($p,p'\gamma$) Angular Correlations in Ti^{48}†

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The angular correlations between the protons inelastically scattered from the first excited state of Ti^{48} and the associated de-excitation gamma rays have been measured as absolute second differential cross sections at bombarding energies of 4.63 and 5.25 MeV. These measurements were made both in the scattering plane and at angles of 30, 45, 60, and 90° relative to the scattering plane. Special attention was directed to studies of the symmetry predictions of the compound nucleus and direct interaction processes, and comparison is made with the statistical-theory computations of Sheldon. For a given proton detector angle, the azimuthal symmetries agree in shape though not in magnitude with statistical-theory predictions. However, results for complementary proton-detector angles are incommensurate with the symmetries predicted by statistical theory.

INTRODUCTION

In recent years there have been a number of advances, both experimental and theoretical, in the use of angular correlations of inelastically scattered nucleons and their associated gamma rays for the study of nuclear reaction mechanisms. Most of the experimental evidence accumulated to date has been at somewhat high energies[1] and in the first instance has, in general, been compared with the predictions of direct interaction (DI) theory. However, in a recent review article by Sheldon,[2] it is demonstrated that much of the data may be fit equally well by the predictions of statistical compound nucleus (CN) theory. Sheldon further points out that the majority of the angular-correlation data have been obtained as relative intensities, whereas the measurement of the absolute double differential cross sections would provide more stringent criteria for diagnosing the mechanism, and also for testing theoretical symmetry predictions.

In this paper we wish to present the results of a series of absolute measurements of double differential cross sections for the correlation between the protons inelastically scattered from the 0.99-MeV first excited state of Ti^{48} and their associated de-excitation gamma rays. The above element was selected because its mass is sufficiently high to offer reasonable assurance that the statistical assumptions of the theory are valid for the compound nucleus V^{49*} at the excitation energies under consideration. At the same time the 4.55-MeV Coulomb barrier of Ti^{48} permitted us to make investigations at incident proton energies appreciably above the barrier as well as in the vicinity of the barrier. These measurements, at bombarding energies of 5.25 and 4.63 MeV, were made not only in the reaction plane but also at a

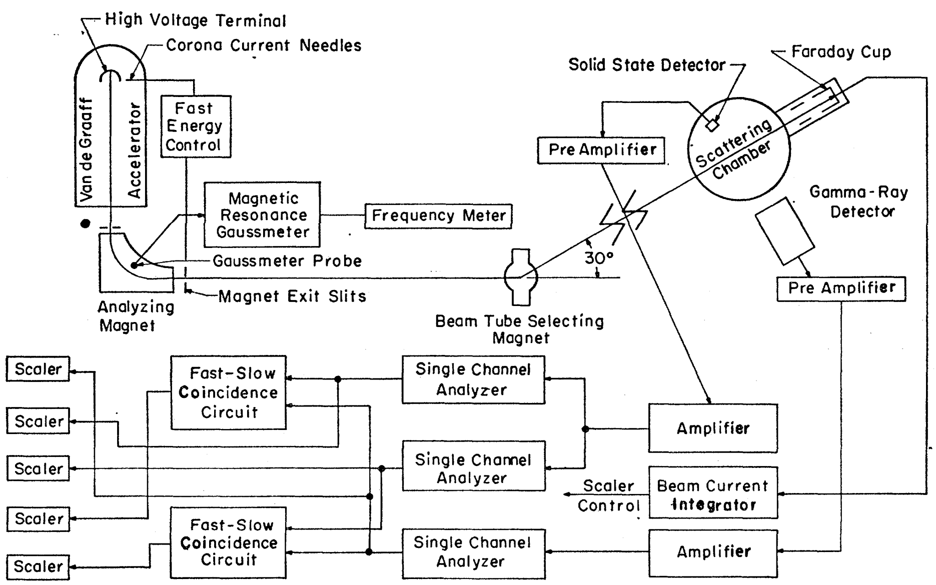

FIG. 1. Experimental arrangement showing the use of the fast-slow coincidence circuit.

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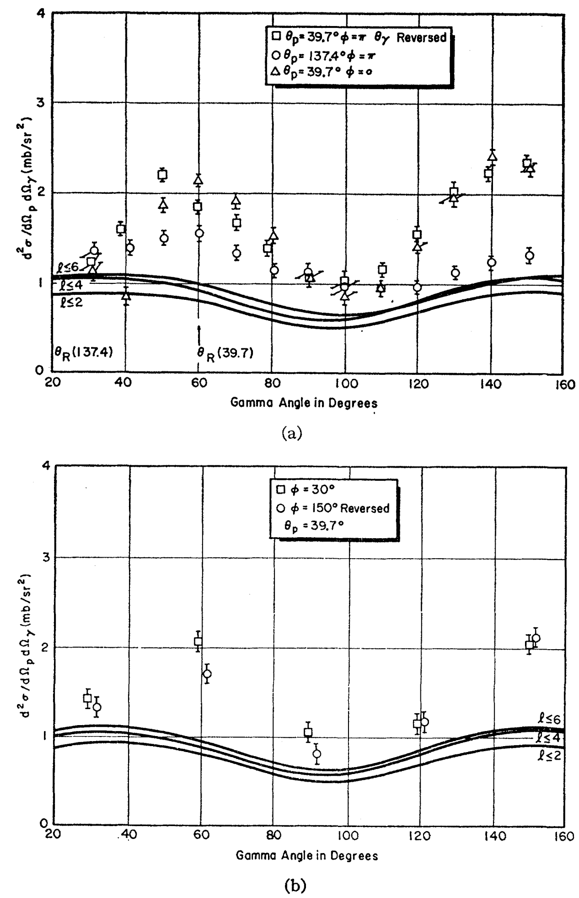
[1] For example: B. Gobbi, R. E. Pixley, and E. Sheldon, Nucl. Phys. **49**, 353 (1963); H. Hulubei *et al.*, Phys. Rev. **126**, 2174 (1962); F. D. Seward, *ibid.* **114**, 514 (1959); H. F. Bowsher, G. F. Dell, and H. J. Hausman, *ibid.* **121**, 1504 (1961).
[2] E. Sheldon, Rev. Mod. Phys. **35**, 795 (1963).



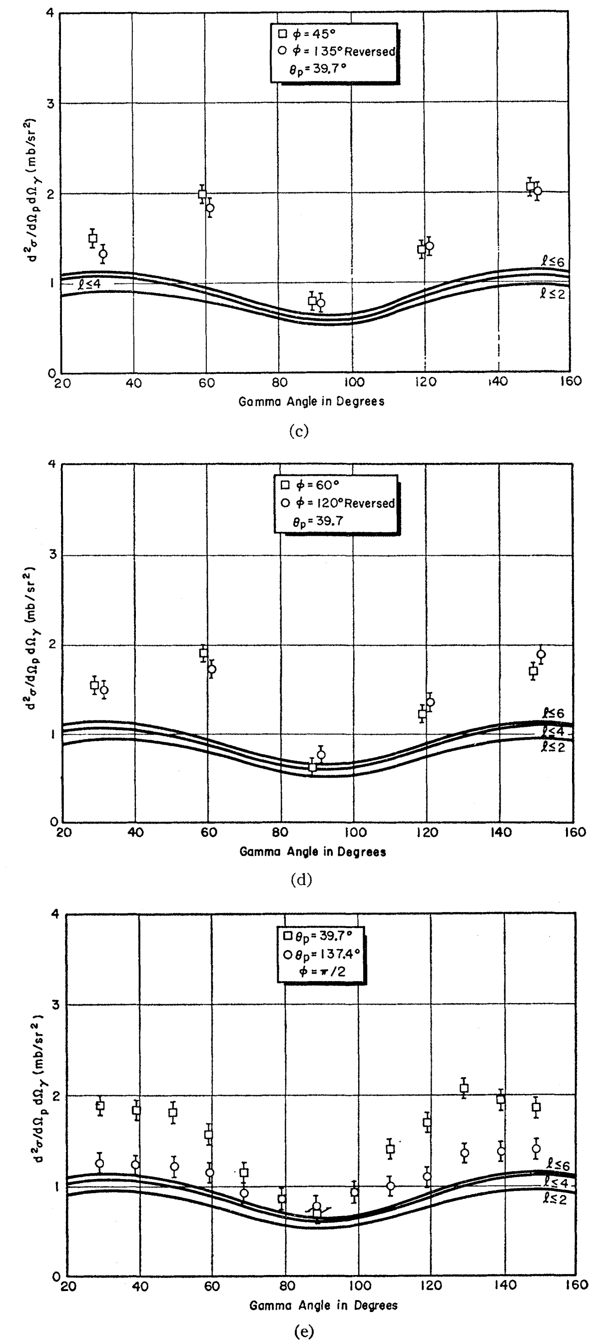


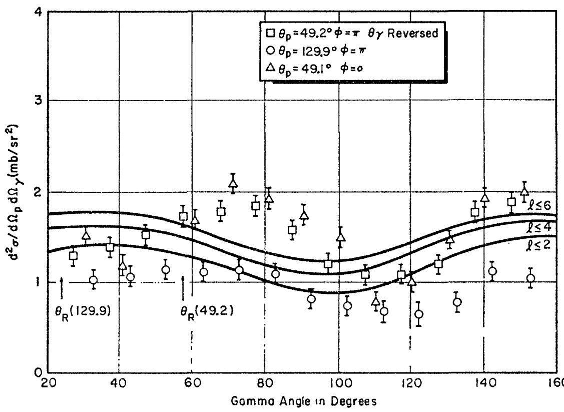
FIG. 2. Angular correlations at 4.63 MeV for gamma-ray angles φ of 0, 30, 45, 60, and 90. The abscissa is reversed for the indicated curves. Figures 2(b)–2(d) are taken from the same data as Fig. 4. The solid curves are the calculated absolute double differential cross sections using Sheldon's statistical compound-nucleus formalism.

number of gamma-ray angles out of the reaction plane in order to test the predicted azimuthal variations.

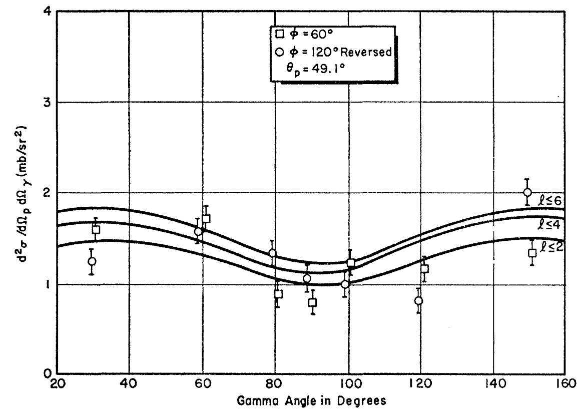
EXPERIMENTAL APPARATUS

The experimental arrangement is shown in Fig. 1. The beam from the O.S.U. 5.5-MeV Van de Graaff accelerator is magnetically analyzed and stabilized to 0.1% energy resolution. The beam is then bent through a switching magnet into the scattering chamber, where the beam is collimated to a spot 0.040 in. in diameter before striking the target. The beam is then collected by the Faraday cup at the rear of the scattering chamber and integrated by a 0.1% current integrator.

The scattering chamber is a 0.063-in.-thick stainless steel cylinder 10 in. in diameter with a hemispherical top to present uniform attenuating thickness to the gamma rays. A solid-state detector which is capable of being remotely positioned to within ±0.1° is contained in the chamber for detection of the scattered charged



(a)



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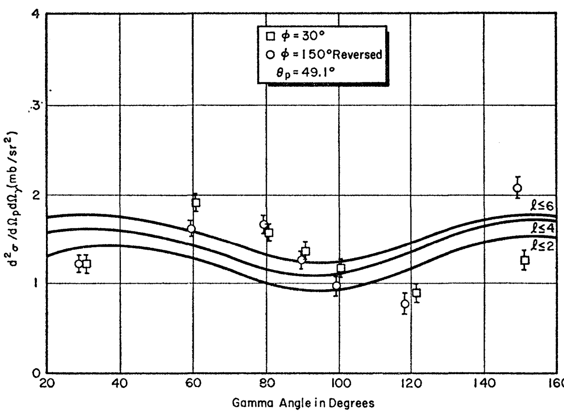
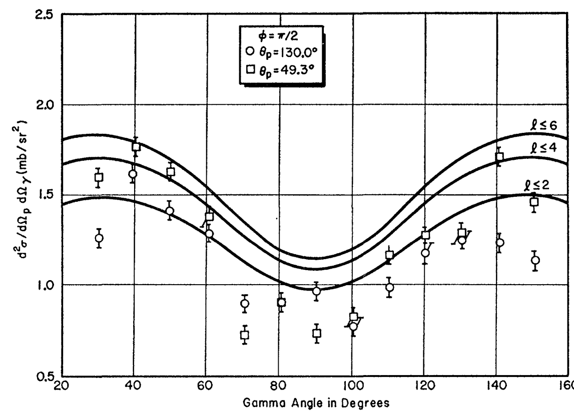
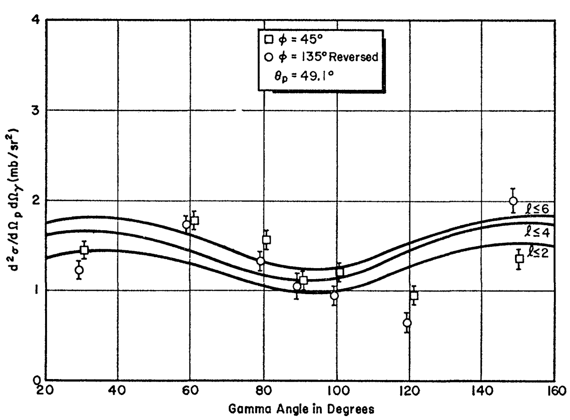


(b)



(e)



(c)

FIG. 3. Angular correlations at 5.25 MeV for gamma-ray angles φ of 0, 30, 45, 60, and 90°. The abscissa is reversed for the indicated curves. Figures 3(b)–3(d) are taken from the same data as Fig. 5. The solid curves are the calculated absolute double differential cross sections using Sheldon's statistical compound nucleus formalism.

particles. This detector is collimated to subtend 1.2×10^{-3} sr. The gamma rays are detected by a remotely positioned 3-in.×3-in. NaI scintillator mounted on an RCA 8054 multiplier phototube located externally to the scattering chamber with the front face 7 in. from the center. The positioning of this detector is accurate to ±0.5°.

After being preamplified at the scattering chamber, the signals from both detectors are further amplified in the accelerator control room by cosmic double delay line amplifiers which are part of the cosmic modular coincidence circuit. With a resolving time of 45 nsec, a mean ratio of true to accidental counts of about 5:1 was obtained.

The target was a self-supporting foil obtained as a separated isotope enriched to 99.36% Ti^{48}. The thickness was determined in two ways: by weighing a representive portion, and by determining the apparent shift in the Li(p,n) threshold when the foil was inserted in the beam. The weighing method gave an average thick-

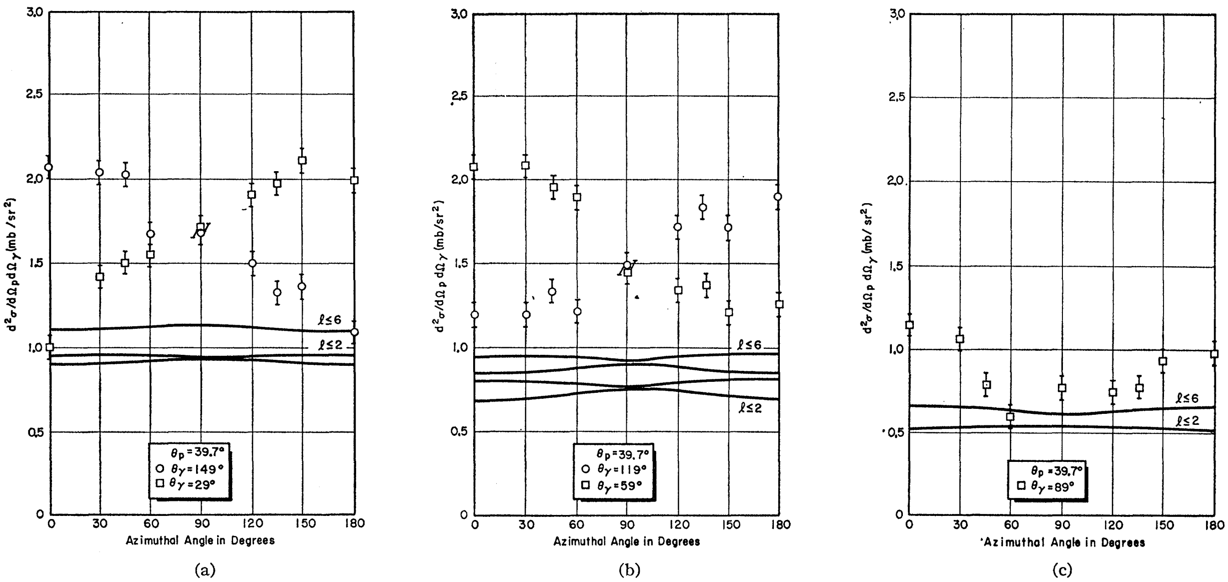


FIG. 4. The φ dependence of the angular correlation at 4.63 MeV. The solid curves are the calculated absolute double differential cross sections using Sheldon's statistical compound-nucleus formalism.

ness of 0.929 ± 0.016 mg/cm² while the threshold shift indicated a thickness of 0.917 ± 0.037 mg/cm². Thus, the thickness was taken as 0.923 ± 0.040 mg/cm² or approximately 40 keV at 4.6 MeV.

THEORY

The basic formulation of the statistical CN correlation theory by BiedenhBiedenharn and Rose[3] and Satchler[4] assumes that a sufficient number of states of a compound nucleus are excited to ensure effective cancellation of interferences between nuclear states as well as between the various incoming and outgoing partial waves, and that the transition probabilities depend only upon the transmission coefficients. Following this formulation and adding the correction that no terms of the form $j_i^{(\pm)}j_i^{(\mp)}$ are allowed, Sheldon[2] has shown that for a transition of the form $J_0(j_1^{(\pm)})J_1(j_2^{(\pm)})J_2(L,L')J_3=J_0$, the correlation function may be written

$$W(\theta_p,\theta_\gamma,\phi)\propto\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}=\text{const}\sum gA_\mu(J_0J_1)R_{\mu\nu\lambda}(J_1J_2)\times A_\lambda(J_2J_3)S_{\mu\nu\lambda}(\theta_p,\theta_\gamma,\phi)\,,\quad(1)$$

where the coordinate system chosen is a right-handed system with the z axis along the incident particle direction and the y axis along the normal to the scattering plane, $\mathbf{k}_i\times\mathbf{k}_f$.

For the inelastic scattering of a proton from the 2^+ excited state of a nucleus possessing a 0^+ ground state, Eq. (1) becomes

$$W(\theta_p,\theta_\gamma,\phi)\propto\frac{d^2\sigma}{d\Omega_p d\Omega_\gamma}=\frac{5\lambda\!\!\!^{-2}}{32\pi}\sum\delta_{(\pm)}NCX\tau S_{\mu\nu\lambda}\ \text{mb-sr}^{-2}\,,\quad(2)$$

where the summation is over μ, ν, λ, J_1, and $j_2^{(\pm)}$, the $\delta_{(\pm)}$ excludes "mixed-j" states, and the other terms are defined as

$$N=(-)^{J_1+j_2^{(\pm)}}(2J_1+1)^2(2j_2^{(\pm)}+1)\,,\quad(3)$$

$$C=(J_1J_1\tfrac{1}{2}-\tfrac{1}{2}|\mu0)(j_2^{(\pm)}j_2^{(\pm)}\tfrac{1}{2}-\tfrac{1}{2}|\nu0)\times(2\,2\,1-1|\lambda0)\,,\quad(4)$$

where $(j_1j_2m_1m_2|jm)$ is the Clebsch-Gordan coefficient,

$$X=X(J_1J_1\mu;\,j_2^{(\pm)}j_2^{(\pm)}\nu;\,2\,2\,\lambda)$$

= the Fano X coefficient which is identical to the Wigner 9-j symbol, (5)

$$\tau=T_{l_1}^{(\pm)}(E_1)T_{l_2}^{(\pm)}(E_2)\sum{}'_{jE}\,T_l^{(\pm)}(E)\,,\quad(6)$$

$S_{\mu\nu\lambda}=$ the Legendre hyperpolynomials.[2] (7)

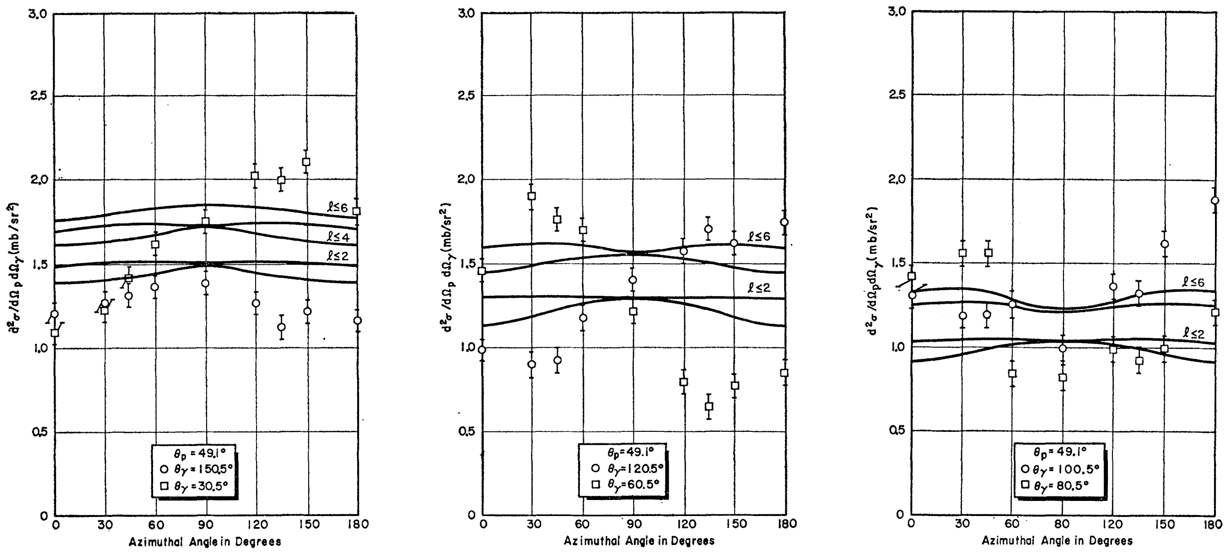
At the energies encountered in this experiment, compound nucleus formation is expected to be predominant. For comparison we may note that the general form of the correlation is

$$W\propto P+Q\sin^2 2(\theta+\epsilon_1)+R\sin^2(\theta+\epsilon_2)\,,\quad(8)$$

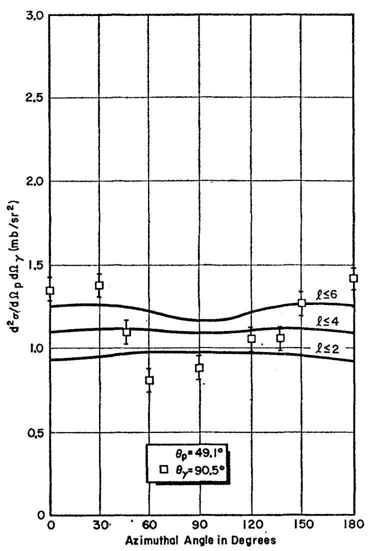
where P, Q, R, ϵ_1, and ϵ_2 depend upon the model. In the DI case we may note that $R=0$ in the absence of spin-flip and $P=R=0$ in the plane-wave approximation.

[3] L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. **25**, 729 (1953).

[4] G. R. Satchler, Phys. Rev. **94**, 1304 (1954); **104**, 1198 (1956); **111**, 1747 (1958).



(a) (b) (c)



(d)

FIG. 5. The φ dependence of the angular correlations at 5.25 MeV. The solid curves are the calculated absolute double differential cross sections using Sheldon's statistical compound-nucleus formalism.

RESULTS

The double-differential cross sections at 4.63 MeV, an energy corresponding to a peak in the excitation curve, are shown in Fig. 2(a). The complementary (in the c.m. system) proton laboratory angles in this case are 39.7° and 137.4° with the gamma-ray detector located in the scattering plane. The results for a laboratory proton angle of 39.7° with the gamma detector at various fixed angles ϕ with respect to the scattering plane are shown in Figs. 2(b)–2(e). The error bars indicate only the statistical uncertainties of the points. In addition, the absolute magnitude contains an uncertainty of $\pm 7\%$ due to uncertainties in target thickness ($\approx 4.3\%$), gamma-ray detector efficiency ($\approx 5\%$), and beam integration ($\approx 0.42\%$), the latter including effects of the integrating device and target-foil scattering of the beam. The solid curves are the theoretical predictions of the IBM 7090 correlation program ERICA.

Figure 3 shows the double differential cross sections at 5.25 MeV, which corresponds to a valley in the excitation curve. The laboratory proton detector angles in this case are 49.2° and 129.9°, corresponding to complementary angles in the center-of-mass system. In addition to statistical uncertainties, the absolute magnitude of the double differential cross sections contains an uncertainty of $\pm 11\%$ at this energy due to the use of an older model current integrator.

In Figs. 4 and 5 are shown the azimuthal variations of the double differential cross sections at a number of gamma-ray angles for protons scattered in the forward direction. The uncertainties in the ordinate in these cases are the same as those for the respective energies in Figs. 2 and 3. Due to the extremely small variation predicted by the CN theory, only the $l_{1\max}=l_{2\max}\leq 2$ and ≤ 6 curves are shown.

The penetrabilities used in computing all of the curves shown were provided by Dr. R. H. Bassel[5] of ORNL using the IBM 7090 optical-model program LEONORA. This program uses a Woods-Saxon potential with a Thomas-type spin-orbit interaction and a Coulomb potential characteristic of a uniformly charged sphere. The parameters used in these calculations were

$$V_0 = 50\text{ MeV}, \qquad R_c = 1.36\text{ F},$$
$$W_0 = 5\text{ MeV}, \qquad a = 0.518\text{ F},$$
$$R_0 = 1.33\text{ F}, \qquad V_S = 3.50\text{ MeV}.$$

DISCUSSION

A cursory examination of Figs. 2 and 3 shows that the predictions of the CN theory are in reasonable

[5] R. H. Bassel, R. Drisko, and G. R. Satchler, Oak Ridge Report ORNL-3240, 1962 (unpublished).

agreement with the experimental results especially for (1) those values in the normal ($\phi=\pi/2$) plane at 5.25 MeV at either proton angle, and (2) the normal ($\phi=\pi/2$) plane values at 4.63 MeV at the backward-scattered proton angle.

Considering the symmetry predictions of the various models[6] in these cases, it can be seen that the model-independent symmetry

$$W(\theta_p,\theta_\gamma,0)=W(\theta_p,\ \pi-\theta_\gamma,\ \pi) \tag{9}$$

holds at both energies, but that neither the CN symmetry

$$W(\theta_p,\theta_\gamma,0)=W(\theta_p-\pi,\ \theta_\gamma,\ 0) \tag{10}$$

nor the DI symmetry

$$W(\theta_p,\theta_\gamma,0)=W(\theta_p,\ \theta_\gamma\pm\pi/2,\ 0) \tag{11}$$

holds true, although the data for checking the latter is (admittedly) rather sparse. Furthermore, the symmetry about the classical recoil angle predicted by the plane wave DI approximation does not hold, although there does appear to be a tendency to have a symmetry axis in the vicinity of the recoil angle for back-scattered protons.

In the planes other than the reaction plane, the issue is not as clear cut since the DI predictions for these cases have not been studied in any detail. However, the symmetry prediction which is known to hold for the CN model

$$W(\theta_p,\theta_\gamma,\phi)=W(\theta_p,\ \pi-\theta_\gamma,\ \pi-\phi) \tag{12}$$

is well obeyed except for the unexplained breakdown involving $\theta_\gamma=30$ and 150° at 5.25 MeV.

The investigations in the $\phi=\pi/2$ plane appear especially interesting owing to the failure of the CN symmetry prediction

$$W(\theta_p,\theta_\gamma,\pi/2)=W(\pi-\theta_p,\ \theta_\gamma,\ \pi/2) \tag{13}$$

at 4.63 MeV while it is upheld at the higher energy of 5.25 MeV. We also see, however, that the CN predictions

$$W(\theta_p,\pi/2,\pi/2)=W(\pi-\theta_p,\ \pi/2,\ \pi/2) \tag{14}$$

and

$$W(\theta_p,\theta_\gamma,\pi/2)=W(\theta_p,\ \pi-\theta_\gamma,\ \pi/2) \tag{15}$$

are fulfilled within statistics. A possible explanation may be offered by noting that in the absence of spin-flip

$$W(\theta_p,\pi/2,\pi/2)=0. \tag{16}$$

This is included in the calculation for the CN model. However, for the DI model it is treated separately. In addition, Eq. (15) is also fulfilled for DI without spin-flip. Since DI predicts a peaking of the proton distribution in the forward quadrant, the DI contribution might be expected to be larger here than for the complementary angle in the backward quadrant. Thus, the difference in the values may be a measure of the contribution of a DI term (see also Gobbi *et al.*[1]). This discrepancy may thus reveal the presence of considerable direct interaction even at a bombarding energy near the Coulomb barrier. This phenomenon is being investigated in more detail and the results of these investigations, along with the excitation curve, will be published in a later paper.

We believe that the over-all agreement between the predictions of the statistical compound-nucleus theory and the measured angular correlations is quite good. The theoretical absolute double differential cross sections are within a factor of two of the measured cross sections despite the failure of the experimental conditions to completely satisfy those assumed in the theory. In particular, the measured yield curves show a resonance-like behavior over the energy range of the experiment which might indicate that a reasonable statistical averaging over compound-nucleus states did not occur. Furthermore, the presence of direct reactions would tend to destroy the predicted symmetries and also affect the absolute cross sections.

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[6] E. Sheldon, Phys. Letters **2**, 178 (1962).