Interpretation of the κ in $K^+p \to K\pi\pi\pi p$*

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The $K\pi$ mass distribution in the reaction $K^+p \to K\pi\pi\pi p$ was computed using a triangle graph. A singularity of the triangle diagram is manifested as a peak in the mass spectrum at a $K\pi$ mass of slightly less than the $K^*\pi$ mass difference. A fit to the data at 3.0 GeV/c is obtained using a p-wave background constant interfering with the triangle amplitude. Some features which favor this description of the κ over a resonance explanation are (1) the absence of the κ bump in three- and four-body final states; (2) its isotropic production distribution; (3) the lack of $\kappa N^*\pi$ events; and (4) the decreasing magnitude of the peak with increasing incident energy. Two suggestions are made for further experimental analysis. Firstly, a study of the above reaction at energies lower than 3.0 GeV/c is predicted to show a similar κ enhancement (while at higher incident energies a diminution is anticipated). Secondly, the $K\pi\pi\pi$ mass distribution in the same five-body final state is predicted to show an enhancement at about 1080 MeV.

I. INTRODUCTION

RECENTLY, some attention has been given to the possibility that triangle singularities can manifest themselves as peaks in cross sections.[1,4–6] Many specific suggestions have been put forward. Anisovich and Dakhno[1] have made an impressive fit to all the curves of the π^-p experiment by Tripp *et al.*,[2] as well as to the pd experiment by Abashian, Booth, and Crowe[3] with a triangle mechanism having a logarithmic singularity. Chang and Tuan[4] have also considered what is essentially a logarithmic effect and have suggested looking at certain three-body final states in Kp and πp reactions. On the other hand, Halpern and Watson[5] emphasize that a singularity of the inverse-square-root type[6] might be more amenable to experimental investigation. They point out various possible four-body final states which should be looked at in this regard.

It is proposed here that the κ, as observed by Ferro-Luzzi *et al.*[7] in the five-body final states of K^+p collisions at 3.0 GeV/c, is a manifestation of a triangle singularity of the inverse-square-root type.

II. CALCULATION

In Fig. 1 is depicted the triangle graph to be computed. All the reactions to be considered can be represented by

$$K^+p \to p\pi\pi\pi K. \quad \text{(II.1)}$$

Let p and p', respectively, be the four-momenta of the incoming and outgoing protons; k is the four-momentum of the incident K^+, while k' and π_3 are those of the K meson and pion resulting from the "decaying κ."[8] π_1 and π_2 are four-momenta of the remaining pions. Define the following invariants:

$$\begin{aligned} W^2 &= -(p+k)^2, \\ V^2 &= -(\pi_1+\pi_2+\pi_3+k')^2, \\ w^2 &= -(\pi_1+\pi_2)^2, \\ v^2 &= -(\pi_3+k')^2. \end{aligned} \quad \text{(II.2)}$$

The masses are labeled μ, μ_K, m, and M for the pion, K meson, proton, and K^*, respectively; the complex K^* mass is

$$M_c = M - iI,$$

where I is the K^* half-width ($I = \Gamma/2 \simeq 25$ MeV).

The inverse square root singularity occurs at[5,6]

$$V = \mu + M_c, \quad \text{(II.3a)}$$

$$w = 2\mu, \quad \text{(II.3b)}$$

$$v = M_c - \mu. \quad \text{(II.3c)}$$

This means that a peak can occur in the cross section near

$$\begin{aligned} V &\gtrsim \mu + M, \\ w &\gtrsim 2\mu, \\ v &\lesssim M - \mu. \end{aligned} \quad \text{(II.3')}$$

It is the condition (II.3c) (which may be referred to as the "difference condition") on the mass v that is the underlying motivation for this work. One might infer from it that a bump can occur in the v mass distribution at a value a little below the $K^*\pi$ mass difference. It is clearly suggestive that this mass difference is about 750 MeV.

The invariant matrix element T for K^+p going to $K\pi\pi\pi p$ (excluding N^*, K^*, and ω events)[9] is thought of

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[1] V. V. Anisovich and L. G. Dakhno, Phys. Letters **10**, 221 (1964).

[2] J. Kirz, J. Schwartz, and R. D. Tripp, Phys. Rev. **130**, 2481 (1963).

[3] N. E. Booth, A. Abashian, and K. M. Crowe, Phys. Rev. Letters **7**, 35 (1961).

[4] Y. F. Chang and S. F. Tuan, Phys. Rev. **136**, B741 (1964).

[5] F. R. Halpern and H. L. Watson, Phys. Rev. **131**, 2674 (1963).

[6] D. B. Fairlie, J. Nuttall, P. V. Landshoff, and J. C. Polkinghorne, J. Math. Phys. **3**, 594 (1962). They refer to the singularity as a second type singularity, somewhat distinct from the logarithmic type. The latter is sometimes referred to as an s_b-type singularity.

[7] M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, G. R. Lynch, F. Muller, and J. M. Perreau, Phys. Letters **12**, 255 (1964).

[8] Any mention of the κ state should not be taken to mean a resonant state. The expression is just used to refer to the bump that is observed in the $K\pi$ mass distribution.

[9] However, in the $K\pi$ distributions calculated here, the N^* and ω events do contribute to the background interfering with the triangle mechanism of Fig. 1.

here as slowly varying (in practice as a constant), except for threshold behavior and for the effect induced by the triangle singularity of the diagram of Fig. 1. Firstly, as far as concerns threshold behavior it can be shown that in the final state created by the triangle mechanism of Fig. 1, the lowest partial waves are s wave between π_1 and π_2, s wave between the systems (π_1,π_2) and (π_3,K), and p wave between π_3 and K. If we retain only these lowest partial waves, the T martix can be written in the form

$$T=f(w,v,V)P_1(\cos\theta_{\pi K}),$$

where $\theta_{\pi K}$ is the polar angle of pion "3" with respect to some set of coordinate axes in the $K\pi_3$ center of mass. Although there is some ambiguity in including the full p-wave effect on f, one choice for a p-wave factor, following Gell-Mann and Watson[10] and Selleri,[11] is

$$A(q_{\pi K})=\frac{q_{\pi K}}{[(q_{\pi K})^2+a^2\mu^2]^{1/2}}, \tag{II.4}$$

where $q_{\pi K}$ is the relative πK momentum in their center-of-mass system. A has the properties

$$A\approx q_{\pi K} \quad \text{as} \quad q_{\pi K}\to 0,$$
$$A\to 1 \quad \text{as} \quad q_{\pi K}\to\infty. \tag{II.5}$$

In practice we use an equivalent form instead of A (obtained by retaining only the threshold behavior of $q_{\pi K}$ in terms of v):

$$\alpha=\left(\frac{v-(\mu+\mu_K)}{[v-(\mu+\mu_K)]+\frac{1}{2}\mu a^2}\right)^{1/2}. \tag{II.6}$$

The function α has the behavior (II.5).

In our calculations a was chosen as unity. (This is similar to the choices made by the above-mentioned authors.)

Finally, f can be written

$$f=\alpha f_\Delta, \tag{II.7}$$

where

$$f_\Delta=N(c+f_\Delta{}^s). \tag{II.8}$$

c is the p-wave background (considered here as a real constant); $f_\Delta{}^s$ is the "singular" part of the triangle

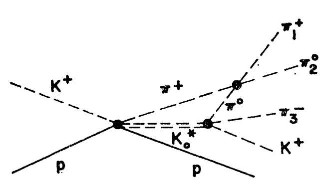

FIG. 1. Triangle mechanism for production of κ_0. Subscripts distinguish the outgoing pions and superscripts designate their charge. In the text, the left vertex is referred to as the "incident vertex," the upper one as the "sum vertex" and the one on the lower right as the "difference vertex."

[10] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

[11] F. Selleri, Phys. Letters 3, 76 (1962).

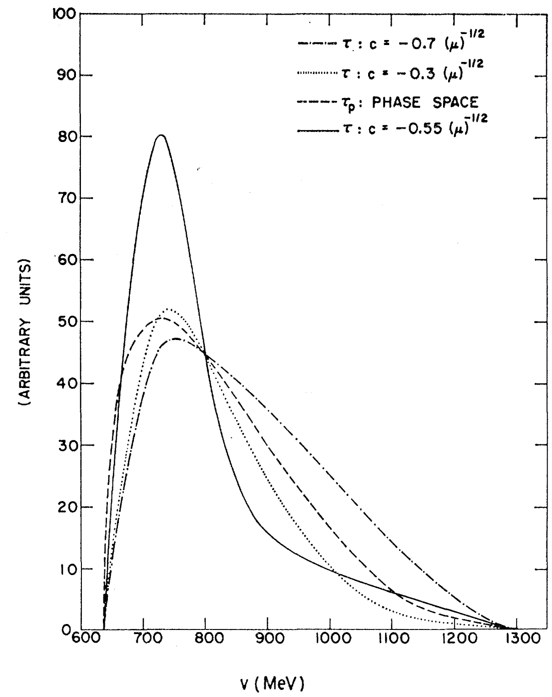

FIG. 2. $K\pi$ mass distributions at 3.0 GeV/c for different c values. τ is the triangle contribution to the $K\pi$ mass spectrum and is defined by Eq. (III.3); τ_P is phase space defined by Eq. (III.4); c is the p-wave background constant defined in Eq. (II.8); and v is the $K\pi$ mass.

diagram; and N is a normalization constant. Specializing the general result of Halpern and Watson[5] to the diagram of Fig. 1, one obtains

$$f_\Delta{}^s=[(w-2\mu)^2+(V-M-\mu)^2+(M-\mu-v)^2+2\Gamma^2]^{-1/4}. \tag{II.9}$$

Using a five-body phase space, the differential cross section in the energy variables is

$$d\sigma\approx|f|^2q\,dwdvdV, \tag{II.10}$$

where f is given by (II.7), (II.8), and (II.9), and

$$q=[(w-2\mu)(v-\mu_K-\mu)\times(V-v-w)(W-m-V)]^{1/2}. \tag{II.11}$$

It must be mentioned that only the threshold behavior that appears in the phase-space factor has been included. No significant alterations ensue if the full-momenta terms are retained.

III. RESULTS

The differential cross section $d\sigma/dv$ as a function of the $K\pi$ mass v and the incident center-of-mass energy W is

$$\frac{d\sigma}{dv}=\tau(v,W)=\frac{|N_T|^2}{4}\int_{v+2\mu}^{W-m}dV\int_{2\mu}^{V-v}dw\, q\alpha^2|c+f_\Delta{}^s|^2, \tag{III.1}$$

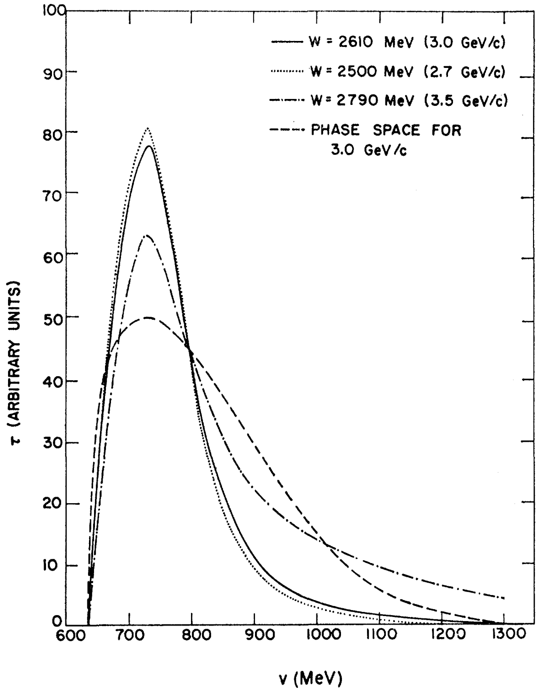

FIG. 3. $K\pi$ mass distributions with $c=-0.5(\mu)^{-1/2}$ at different incident energies. τ, c, and v are identified in the caption of Fig. 2. W is the total energy in the center-of-mass system.

where N_T is a normalizing factor. With the transformations

$$V-(W-m)=y^2[v+2\mu-(W-m)],\qquad w-2\mu=z^2(W-m-v-2\mu)\,, \tag{III.2}$$

τ can be written

$$\tau(v,W)=|N_T|^2(W-m-v-2\mu)^{7/2}\times(v-\mu-\mu_K)^{1/2}\alpha^2\int_0^1 y^2dy\times\int_0^{(1-y^2)^{1/2}} z^2dz(1-y^2-z^2)^{1/2}|c+f_\Delta{}^s|^2. \tag{III.3}$$

We intend, with this cross section, to fit the $K\pi$ mass distribution (excluding the K^*) for reaction (II.1). Because of the approximations made in the kinematics, the appropriate phase space for comparison is

$$\tau_P(v,W)=|N_P|^2(W-m-v-2\mu)^{7/2}(v-\mu-\mu_K)^{1/2}, \tag{III.4}$$

where N_P is a normalization constant.[12]

Figure 2 is a plot of v distributions for τ and τ_P at a center of mass energy of 2610 MeV, corresponding to an incident K^+ lab momentum of 3.0 GeV/c. The curves for τ are drawn for a few c values and all, including τ_P, are normalized to 45 events at 800 MeV. The peaking above phase space diminishes on either side of the value of $c=-0.55(\mu)^{-1/2}$. It is clear from these curves that with an appropriate choice of c an enhancement in the κ region can be obtained.

Also considered is the dependence of τ on the incident energy W. Figure 3 shows the τ curves for three W values, corresponding to K^+ lab momenta of 2.7, 3.0, and 3.5 GeV/c. The optimum c value is not too sensitive to W and for purposes of comparison the value $c=-0.5(\mu)^{-1/2}$ is used. The energy variation apparent in these curves is typical of triangle graphs, the peaking effect disappearing with increasing energy. There are two specific points: at 2.7 GeV/c the v distribution is not dissimilar to that at 3.0 GeV/c. There is a slight enhancement of the peaking. It is suggested thus that $K\pi\pi\pi p$ final states in K^+p interactions at energies below 3.0 GeV/c be adequately studied in an effort to observe the κ effect. Secondly, the peak position (at a v value of about 730 MeV) is 60% above phase space in the 3.0 GeV/c case whereas at 3.5 GeV/c this is reduced to about 25% above phase space. This result is consistent with data observed by the Wisconsin group[13] where no obvious evidence of the κ is found.

Finally in Fig. 4 a fit to the complete data of Ferro-

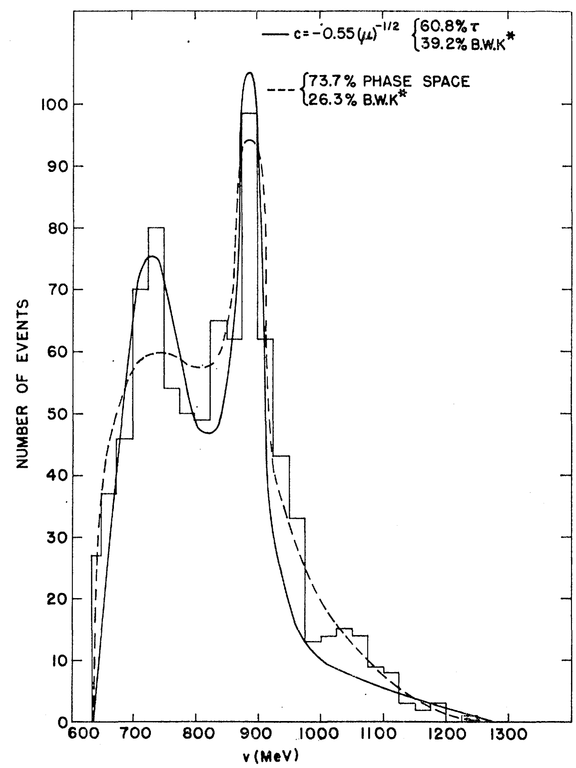

FIG. 4. Comparison with experimental data at 3.0 GeV/c. The experimental results are those of Ferro-Luzzi *et al.* (Ref. 7.) The dashed curve is composed of 73.7% (phase space) and 26.3% (Breit-Wigner for the K^*). The solid curve consists of 60.8% (triangle contribution) and 39.2% (Breit-Wigner for the K^*). c is the p-wave background constant defined in Eq. (II.8), and v is the $K\pi$ mass.

[12] In calculations performed here this phase space is very close to (less than 4% deviation from) the correct phase space.

[13] A. T. Goshaw, A. R. Erwin, W. D. Walker, and A. Weinberg, Bull Am. Phys. Soc. **10**, 90 (1965).

Luzzi *et al.*[7] is made with about 60% τ and 40% Breit-Wigner for the K^*. The κ enhancement is adequately described. On the other hand, if τ is replaced by a pure phase space term the best fit is very poor in the κ region.

IV. CONCLUSIONS

The $K\pi$ mass spectrum in the 3.0 GeV/c experiment has been fitted with a p-wave background constant interfering with the "singular" part of the triangle graph of Fig. 1. It is concluded that the κ peaking may not be a resonance, but rather the dynamical effect of a triangle singularity.

Although not a necessary consequence of the triangle graph, the absence of an isospin $\frac{3}{2}$ $K\pi$ enhancement can be fit by a suitable choice of the parameters at the "difference vertex." Further, the κ_+/κ_o production ratio of $\frac{4}{3}$ is obtained with either of the two choices

$$a_2/a_0 \sim +1.7 \quad \text{or} \quad -0.6,$$

where a_2 and a_0 are the $\pi\pi$ isospin 2 and 0 scattering lengths, respectively.

We now review some of the considerations, other than curve fitting, that have contributed to our interpretation of the κ peaking. What alternatives are open to us at present? As a basis for discussion, consider that either the κ is as we have suggested or it is a resonance. To choose between the two, we contrast the κ with the K^* (an undoubted resonance) in the reaction (II.1). We find that if it is a resonant state, the κ possesses at least four unusual characteristics:

(1) The κ is not observed in three- and four-body final states.[7,14] The K^*, on the other hand, provides a striking contrast, being copiously produced.[14]

(2) The production angular distribution for the κ is isotropic. This is in direct opposition to K^* production in the same final state. The K^* is produced mainly at low momentum transfers.[7]

(3) The κ is not produced together with an N^*. This is not true for the K^*, which is predominantly produced in association with an N^*.[7]

(4) At 3.5 GeV/c the Wisconsin group[13] fails to see the κ, while the K^* is abundantly produced.

Taken together these features give to the κ resonance, if indeed it is a resonance, a most peculiar nature. However, all these discords to the resonance conception for the κ become consistent details when the triangle mechanism is considered to be responsible for the peaking:

(1) Our triangle mechanism requires at least five particles in the final state. This can be seen in the following way. Firstly, in order that the κ shall be a manifestation of the $K^*\pi$ mass difference, the $K\pi$ mass must appear at the "difference vertex." Secondly, at around 3 GeV/c all of the incident energy cannot go into the triangle. This means that there must be an extra outgoing particle whose purpose it is to carry off some of the incoming energy. (This role is served by the outgoing proton in Fig. 1.) Thus, two particles are needed at the "difference vertex"; two are of course required at the "sum vertex"; and one more is needed to carry off some of the incident energy. Required then is a minimum of five particles.

(2) The triangle diagram is independent of the momentum transfer to the κ state. Thus this mechanism is consistent with the observed isotropic "production" distribution.

(3) As far as concerns the non-existence of $N^*\kappa\pi$ states, it is clear from an observation of Fig. 1 that there is no allowance made for πp interaction in the final state: The two pions at the sum vertex are "produced" separately from the proton. Generally speaking, then, with this mechanism, the number of πp combinations in the N^* range that appear together with a κ should be dictated by phase space alone. That this is in fact the case can be seen from the graphs of Fig. 2 of Ref. 7.

(4) The fourth point has been previously discussed. It has been shown that the κ effect as calculated here tends to disappear with increasing incident energy. As was previously remarked, this is consistent with the results of Goshaw *et al.*[13]

Another, perhaps critical prediction can be made. It is proposed that the four-meson mass (V) distributions should be studied with particular emphasis placed on the region $1020\text{ MeV} < V < 1120$ MeV. According to preliminary computations, involving a simple extension of formula (II.10), a noticeable enhancement above phase space may be observed at approximately 1080 MeV.

Finally, we comment on two points. Firstly, as has already been expressed by Halpern and Watson,[5] the triangle effect is distinct from a resonance in that it is observed as a "peak" and not a "band." By this is meant that a Dalitz-type plot of, say, $M(K\pi)$ versus $M(K\pi\pi\pi)$ should exhibit a cluster at a point rather than along a line. Specifically, we expect an enhancement simultaneously at $M(K\pi)\simeq 725$ MeV and $M(K\pi\pi\pi)\simeq 1080$ MeV. Similarly, one might look at such a plot with one of the axes being the mass of the two spectator pions, $M(\pi\pi)$—that is, those pions not involved in formation of the κ. From relation (II.3b), one expects to observe an effect for $M(\pi\pi)$ just above threshold. However, it should be remembered that in looking at the differential cross section as a function of $M(\pi\pi)$, the effect might be masked as a result of the falling phase space.[4,5]

Secondly, it is perhaps in order to say a few words about the influence of the K^* spin on the triangle graph.[15] For simplicity, consider the matrix element,

[14] G. R. Lynch, M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, F. Muller, and J. M. Perreau, Phys. Rev. Letters **9**, 359 (1964); M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, G. R. Lynch, F. Muller, and J. M. Perreau (to be published).

[15] A further study of this problem will appear in a forthcoming paper. The author would like to express his thanks to Professor C. Goebel for emphasizing this point.

apart from the propagators and an implied integration over d^4k, to be

$$\epsilon_{\mu\nu\alpha\beta}A_\mu p_\nu r_\alpha k_\beta,$$

where A is a 4-vector related to the incident state and the outgoing proton, p is the 4-momentum of the outgoing κ state, and k is the 4-momentum of the internal pion at the $K^* \to K\pi\pi$ vertex. Using the method of Feynman parametrization and symmetric integration (note that the integral over the internal loop is convergent), the amplitude can be shown to be

$$\epsilon_{\mu\nu\alpha\beta}A_\mu p_\nu r_\alpha q_\beta I,$$

where q is the 4-momentum of the two spectator pions, and I is essentially the scalar triangle graph. In the center-of-mass system of the κ, this becomes

$$p_0\mathbf{A}\cdot\mathbf{p}_K\times\mathbf{p}_{2\pi}I,$$

where $\mathbf{A}$ is a 3-vector (perhaps the momentum of one of the protons); and $\mathbf{p}_K$ and $\mathbf{p}_{2\pi}$ are, respectively, the 3-momentum of the outgoing K meson and the 3-momentum of the two spectator pions. Hence, the effect of the K^* spin can be included with a kinematical factor (a function of the external invariants) multiplying the scalar triangle graph. A further treatment will appear elsewhere.[15] In our actual calculations we have taken account of the scalar graph I and the momentum factor $\mathbf{p}_K$. Their effects are included in our expression for the amplitude (II.7).

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Strange-Particle Production in π^+-p Collisions*

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Strange particles produced in interactions of positive pions with protons have been studied with the Brookhaven 20-in. bubble chamber, which was exposed to π^+ beams of 2.35, 2.62, and 2.90 BeV/c. Cross sections are presented and the production of resonances is discussed. The outstanding feature of the multiparticle final states is that they are dominated by K-π, K-$\bar{K}$, and Y-π resonances. The isotopic spin of the φ is confirmed to be zero and no evidence is found for a φ decay into three pions.

APPROXIMATELY 600 strange-particle events[1] were found in an exposure of the Brookhaven 20-in. hydrogen bubble chamber to a π^+ beam at the AGS. There were 30 000 pictures taken at an incident π^+ momentum of 2.35 BeV/c, fifteen thousand at 2.62 BeV/c, and thirty thousand at 2.90 BeV/c. The reactions which were studied[2] are shown in Table I.

SCANNING, MEASURING, AND CLASSIFICATION TECHNIQUE

The film was scanned for all interactions in which a kink appeared in a track and/or one or more V's appeared. To each track an ionization number was assigned which designated whether the bubble density was light, medium, or heavy. From the ionization number and measured momentum alone, a π and K could be distinguished up to a momentum of 500 MeV/c and a π and proton could be distinguished up to 1 BeV/c. The details of this technique are described elsewhere.[3] Each of the events was digitized and processed through track reconstruction and kinematic fitting programs which tested for likely interpretations.

After imposing the conditions that the χ^2 probability for a hypothesis be greater than 5% and that the ionization code number be consistent with the momentum and mass, a logic program cataloged the events. The identification of an event was said to be ambiguous when it fit two or more hypotheses and one was not more than three times more likely than any other. Otherwise the identification was considered unambiguous and the most likely hypothesis was assumed to be

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[1] N. Gelfand and D. Berley, post deadline paper, American Physical Society, Washington meeting, 1963 (unpublished).

[2] Other studies of strange-particle production in π^+-p interactions are C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill, J. Sandweiss, J. R. Sanford, D. L. Stonehill, and H. D. Taft, Rev. Mod. Phys. **33**, 374 (1961); F. Grard and G. A. Smith, Phys. Rev. **127**, 607 (1962); F. Crawford, F. Grard, and G. A. Smith, *ibid.* **128**, 368 (1962); H. W. Foelsche, H. L. Kraybill, and J. R. Sanford, Bull. Am. Phys. Soc. **8**, 342 (1963); S. S. Yamamoto, L. Bertanza, G. C. Moneti, D. C. Rahm, and I. O. Skillicorn, Phys. Rev. **134**, B383 (1964).

[3] C. Alff, D. Berley, D. Colley, N. Gelfand, U. Nauenberg, D. Miller, J. Schultz, J. Steinberger, T. H. Tan, H. Brugger, P. Kramer, and R. Plano, Phys. Rev. Letters **9**, 322 (1962). N. Gelfand, thesis, Columbia University (unpublished).