Hyperfine Structure and Weak Interactions*

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The contribution of possible four-fermion $V-A$ weak interactions to the hyperfine splitting is considered for the hydrogen-like bound systems (e^-,p^+), (μ^-,p^+), (e^-,μ^+), and (e^-,e^+). The effect might have experimental significance (19 parts per million) for the (μ^-,p^+) system.

IN nature we are presented with few phenomena in which we can observe weak interactions, and in the search for better understanding these should be fully explored. A striking feature of the successful $V-A$ theory of four-fermion interactions is its extreme dependence on the relative orientation of the spins of the fermions. Under this simple theory muon capture $(\mu+p\rightarrow\nu+n)$ proceeds only from the singlet state of the μ,p system and not from the triplet state. One might suppose then that the weak perturbations of the hyperfine levels of a bound two-fermion system might exhibit this same spin dependence. The great accuracy with which hyperfine splittings can be measured leads one to wonder whether the effect would be within the range of possible experimental observation.

The most obvious candidate for study is the bound (e^-,p^+) system, hydrogen. However, there are others which can be considered: (μ^-,p^+), (e^-,μ^+), (e^-,e^+). We will suppose that to a good approximation the effective weak-interaction Hamiltonian density can be written as

$$\mathcal{H}=(G_W/\sqrt{2})\bar{\psi}_4\gamma_\mu(1+\gamma_5)\psi_2\bar{\psi}_3\gamma^\mu(1+\gamma_5)\psi_1 \tag{1}$$

which we will abbreviate as (42) (31). The interactions which could give rise to modification of hyperfine splitting in the various systems are then (ee) (pp) or equivalently by Fierz transformation (ep) (pe), and $(\mu\mu)$ (pp) or (μp) $(p\mu)$, (ee) $(\mu\mu)$ or $(e\mu)$ (μe), (ee) (ee). None of these interaction forms has great theoretical appeal. Hamiltonian densities of the form (ep) (pe) and (μp) $(p\mu)$ would involve coupling of charge-2 currents which violate both the conservation of baryons and the conservation of leptons. The other forms would involve weak coupling of neutral currents. These might be mediated by a neutral boson. However, neutrino beam experiments have shown that the interaction $(\nu_\mu\nu_\mu)$ (pp) does not exist.[1] This would make it seem unlikely that weak coupling of neutral currents exists.[2] Nonetheless, it would be worth some experimental effort to test further this conjecture.

We will compute as simply as possible the shift due to interaction of the form (1) in a hyperfine level of a hydrogen-like system in its ground state. The Schrödinger solution for the hydrogen-like system and the nonrelativistic limit (NRL) of (1) will be used. Consider the matrix element of the Hamiltonian

$$H_{mn}=\langle\mathbf{p}_3,\mathbf{p}_4|H|\mathbf{p}_1,\mathbf{p}_2\rangle$$
$$=\langle\mathbf{p}_3,\mathbf{p}_4|\int_{(\infty)}d^3x\,\mathcal{H}(x)|\mathbf{p}_1,\mathbf{p}_2\rangle,$$

where $|\mathbf{p}_1,\mathbf{p}_2\rangle$ is a state in which the particles 1 and 2 are plane waves with three-momenta $\mathbf{p}_1$, $\mathbf{p}_2$. The nonrelativistic limit is

$$H_{mn}\underset{\text{NRL}}{\rightarrow}(G_W/\sqrt{2})(2\pi)^{-3}\delta^3(\mathbf{K}-\mathbf{K}')(4P_s),$$

where

$$\mathbf{K}'=\mathbf{p}_3+\mathbf{p}_4,\quad \mathbf{K}=\mathbf{p}_1+\mathbf{p}_2,$$
$$4P_s=\chi_4^\dagger\chi_2\chi_3^\dagger\chi_1-(\chi_4^\dagger\boldsymbol{\sigma}\chi_2)\cdot(\chi_3^\dagger\boldsymbol{\sigma}\chi_1),$$

and χ_1 is the two-component Pauli spinor of particle 1. If we had quantized the system so that the center-of-mass coordinate satisfied periodic boundary conditions in a volume V, then

$$(H_{m,n})_{\text{NRL}}=(G_W/\sqrt{2})\delta_{\mathbf{K},\mathbf{K}'}(4P_s).$$

For the hydrogen-like bound state $|B\rangle$ we use the wave function

$$\langle\mathbf{r},\mathbf{R}|B\rangle=V^{-1/2}e^{-i\mathbf{K}\cdot\mathbf{R}}\psi(\mathbf{r}),$$

where $\mathbf{r}$ is the relative coordinate, $\mathbf{R}$ is the center-of-mass coordinate, and $\mathbf{K}$ is the total momentum quantized in the volume V again. The shift in an energy level due to the weak interaction is, to first order,

$$\delta E=\langle B|H|B\rangle=(G_W/\sqrt{2})|\psi(0)|^2(4P_s).$$

Now $4P_s$ is $+4$ for the singlet hyperfine state and zero for the triplet and shows the extreme spin dependence expected. The weak interaction raises the singlet energy level with respect to the triplet by an amount ΔE_W:

$$\Delta E_W=\delta E_s-\delta E_t,$$
$$=(G_W/\sqrt{2})4|\psi(0)|^2,$$
$$=(4/\pi)(G_W/\sqrt{2})e^6M^3,$$

where M is the reduced mass of the system and units are $\hbar=c=1$. Direct measurement of the sign of the

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[1] J. S. Bell, J. Løvseth, and M. Veltman, *Proceedings of the Sienna International Conference on Elementary Particles, Sienna 1963*, edited by G. Bernadini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963).

[2] A weak interaction which could give rise to an atomic electric dipole moment, such as may have been observed by P. G. H. Sandars and E. Lipworth, Phys. Rev. Letters **13**, 718 (1964), could not be of the form (1), which is time-reversal invariant.

coupling constant in most weak interactions is impossible because quantities measured usually depend on the square of the coupling constant. The intermediate vector boson theory however predicts that G_W is positive, and preliminary results indicate that this is true for the interaction (pn) (np).[3]

Now the normal electromagnetic hyperfine splitting of a hydrogen-like system is given by[4]

$$\Delta E_{EM}=(32/3)e^6M^3\mu_1\mu_2$$

where

$$\mu_i=\frac{e}{2m_i}g_i$$

is the magnetic moment of particle i, and g_i is a factor to account for anomalies of moments: $g_i=+1$ for leptons and 2.79 for protons. The ratio of the modification of the hyperfine splitting due to the weak interaction to the hyperfine splitting due to electromagnetic effects is

$$\frac{\Delta E_W}{\Delta E_{EM}}=\frac{3G_W}{2\pi\sqrt{2}e^2}\frac{m_1m_2}{g_1g_2}.$$

Table I presents, for the various systems, numerical values for the change in hyperfine splitting due to the weak interactions and the ratio of this splitting to the fundamental electromagnetic splitting, together with a figure for the best ratio of experimental uncertainty to experimental value of the hyperfine splitting measured at this time.[5] The remarkable accuracy of the experiment on the (e^-,p^+) system indicates that, if this weak effect exists, it has already been measured. However, there exists an uncertainty of approximately 30 parts per million in the theoretical prediction of ΔE_{EM} due to difficulty in computing the effects of nucleon structure on the hyperfine splitting of the (e^-,p^+) system,[5,6] and so it is impossible to use this as a test for the weak effect. The finite lifetime of the (e^-,μ^+) and (e^+,e^-) imposes a minimum limit on the line breadth of the hyperfine transition. As a practical matter this limitation would seem to exclude the possibility of improving the accuracy of these experiments so that they could test theories of weak interactions.

For the mesonic atom (μ^-,p^+) the size of the effect, 19 parts per million, might lie within the realm of the experimentally measurable. The experiment, while difficult, is apparently being seriously considered.[7] It is hoped that the nucleon structure correction in the (e^-,p^+) and (μ^-,p^+) systems can be related. This would allow the (e^-,p^+) structure correction to be evaluated from a measure of the (μ^-,p^+) hyperfine splitting where nucleon structure corrections are expected to be important because of the smallness of the μ's Bohr radius. An experimental accuracy only somewhat better than that achieved for the (e^-,μ^+) system would be needed. Such an analysis should include the possibility of a contribution from a weak interaction and might be a clear test of the weak interaction $(\mu\mu)$ (pp).

TABLE I. The weak hyperfine effect, its importance relative to the electromagnetic splitting, and the current experimental accuracy of hyperfine splitting.

System	ΔE_W (cps)	$\Delta E_W/\Delta E_{EM}$	Expt. uncertainty/ expt. value
ep	131	0.92×10^{-7}	2×10^{-11} [a]
μp	8.4×10^{7}	1.91×10^{-5}	Not done
$e\mu$	129	2.09×10^{-8}	1.3×10^{-5} [b]
ee	16.4	1.40×10^{-10}	2×10^{-4} [c]

[a] S. B. Crampton, D. Kleppner, and N. F. Ramsey, Phys. Rev. Letters **11**, 338 (1963).
[b] W. E. Cleland, J. M. Bailey, M. Eckhause, V. W. Hughes, R. M. Mobley, R. Prepost, and J. E. Rothberg, Phys. Rev. Letters **13**, 202 (1964).
[c] V. W. Hughes, S. Marder, and C. S. Wu, Phys. Rev. **106**, 934 (1957).

[3] F. Boehm and E. Kankeleit, Phys. Rev. Letters **14**, 312 (1965).

[4] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic Press Inc., New York, 1957), p. 107 ff.

[5] For a general review of the sources of uncertainty in the predictions and experiments see the review article of V. W. Hughes in *Nucleon Structure, Proceedings of the International Conference at Stanford University*, 1963, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1964), p. 235.

[6] C. K. Iddings, Phys. Rev. **138**, B446 (1965).

[7] M. M. Sternheim, Phys. Rev. **138**, B430 (1965), and unpublished.