Suggested Measurements of Neutral-Kaon Decay Rates

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A method which involves the use of K mesons produced in both the K^0 state and the $\bar{K}^0$ state is proposed to measure the decay rates of the short-lived and long-lived components of neutral kaons. A phenomenological analysis of these decay rates is carried out in the leptonic case, and a new test for CPT invariance is suggested.

I. INTRODUCTION

According to standard quantum mechanics, there are two states of the K^0, $\bar{K}^0$ complex which decay exponentially in vacuum.[1] Let us call them K^L and K^S, where S stands for short-lived, and L for long-lived. The masses, lifetimes, and time-independent decay rates of K^S and K^L are fundamental constants to be measured. However, the neutral K-mesons are not produced in the K^S and K^L states, but in the K^0 and $\bar{K}^0$ states, which have definite strangeness. Up to now, experiments have mainly been performed using neutral kaons produced in the K^0 state. But more information could be obtained through the experimental study of K mesons produced in the $\bar{K}^0$ state as well.

In Sec. II we propose a method to measure the ratio $R(n)$ which is defined for any decay mode n as

$$R(n)=[\Gamma^S(n)]/[\Gamma^L(n)], \tag{1}$$

where $\Gamma^S(n)$ and $\Gamma^L(n)$ are the decay rates, into the final state n, of the short-lived and long-lived neutral K mesons, respectively.

In Sec. III we use the example of leptonic decays to show some of the conclusions that can be drawn from the knowledge of these decay rates.

We use, hereafter, the description of the K^0, $\bar{K}^0$ complex given by Lee, Oehme, and Yang.[2] According to them, if CPT invariance holds,

$$|K^S\rangle=\alpha|K^0\rangle+\beta|\bar{K}^0\rangle, \tag{2a}$$

$$|K^L\rangle=\alpha|K^0\rangle-\beta|\bar{K}^0\rangle, \tag{2b}$$

with $|\alpha|^2+|\beta|^2=1$. In the notation of Ref. 2,

$$\alpha=p/(|p|^2+|q|^2)^{1/2} \quad\text{and}\quad \beta=q/(|p|^2+|q|^2)^{1/2}.$$

(α and β can be multiplied by the same arbitrary phase.)

II. GENERAL DECAYS

The states that we shall be concerned with will be denoted by $|K^0(t)\rangle$ and $|\bar{K}^0(t)\rangle$. At time $t=0$, $|K^0(0)\rangle=|K^0\rangle$ and $|\bar{K}^0(0)\rangle=|\bar{K}^0\rangle$. The corresponding time dependences are given by

$$|K^0(t)\rangle=(1/2\alpha)[a(t)|K^S\rangle+b(t)|K^L\rangle], \tag{3}$$

and

$$|\bar{K}^0(t)\rangle=(1/2\beta)[a(t)|K^S\rangle-b(t)|K^L\rangle], \tag{4}$$

with

$$\begin{aligned} a(t)&=\exp(-im_St-t/2\tau_S),\\ b(t)&=\exp(-im_Lt-t/2\tau_L), \end{aligned} \tag{5}$$

where m_S and τ_S are the mass and lifetime of the component K^S, and m_L and τ_L are those of the component K^L.

Let us call $A^S(n)$ the amplitude for K^S to go into the final state n, and $A^L(n)$ that for K^L to go into the same final state n. The corresponding amplitudes for $K^0(t)$ and $\bar{K}^0(t)$ are given in terms of $A^S(n)$ and $A^L(n)$ thus:

$$\begin{aligned} A^0(n;t)&=(1/2\alpha)[a(t)A^S(n)+b(t)A^L(n)],\\ \bar{A}^0(n;t)&=(1/2\beta)[a(t)A^S(n)-b(t)A^L(n)]. \end{aligned} \tag{6}$$

The decay rates are then

$$\Gamma^0(n;t)=(L/4|\alpha|^2)\Big\{|a(t)|^2\Gamma^S(n)+|b(t)|^2\Gamma^L(n) +2\int \mathrm{Re}[a(t)b^*(t)A^S(n)A^{L*}(n)]d\rho\Big\}, \tag{7a}$$

$$\bar{\Gamma}^0(n;t)=(1/4|\beta|^2)\Big\{|a(t)|^2\Gamma^S(n)+|b(t)|^2\Gamma^L(n) -2\int \mathrm{Re}[a(t)b^*(t)A^S(n)A^{L*}(n)]d\rho\Big\}, \tag{7b}$$

where $\Gamma^S(n)=\int|A^S(n)|^2d\rho$ and $\Gamma^L(n)=\int|A^L(n)|^2d\rho$ are the decay rates into the mode n of the short-lived and long-lived components, and $d\rho$ is the phase-space volume element.

The decay rate $\Gamma^L(n)$ can be measured directly. Indeed, starting from a neutral kaon beam, one gets in vacuum a pure K^L beam, just by waiting for a time long relative to the lifetime of the short-lived kaon. To measure the ratio $R(n)$, we propose the following method:

(a) Experiments should yield the time dependence of the rates $\Gamma^0(n;t)$ and $\bar{\Gamma}^0(n;t)$, but not necessarily their absolute calibration. Also, it is enough to know these curves in two regions: one in which the time t is of the same order as the lifetime τ_S, or smaller, the other in

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[2] T. D. Lee, R. Oehme and C. N. Yang, Phys. Rev. **106**, 340 (1957).

which the time t is far greater than the lifetime τ_S. We denote the time variable in the latter region by T.

(b) One plots the quantities

$$\Gamma^{0\prime}(n;t)=\gamma\Gamma^0(n;t), \quad \text{and} \quad \bar{\Gamma}^{0\prime}(n;t)=\delta\bar{\Gamma}^0(n;t),$$

where γ and δ are only scale factors chosen in such a way that

$$\frac{\Gamma^{0\prime}(n;t)}{\bar{\Gamma}^{0\prime}(n;t)}\to 1 \quad \text{when} \quad t\to\infty. \tag{8a}$$

The practical requirement will be

$$\Gamma^{0\prime}(n;T)=\bar{\Gamma}^{0\prime}(n;T). \tag{8b}$$

We require relations (8) in order to replace the factors $1/|\alpha|^2$ and $1/|\beta|^2$ in Eqs. (7) by a common one.[3] This is possible, because for $T\gg\tau_S$

$$\Gamma^0(n;T)\approx 1/4|\alpha|^2\times e^{-T/\tau_L}\Gamma^L(n),$$

and

$$\bar{\Gamma}^0(n;T)\approx 1/4|\beta|^2\times e^{-T/\tau_L}\Gamma^L(n).$$

Taking into account Eqs. (7) and (8), and using the values (5) for $a(t)$ and $b(t)$, we get the following relation for any time t and any other time $T\gg\tau_S$:

$$\frac{\Gamma^{0\prime}(n;t)+\bar{\Gamma}^{0\prime}(n;t)}{\Gamma^{0\prime}(n;T)}=\frac{2(\Gamma^S(n)e^{-t/\tau_S}+\Gamma^L(n)e^{-t/\tau_L})}{\Gamma^L(n)e^{-T/\tau_S}}. \tag{9}$$

From Eq. (9) we deduce the third step.

(c) One plots the following quantity as a function of time t, having chosen a fixed time T:

$$P(t)=e^{t/\tau_S-T/\tau_L}\left[\frac{\Gamma^{0\prime}(n;t)+\bar{\Gamma}^{0\prime}(n;t)}{2\Gamma^{0\prime}(n;T)}-e^{-(t-T)/\tau_L}\right]. \tag{10}$$

This plot should be compatible with a straight line parallel to the time axis. This is a test for the exponential decay; in the model of Laurent and Roos[4] for instance, one should find a serious discrepancy in the region $6\tau_S\lesssim t\lesssim 8\tau_S$. If $P(t)$ fits a straight line, then the ratio $R(n)$ is given by

$$R(n)=P(t).$$

An advantage of this method is that it is independent of CP violation, and of any hypothesis on the interaction responsible for the decay. Note that this is not the case with the method used by Ely *et al.*[5] to measure the leptonic charge asymmetries of K^S and K^L. A further advantage is that one does not need to know the exact number of K^0 and $\bar{K}^0$ produced; it is only necessary to count the number of events where K^0 or $\bar{K}^0$ goes into the final state n after time t. If one could know the numbers of K^0 and $\bar{K}^0$ produced, then one could measure the ratio

$$|\alpha|^2/|\beta|^2=\bar{\Gamma}^0(n;T)/\Gamma^0(n;T) \quad \text{for} \quad T\gg\tau_S. \tag{11}$$

In a recent report, Gaillard[6] has suggested the use of neutral kaons produced in the $\bar{K}^0$ state, but she assumes the equality of α and β up to the precision of the measurements that she considers.

III. LEPTONIC DECAYS

As an example, we study the case of leptonic[7] decays. They are interesting because they yield a test of CPT invariance. Furthermore, since the $\Delta S=\Delta Q$ rule seems to hold to a good accuracy,[8] they can provide us with a good measurement of the ratio $|\alpha|^2/|\beta|^2$.

We denote the $\Delta S=\Delta Q$ amplitude by

$$f=A(K^0\to\Pi^-l^+\nu)=[A(\bar{K}^0\to\Pi^+l^-\bar{\nu})]^*, \tag{12}$$

and the $\Delta S=-\Delta Q$ amplitude by

$$g=A(\bar{K}^0\to\Pi^-l^+\nu)=[A(K^0\to\Pi^+l^-\bar{\nu})]^*. \tag{13}$$

The relations (12) and (13) are consequences of CPT invariance.[9] Writing $(-+)$ for $(\pi^-l^+\nu)$, and $(+-)$ for $(\pi^+l^-\nu)$, we have

$$A^S(-+)=\alpha f+\beta g, \tag{14a}$$

$$A^L(-+)=\alpha f-\beta g, \tag{14b}$$

$$A^S(+-)=\alpha g^*+\beta f^*, \tag{14c}$$

$$A^L(+-)=\alpha g^*-\beta f^*. \tag{14d}$$

Then the decay rates of K^S and K^L into the various leptonic modes are given by

$$\begin{aligned}
\Gamma^S(-+)&=\int[|\alpha|^2|f|^2+|\beta|^2|g|^2+2\,\mathrm{Re}(\alpha\beta^*fg^*)]d\rho,\\
\Gamma^S(+-)&=\int[|\alpha|^2|g|^2+|\beta|^2|f|^2+2\,\mathrm{Re}(\alpha\beta^*fg^*)]d\rho,\\
\Gamma^L(-+)&=\int[|\alpha|^2|f|^2+|\beta|^2|g|^2-2\,\mathrm{Re}(\alpha\beta^*fg^*)]d\rho,\\
\Gamma^L(+-)&=\int[|\alpha|^2|g|^2+|\beta|^2|f|^2-2\,\mathrm{Re}(\alpha\beta^*fg^*)]d\rho.
\end{aligned} \tag{15}$$

From these expressions, we deduce the relation

$$\Gamma^S(-+)+\Gamma^L(+-)-\Gamma^S(+-)-\Gamma^L(-+)=0. \tag{16a}$$

[3] At the same time, it allows us to work with a number of K^0 different from that of $\bar{K}^0$, even without knowing these numbers.

[4] B. Laurent and M. Roos, Phys. Letters **13**, 269 (1964).

[5] R. P. Ely *et al.*, Phys. Rev. Letters **8**, 132 (1962).

[6] M. K. Gaillard (unpublished).

[7] Leptonic decay here means either electronic or muonic decay, therefore l stands for either e or μ.

[8] Lagarrigue, communication in a seminar (unpublished).

[9] R. G. Sachs, Phys. Rev. **129**, 2280 (1963).

The quantities which can be precisely measured are the ratios $R(n)$ [see Eq. (1)] and $r=\Gamma^L(+-)/\Gamma^L(-+)$. In terms of these ratios, relation (16) reads

$$R(-+)-1+r[1-R(+-)]=0. \tag{16b}$$

If relations (16a) and (16b) are not satisfied, then CPT invariance does not hold in leptonic decays of neutral kaons. Indeed, we have taken into account the CPT invariance in the definition (2) of K^S and K^L,[2] and in the derivation of Eqs. (12) and (13).[9]

This test suffers from the following drawbacks:

(a) The CP invariance alone implies the relations

$$rR(+-)-R(-+)=0$$

and

$$r-1=0,$$

which relations have (16b) as a consequence. Hence, because of the weakness of CP violation,[10] any discrepancy from relation (16b) should be small.

(b) The phenomenological description of unstable particles through which these conclusions have been reached might be inadequate for dealing with a fundamental question such as CPT invariance.

In what follows, we assume CPT invariance to hold. Then, using Eqs. (15), we get the following relations:

$$\frac{2\Gamma^S(-)}{\Gamma^S(l)+\Gamma^L(l)}=\frac{2\Gamma^L(-)}{\Gamma^S(l)+\Gamma^L(l)}=\frac{(|\alpha|^2-|\beta|^2)(F-G)}{F+G}, \tag{17}$$

where

$$\Gamma^{S/L}(-)=\Gamma^{S/L}(-+)-\Gamma^{S/L}(+-),$$

$$\Gamma^{S/L}(l)=\Gamma^{S/L}(-+)+\Gamma^{S/L}(+-), \quad F=\int|f|^2d\rho,$$

and

$$G=\int|g|^2d\rho.$$

[10] J. H. Christenson *et al.*, Phys. Rev. Letters **13**, 138 (1964).

If the $\Delta S=\Delta Q$ rule is strictly valid, then $G=0$, and Eq. (17) provides us with a rigorous value of the scalar product $\langle K^S|K^L\rangle$; this value is given in terms of the ratios $R(n)$ and r by

$$\langle K^S|K^L\rangle=|\alpha|^2-|\beta|^2=\frac{2[R(-+)-rR(+-)]}{R(-+)+1+r[R(+-)+1]}=\frac{2(1-r)}{R(-+)+1+r[R(+-)+1]}. \tag{18}$$

Even in the case where the $\Delta S=\Delta Q$ rule is only approximate, but $G/F\ll 1$, Eq. (18) will still provide us with a very good value for the scalar product $\langle K^S|K^L\rangle$, since the relative error is equal to $2G/F$. More precisely, the ratio G/F may be as much as ten times bigger than the CP-violating term $|\alpha|^2-|\beta|^2$, the latter being of the order of 10^{-3}.[10] Note that, from this point of view, the situation is much less favorable when one measures the ratios of the decay rates into two pions. Indeed, the study of Wu and Yang[11] shows that a measurement of $R(\pi^+\pi^-)$ gives a good value of $|\epsilon|=|\alpha-\beta|/|\beta|$ only if $\mathrm{Im}A_2/A_0$ is much smaller than ϵ itself [the notation here is that of Ref. (11)]. Unfortunately our method gives only the real part of ϵ. Because of the weakness of the $\Delta S=\Delta Q$ violation, a measurement of $\mathrm{Im}(\epsilon)=2\,\mathrm{Im}(\alpha\beta^*)$ would be very difficult in leptonic decays.

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[11] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).