\sigma}\cdot\mathbf{q})_i{}^j}{\mu_{00}} P_A{}^B + \delta_i{}^j \epsilon_0 V_A{}^B. \tag{2.29}$$

Note that

$$M_\alpha{}^\beta(0) = M_\alpha{}^\beta, \quad N_\alpha{}^\beta(0) = 0. \tag{2.30}$$

Equation (2.30) contains the implementation required by $SU(6)$.

Let us now go to the no-fermion-recoil approximation. Here we still use Eq. (2.21) and (2.24) but we now put

$$u_+{}^{iaA}(\mathbf{p}) \to u_+{}^{iaA}(0). \tag{2.31}$$

Thus in this approximation the interaction proceeds exclusively via $N_\alpha{}^\beta(\mathbf{q})$, which contains the s-wave vector interaction and the p-wave pseudoscalar interaction *in a prescribed mixture*. This comes about because of the interplay of $SU(6)$ and the Lorentz group. The former prescribes the mixture of V and PS in the 6×6 matrix $M_\alpha{}^\beta$. The latter requires us to boost the **35** by introducing the $N_\alpha{}^\beta$ matrix. The same is true for the interaction with the **56** which can be written as[17]

$$B_{\lambda\mu\nu}{}^\dagger(\mathbf{p}_2) B^{\lambda\mu\epsilon}(\mathbf{p}_1) \mathfrak{M}_\epsilon{}^\nu(\mathbf{q}); \quad \lambda, \mu, \nu, \epsilon = 1, 2, \cdots 12. \tag{2.32}$$

It may also be noted that the odd parity of the mesons makes the no-fermion recoil approximation go entirely via $N_\alpha{}^\beta(\mathbf{q})$. For an even parity **35** the interaction would have gone via $M_\alpha{}^\beta(\mathbf{q})$.

It is quite instructive to consider the meson matrix also in a different representation defined by

$$w^\lambda(\mathbf{p})' = (Sw(\mathbf{p}))^\lambda, \tag{2.33}$$

$$S = 2^{-1/2}(\gamma_4 + \gamma_5). \tag{2.34}$$

Correspondingly,

$$\mathfrak{M}_\mu{}^\lambda(\mathbf{q})' = (\bar{S}\mathfrak{M}(\mathbf{q})S)_\mu{}^\lambda, \tag{2.35}$$

$$\bar{S} = \gamma_4 S^\dagger \gamma_4. \tag{2.36}$$

We get

$$\mathfrak{M}_\mu{}^\lambda(\mathbf{q})' = \begin{bmatrix} M^{(+)}{}_\alpha{}^\beta(\mathbf{q}), & 0 \\ 0 & M^{(-)}{}_\alpha{}^\beta(\mathbf{q}) \end{bmatrix}. \tag{2.37}$$

In this equation, 0 is a 6×6 null matrix, while

$$M^{(\pm)}{}_\alpha{}^\beta(\mathbf{q}) = \mp i \frac{(\sigma_\mu{}^{(\pm)} q_\mu)_i{}^j}{\mu_{00}} P_A{}^B + (\sigma_\mu{}^{(\pm)}\epsilon_\mu)_i{}^j V_A{}^B. \tag{2.38}$$

Here

$$\sigma_\mu{}^{(\pm)} = \boldsymbol{\sigma}, \quad \pm i.1. \tag{2.39}$$

We have

$$M^{(+)}{}_\alpha{}^\beta(0) = M^{(-)}{}_\alpha{}^\beta(0) = M_\alpha{}^\beta. \tag{2.40}$$

In the primed representation one can introduce the adjoint of $\mathfrak{M}'$ as follows.[18]

$$\bar{\mathfrak{M}}_\mu{}^\lambda(\mathbf{q})' = [\gamma_4' \mathfrak{M}(\mathbf{q})' \gamma_4']_\mu{}^\lambda; \quad \gamma_4' = \gamma_5. \tag{2.41}$$

Equation (2.41) is equivalent to

$$\bar{M}^{(\pm)}{}_\alpha{}^\beta(\mathbf{q}) = M^{(\mp)}{}_\alpha{}^\beta(\mathbf{q}). \tag{2.42}$$

One checks [using also Eq. (2.40)] that this definition of adjoint reduces to the one employed in Eq. (2.26) for $\mathbf{q}=0$. We now have

$$\tfrac{1}{2}\bar{M}^{(+)}{}_\alpha{}^\beta(\mathbf{q}) M^{(+)}{}_\beta{}^\alpha(\mathbf{q}) = \tfrac{1}{2}\bar{M}^{(-)}{}_\alpha{}^\beta(\mathbf{q}) M^{(-)}{}_\beta{}^\alpha(\mathbf{q})$$
$$= (\epsilon_\mu V_A{}^B)(\epsilon_\mu V_B{}^A) - \frac{q^2}{\mu_{00}{}^2} P_A{}^B P_B{}^A. \tag{2.43}$$

This is the boosted $SU(6)$ invariant encountered in Eq. (2.26); and it is a Lorentz scalar. On the mass shell the right-hand side of Eq. (2.43) reduces to Eq. (2.26).

(c) Vertices in the General Case

We next discuss the vertices without restricting the behavior of $\mathfrak{M}_\mu{}^\lambda(q)$ under the γ_5 transformation. This we call the general case. Thus we start again with Eq.

[16] Apart from some slight differences in the phase conventions, Eq. (2.9) is identical with I Eq. (27) with $g_1=g_2=g_3=1$.

[17] *Note added in manuscript.* The meaning of $B^{\lambda\mu\epsilon}$ and its adjoint is given in detail in M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965).

[18] This corresponds to $\mathfrak{M}_\mu{}^\lambda(\mathbf{q}) = [\gamma_4 \mathfrak{M}(\mathbf{q})\gamma_4]_\mu{}^\lambda$.

(2.19)–(2.25), but now also admit, in addition to $V(V)$ and $PS(PV)$ vertices, $V(T)$ and $PS(PS)$ vertices for the *same* set of mesons. The procedure followed is a straight extension of the one used in I Sec. III. We shall first consider the vertex for the sextet in interaction with our set of mesons.

The effective coupling again takes the form as in Eq. (2.24) but where now

$$\mathfrak{M}_\mu{}^\lambda(\mathbf{q})=\begin{pmatrix} A_\alpha{}^\beta(\mathbf{q}), & B_\alpha{}^\beta(\mathbf{q}) \\ C_\alpha{}^\beta(\mathbf{q}), & D_\alpha{}^\beta(\mathbf{q}) \end{pmatrix}, \tag{2.44}$$

$$A_\alpha{}^\beta(\mathbf{q})=\left[-if^A\frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{\mu_{00}}P_A{}^B-f^V\epsilon_0 V_A{}^B +if^T\frac{\boldsymbol{\sigma}\cdot(\mathbf{q}\times\boldsymbol{\varepsilon})}{\mu_{00}}V_A{}^B\right]_i^j, \tag{2.45}$$

$$B_\alpha{}^\beta(q)=\left[if^A\frac{q_0}{\mu_{00}}P_A{}^B+f^V(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})V_A{}^B+if^PP_A{}^B +\frac{f^T}{\mu_{00}}\{q_0(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})V_A{}^B-\epsilon_0(\boldsymbol{\sigma}\cdot\mathbf{q})V_A{}^B\}\right]_i^j, \tag{2.46}$$

$$C_\alpha{}^\beta(\mathbf{q})=B_\alpha{}^\beta(\mathbf{q})\text{, but with } f^V\to -f^V;\ f^A\to -f^A, \tag{2.47}$$

$$D_\alpha{}^\beta(\mathbf{q})=A_\alpha{}^\beta(\mathbf{q})\text{, but with } f^V\to -f^V;\ f^T\to -f^T. \tag{2.48}$$

A more transparent form of the meson matrix is obtained if one again makes the unitary transformation given in Eq. (2.33)–(2.36). This yields

$$\mathfrak{M}_\mu{}^\lambda(\mathbf{q})'=\begin{pmatrix} K^{(+)}{}_\alpha{}^\beta(\mathbf{q}), & L^{(+)}{}_\alpha{}^\beta(\mathbf{q}) \\ L^{(-)}{}_\alpha{}^\beta(\mathbf{q}), & K^{(-)}{}_\alpha{}^\beta(\mathbf{q}) \end{pmatrix}, \tag{2.49}$$

$$K^{(\pm)}{}_\alpha{}^\beta(\mathbf{q})=\left[\mp if^A\frac{\sigma_\mu{}^{(\pm)}q_\mu}{\mu_{00}}P_A{}^B +f^V(\sigma_\mu{}^{(\pm)}\epsilon_\mu)V_A{}^B\right]_i^j, \tag{2.50}$$

$$L^{(\pm)}{}_\alpha{}^\beta(q)=\left[\pm if^PP_A{}^B+if^T\frac{\sigma_{\mu\nu}{}^{(\pm)}q_\mu\epsilon_\nu}{\mu_{00}}V_A{}^B\right]_i^j, \tag{2.51}$$

$$\sigma_{ik}{}^{(\pm)}=\sigma_l,\quad i,k,l=1,2,3,\text{ cycl};\qquad \sigma_{i4}{}^{(\pm)}=\pm\sigma_i. \tag{2.52}$$

We have

$$K^{(\pm)}{}_\alpha{}^\beta(\mathbf{q})=f_1M^{(\pm)}{}_\alpha{}^\beta(\mathbf{q}), \tag{2.53}$$

if

$$f^A=f^V=f_1. \tag{2.54}$$

Thus, if Eq. (2.54) is fulfilled, we have in particular

$$K^{(\pm)}{}_\alpha{}^\beta(0)=f_1M_\alpha{}^\beta. \tag{2.55}$$

Next we note that

$$L^{(\pm)}{}_\alpha{}^\beta(0)=f_2M_\alpha{}^\beta, \tag{2.56}$$

if

$$f^P=f^T=f_2. \tag{2.57}$$

With the help of Eq. (2.49)–(2.57) we now give the following interpretation to the general meson matrix in $SU(6)$. Each of the four 6×6 matrices $K^{(\pm)}{}_\alpha{}^\beta(\mathbf{q})$; $L^{(\pm)}{}_\alpha{}^\beta(\mathbf{q})$ are boosted **35**'s with respect to the *proper* Lorentz group, as long as the Eqs. (2.54) and (2.57) are satisfied. For then we have the zero-momentum-limit property for both the K's and the L's given in Eqs. (2.55) and (2.56), which is what we mean by $SU(6)$. While this identification is particularly transparent in the representation Eq. (2.49) for the meson matrix, this interpretation is actually independent of any particular representation. The same results would have been found had we applied the reasoning to the separate matrices A, B, C, D in Eq. (2.44).

We again define[18] the adjoint of $\mathfrak{M}'$ by Eq. (2.41). Hence,

$$\bar{K}^{(\pm)}{}_\alpha{}^\beta(\mathbf{q})=K^{(\mp)}{}_\alpha{}^\beta(\mathbf{q}), \tag{2.58}$$

$$\bar{L}^{(\pm)}{}_\alpha{}^\beta(\mathbf{q})=L^{(\mp)}{}_\alpha{}^\beta(\mathbf{q}). \tag{2.59}$$

Thus,

$$\tfrac12\bar{K}^{(+)}{}_\alpha{}^\beta(\mathbf{q})K^{(+)}{}_\beta{}^\alpha(\mathbf{q})=\tfrac12\bar{K}^{(-)}{}_\alpha{}^\beta(\mathbf{q})K^{(-)}{}_\beta{}^\alpha(\mathbf{q}) =|f_1|^2\left[(\epsilon_\mu V_A{}^B)(\epsilon_\mu V_B{}^A)-\frac{q^2}{\mu_{00}{}^2}P_A{}^BP_B{}^A\right], \tag{2.60}$$

$$\tfrac12\bar{L}^{(+)}{}_\alpha{}^\beta(\mathbf{q})L^{(+)}{}_\beta{}^\alpha(\mathbf{q})=\tfrac12\bar{L}^{(-)}{}_\alpha{}^\beta(\mathbf{q})L^{(-)}{}_\beta{}^\alpha(\mathbf{q}) =|f_2|^2\left[-\frac{q^2}{\mu_{00}{}^2}(\epsilon_\mu V_A{}^B)(\epsilon_\mu V_B{}^A)+P_B{}^AP_A{}^B\right]. \tag{2.61}$$

Equation (2.60) is the same as Eq. (2.43). Equation (2.61) represents a second Lorentz scalar which is a boosted $SU(6)$ invariant. On the mass shell, Eqs. (2.60) and (2.61) are of course identical apart from a constant factor.

Note that the case $f_2=0$ corresponds to the notion of "minimal vertex" introduced earlier.[3] We now must ask in what respects the case $f_1\neq0$, $f_2\neq0$ differs from the restricted case. This is done in the next section.

III. RECOIL EFFECTS

The meson matrix Eq. (2.45) may be used to write down an effective meson-baryon coupling[17] with completed $SU(6)$ structure and more general than the coupling exhibited in Eq. (2.32),

$$3\sqrt{2}B^\dagger{}_{\lambda\mu\nu}(\mathbf{p}_2)B^{\lambda\mu\epsilon}(\mathbf{p}_1)\mathfrak{M}_\epsilon{}^\nu(\mathbf{q}). \tag{3.1}$$

It is instructive to investigate this coupling in the brick-wall frame defined by $\mathbf{p}_1+\mathbf{p}_2=0$. In this frame the vertex depends only on the single momentum $\mathbf{q}$ and the v/c limit is uniquely specified by the requirement that we retain terms only up to the first power in $|\mathbf{q}|$. On dimensional grounds, these v/c terms occur either in the form $|\mathbf{q}|/\mu_{00}$ or $|\mathbf{q}|/M_{00}$. Terms of the latter set are fermion recoil terms which do not survive in a purely static theory.

Our interpretation of $SU(6)$ theory[1] is not complete without a discussion of these recoil terms. In order to bring out their structure we reduce Eq. (3.1) and exhibit the relevant couplings through a phenomenological static interaction density. It is sufficient to consider the couplings of protons to π_0 and ρ_0; other couplings are easily obtained by the methods of Refs. 4 and 5. For these couplings we have

$$g_{\rm el}p^\dagger p\rho_0{}^0+(1/\mu_{00})g_{\rm mag}p^\dagger\boldsymbol{\sigma}p\cdot(\nabla\times\boldsymbol{\rho}_0)+(1/\mu_{00})g_Ap^\dagger\boldsymbol{\sigma}p\cdot\nabla\pi_0\,, \quad (3.2)$$

which contains three strong-interaction coupling constants given by

$$g_{\rm el}=[f^V+(\mu_{00}/2M_{00})f^T]\,, \quad (3.3)$$

$$g_{\rm mag}=5/3[(\mu_{00}/2M_{00})f^V+f^T]\,, \quad (3.4)$$

$$g_A=5/3[f^A+(\mu_{00}/2M_{00})f^P]\,. \quad (3.5)$$

(a) Note that since $f^A=f^V$ and $f^P=f^T$, one retains the relationship[3]

$$g_A=5g_{\rm el}/3 \quad (3.6)$$

also in the presence of fermion recoil. However, the ratio f^V/f^T is not determined by $SU(6)$ and thus $g_{\rm mag}$ is a parameter independent of $g_{\rm el}$.

(b) The arbitrariness of $g_{\rm mag}/g_{\rm el}$ in the $SU(6)$ scheme is essentially the same as the arbitrariness of the isovector magnetic-moment-to-charge ratio, i.e. $[\mu(p)-\mu(n)]/e=5\mu(p)/3e$. (Of course, the vanishing photon mass and considerations of gauge invariance make the two problems distinct in other respects.)

Unless $SU(6)$ is supplemented by further dynamical assumptions, the theory does not give a unique prediction for the magnitude of $\mu(p)$. This is precisely why it was not assumed in earlier work that the effective electric-charge operator and the effective magnetic-moment operator are members of the *same* **35** representation[4] of $SU(6)$; and similarly for weak currents.[5]

IV. COMPLETION FOR OTHER REPRESENTATIONS

(1) Completed $SU(6)$ and the Representation $\mathbf{70}^-$

As a further application of completion we consider the $\mathbf{70}^-$. This case is of interest for two reasons. First, the $\mathbf{70}^-$ is a likely candidate[19] for the assignment of higher baryon resonances. Secondly, it is of interest to see how our methods apply to the $(\mathbf{70}^-, \mathbf{56}^+; \mathbf{35}^-)$ vertex which describes the decay of the $\mathbf{70}^-$. In particular we shall see how the meson matrix has to be handled for a coupling to fermion supermultiplets of opposite relative parity.

At zero three-momentum the **70** baryons are described by an $SU(6)$ tensor $\Psi^{[\alpha\beta]\gamma}$. Here and below, the bracket [ ] denotes antisymmetry with regard to the enclosed indices. Ψ must satisfy

$$\Psi^{[\alpha\beta]\gamma}+\Psi^{[\gamma\alpha]\beta}+\Psi^{[\beta\gamma]\alpha}=0\,. \quad (4.1)$$

This representation has the $SU(3)\otimes SU(2)$ content[9] $\mathbf{70}=(\mathbf{1},\mathbf{2})+(\mathbf{8},\mathbf{2})+(\mathbf{10},\mathbf{2})+(\mathbf{8},\mathbf{4})$. We find the following explicit form:

$$\begin{aligned}\Psi^{[\alpha\beta]\gamma}=\frac{1}{\sqrt6}\epsilon^{ABC}\chi^{(ij)k}+\frac{\sqrt2}{3}[2\psi^{[AB]C}\chi^{(ij)k}\\ -\psi^{[BC]A}\chi^{(jk)i}-\psi^{[CA]B}\chi^{(ki)j}]\\ +\phi^{(ABC)}\chi^{[ij]k}+\xi^{[AB]C}\chi^{(ijk)}\,,\end{aligned} \quad (4.2)$$

$$\alpha=iA\,,\quad \beta=jB\,,\quad \gamma=kC\,.$$

The first line gives the $(\mathbf{1},\mathbf{2})$ part. ϵ^{ABC} is the $SU(3)$ Levi-Civita symbol, and

$$\chi^{(ij)k}=\frac{1}{\sqrt6}(\epsilon^{ik}\chi^j+\epsilon^{jk}\chi^i) \quad (4.3)$$

is the spin-$\frac12$ function with the appropriate symmetry; ϵ^{ij} is the $SU(2)$ Levi-Civita symbol, and χ^i is a Pauli spinor. The second line describes $(\mathbf{8},\mathbf{2})$. $\psi^{[AB]C}$ is an $SU(3)$ octet tensor and satisfies a relation analogous to Eq. (4.1). The third line is $(\mathbf{10},\mathbf{2})$, and $\phi^{(ABC)}$ is totally symmetric, while

$$\chi^{[ij]k}=\frac{1}{\sqrt2}\epsilon^{ij}\chi^k\,. \quad (4.4)$$

Finally, the last line of Eq. (4.2) gives the $(\mathbf{8},\mathbf{4})$. $\xi^{[AB]C}$ has the same $SU(3)$ properties as does $\psi^{[AB]C}$, and $\chi^{(ijk)}$ is totally symmetric. Our definitions satisfy the right relative normalization conditions; we have

$$\begin{aligned}&\sum\|\Psi^{[\alpha\beta]\gamma}\|=70\,,\\ &\sum\|\chi^{(ij)k}\|=\sum\|\chi^{[ij]k}\|=2\,,\\ &\sum\|\psi^{[AB]C}\|=\sum\|\xi^{[AB]C}\|=8\,,\\ &\sum\|\chi^{(ijk)}\|=4;\quad \sum\|\phi^{(ABC)}\|=10\,,\end{aligned} \quad (4.5)$$

where the summations go in each case over the range of all tensor indices.

We can now go from $\Psi^{[\alpha\beta]\gamma}$ to the completed description by a procedure similar to that[17] for the **56**. The vertex is given by

$$\Psi^\dagger{}_{[\lambda\mu]\nu}(\mathbf{p}_2)B^{(\lambda\mu\tau)}(\mathbf{p}_1)(\gamma_5\mathfrak{M}(q))_\tau{}^\nu\,. \quad (4.6)$$

Here $(\gamma_5\mathfrak{M}(q))_\tau{}^\nu$ is defined with the help of γ_5 as given in Eq. (2.21) and $\mathfrak{M}$ as given in Eq. (2.27). Thus

$$(\gamma_5\mathfrak{M}(q))_\tau{}^\nu=\begin{bmatrix}C_\alpha{}^\beta(q)\,, & D_\alpha{}^\beta(q)\\ A_\alpha{}^\beta(q)\,, & B_\alpha{}^\beta(q)\end{bmatrix}. \quad (4.7)$$

[19] See Ref. 9 and M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 509 (1964); I. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965); F. J. Dyson (private communication).

[20] Here again $\lambda, \mu, \nu, \tau=1, \cdots 12$.