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$K_{\mu 3}$ Branching Ratio and μ^+ Energy Spectrum

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The muon energy spectrum of $K_{\mu 3}$ decay was studied in heavy liquid and H_2 bubble chambers. The results favor vector coupling with a small negative value of the real part of the ratio ξ of the two vector form factors. The $K_{\mu 3}$ branching ratio obtained is $3.45 \pm 0.20\%$.

I. INTRODUCTION

In the usual representation of leptonic decays, the matrix element is written as a product of two currents. In the case of K_{l3} decay we expect a product of the leptonic current with the scalar, vector, and tensor components of the strangeness-nonconserving hadron current. Previous experimental results have been compatible with the presence of the vector component with little or no mixing of the scalar component. The tensor component has not been detected experimentally. Keeping only the vector component, the matrix element can be written as[1]

$$[f_+(p_K+p_\pi)+f_-(p_K-p_\pi)]_\lambda u(p_\nu)\gamma_\lambda(1+\gamma_5)v(p_l)\,,$$

where the f_+ and f_- are the form factors, scalar functions of $q^2=(p_K-p_\pi)^2$, and p_K (p_π,p_ν,p_l) is the four-vector of the kaon (pion, neutrino, lepton). If the K_{l3} decay is invariant under time reversal, both f_+ and f_- are relatively real, i.e., have the same phase. Theoretically, it is expected that f_+ and f_- vary little over the pion energy range.[1,2] If these form factors are assumed constant the spectra can be written explicitly in terms of their ratio $\xi=f_-/f_+$. By assuming electron-muon universality and neglecting the small electron mass, a particularly simple expression then gives the ratio of the $K_{\mu 3}$ to K_{e3} branching ratio R[1]:

$$R=0.65+0.124\,\mathrm{Re}\xi+0.019|\xi|^2. \tag{1}$$

The recent discovery of the decay mode[3] $K_2{}^0 \to \pi^+\pi^-$ implies the possibility of noninvariance under time reversal of certain types of weak interactions. In the K_{l3} decay f_+ and f_-, and thus ξ, can be complex. The imaginary part of ξ can be directly measured by determining the muon polarization transverse to the decay plane. The spectra are less sensitive since $\mathrm{Im}\xi$ occurs only in $|\xi|^2$. However, Cabibbo[4] has pointed out that in the SU_3 limit, ξ should be purely imaginary.

In a previous paper[5] we presented a study of the μ^+ energy spectrum in the decay $K^+ \to \mu^+\pi^0\nu$. The lower part of the spectrum was obtained with K^+ stopping in the Saclay 81-cm hydrogen bubble chamber, the upper part from stops of K^+ re-emitted in interactions of an 800-MeV/c K^+ beam in the École Polytechnique 1-m chamber filled with a propane-freon (PF) mixture. The μ^+ spectrum was found to be consistent with vector coupling and a small value of the ratio of the two vector form factors, assumed constant and real. In the present paper we add significantly to the upper part of the spectrum and present a more detailed analysis of all the data. Possible contributions from $\mathrm{Im}\xi \neq 0$ and from variation of $\mathrm{Re}\xi$ with q^2 are checked. For the sake of completeness we discuss also the possibility of mixed scalar-vector couplings.

In Sec. V we compare our results with the previous experiments.

II. EXPERIMENTAL DETAILS

The new data added to the spectrum were obtained from photos of the same heavy-liquid bubble chamber as in Ref. 5, with the same entering beam, filled with a different mixture (30% in volume CF_3Br and 70% C_2F_5Cl, hereafter referred to as Freon 30–70).

The scanning criteria were:

(a) The K^+ decay apparently occurred at rest.

(b) The entire track of the secondary and the beginning of the decay-electron track were visible in three views.

(c) The charged secondary had a projected length in at least one view of at least 1 cm, to aid in distinguishing decays from scatters.

(d) No kink with a projected angle greater than 20° was visible along the secondary track in order to reduce the background of π scatters and decays in flight.

For each decay we recorded the number of correlated γ conversions and whether a π-μ decay was clearly visible at the stopping point. Very often the μ track is not distinguishable because it is very short (2.0 mm in PF and 1.7 mm in Freon 30–70). All the films were scanned at the same time for τ decays needed for normalization. All events were checked by a physicist.

The geometrical reconstruction, made with the RANGE

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program,[6] required that the entire track be within the fiducial volume. Geometrical corrections due to the finite size of the chamber were carried out with a Monte Carlo calculation, which evaluates for each event the probability that the μ^+ track is entirely contained in the fiducial volume. The inverse of this probability was the weight assigned to each event. The detailed description of the method is given in the Appendix of Ref. 7. It must be emphasized that this correction method is directly applied to the experimental spectrum and keeps account of stopping point and effects of multiple scattering and local magnetic field on the curvature.

The scan and measurement results are summarized in Table I.

TABLE I. Summary of scan and measurement results.

	Propane-Freon	Freon 30-70
No. of events that satisfy scan criteria	2080	4754
No. of above events inside fiducial volume with π-μ not visible and with range corresponding to muon kinetic-energy intervals 25–95 and 105–120 MeV	745	1430
Counted τ	2427	3490
τ measured to determine fraction in fiducial volume	333	375
Recognized and measured τ' in fiducial volume	289	468
Recognized $K_{\pi 2}$	519	1170
Recognized and measured $K_{\pi 2}$ in fiducial volume	376	361

III. BACKGROUND EVENTS

The accepted $K_{\mu 3}$ events are listed in Table I as the second entry. Corrections were made for the following: (1) $K_{\pi 2}$ and τ' decays at rest and in flight with no visible π-μ decay at the stopping point; (2) $K_{\pi 2}$ decays with the positive pion decaying in flight with a small projected angle; (3) $K_{\mu 2}$ decays in flight. The subtractions of these respective backgrounds were effected in the following ways:

(1) All the τ' and a large fraction (72% in PF and 31% in Freon 30–70) of $K_{\pi 2}$ events with a recognized positive pion were measured. The probability that the π-μ decay is visible was determined through inspection of the π^+ endings in the τ decays and was found to be 0.590±0.035 in Freon 30-70 and 0.665±0.019 in PF. From these figures,[8] and from the number of observed events with a visible π-μ decay, the contamination of τ' and $K_{\pi 2}$ events was directly determined. We excluded from the analysis the μ^+ energy interval between 95 and 105 where the $K_{\pi 2}$ background is excessive.

(2) This subtraction was analytically calculated as a function of the total π^+-plus-μ^+ range: the uncertainty due to the decay cut-off angle and to the probability that the pion interacts before decaying (~30%)[9] was taken into account in the errors which affect the subtraction.

(3) By inspection of the ionization and gap structure of K^+ decaying into an apparently momentum-balanced τ, we estimate that we cannot distinguish between a K^+ at rest and a K^+ with (5±2) cm of residual range. Thus the fraction of K^+ decaying in flight is 4.0±1.0% in Freon 30-70 and 4.2±1.1% in PF. This fraction is in good agreement with that determined from the shape of the range spectrum of the recognized $K_{\pi 2}$ decays. The spectrum of the μ^+ from $K_{\mu 2}$ decays was accordingly calculated and subtracted. We excluded from the analysis the μ^+ energy interval above 120 MeV, where the $K_{\mu 2}$ background is excessive.

A small correction was applied to the resulting spectrum to take into account the probability that a μ^+ suffered a large Coulomb scatter (>20° projected angle) and was thus rejected [scan criterion (d)].

IV. FORM-FACTOR ANALYSIS

The final corrected spectrum is shown in Fig. 1. The two heavy-liquid spectra (Table I) have been added. The low part of the spectrum (below 25-MeV μ^+ energy) is not well determined in the heavy liquids because of the τ' subtraction and the distortion due to scan criterion (c). We then prefer to use for the first intervals the data obtained in hydrogen (based on 670 accepted events with very small corrections) and published in Ref. 5. The normalization was based on the τ counts in H_2 and in the two heavy liquids.

We have fitted our experimental spectrum shape to different theoretical predictions with a standard least-squares method. In the fitting procedure we fix the parameter values and determine the best-fit normalization of the experimental to the corresponding theoretical spectrum. We obtain χ^2 values and best-fit normalization as a function of the parameters considered. The normalization values are then proportional to the $K_{\mu 3}$ decay rate. From this ratio and from the τ count in the film we then calculate, as a function of the parameters, the branching ratio $K_{\mu 3}/K_{e3}$ and compare it with the theoretical ratios obtained with the assumption of μ-e universality.

The τ and K_{e3} branching ratios used were 5.46 ±0.09%[10] and 4.7±0.3%.[11]

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[8] Slightly higher values were used for the $K_{\pi 2}$ subtraction because the π track in the $K_{\pi 2}$ dedays is on the average less dipping than in the τ decays, and thus the visibility of the π-μ decay is higher.

[9] The interaction cross section for a π^+ in the energy interval of a $K_{\pi 2}$ decay was evaluated with an optical model using the published values of $\pi^+ + P$ and $\pi^+ + N$ cross sections; the error on the cross section in our heavy liquid mixtures was estimated ~30%. The $K_{\pi 2}$ branching ratio used was 22.4±0.8%.

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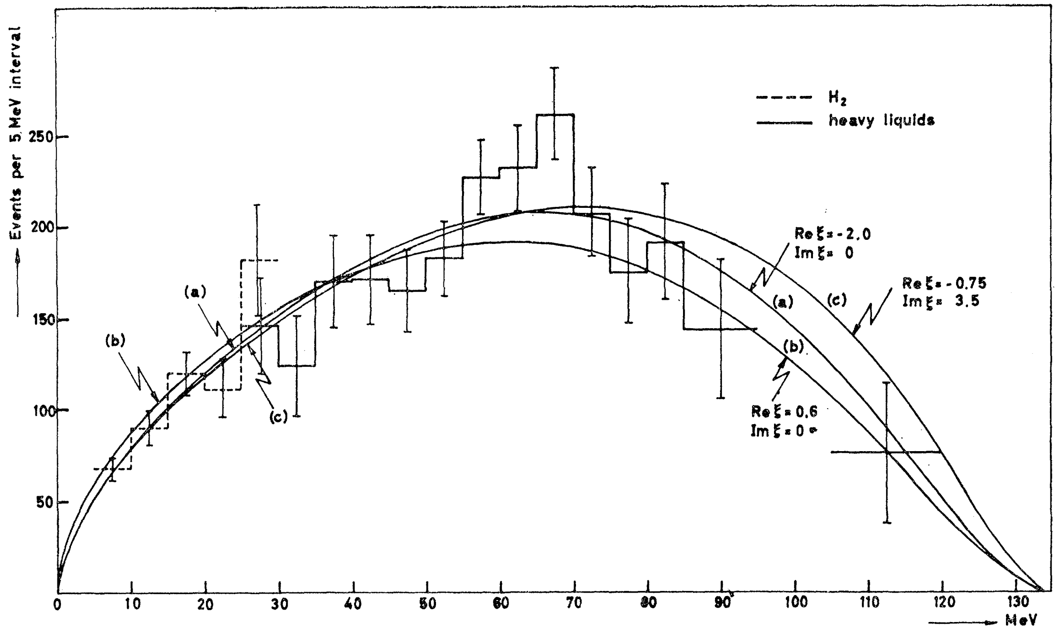

FIG. 1. Corrected muon kinetic-energy spectrum obtained in heavy liquid (solid line) and in hydrogen (dashed line). The curves are the best fits for the indicated real and imaginary parts of the ratio of constant vector form factors.

A. Vector Coupling. Constant Complex Form Factors

With complex values of the ratio $\xi = f_-/f_+$ the theoretical spectrum is[1]

$$F(E_\mu)dE_\mu = A(E_\mu) + B(E_\mu)\,\mathrm{Re}\xi + C(E_\mu)(\mathrm{Re}\xi^2 + \mathrm{Im}\xi^2)dE_\mu,$$

where E_μ is the μ^+ energy; A, B, C are functions of E_μ. For each set of values of the parameters Reξ and Imξ, the fitting process gives a χ^2 value. In Fig. 2 curves of constant χ^2 probability are shown as a function of the two parameters.

The branching ratios $K_{\mu3}/K_{e3}$ are shown in Fig. 3 for different values of Imξ and compared with the theoretical parabola given in Eq. (1). Because these ratios are proportional to the area under the corresponding best-fit spectral curves, they vary only slightly with parameter variation. The experimental error includes the errors on the τ count, on the τ and K_{e3} branching ratios and the error in the best-fit spectrum normalization. The values of Reξ and Imξ for which the theoretical parabolas and the experimental curves agree are shown in the shaded area of Fig. 2.

The highest χ^2 probability is obtained for Re$\xi = -2$ and Im$\xi = 0$ [curve (a) in Fig. 1]. The constraint of the $K_{\mu3}/K_{e3}$ branching ratio favors, however, for Im$\xi = 0$, Re$\xi = 0.6 \pm 0.3$ [curve (b) in Fig. 1] for which the χ^2 probability is 5–10%. This effect was already seen in

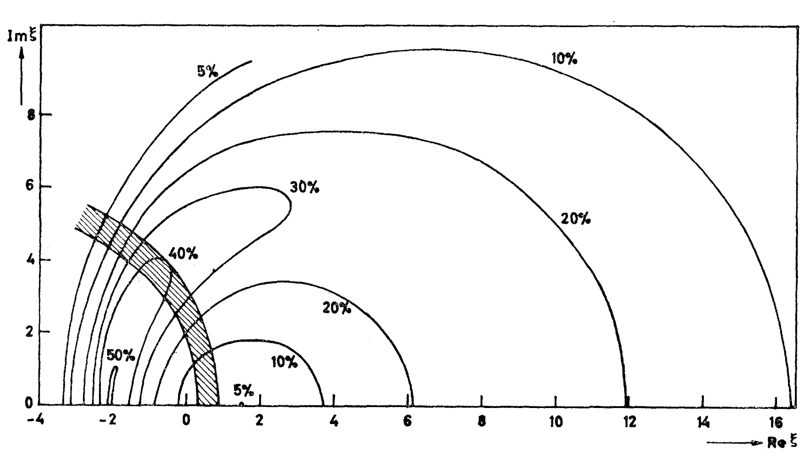

FIG. 2. Curves of constant χ^2 probability obtained from the fits to the experimental data as a function of Reξ and Imξ, both assumed constant. In the shaded zone the corresponding $K_{\mu3}/K_{e3}$ branching ratio is in agreement with μ-e universality.

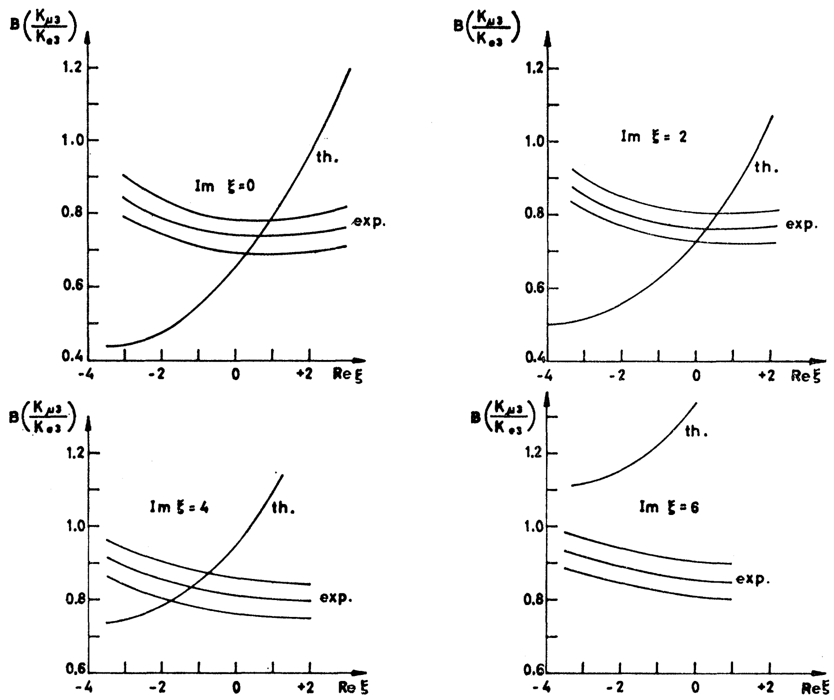

FIG. 3. Curves showing agreement of the $K_{\mu 3}/K_{e3}$ branching ratios as determined experimentally (exp) from the fit to the spectrum and theoretically (th) from μ-e universality [Eq. (1)] as a function of Reξ for several values of Imξ. The outer exp curves correspond to one standard deviation.

the previous paper.[5] The region in which at the same time the χ^2 probability is $\geq 40\%$ and the $K_{\mu 3}/K_{e3}$ branching ratio is compatible with the μ-e universality hypothesis, gives the following values: Re$\xi = -0.75 \pm 0.50$ and Im$\xi = 3.5 \pm 0.50$ [curve (c) in Fig. 1]. We feel, however, that this indication of a finite value of Imξ is not a strong one, because of the uncertainty in the branching ratios and the fact that, as already mentioned in the Introduction and as is evident from Fig. 1, the spectrum shape is not very sensitive to a small variation of the ξ values.

B. Vector Coupling. Energy-Dependent Form Factors

Assuming real form factors, we have tried to improve the χ^2 probability in the fit to the spectrum shape and the agreement with the branching ratio $K_{\mu 3}/K_{e3}$ by introducing an energy dependence of the form factors. This dependence was taken of the form[2]

$$f_{\pm}(q^2) = f_{\pm}(0)(1 + \lambda_{\pm} q^2/m_K^2), \qquad (2)$$

where $q^2 = (p_K - p_\pi)^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi$.

The branching ratio $K_{\mu 3}/K_{e3}$ was expected to be[2]

$$B(K_{\mu 3}/K_{e3}) = 0.65 + 0.13\xi + 0.019\xi^2 + 0.007\lambda_+ + 0.033\xi(\lambda_+ + \lambda_-) + 0.012\xi^2\lambda_-,$$

where now $\xi = f_-(0)/f_+(0)$.

With reasonably small values (≤ 0.2) for $\lambda_\pm$, we found no significant variation either in the χ^2 probability or in the agreement with the branching ratio with respect to the previous analysis.

C. Mixed Couplings

The pure scalar coupling with constant form factors is excluded with a high probability ($>99\%$).

We tried to determine the possible presence of a certain amount of scalar mixture with the vector coupling,[1] assuming real constant form factors. The ratio of the scalar (f_s) to the vector form factor f_+ and the ratio $\xi = f_-/f_+$ were used as the two independent parameters in the fit. The best solution is $f_s/f_+ = -1.1 \pm 0.3$ with $\xi = -1.0 \pm 0.5$ with 20% χ^2 probability for the spectral shape and in agreement with published K_{e3} decay rates. Good values of the χ^2 probability ($\sim 50\%$) were also found for f_s/f_+ varying between 0 and +0.5 and ξ between -2 and $+2$. For these values of the parameters, however, the $K_{\mu 3}/K_{e3}$ branching ratio is more than 4 standard deviations away from the values expected from μ-e universality.

D. Branching Ratio

From the observed number of $K_{\mu 3}$ (linear interpolation was used for the missing intervals), the $K_{\mu 3}$ branching ratio independent of spectral fit is found to be 3.45±0.20%.

Assuming for the K_{e3} branching ratio the value 4.7±0.3%, from Eq. (1) we obtain

$$\xi = 0.6 \pm 0.5 \quad \text{or} \quad \xi = -7.25_{-0.40}^{+0.50}.$$

V. CONCLUSIONS

In Table II we summarize the recent measurements of ξ in K^+ lepton decays based on the hypothesis of real, constant form factors and vector coupling.

TABLE II. Summary of recent determinations of ξ.

	Method	ξ value
Our preliminary result[a]	Branching ratio	$+0.3\pm0.8$ (or -7.1 ± 0.8)
	Spectrum	>-3
This experiment	Branching ratio	$+0.6\pm0.5$ (or -7.3 ± 0.5)
	Spectrum (for χ^2 probability $>5\%$)	>-3.3
Gidal *et al.*[b]	Polarization	-0.15 ± 0.90 (or -4.05 ± 0.75)
Jensen *et al.*[c]	Branching ratio	-0.2 ± 0.8 (or -6.5 ± 0.8)
	Spectra and angular correlation	$+0.6\pm2.0$
	Combined result	-0.08 ± 0.7
Brown *et al.*[d]	Combined result	$+1.8\pm0.6$
Boyarski *et al.*[e]	Spectrum	-7.6
	Polarization	>-4
Giacomelli *et al.*[f]	Spectrum up to 25 MeV and branching ratio	$+0.7\pm0.5$

[a] See Ref. 5.

[b] G. Gidal, W. M. Powell, R. March, and S. Natali, Phys. Rev. Letters **13**, 95 (1964).

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The results of our analysis for real, constant form factors, in agreement with most of the previous results, indicate clearly a ξ value very near to zero. The high positive ξ values allowed by the spectrum shape are excluded by the branching-ratio value, while the spectrum shape excludes the high negative value compatible with the branching ratio.

For the imaginary part of ξ, no published results are available up to now. Our data are not selective enough to draw any conclusion about *CP* or *T* violation.

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Quartet Scheme and Weak Interactions*

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A unified scheme of weak interactions has been given within the framework of $SU(4)$ by means of intermediate bosons. We assume that the fundamental entities—leptons, intermediate bosons, and quarks—are all quartets. The damping of the $|\Delta S|=1$ transitions has been introduced either through the breakdown of the strong-interaction symmetry or through a certain mixing of unphysical neutrinos. The present theory possesses a complete symmetry between leptons and hadrons and accounts for (i) the $|\Delta I|=\frac{1}{2}$ rule and (ii) the weak-interaction Hamiltonian transforming like the sixth component of the unitary octet for the $|\Delta S|=1$ part of the nonleptonic (noncharmed) decays. The problem of the absence of neutral lepton currents remains.

I. INTRODUCTION

THE group $SU(4)$ has been introduced recently as a possible basis for the eightfold way which avoids the puzzle of nonintegral charges.[1,2] It has also been proposed as a symmetry for strong interactions.[3–6] So far, the latter proposal does not seem to be successful, but it suggests a possible "hadron-lepton symmetry.[1,2,5,6]" It could be that the $SU(4)$ symmetry is so badly broken that only the noncharmed parts[5] (i.e., the parts with charm number zero) of its $SU(3)$ subgroups correspond to the actual dominant symmetry of the strongly interacting particles (hadrons); and therefore the $SU(4)$ group will not be a reasonable scheme for strong interactions. On the other hand, since there are four distinct kinds of leptons and the minimum number of "the elementary hadrons" will be four if the charges of the quarks[1] are nonfractional, it is quite plausible that the $SU(4)$ symmetry will give a unified scheme for weak interactions.

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