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Coherence Properties and Photon Correlation

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An intensity-correlation experiment is proposed to test the second-order coherence of laser light from a cw-operated laser based on the Glauber formalism. The calculation utilizes the known form of the density matrix for radiation from a randomly excited source such as a discharge tube which is equivalent to filtered black-body radiation, and represents the laser radiation field by a coherent state. The intensity correlation calculated exhibits the standard Hanbury Brown–Twiss term and an extra term due to intensity interference between the Fourier components of the thermal field and the laser mode.

THE quantitative interpretation of the impressive amount of experimental data accumulated in the last four years in the laser field is presently not always free from doubt. Although much of the theoretical work[1] uses classical methods with little inhibition, citing the usual argument (high field intensities equal classical limit), some authors have attempted to formulate the problem on its proper ground, namely on a microscopic, quantum-mechanical level with due respect for the statistics underlying the electromagnetic field. R. Glauber[2] in particular has formulated a theory of coherence which is founded on quantum electrodynamics and gives a framework within which to discuss the particular structure of the electromagnetic field originating in a laser cavity, a phenomenon which is intrinsically quantum mechanical. In particular a complete set of eigenfunctions of the photon-destruction operator is introduced, and a new definition of coherence in terms of separable correlation functions of all orders is given. The question arises then: To what extent is a laser, and for simplicity we will always mean by that a single-mode system such as a He-Ne maser, represented quantum-statistically by a coherent state? It would require an infinite set of increasingly and finally forbiddingly difficult experiments to test the coherent-state aspect of a laser. Instead, we propose a more modest test. As we know, the originally striking, but now familiar results of the Hanbury Brown–Twiss[3] experiments follow unambiguously from the Gaussian nature of the chaotic radiation field, which in Glauber's terminology is only first-order coherent. How about lasers?

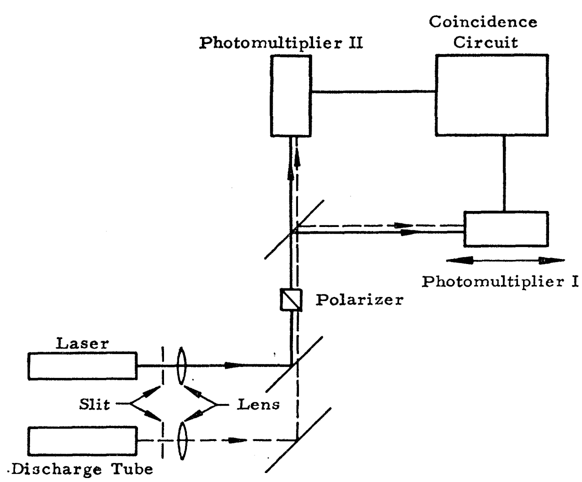

FIG. 1. Schematic arrangement of proposed experiment.

In the following, an intensity-correlation experiment is proposed utilizing the superposition of a laser source and the light produced by a discharge tube (see Fig. 1). The result to be expected on the basis of the assumption that the radiation field produced by the laser is described by a coherent state is calculated. Comparison of experimental results with the theory should then allow one to support the coherent-state nature of the laser light to second order or dismiss it. The assumption for the electromagnetic field produced by the discharge tube is the standard Gaussian one, which is the only well-established one. In the following we use the notation introduced by Glauber.[1]

In the P representation, the density matrix for the radiation field of a chaotic origin is

$$\rho_{th}=\int P(\{\alpha_k\})\,|\{\alpha_k\}\rangle\langle\{\alpha_k\}|\prod_k d^2\alpha_k$$
$$P(\{\alpha_k\})=\prod_k \frac{1}{\pi\langle n_k\rangle}\exp-|\alpha_k|^2/\langle n_k\rangle. \quad (1)$$

The laser is described by

$$\rho_L(\alpha_K)=\int \delta^{(2)}(\alpha_j-\alpha_K)\,|\alpha_j\rangle\langle\alpha_j|\,d^2\alpha_j=|\alpha_K\rangle\langle\alpha_K| \quad (2)$$

since it is assumed to be in a purely coherent state.[4] Superposing the radiation fields due to both sources, we form the convolution integral to find the joint P

[1] L. Mandel, Progr. Opt. **2**, 183 (1963).
[2] R. Glauber, Phys. Rev. **130**, 2529 (1963); **131**, 2766 (1963).
[3] R. Hanbury Brown and R. Q. Twiss, Nature **177**, 27 (1956).
[4] We could also use a mixture of coherent states over various phases. For simplicity we will confine ourselves to the simplest assumption. The result of the calculation would remain unchanged for the more general assumption.

representation

$$P_{\mathrm{joint}}(\{\beta_k\})$$
$$=\int P(\{\alpha_k\})P(\alpha_j)\prod_k \delta^{(2)}(\beta_k-\alpha_k-\alpha_j)d^2\alpha_j\prod_k d^2\alpha_k$$
$$=\frac{1}{\pi\langle n_j\rangle}\exp-|\beta_j-\alpha_K|^2/\langle n_j\rangle$$
$$\times\prod_{k\neq j}\frac{1}{\pi\langle n_k\rangle}\exp-|\beta_k|^2/\langle n_k\rangle. \quad (3)$$

The density matrix for the superposed fields is

$$\rho_{\mathrm{joint}}=\int P(\{\beta_k\})|\{\beta_k\}\rangle\langle\beta_k\}|\prod_k d^2\beta_k. \quad (4)$$

We propose to calculate the second-order correlation function, which is a measure of the intensity correlations at different space-time points $\{x_i(\mathbf{x}_i,t_i),x_j(\mathbf{x}_j,t_j)\}$. By definition

$$G_{\mu_1\mu_2\mu_3\mu_4}{}^{(2)}(x_1,x_2,x_3,x_4)$$
$$=\mathrm{Tr}\{\rho E_{\mu_1}{}^{(-)}(x_1)E_{\mu_2}{}^{(-)}(x_2)E_{\mu_3}{}^{(+)}(x_3)E_{\mu_4}{}^{(+)}(x_4)\}. \quad (5)$$

Expanding the $E_{\mu_i}{}^{(+)}(x_i)$ in terms of destruction operators a_{k_i} we have

$$E_{\mu_i}{}^{(+)}(x_i)=-c\sum_{k_i}\left(\frac{\hbar\omega_i}{2}\right)^{1/2}u_{\mu_i}{}^{k_i}(\mathbf{x}_i)a_{k_i}e^{-i\omega_{k_i}t_i}, \quad (6)$$

where the $u_{\mu_i}{}^{k_i}(\mathbf{x}_i)$ are solutions of the wave equation and form a complete set. Substituting in Eq. (5) we find

$$G_{\mu_1\mu_2\mu_3\mu_4}{}^{(2)}(x_1,x_2,x_3,x_4)=\sum_{k_1k_2k_3k_4}c^4\frac{\hbar^2}{4}(\omega_{k_1}\omega_{k_2}\omega_{k_3}\omega_{k_4})^{1/2}u_{k_1}{}^{*\mu_1}(\mathbf{x}_1)u_{k_2}{}^{*\mu_2}(\mathbf{x}_2)$$
$$\times u_{k_3}{}^{\mu_3}(\mathbf{x}_3)u_{k_4}{}^{\mu_4}(\mathbf{x}_4)\exp\{i(\omega_{k_1}t_1+\omega_{k_2}t_2-\omega_{k_3}t_3-\omega_{k_4}t_4)\}$$
$$\times\frac{1}{\pi\langle n_j\rangle}\int d^2\alpha_j\exp-|\alpha_j-\alpha_K|^2/\langle n_j\rangle\int\prod_k\frac{d^2\alpha_k}{\pi\langle n_k\rangle}\exp-|\alpha_k|^2/\langle n_k\rangle\alpha_{k_1}{}^*\alpha_{k_2}{}^*\alpha_{k_3}\alpha_{k_4}, \quad (7)$$

where we have used the cyclic invariance of the trace and the defining property of the coherent states $|\{\alpha_j\}\rangle$

$$a_{k_i}|\{\alpha_j\}\rangle=\alpha_{k_i}|\{\alpha_j\}\rangle \quad (8)$$

as well as the normalization condition

$$\mathrm{Tr}\rho=1. \quad (9)$$

To evaluate Eq. (7) we note that

$$\int P(|\alpha_k|)\alpha_k{}^{*l}\alpha_k{}^m d^2\alpha_k=C\delta_{lm}$$

$$\int d^2\alpha_jP(|\alpha_j-\alpha_K|)\int\prod_k P(|\alpha_k|)d^2\alpha_k\,\alpha_{k_1}{}^*\alpha_{k_2}{}^*\alpha_{k_3}\alpha_{k_4}=\int d^2\alpha_jP(|\alpha_j-\alpha_K|)\int\prod_k P(|\alpha_k|)d^2\alpha_k$$
$$\times\alpha_{k_1}{}^*\alpha_{k_2}{}^*\alpha_{k_3}\alpha_{k_4}(\delta_{k_1k_2}\delta_{k_3k_4}+\delta_{k_2k_4}\delta_{k_2k_3})(1+\delta_{k_1K}+\delta_{k_2K}+\tfrac{1}{2}\delta_{k_1K}\delta_{k_2K}). \quad (10)$$

Specifying $\mathbf{x}_1=\mathbf{x}_3$, $t_1=t_3$; $\mathbf{x}_2=\mathbf{x}_4$, $t_2=t_4$; and $\mu_1=\mu_3=\mu$; $\mu_2=\mu_4=\nu$, we find

$$G_{\mu\nu\mu\nu}{}^{(2)}(\mathbf{x}_1,\mathbf{x}_2;t_1,t_2)=\sum_{k_1,k_2}{}''c^4\frac{\hbar^2}{4}\omega_{k_1}\omega_{k_2}\langle n_{k_1}\rangle\langle n_{k_2}\rangle[u_{k_1}{}^{*\mu}(\mathbf{x}_1)u_{k_2}{}^{*\nu}(\mathbf{x}_2)u_{k_1}{}^{\mu}(\mathbf{x}_1)u_{k_2}{}^{\nu}(\mathbf{x}_2)$$
$$+u_{k_1}{}^{*\mu}(\mathbf{x}_1)u_{k_2}{}^{*\nu}(\mathbf{x}_2)u_{k_2}{}^{\mu}(\mathbf{x}_1)u_{k_1}{}^{\nu}(\mathbf{x}_2)\exp\{i(t_1-t_2)(\omega_{k_1}-\omega_{k_2})\}]+\sum_{k_2}{}'c^4\frac{\hbar^2}{4}\omega_{k_2}\omega_K\langle n_{k_2}\rangle(\langle n_K\rangle+|\alpha_K|^2)$$
$$\times[u_K{}^{*\mu}(\mathbf{x}_1)u_{k_2}{}^{*\nu}(\mathbf{x}_2)u_K{}^{\mu}(\mathbf{x}_1)u_{k_2}{}^{\nu}(\mathbf{x}_2)+u_K{}^{*\mu}(\mathbf{x}_1)u_{k_2}{}^{*\nu}(\mathbf{x}_2)u_{k_2}{}^{\mu}(\mathbf{x}_1)u_K{}^{\nu}(\mathbf{x}_2)\exp\{i(t_1-t_2)(\omega_K-\omega_{k_2})\}]$$
$$+\sum_{k_1}{}'c^4\frac{\hbar^2}{4}\omega_{k_1}\omega_K\langle n_{k_1}\rangle(\langle n_K\rangle+|\alpha_K|^2)[u_{k_1}{}^{*\mu}(\mathbf{x}_1)u_K{}^{*\nu}(\mathbf{x}_2)u_{k_1}{}^{\mu}(\mathbf{x}_1)u_K{}^{\nu}(\mathbf{x}_2)+u_{k_1}{}^{*\mu}(\mathbf{x}_1)u_K{}^{*\nu}(\mathbf{x}_2)u_K{}^{\mu}(\mathbf{x}_1)u_{k_1}{}^{\nu}(\mathbf{x}_2)$$
$$\times\exp\{i(t_1-t_2)(\omega_{k_1}-\omega_K)\}]+c^4\frac{\hbar^2}{4}\omega_K{}^2(2\langle n_K\rangle^2+4\langle n_K\rangle|\alpha_K|^2+|\alpha_K|^4)u_K{}^{*\mu}(\mathbf{x}_1)u_K{}^{*\nu}(\mathbf{x}_2)u_K{}^{\mu}(\mathbf{x}_1)u_K{}^{\nu}(\mathbf{x}_2), \quad (11)$$

where $\sum'$ denotes the fact that we do not sum over K in the Fourier expansions of the field. Completing the sums by adding the K mode in the Fourier expansions for the chaotic field, we can rewrite Eq. (11) in terms of the first-order correlation functions $G_{\mu\nu}{}^{(1)\mathrm{ch},\mathrm{L}}(x_1,x_2)$ for the radiation field of random and coherent origin, respectively, to arrive after some algebra at

$$G_{\mu\nu\mu\nu}^{(2)}(x_1,x_2)=G_{\mu\mu}^{(1)\mathrm{ch}}(x_1)G_{\nu\nu}^{(2)\mathrm{ch}}(x_2)+G_{\mu\nu}^{(1)\mathrm{ch}}(x_1,x_2)G_{\nu\mu}^{(1)\mathrm{ch}}(x_2,x_1)$$
$$+G_{\mu\mu}^{(1)\mathrm{L}}(x_1)G_{\nu\nu}^{(1)\mathrm{ch}}(x_2)+G_{\nu\nu}^{(1)\mathrm{L}}(x_2)G_{\mu\mu}^{(1)\mathrm{ch}}(x_1)+G_{\mu\mu}^{(1)\mathrm{L}}(x_1)G_{\nu\nu}^{(1)\mathrm{L}}(x_2)+G_{\mu\nu}^{(1)\mathrm{L}}(x_1,x_2)G_{\nu\mu}^{(1)\mathrm{ch}}(x_2,x_1)$$
$$+G_{\nu\mu}^{(1)\mathrm{L}}(x_2,x_1)G_{\mu\nu}^{(1)\mathrm{ch}}(x_1,x_2)=[G_{\mu\mu}^{(1)\mathrm{ch}}(x_1)+G_{\mu\mu}^{(1)\mathrm{L}}(x_1)][G_{\nu\nu}^{(1)\mathrm{ch}}(x_2)+G_{\nu\nu}^{(1)\mathrm{L}}(x_2)]$$
$$+G_{\mu\nu}^{(1)\mathrm{ch}}(x_1,x_2)G_{\nu\mu}^{(1)\mathrm{ch}}(x_2,x_1)+2\,\mathrm{Re}G_{\mu\nu}^{\mathrm{L}}(x_1,x_2)G_{\nu\mu}^{(1)\mathrm{ch}}(x_2,x_1)\,. \quad (12)$$

Summing over polarization indices μ, ν we find

$$\sum_{\mu,\nu} G^{(2)}(x_1,x_2)=\langle I_{\mathrm{tot}}(x_1)I_{\mathrm{tot}}(x_2)\rangle=[I_{\mathrm{ch}}(x_1)+I_L(x_1)][I_{\mathrm{ch}}(x_2)+I_L(x_2)]$$
$$+\sum_{\mu,\nu} G_{\mu\nu}^{(1)\mathrm{ch}}(x_1,x_2)G_{\mu\nu}^{(1)\mathrm{ch}}(x_2,x_1)+2\sum_{\mu,\nu}\mathrm{Re}G_{\mu\nu}^{(1)\mathrm{L}}(x_1,x_2)G_{\nu\mu}^{(1)\mathrm{ch}}(x_2,x_1)\,. \quad (13)$$

$I_{\mathrm{ch,L}}$ denotes the instantaneous intensity of the light of chaotic or laser origin. From the Hermiticity of ρ it follows that

$$G_{\nu\mu}^{(1)\mathrm{ch}}(x_2,x_1)=G_{\mu\nu}^{*(1)\mathrm{ch}}(x_1,x_2)\,. \quad (14)$$

Next we introduce the normalized first-order correlation functions $\rho_{\mu\nu}^{(1)\mathrm{ch}}(x_1,x_2)$, $\lambda_{\mu\nu}^{(1)\mathrm{L}}(x_1,x_2)$ by defining

$$\rho_{\mu\nu}^{(1)\mathrm{ch}}(x_1,x_2)=\frac{G_{\mu\nu}^{(1)\mathrm{ch}}(x_1,x_2)}{[G_{\mu\mu}^{(1)\mathrm{ch}}(\mathbf{x}_1)G_{\nu\nu}^{(1)\mathrm{ch}}(\mathbf{x}_2)]^{1/2}}$$
$$\lambda_{\mu\nu}^{(1)\mathrm{L}}(x_1,x_2)=\frac{G_{\mu\nu}^{(1)\mathrm{L}}(x_1,x_2)}{[G_{\mu\mu}^{(1)\mathrm{L}}(\mathbf{x}_1)G_{\nu\nu}^{(1)\mathrm{L}}(\mathbf{x}_2)]^{1/2}}\,. \quad (15)$$

The above procedure is for unpolarized light. For polarized light we have only one component of polarization nonvanishing, hence $G_{\mu\mu}^{(1)\mathrm{ch,L}}(x_1)=I_{\mathrm{ch,L}}(x_1)$. The second-order cross-correlation function is by construction the ensemble average of the intensities at x_1, t_1 and x_2, t_2; subtracting the product of instantaneous intensities due to the laser and chaotic sources, and working with polarized light for both chaotic and laser radiation as well as taking $\mathbf{x}_1=\mathbf{x}_2$ to maximize the effect we find the intensity fluctuation

$$\langle I_{\mathrm{tot}}(\mathbf{x}_1,t_1)I_{\mathrm{tot}}(\mathbf{x}_1,t_2)\rangle-I_{\mathrm{tot}}(\mathbf{x}_1)I_{\mathrm{tot}}(\mathbf{x}_1)$$
$$=I_{\mathrm{tot}}(\mathbf{x}_1)I_{\mathrm{tot}}(\mathbf{x}_1)\{[I_{\mathrm{ch}}(\mathbf{x}_1)I_{\mathrm{ch}}(\mathbf{x}_1)/I_{\mathrm{tot}}(\mathbf{x}_1)I_{\mathrm{tot}}(\mathbf{x}_1)]$$
$$\times|\rho^{\mathrm{ch}}(t_1,t_2)|^2+[I_{\mathrm{ch}}^{(1)}(\mathbf{x}_1)I_{\mathrm{L}}^{(1)}(\mathbf{x}_1)/I_{\mathrm{tot}}(\mathbf{x}_1)I_{\mathrm{tot}}(\mathbf{x}_1)]$$
$$\times 2\,\mathrm{Re}\,\mathrm{Tr}\lambda^{(1)\mathrm{L}}(t_1,t_2)\rho^{*(1)\mathrm{ch}}(t_1,t_2)\}\,. \quad (16)$$

The first term is already present in a chaotic beam intensity correlation experiment (Brown-Twiss), and is due to interference between the different Fourier components of the randomly produced field. The second term expresses an interference effect between the laser field and the various Fourier components of the chaotic radiation field. In order to express our result in terms of an experimentally measurable quantity, e.g., the excess coincidence rate in the output of two photon detectors, we have to consider the absorption of photons by an atomic system (photoionization) and relate the rate of absorption ($\equiv$rate of emission of photoelectrons) to the second-order correlation function of the radiation field. This has been done by R. Glauber,[5] and we follow his treatment. In particular, he shows that under certain approximations the probability for absorption of n photons by n atoms, each atom absorbing one photon, in a time interval t is given by

$$p^n(t)=\int_0^t dt_1'\cdots\int_0^t dt_n'$$
$$\times\int_0^t dt_n''\cdots\int_0^t dt_1''\prod_{j=1}^{n}S(t_j'-t_j'')G^{(n)}$$
$$\times(\mathbf{r}_1't_1'\cdots\mathbf{r}_n't_n'\mathbf{r}_n''t_n''\cdots\mathbf{r}_1''t_1'') \quad (17)$$

with

$$S(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S(\omega)e^{i\omega t}d\omega;$$
$$S(\omega)=2\pi\left(\frac{e}{\hbar}\right)^2\sum_f R(f)M_{fg}M_{fg}^*\delta(\omega-\omega_{fg})\,. \quad (17\mathrm{a})$$

The factors $S(t)$ originate from the atomic part of the photoionization transition probability, M_{fg} is the dipole matrix element evaluated between the ground state and final ionized (continuum) state of the jth atom, and $\omega_{fg}=E_f-E_g/\hbar$. The sum over f with the weighing factor $R(f)$ takes account of the continuum of states available to the emitted electron for $\omega_{fg}\geq\omega_{\mathrm{ionization}}$. Since the counter consists of a large number of atoms ($\sim10^{20}$), exceeding by many orders of magnitude the number of absorbed photons, we have to consider the ways in which n photons can be absorbed by N atoms, with $N\gg n$.

We consider, therefore, the generating function[5] $Q(\lambda,t)$ defined as

$$Q(\lambda,t)=\langle(1-\lambda)^C\rangle\,, \quad (18)$$

where $0\leq\lambda\leq1$, C is the number of photon counts in a time interval t (which we shall later take to be the resolution time of the detecting system) and the brackets denote ensemble averaging. Expanding $Q(\lambda,t)$ in terms of the n-photon-absorption probability $p^{(n)}(t)$ and summing over the various ways in which n photons can be absorbed by N atoms, as well as going to the continuum

[5] R. Glauber, *Quantum Optics and Electronics* (Gordon and Breach Science Publishers, Inc., New York, 1964).

limit ($N\to\infty$), we arrive at the expression

$$Q(\lambda,t)=\sum_{n=0}^{\infty}\frac{(-\lambda)^n}{n!}\times\int\cdots\int G^{(n)}(x_1',\cdots,x_n';x_n'',\cdots,x_1'')\times\prod_{j=1}^{n}V(x_j',x_j'')d^4x_j',d^4x_j'',\quad(19)$$

where

$$V(x',x'')=\sigma(r')\delta^3(r'-r'')S(t''-t')\qquad(19a)$$

and $\sigma(r)$ is the number of atoms per unit volume. The integrals are taken over the counter volumes, and from 0 to t along the time axis. By filtering the radiation field we can achieve $S(\omega)=\text{const}$ over the frequency range of the field and, therefore, $S(t''-t')=s\delta(t''-t')$. To find the second moment of the photon-counting distribution, we differentiate Eqs. (18) and (19) twice with respect to λ and set $\lambda=0$ to arrive at

$$\langle C(C-1)\rangle=\underset{\substack{\text{counter volume}\\\text{resolution time}}}{\int\cdots\int}G^{(2)}(x_1',x_2',x_2'',x_1'')\times\prod_{j=1}^{2}V(x_j',x_j'')\prod_{j=1}^{2}d^4x_j'd^4x_j''=s'^2\int\int d^4x_1'd^4x_2'G^{(2)}(x_1',x_2',x_2',x_1'),\quad(20)$$

where we assumed the atoms to be distributed uniformly and lumped $\sigma(r'')=\sigma(r')=a$ into the new sensitivity parameter $s'=sa$.

Noting that

$$\langle C\rangle=\int\int G^{(1)}(x_1',x_1'')V(x_1',x_1'')d^4x_1'd^4x_1''=s'\underset{\substack{\text{counter volume}\\\text{resolution time}}}{\int}G^{(1)}(x_1',x_1')d^4x_1'\qquad(21)$$

and substituting for $G^{(2)}(x_1',x_2',x_2',x_1')$ from Eq. (16), we find

$$\langle C^2\rangle-\langle C\rangle=\langle C\rangle^2\Bigg[1+\frac{\langle C^{\text{ch}}\rangle^2}{T_R^2\langle C\rangle^2}\int_0^{T_R}dt_1\int_0^{T_R}dt_2|\rho^{\text{ch}}(t_1,t_2)|^2+\frac{\langle C^{\text{ch}}\rangle\langle C^{\text{L}}\rangle}{T_R^2\langle C\rangle^2}2\,\text{Re}\int_0^{T_R}dt_1\int_0^{T_R}dt_2\lambda(t_1,t_2)\rho^*(t_1,t_2)\Bigg].\quad(22)$$

Note that $\langle C\rangle=\langle C^{\text{ch}}\rangle+\langle C^{\text{L}}\rangle$ and $\langle C^{\text{ch}}\rangle$, $\langle C^{\text{L}}\rangle$ are the photon mean numbers for the chaotic and laser fields, respectively. The spatial integrations over counter volume are trivial since assuming a plane-wave representation for the field, the intensity is constant over the surface of the two counters placed perpendicular to the propagation direction of the field and the remaining integration in the z direction just averages the intensity over a thin layer of atoms. To get counts per unit time, we divide by T_R^2.

The variance of the photon-counting distribution is then given by

$$\langle C^2\rangle-\langle C\rangle^2=\langle C\rangle+\frac{\tau_{\text{coh}}^{\,l}}{2T_R}\langle C^{\text{ch}}\rangle^2\left[1+4\frac{\langle C^{\text{L}}\rangle}{\langle C^{\text{ch}}\rangle}\kappa\right],\quad(23)$$

where the remaining time integrals in Eq. (22) are elementary recalling that the first-order correlation function $\rho^{\text{ch}}(t_1,t_2)$ is related to the Fourier transform of the spectral profile via

$$\rho(t_1,t_2)=\frac{1}{2\pi}\int_{-\infty}^{\infty}g(\omega)e^{-i\omega(t_1-t_2)}d\omega\qquad(24)$$

and we calculate for square pulse, Gaussian and Lorentzian lineshape. The forms of $g(\omega)$ are

$$\begin{aligned}g(\omega)&=2\pi/2\Gamma\quad \omega_\kappa-\Gamma\le\omega\le\omega_\kappa+\Gamma &&\text{(square pulse)}\\&=0\quad\text{otherwise}\\g(\omega)&=2\pi/\pi^{1/2}\Gamma\exp-(\omega-\omega_\kappa)^2/\Gamma^2&&\text{(Gaussian)}\qquad(24a)\\g(\omega)&=2\Gamma/[(\omega-\omega_\kappa)^2+\Gamma^2]&&\text{(Lorentzian)}\\g(\omega)&=2\pi\delta(\omega-\omega_\kappa)&&\text{(Laser)}.\end{aligned}$$

κ is $\frac{1}{2}$, $1/\sqrt{2}$, 2 for the square pulse, Gaussian, and Lorentzian, respectively. The coherence time is defined by $\tau^l{}_{\text{coh}}=1/\Gamma$, and we have assumed the same Γ for the various lineshapes, so that $\tau^l{}_{\text{coh}}=2\pi\tau^s{}_{\text{coh}}=\tau^g{}_{\text{coh}}2(2\pi)^{1/2}$ where the superscripts l, s, g correspond to Lorentzian, square, and Gaussian lineshape, respectively. T_R is the resolution time of the detecting system. The expression Eq. (23) for the variance contains the familiar Poisson distribution for the photoelectrons (first term) as well as the Hanbury Brown–Twiss correlation term for the chaotic field and a new term which arises from intensity interference of the various Fourier components of the chaotic field with the laser field. This term is the crucial one to detect in the proposed experiment testing the coherent-state nature of the laser field to second order.

In the above form Eq. (23) is applicable to the recent measurements of J. Armstrong and A. Smith[6] on the intensity correlation of the light produced by a GaAs laser below and above lasing threshold, under the assumption that the radiation is described by the superposition of a coherent, amplitude stabilized part (above threshold) and a noise field corresponding to black-body radiation.

The suggested experiment tests the coherent-state nature of a laser beam to second order by exploiting the known density matrix for a chaotic field. Higher order coherence can be tested in measurements based on nonlinear effects of higher than second order, and this topic will be taken up in a future publication.

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[6] J. Armstrong and A. Smith, Phys. Rev. Letters **14**, 68 (1965).