Meson States in a Nonlinear Spinor Model of Elementary-Particle Theory with Spontaneous Symmetry Breakdown*

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(Received 8 April 1965)

We consider a self-coupled spinor model of elementary particles having $SU_2\otimes SU_2$ symmetry. We demonstrate that for the case of complete spontaneous breakdown of the isospin symmetry the bubble approximation gives a method for treating the meson states that is compatible with symmetries of the model in that it gives the Goldstone bosons and Goldberger-Treiman relations exactly. We also show that by a continuation in the parameter characterizing the isospin breakdown we can transform from a solution having isospin degeneracy to solutions in which the isospin, as well as the chiral symmetry is spontaneously broken. Calculational techniques for our approximation are explicitly given.

IN the preceding paper we consider the spontaneous breakdown of unitary octet symmetry in a nonlinear spinor model of elementary-particle theory.[1] That model incorporated the symmetry $SU_3\otimes SU_3$. We gave simple arguments for the existence of various bosons. The present paper is a more detailed study of the meson states in such a theory using a simpler but analogous Lagrangian which is invariant under $SU_2\otimes SU_2$.

We consider nonperturbative self-consistent solutions having a high degree of symmetry breakdown and identify as mesons the freely propagating collective excitations of zero baryon number. In calculating the properties of these collective excitations, we need a simple approximation which is compatible with the over-all self-consistency of our solutions. We want to demonstrate that the description of the meson states as chains of baryon-antibaryon *bubbles* in the lowest order dynamical approximation in the Bethe-Salpeter equation is the appropriate method for treating the mesons so as to maintain the over-all self-consistency of solutions of our model where the fermion mass is treated in the lowest order dynamical approximation. Nambu and Jona-Lasinio showed this for the case of incomplete spontaneous breakdown of $SU_2\otimes SU_2$ symmetry where the chiral (γ_5) but not the unitary (isospin) symmetry was broken.[2] We will demonstrate this ***bubble approximation*** for the case of more complete symmetry breakdown where the unitary symmetry, as well as the chiral symmetry, suffers spontaneous breakdown. The generalization is straightforward but not entirely trivial.

If the results of our calculations are to represent solutions of our model that are generally valid beyond the restrictions of our particular dynamical approximation, in that higher order corrections will not alter them in a drastic, qualitative way, then our calculations should preserve exactly the over-all symmetry of the model. They should give exactly the zero-mass bosons predicted by the Goldstone theorem.[3] The essential point of this theorem is that in order to conserve the current that is associated with the symmetry of the Lagrangian under a certain group of gauge transformations a specific set of zero-mass boson states must accompany the occurrence of the symmetry-breaking states in the nonperturbative solutions. In our case the asymmetric states are the physical baryons which do not have the full symmetry of the Lagrangian because of the dynamical masses they acquire through the nonlinear self-interaction. We will confirm the actual presence in our solutions of all the zero-mass bosons predicted by the Goldstone theorem by explicit calculations, thereby justifying our lowest order bubble approximation and our assumption that the theorem and its proof are relevant and applicable to our model.

Our solutions, although nonperturbative in nature, are continuous functions of the parameters of the problem. We will show that we can pass continuously from one solution to an adjacent solution. In particular, we will discuss how we can continue away from the solution of incomplete symmetry breakdown where the chiral, but not the unitary symmetry, is spontaneously broken to the case of complete spontaneous breakdown of both the chiral and the unitary symmetries. This will allow us to solve for the meson states of our model in the limit of a particular unitary symmetric solution and then transform to solutions with complete symmetry breakdown by continuation in the parameter which characterizes the unitary symmetry breakdown, i.e., the baryon mass splitting. This process is equivalent to using degenerate perturbation theory for the meson mass splittings in terms of the baryon mass splittings.

To simplify our demonstration of the bubble approximation for the meson states and its consistency for the case of complete spontaneous symmetry breakdown we shall consider a model in which the basic fermion field has

* Supported in part by the U. S. Air Force through Air Force Office of Scientific Research Contract AF 49(638)-1389. This material is taken, in part, from the doctoral dissertation of N. Byrne, Physics Department, Stanford University, 1965 (unpublished).

[1] N. Byrne, C. Iddings, and E. Shrauner, preceding paper, Phys. Rev. **139**, B918 (1965). The results of the present paper are applied in Sec. 4 of the accompanying paper, which will be referred to as I.

[2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[3] J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1961); S. Bludman and A. Klein, *ibid.* **131**, 236 (1963).

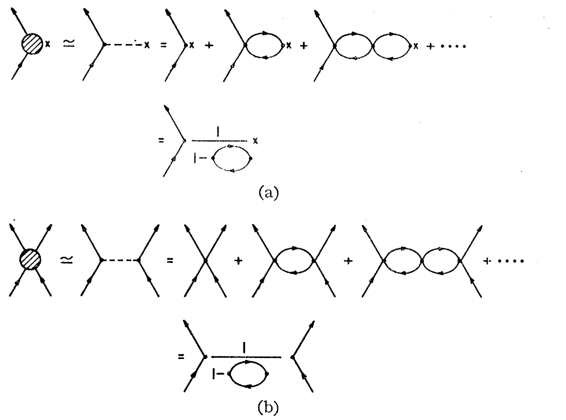

FIG. 1. (a) The baryon interaction with a weak external probe field. The bare coupling with the external field is modified by iterations of the strong four-fermion interaction of our model Lagrangian Eq. (1) to form in our approximation simple chains of closed baryon "bubbles" effecting an equivalent phenomenological coupling that is mediated by exchange of a meson which could propagate freely with a definite mass. The small cross indicates the (bare) coupling to the external field. (b) The baryon-baryon scattering at momentum transfers near that corresponding to the virtual exchange of a meson as effected by the iteration of our four-fermion interaction in a chain of baryon-antibaryon "bubbles."

two isotopic spin components $\psi=(p,n)$; and we shall consider self-consistent solutions in which the masses of these two components are not degenerate $m_p\neq m_n$. This is the two-component analog of the case discussed in Sec. 3 of I where the unitary spin octet $\psi^\alpha{}_\beta$ had different self-consistent mass solutions for different isomultiplets.[4] Our Lagrangian, constructed by excatly the same methods as are discussed in Sec. 2 of I, is

$$\mathcal{L}=-\bar{\psi}\gamma\cdot\partial\psi+\tfrac{1}{2}\alpha[(S^2-P^2)(\mathbf{1}^2+\boldsymbol{\tau}\cdot\boldsymbol{\tau})-(V^2+A^2)(\mathbf{1}^2)] +\tfrac{1}{2}\beta(S^2+P^2+\tfrac{1}{3}T^2)(\mathbf{1}^2-\boldsymbol{\tau}\cdot\boldsymbol{\tau}) +\tfrac{1}{2}h(A^2-V^2)(3\mathbf{1}^2+\boldsymbol{\tau}\cdot\boldsymbol{\tau}), \quad (1)$$

or

$$\mathcal{L}=-\bar{\psi}\gamma\cdot\partial\psi+\tfrac{1}{2}S^2(g\mathbf{1}^2+f\boldsymbol{\tau}\cdot\boldsymbol{\tau})+\tfrac{1}{2}P^2(-f\mathbf{1}^2-g\boldsymbol{\tau}\cdot\boldsymbol{\tau}) -\tfrac{1}{2}\alpha(V^2+A^2)\mathbf{1}^2+\tfrac{1}{2}h(A^2-V^2)(3\mathbf{1}^2+\boldsymbol{\tau}\cdot\boldsymbol{\tau}) +\tfrac{1}{6}\beta T^2(\mathbf{1}^2-\boldsymbol{\tau}\cdot\boldsymbol{\tau}), \quad (1')$$

where

$$g\equiv(\alpha+\beta), \quad f\equiv(\alpha-\beta). \quad (1'')$$

S^2, V^2, T^2, A^2, P^2 here are the same Dirac couplings that were described in Sec. 2 of article I; $\mathbf{1}^2$ and $\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ represent the analogous scalar and vector isospin couplings, respectively. Equation (1) is thus the analog in this two-component isospin model to Eq. (2.13) of I for the eight-component unitary spin model. Equation (1) is the most general relativistic Lagrangian with the four-fermion interaction that is invariant under the following groups of gauge transformations[5]:

$$\mathcal{A}:\quad \psi\rightarrow e^{i\nu}\psi, \qquad \bar{\psi}\rightarrow\bar{\psi}e^{-i\nu}, \quad (2)$$

$$\mathcal{B}:\quad \psi\rightarrow e^{i\frac{1}{2}\theta\cdot\tau}\psi, \qquad \bar{\psi}\rightarrow\bar{\psi}e^{-i\frac{1}{2}\theta\cdot\tau}, \quad (3)$$

$$\mathcal{C}:\quad \psi\rightarrow e^{i\frac{1}{2}\gamma_5\varphi\cdot\tau}\psi, \quad \bar{\psi}\rightarrow\bar{\psi}e^{i\frac{1}{2}\gamma_5\varphi\cdot\tau}. \quad (4)$$

We consider solutions of the model (1) for which the physical baryon masses have the form

$$m=m_1\mathbf{1}+m_3\boldsymbol{\tau}_3=\tfrac{1}{2}(m_p+m_n)\mathbf{1}+\tfrac{1}{2}(m_p-m_n)\boldsymbol{\tau}_3. \quad (5)$$

By the method discussed in Sec. 3 of I, we obtain self-consistent mass equations analogous to Eqs. (3.43)

$$\begin{aligned}\tfrac{1}{8}m_p&=\alpha m_p\Delta_F(0,m_p)+\beta m_n\Delta_F(0,m_n),\\ \tfrac{1}{8}m_n&=\alpha m_n\Delta_F(0,m_n)+\beta m_p\Delta_F(0,m_p),\end{aligned} \quad (6)$$

or

$$\begin{aligned}m_p+m_n&=8g[m_p\Delta_F(0,m_p)+m_n\Delta_F(0,m_n)],\\ m_p-m_n&=8f[m_p\Delta_F(0,m_p)-m_n\Delta_F(0,m_n)],\end{aligned} \quad (6')$$

where

$$\Delta_F(x,m)=\frac{-2i}{(2\pi)^4}\int d^4p\frac{e^{ixp}}{p^2+m^2-i\epsilon}.$$

Since m_1 in Eq. (5) represents the spontaneous breakdown of the symmetry group $\mathcal{C}$ above, then by the Goldstone theorem there will occur stable pseudoscalar, isovector collective modes of zero mass. These states we call $\pi^\pm$, π^0 mesons. Similarly, since m_3 in Eq. (5) represents a spontaneous breakdown of the isospin symmetry group $\mathcal{B}$ for rotations perpendicular to the third isospin axis, then, according to the theorem, there will also occur charged scalar meson states of zero mass. We designate these mesons as $\varphi^\pm$. With our particular choice of the axis of quantization the mass operator (5) is still invariant under rotations about the $\boldsymbol{\tau}_3$ isospin axis, so the existence of a massless neutral scalar meson is not required by the theorem. The neutral scalar may turn out to be bound, but, in general, its mass is not required to be zero.

We consider the response of our system to a charged,

[4] Nambu and Joan-Lasinio studied the two-component model and emphasized solutions in which m_p and m_n were equal. They also considered the case of a bare-mass term of the form $m_0=m_{01}\mathbf{1}+m_{03}\tau_3$ in the Lagrangian, which split m_p and m_n. However, they showed only that in the presence of such a bare mass the π-meson mass ($J^P=0^-$ isovector collective state) is the same for $\tau_3=-1$, 0, 1 states if terms of the order of $(m_{03}/m_{01})^2$ are neglected. We shall consider a case where the bare mass of ψ is zero, so that the complete symmetry of the Lagrangian is retained, but in the nonperturbative solutions the two components of ψ acquire nondegenerate dynamical masses $m_p\neq m_n$; i.e., complete breakdown of the isospin symmetry, as well as of the chiral symmetry. For our case the Goldstone theorem applies rigorously. We assume the validity and relevance of its proof. See M. Baker, K. Johnson, and B. Lee, Phys. Rev. **133**, B209 (1964). The theorem does not apply in the presence of any bare term in the Lagrangian. A situation similar to ours has been discussed by M. Baker and S. Glashow, Phys. Rev. **128**, 2462 (1962).

[5] If $\beta\equiv0$ in Eq. (1), then $\mathcal{L}$ is also invariant under the ordinary γ_5-transformations

$$\mathcal{D}\psi\rightarrow e^{i\gamma_5\xi}\psi, \quad \bar{\psi}\rightarrow\bar{\psi}e^{i\gamma_5\xi};$$

however, we shall not impose this symmetry requirement. Note that since the scalar and pseudoscalar Dirac couplings S^2 and P^2 do not involve the parameter h, it will have no effect on the baryon mass in the lowest order dynamical approximation (due to Lorentz invariance). Also, we shall see that it cannot have any effect on the presence of zero-mass scalar and pseudoscalar meson states, since their existence is implied by the nonzero baryon mass solutions and the Goldstone theorem.

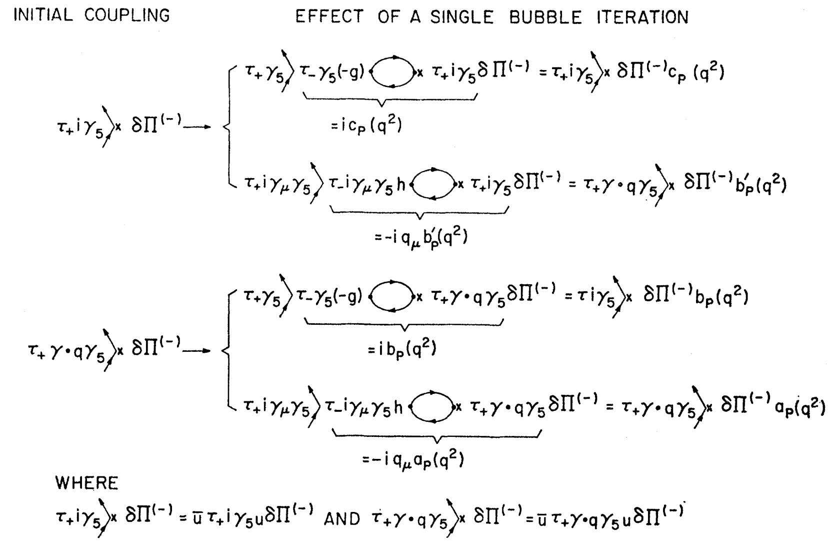

FIG. 2. Diagrammatic representation of the first-order corrections in the bubble approximation toward the renormalization of the vertex for coupling of the physical baryons of our model to a weak external probe field $\delta\Pi^{(-)}$. Here $\delta\Pi^{(-)}$ transforms as a Lorentz pseudoscalar and as the negative component of an isovector. The elements of the bubble matrix a_P, b_P, b_P', c_P are those given in Eqs. (12)–(15).

pseudoscalar, external field $\delta\Pi$ which acts only in first order. The most general coupling allowed by Lorentz invariance is

$$\delta\mathcal{L}_P{}^{(-)}=V^0\bar{\psi}\tau_+i\gamma_5\psi\delta\Pi^{(-)}+W^0\bar{\psi}\tau_+i\gamma_\mu\gamma_5\psi\partial_\mu(\delta\Pi^{(-)}). \quad (7)$$

We want to calculate the phenomenological matrix element of the vertex for scattering of a physical baryon of our model from state 1 to state 2 by this external field:

$$\langle 2|\delta\mathcal{L}_P{}^{(-)}|1\rangle=\bar{u}_2\tau_+[V^R(q^2)i\gamma_5+W^R(q^2)\gamma\cdot q\gamma_5]u_1\delta\Pi^{(-)}. \quad (8)$$

Here $V^R(q^2)$ and $W^R(q^2)$ are the phenomenological form factors we get by the renormalization of the couplings (7) with the bare strength parameters V^0 and W^0. In renormalizing such couplings we will consider only the class of diagrams shown in Fig. 1(a). We shall define the bubble-amplitude matrix $B(q^2)$ as that matrix which gives the effect of adding one more closed baryon loop to a chain of N loops.

$$\begin{pmatrix} V_{N+1}(q^2) \\ W_{N+1}(q^2) \end{pmatrix}=B(q^2)\begin{pmatrix} V_N(q^2) \\ W_N(q^2) \end{pmatrix}. \quad (9)$$

The phenomenological form factors $V^R(q^2)$ and $W^R(q^2)$, as in Eq. (8), include in our approximation the sum of the effects of all possible lengths of simple chains of closed baryon bubbles, as shown in Fig. 1,

$$\begin{pmatrix} V^R(q^2) \\ W^R(q^2) \end{pmatrix}=\sum_{N=0}^{\infty}[B(q^2)]^N\begin{pmatrix} V^0 \\ W^0 \end{pmatrix}=[1-B(q^2)]^{-1}\begin{pmatrix} V^0 \\ W^0 \end{pmatrix}. \quad (10)$$

The derivative couplings are included in Eq. (7) ($W^0\neq 0$), since the chain graphs of Fig. 1 will induce such terms in the renormalized vertex (8), even if only the direct coupling term was present in the original interaction; thus, they must be accounted for by the bubble matrix $B(q^2)$.

In the bubble approximation to the inhomogeneous Bethe-Salpeter equation for the $\delta\Pi$ vertex, as diagrammed in Fig. 1, the direct coupling of Eq. (7) is modified by a single iteration of the Lagrangian (1′) in which the baryon pair interacts at the closure of the first bubble through the axial-vector coupling term $h/2A^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$, or through the pseudoscalar coupling term $-g/2P^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$, of the Lagrangian (1′). Likewise, the gradient coupling in (7) can be modified by the baryon pair interacting through either of these two terms of the Lagrangian. The (four) possible ways in which a single iteration of the Lagrangian acts in the first-order bubble corrections to the couplings (7) is illustrated in Fig. 2. Thus, the effect of a single bubble correction is that the direct coupling,

$$\bar{u}\tau_+i\gamma_5u\delta\Pi^{(-)}, \quad \text{becomes } \bar{u}\tau_+[i\gamma_5c_P(q^2)+\gamma\cdot q\gamma_5b_P'(q^2)]u\delta\Pi^{(-)}$$

and the gradient coupling

$$\bar{u}\tau_+\gamma\cdot q\gamma_5u\delta\Pi^{(-)}, \quad \text{becomes } \bar{u}\tau_+[i\gamma_5b_P(q^2)+\gamma\cdot q\gamma_5a_P(q^2)]u\delta\Pi^{(-)},$$

where a_P, b_P, b_P', c_P are the elements of the bubble matrix

$$B(q^2)=\begin{pmatrix} c(q^2) & b(q^2) \\ b'(q^2) & a(q^2) \end{pmatrix} \quad (11)$$

for the case of pseudoscalar coupling.

$$a_P(q^2)=\frac{-2i(ih)}{q^2(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)i\gamma\cdot q\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\gamma\cdot q\gamma_5\tau_+], \quad (12)$$

$$b_P'(q^2)=\frac{-2i(ih)}{q^2(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)i\gamma\cdot q\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\gamma_5\tau_+],\tag{13}$$

$$b_P(q^2)=\frac{-2i(ig)}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\gamma\cdot q\gamma_5\tau_+],\tag{14}$$

$$c_P(q^2)=\frac{-2i(ig)}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)e\gamma_5\tau_+],\tag{15}$$

where

$$S_F(p)=2i\left[\frac{1}{2}(1+\tau_3)\frac{1}{i\gamma p+m_p}+\tfrac{1}{2}(1-\tau_3)\frac{1}{i\gamma p+m_n}\right]\tag{16}$$

and

$$\tau_\pm=2^{-1/2}(\tau_x\pm i\tau_y)$$

and Sp indicates the spur of any Dirac and/or isospin matrices which fall under this sign.

The matrix elements (12)–(15) reduce to forms depending on just a single integral,

$$\begin{aligned}a_P(q^2)&=-q^{-2}(m_p+m_n)^2h[I(q^2)-I(0)],\\ b_P'(q^2)&=(h/g)(1/q^2)b_P(q^2)=-q^{-2}(m_p+m_n)h[I(q^2)-I(0)],\\ c_P(q^2)&=-gI(q^2),\end{aligned}\tag{17}$$

where

$$I(q^2)=\frac{4i}{(2\pi)^4}\int d^4p\,\mathrm{Sp}\left[\frac{1}{i\gamma(p+\frac{1}{2}q)+m_p}\gamma_5\frac{1}{i\gamma(p-\frac{1}{2}q)+m_n}\gamma_5\right]\tag{18}$$

and

$$I(q^2=0)=\frac{4i}{(2\pi)^4}\int d^4p\,\mathrm{Sp}\left[\frac{1}{i\gamma p+m_p}+\frac{1}{-i\gamma p+m_n}\right](m_p+m_n)^{-1}\tag{19}$$

$$=-8[m_p\Delta_F(0,m_p)+m_n\Delta_F(0,m_n)](m_p+m_n)^{-1}.\tag{20}$$

The integral $I(q^2)$ is a function only of the square of the four-momentum q, by Lorentz invariance; so we can evaluate the integral $I(q^2=0)$ by setting q identically equal to zero. In this case the integrand may be separated into partial fractions, as in Eq. (19), and $I(q^2=0)$ turns out to be given by the self-consistent mass equation (6′),

$$I(q^2=0)=-g^{-1}.\tag{21}$$

Reduction of the matrix elements (12)–(15) to the simple dependence on the single integral $I(q^2)$ is accomplished by using the identity

$$i\gamma\cdot q=[i\gamma(p+\tfrac{1}{2}q)+m_p]+[-i\gamma(p-\tfrac{1}{2}q)+m_n]-(m_p+m_n)\tag{22}$$

in the numerators of the integrands, and taking advantage of the Lorentz invariance of our model and the convention of symmetric integration.[6] These relations are derived by considering the formal expressions for the covariant integrals of our model without evaluating these integrals, which are, in fact, divergent. The relations derived this way are not strictly correct in an arbitrary Lorentz frame when a simple cutoff procedure is used. The convention that we have adopted for the reduction of the relations among the (divergent) integrals of our model corresponds to the adoption of particular subtractions in the dispersion integrals of Nambu and Jona-Lasinio. [See Eq. (13) of Ref. 2.] We hope to convince the reader that fixing the subtractions according to our convention sustains the self-consistency of our solutions.[7]

In the case of the pseudoscalar coupling (7) and (8) the matrix which enters Eq. (10) for the total correction in our bubble approximation is

$$[1-B_P(q^2)]^{-1}=\frac{1}{\Delta_P(q^2)}\begin{pmatrix}(1-a_P(q^2)) & b_P(q^2)\\ b_P(q^2) & (1-c_P(q^2))\end{pmatrix},\tag{23}$$

where

$$\Delta_P(q^2)=\det[1-B_P(q^2)]=1-c_P(q^2)$$

follows immediately from Eqs. (17) and (21) for the matrix elements. Using the matrix (23) in Eq. (10),

[6] See J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955). The appendices discuss the evaluation of integrals of the type we are concerned with here. In particular, we take

$$\int d^4p\,\mathrm{Sp}\frac{1}{i\gamma(p-q)+m}=\int d^4p\,\mathrm{Sp}\frac{1}{i\gamma p+m}$$

and

$$\int d^4p\,\mathrm{Sp}\left[\frac{\gamma q}{}\frac{1}{i\gamma(p-q)+m}\right]=0.$$

We wish to thank Professor J. D. Bjorken for calling our attention to the usefulness of Eq. (22).

[7] Our subtraction method is consistent with that used in quantum electrodynamics, although there covariance complications are minimized by taking the limit of no cutoff.

we find that the direct pseudoscalar coupling $(V^0,W^0)=(1,0)$, renormalizes into

$$\langle 2|\bar{\psi}\tau_+ i\gamma_5\psi|1\rangle=\bar{u}_2\tau_+\left\{\left[\frac{1}{g[I(q^2)-I(0)]}+\frac{(m_p+m_n)^2h}{q^2g}\right]i\gamma_5-\frac{(m_p+m_n)h}{q^2g}\gamma\cdot q\gamma_5\right\}u_1. \quad (24)$$

The first term on the left-hand side of this expression has a singularity at $q^2=0$ corresponding to a π^- meson of zero mass. The other terms (proportional to h) have nothing to do with the fact that these Goldstone bosons must occur.[8]

The renormalization of the gradient coupling $(V^0,W^0)=(0,1)$ in Eq. (10) gives

$$\langle 2|\bar{\psi}\tau_+\gamma\cdot q\gamma_5\psi|1\rangle=\bar{u}_2\tau_+[\gamma\cdot q\gamma_5-(m_p+m_n)i\gamma_5]u_1. \quad (25)$$

This is more clearly understood by considering that the gradient coupling in the pseudoscalar case is equivalent to a direct coupling to a longitudinal axial vector, the gradient of the pseudoscalar test field, $q_\mu\delta\Pi^{(-)}=\delta A_\mu{}^{L(-)}$. Thus, the same bubble matrix applies, giving

$$\langle 2|\bar{\psi}\tau_+ i\gamma_\mu\gamma_5\psi|1\rangle\delta A_\mu{}^{L(-)}=\bar{u}_2\tau_+[i\gamma_\mu\gamma_5+((m_p+m_n)/q^2)q_\mu\gamma_5]u_1\delta A_\mu{}^{L(-)}, \quad (26)$$

which is the matrix element of a conserved axial current, the "induced pseudoscalar" gradient coupling being just that required for the Goldberger-Treiman relation.[9]

All that is said of one charged meson state applies directly to the state of opposite charge, because our solutions, as well as our Lagrangian, are symmetric under rotations about the T_3 isospin axis. This symmetry is manifest in our bubble approximation in that $\mathrm{Sp}(\tau_+\tau_-)=\mathrm{Sp}(\tau_+\tau_-)$ in the basic bubble elements (12)–(15), so they are identical for the positive and negative channels.

Next, we consider the neutral pseudoscalar meson channel. The $T=1$, $T_3=0$ mode, π^0, may mix with the $T=T_3=0$ mode, η, in our solution because the physical baryon mass, Eq. (5), does not commute with the total isospin T (although T_3 is still conserved in the single-baryon states). The most general coupling of the neutral pseudoscalar channel to the baryons of our model is

$$\delta\mathcal{L}_P{}^{(0)}=V^0\bar{\psi}\tau_3 i\gamma_5\psi\delta\Pi^{(0)}+X^0\bar{\psi}\tau_3 i\gamma_\mu\gamma_5\psi\partial_\mu(\delta\Pi^{(0)})+W^0\bar{\psi}i\gamma_5\psi\delta H+Y^0\bar{\psi}i\gamma_\mu\gamma_5\psi\partial_\mu(\delta H), \quad (27)$$

where $\delta\Pi^{(0)}$ is an external, neutral pseudoscalar test field having isospin $T=1$ and δH is a similar isoscalar test field. The full bubble matrix that corresponds to Eq. (11) is a 4×4 matrix in this case. To avoid the complications of a four-channel problem we anticipate the masslessness predicted by the Goldstone theorem and treat the coupling of the π^0 and η modes only at $q_\mu=0$, where the gradient couplings vanish. In the neutral pseudoscalar channel the baryon pair which interacts at one end of the bubble through the isovector part $-\frac{1}{2}gP^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ of the Lagrangian (1) can interact at the other end of the bubble through the term $-\frac{1}{2}fP^2\mathbf{1}^2$. So the "bubble" in the $q_\mu=0$ case is a 2×2 matrix which mixes the $T=1$ and $T=0$ modes. For the matrix elements we have

$$c(\pi^0,q^2)=\frac{-2i(ig)}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\gamma_5\tau_3\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\gamma_5\tau_3]\xrightarrow[q_\mu=0]{}\frac{-2ig}{(2\pi)^4}\int d^4p\,4\left[\frac{1}{p^2+m_p{}^2}+\frac{1}{p^2+m_n{}^2}\right] \quad (28)$$

$$=4g[\Delta_F(0,m_p)+\Delta_F(0,m_n)], \quad (29)$$

and similarly

$$a(\eta,q^2)=\frac{-2i(if)}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\gamma_5\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\gamma_5]\xrightarrow[q_\mu=0]{}4f[\Delta_F(0,m_p)+\Delta_F(0,m_n)] \quad (31)$$

and

$$b(\pi^0,\eta,q^2)=\frac{-2i(ig)}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}\pi)\gamma_5\tau_3\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\gamma_5], \quad (32)$$

$$\xrightarrow[q_\mu=0]{}4g[\Delta_F(0,m_p)-\Delta_F(0,m_n)], \quad (33)$$

and

$$b'(\eta,\pi^0,q^2=0)=(f/g)b(\pi^0,\eta,q^2=0). \quad (34)$$

The eigenvalues of the neutral pseudoscalar bubble matrix at $q^2=0$ are

$$\lambda=1$$

$$\lambda=\frac{\alpha^2+\beta^2}{\alpha^2-\beta^2}-\frac{\alpha\beta}{\alpha^2-\beta^2}\left(\frac{m_n}{m_p}+\frac{m_p}{m_n}\right). \quad (35)$$

So, there is a triplet of pseudoscalar meson states with mass zero, as predicted by the theorem. The neutral

[8] The h/q^2 factors are due to our use of a longitudinal projection operator on the axial-vector coupling. A careful treatment that avoids them is given by N. Byrne, thesis, Stanford University, 1965 (unpublished).

[9] See P. G. O. Freund and Y. Nambu, Phys. Rev. Letters **13**, 221 (1964); which contains further references.

component does not have definite isospin because in the solutions we are considering the baryon mass (5) breaks the isospin symmetry $\mathcal{B}$ as well as the chiral symmetry $\mathcal{C}$. We shall assume, however, that this isospin breakdown is small enough so that we need not calculate here the coupling for the π^0 separately, but instead can use the same coupling as the charged pions in an approximate charge independence. There may also be some other value of q^2 for which the η mode propagates as a free particle, i.e., another root of $\det[1-B(q^2)]=0$. We shall ignore it for now.

Following the method of Nambu and Jona-Lasinio, we can also obtain the effective coupling constants for the phenomenological interactions of the mesons of our model with the baryons. The scattering amplitude near the pole corresponding to the virtual exchange of, say, a π^+ meson, is

$$F=\tau_-\gamma_5 \frac{(-g)}{1-c_P(q^2)}\tau_+\gamma_5 \tag{36}$$

and in the neighborhood of the pole $[1-c_P(q^2)]$ is just[10]

$$1-c_P(q^2)=1-c_P(q^2=-\mu_\pi^2) -(q^2+\mu_\pi^2)[dc_P(q^2)/dq^2]_{q^2=-\mu k^2} \tag{37}$$

so

$$F\cong\tau_-\gamma_5\frac{(-g)}{-(q^2+\mu_\pi^2)[dc_P(q^2)/dq^2]_{q^2=-\mu_\pi^2}}\tau_+\gamma_5$$
$$\equiv G_P\tau_-\gamma_5(1/(q^2+\mu_\pi^2))\tau_+\gamma_5 G_P\,, \tag{38}$$

and, thus, the phenomenological coupling constant is

$$G_P{}^2=\frac{g}{[dc_P(q^2)/dq^2]_{q^2=-\mu_\pi^2=0}} =\frac{-1}{[dI(q^2)/dq^2]_{q^2=-\mu_\pi^2=0}}=-1/I'(0)\,. \tag{39}$$

We shall defer evaluation of $G_P{}^2$ until we have treated also the zero-mass scalar mesons of our model.[11]

The Goldstone theorem also predicts a pair of charged scalar mesons, corresponding to the spontaneous breakdown of the isospin symmetry $\mathcal{B}$ by the isovector component of the baryon mass m_3. We will confirm the presence of these zero-mass scalar states in our theory. We will also confirm that the phenomenological matrix element of the charged components of the longitudinal vector, isovector vertex is that of a conserved current, conforming to a "generalized Goldberger-Treiman relation" in its induced gradient term. The same techniques apply here as were used in treating the pseudoscalar mesons and the charged components of the longitudinal axial-vector current.

The bare coupling of our baryon system to a charged, scalar, test field $\delta\phi^{(-)}$ is

$$\delta\mathcal{L}_S{}^{(-)}=V^0\bar\psi\tau_+\psi\delta\phi^{(-)}+W^0\bar\psi\tau_+i\gamma_\mu\psi\partial_\mu(\delta\phi^{(-)})\,, \tag{40}$$

and the phenomenological matrix element of this vertex that we want to calculate is

$$\langle 2|\delta\mathcal{L}_S{}^{(-)}|1\rangle=\bar u_2\tau_+[V^R(q^2)+W^R(q^2)\gamma\cdot q]u_1\delta\phi^{(-)}\,. \tag{41}$$

In the charged scalar (or longitudinal vector) case the bubble corrections are accomplished by the strong interactions of baryon pairs through the two terms $\frac{1}{2}hV^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ and $\frac{1}{2}fS^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ of the Lagrangian (1′). The elements of the bubble matrix in this case are

$$a_S(q^2)=\frac{-2i(-h)}{q^2(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\tau_-\gamma\cdot q\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\tau_+\gamma\cdot q]\,, \tag{42}$$

$$b_S{}'(q^2)=\frac{-2i(-h)}{q^2(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\tau_-\gamma\cdot q\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\tau_+]\,, \tag{43}$$

$$b_S(q^2)=\frac{-2i(f)}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\tau_+\gamma\cdot q]\,, \tag{44}$$

$$c_S(q^2)=\frac{-2i(f)}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\tau_+]\,. \tag{45}$$

Diagrams analogous to those of Fig. 4 for the pseudoscalar case can also be used to illustrate the single-bubble process which these matrix elements describe. These matrix elements also reduce to a simple dependence on a single integral:

$$\begin{aligned} a_S(q^2)&=q^{-2}(m_p-m_n)^2h[J(q^2)-J(0)]\,,\\ b_S(q^2)&=(-f/h)q^2b'(q^2)=-i(m_p-m_n)f[J(q^2)-J(0)]\,,\\ c_S(q^2)&=fJ(q^2)\,, \end{aligned} \tag{46}$$

[10] A careful treatment of the baryon-baryon scattering requires the use of the full matrix B. See the reference given in footnote 8.

[11] In general, the equivalent meson nucleon coupling constants must be obtained by summing nucleon-nucleon scattering graphs and comparing the results with perturbation theory for one meson exchange, cf. Fig. 1b. For the case of 1^- mesons for example, both the γ_μ and $\sigma_{\mu\nu}q_\nu$ couplings can be uniquely determined.

where

$$J(q^2)=\frac{4i}{(2\pi)^4}\int d^4p\ \mathrm{Sp}\left[\frac{1}{(i\gamma(p+\frac{1}{2}q)+m_p)}\frac{1}{(i\gamma(p-\frac{1}{2}q)+m_n)}\right] \tag{47}$$

and

$$J(0)=\frac{4i}{(2\pi)^4}\int d^4p\ \mathrm{Sp}\left[\frac{1}{i\gamma p+m_p}-\frac{1}{i\gamma p+m_n}\right](m_n-m_p)^{-1}$$
$$=8[m_p\Delta_F(0,m_p)-m_n\Delta_F(0,m_n)](m_p-m_n)^{-1}. \tag{48}$$

The same methods as those used for handling the integral $I(q^2)$ in Eqs. (18)–(20) apply here to $J(q^2)$. The self-consistent mass equation (6′) gives

$$J(q^2=0)=f^{-1} \tag{49}$$

corresponding to Eq. (21) for $I(q^2=0)$. Again we have

$$\Delta_S(q^2)=\det[1-B_S(q^2)]=1-c_S(q^2)\,, \tag{50}$$

and the bubble denominator matrix follows the same way as in Eq. (23) for the pseudoscalar case. Applying the matrix $[1-B_S(q^2)]^{-1}$ to the column vector $(V^0,W^0)=(1,0)$ we get the effect of the full strong-interaction corrections in the bubble approximation to a direct coupled, charged scalar probe

$$\langle 2|\bar\psi\tau_+\psi|1\rangle=\bar u_2\tau_+\left\{\left[\frac{1}{f[J(0)-J(q^2)]}+\frac{(m_p-m_n)^2h}{q^2f}\right]-\frac{(m_p-m_n)h}{q^2f}i\gamma\cdot q\right\}u_1. \tag{51}$$

This expression clearly indicates that there is a pole at $q^2=0$ in the charged scalar channel of zero baryon number.[12] The positive and negative channels are the same throughout our treatment, and the two stable meson states implied by these poles we call $\varphi^{\pm}$.

We get the renormalized gradient coupling in the scalar channel by applying $[1-B_S(q^2)]^{-1}$ to the column vector $(V^0,W^0)=(0,1)$

$$\langle 2|\bar\psi\tau_+\gamma\cdot q\psi|1\rangle=\bar u_2\tau_+[(m_p-m_n)i+\gamma\cdot q]u_1. \tag{52}$$

The gradient coupling to a scalar probe is equivalent to the direct coupling to a longitudinal vector probe, which may be taken as the gradient of the scalar probe field, $\delta V_\mu{}^{L(-)}=\partial_\mu(\delta\phi^{(-)})$. The bubble matrix for these two cases is readily seen to be the same, and we get

$$\langle 2|\bar\psi\tau_+i\gamma_\mu\psi|1\rangle\delta V_\mu{}^{L(-)}=\bar u_2\tau_+[i\gamma_\mu-((m_p-m_n)/q^2)q_\mu]u_1\delta V_\mu{}^{L(-)} \tag{53}$$

which is the familiar form of a conserved current matrix. The induced scalar gradient coupling term here is just that which conforms with a generalized Goldberger-Treiman relation.[9]

[12] The h/q^2 terms do not represent φ^- mesons. The situation is analogous to that discussed in Ref. 8.

These charged, massless scalar mesons, which are predicted by the Goldstone theorem, are not the scalar meson states that Nambu and Jona-Lasinio found in their calculation. They found scalar states of baryon-antibaryon systems of zero binding, i.e., instead of zero mass they found scalar collective modes of mass $2m$. This difference is due to the different self-consistency conditions which we imposed in obtaining our solution. They considered nonperturbative solutions for which the isovector component m_3 of the baryon mass (5) was zero.

What about the neutral scalar mesons? As we discussed above, the isospin symmetry breakdown, as represented in the baryon mass expression (5), must be accompanied by the *charged*, massless scalar mesons, but the Goldstone theorem does not require the presence of the *neutral* scalar mesons of zero mass. However, as m_p and m_n are allowed to become degenerate the spontaneous breakdown of the isospin symmetry is removed, and the theorem no longer requires any massless scalar mesons. How is this symmetric limit approached? Do the charged Goldstone mesons uncouple, or vanish, or do they remain coupled while the isospin symmetry is restored by the occurrence of a zero-mass neutral scalar meson to complete the isomultiplet? It turns out that the latter is the case, and so we shall demonstrate this interesting effect rather than trying to find the neutral's mass and coupling in the more general non-symmetric solution.

We consider the neutral scalar meson channel. The $T=1$, $T_3=0$ mode, φ^0, may mix with the $T=T_3=0$ mode, σ, in our solution. The most general coupling of the neutral scalar channel to the baryons of our model is

$$\delta\mathfrak{L}_S{}^{(0)}=V^0\bar\psi\tau_3\psi\delta\phi^{(0)}+X^0\bar\psi\tau_3i\gamma_\mu\psi\partial_\mu(\delta\phi^{(0)})+W^0\bar\psi\psi\delta\Sigma+Y^0\bar\psi i\gamma_\mu\bar\psi\partial_\mu(\delta\Sigma)\,, \tag{54}$$

where $\delta\phi^{(0)}$ is an external scalar test field having isospin $T=1$ and $\delta\Sigma$ is the isoscalar test field that is like a classical field of the σ mesons.

As we are interested in a massless meson, we avoid the complications of a four-channel problem by going directly to treat the coupling of the neutral scalar channel at zero momentum transfer $q_\mu=0$, so that the gradient couplings need not be considered. In the neutral scalar channel the baryon pair which interacts at one end of the bubble through the isovector part $\frac{1}{2}fS^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ of the Lagrangian (1′) can interact at the other end of the bubble through the isoscalar term $\frac{1}{2}gS^2\mathbf{1}^2$. So the bubble in the $q_\mu=0$ case is a 2×2 matrix which mixes the $T=1$, $T_3=0$ φ^0 mode with the $T=T_3=0$ σ mode, in direct analogy with Eq. (27) in the pseudoscalar case. The matrix elements are

$$c(\varphi^0,q^2)=\frac{-2if}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\tau_3\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\tau_3]\xrightarrow[q_\mu=0]{}\frac{2if}{(2\pi)^4}\int d^4p\,\mathrm{Sp}\left[\frac{1}{(i\gamma p+m_p)^2}+\frac{1}{(i\gamma p+m_n)^2}\right]$$
$$=4f\left\{\frac{\partial}{\partial m_p}[m_p\Delta_F(0,m_p)]+\frac{\partial}{\partial m_n}[m_n\Delta_F(0,m_n)]\right\} \tag{55}$$

and

$$a(\sigma,q^2)=\frac{-2ig}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)]\xrightarrow[q_\mu=0]{}a(\sigma,0)=\frac{g}{f}c(\varphi^0,0) \tag{56}$$

and

$$b(\varphi^0,\sigma,q^2)=\frac{-2if}{(2\pi)^4}\int d^4p\,\mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\tau_3\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)]\xrightarrow[q_\mu=0]{}4f\left\{\frac{\partial}{\partial m_p}[m_p\Delta_F(0,m_p)]-\frac{\partial}{\partial m_n}[m_n\Delta_F(0,m_n)]\right\}$$
$$=b(\varphi^0,\sigma,0)=b'(\sigma,\varphi^0,0)(f/g). \tag{57}$$

As we approach the limit $m_p\to m_n$, in which the isospin symmetry $\mathcal{B}$ is no longer broken, we get

$$\begin{aligned}c(\varphi^0,\,q^2=0)&=\frac{f}{g}a(\sigma,\,q^2=0)\xrightarrow[m_p\to m_n]{}8f\frac{\partial}{\partial m_n}[m_n\Delta_F(0,m_n)]\\ b(\varphi^0\,\sigma,0)&=(f/h)b'(\sigma,\varphi^0,0)\xrightarrow[m_p\to m_n]{}0.\end{aligned} \tag{58}$$

So, in the symmetric limit $m_p\to m_n$, the $T=1$ and $T=0$ isospin modes uncouple. Furthermore Eq. (58) represents the emergence of a stable φ^0 meson because in the symmetric limit $m_p\to m_n$ the self-consistent mass equations (6′) become

$$g^{-1}=8\Delta_F(0,m_n) \tag{59}$$

$$f^{-1}=8\frac{\partial}{\partial m_n}[m_n\Delta_F(0,m_n)], \tag{60}$$

whereby $c(\varphi^0,\,q^2=0)=1$ in Eq. (58) and we get a pole in the φ^0 channel. So in the symmetric limit $m_p\to m_n$ we have a complete isospin triplet of scalar mesons of zero mass, free of any coupling with the $T=0$, σ channel. In the general case where $m_p\neq m_n$ the φ^0 channel may have a stable state of nonzero mass.

The phenomenological coupling constant for these massless scalar mesons is given by the analog of Eq. (39) for the pseudoscalar mesons

$$G_S{}^2=\frac{f}{[dc_S(q^2)/dq^2]_{q^2=-\mu_\varphi{}^2=0}}=\frac{1}{[dJ(q^2)/dq^2]_{q^2=0}}=1/J'(0). \tag{61}$$

We can consider these coupling constants for the massless scalar and pseudoscalar mesons simultaneously by writing [from Eqs. (18) and (47)]

$$\left.\begin{matrix}I'(0)\\J'(0)\end{matrix}\right\}=\frac{4i}{(2\pi)^4}\int d^4p\frac{d}{dq^2}\left[\frac{4m_pm_n\pm(4p^2-q^2)}{[(p+q/2)^2+m_p{}^2][(p-q/2)^2+m_n{}^2]}\right]_{q^2=0}$$
$$=\mp4\left[\frac{\Delta_F(0,m_p)-\Delta_F(0,m_n)}{m_p{}^2-m_n{}^2}\right]-\frac{2}{m_n\pm m_p}\left[m_p\frac{d\Delta_F(0,m_p)}{dm_p{}^2}\pm m_n\frac{d\Delta_F(0,m_n)}{dm_n}\right] \tag{62}$$

so in the limit $m_p\to m_n$

$$I'(q^2=0)\xrightarrow[m_p\to m_n]{}-6\frac{\partial\Delta_F(0,m_n)}{\partial m_n}J'(q^2=0)\xrightarrow[m_p\to m_n]{}6\frac{\partial\Delta_F(0,m_n)}{\partial m_n}+4m_n{}^2\left(\frac{\partial}{\partial m_n{}^2}\right)^2\Delta_F(0,m_n). \tag{63}$$

Assuming that the higher the derivative with respect to m_n of the (divergent) integral $\Delta_F(0,m_n)$, the more convergent is the resultant integral, we might ignore the second term in the expression (63) for $J'(0)$. In that case it becomes apparent that the renormalized phenomenological coupling constants of the massless scalar and pseudoscalar mesons in our model remain more-or-less equal in the limit $m_p=m_n$, in which the isospin symmetry $\mathcal{B}$ is no longer broken:

$$G_S^2 \simeq G_p^2 \simeq \left[6\frac{d\Delta_F(0,m_n)}{dm_n^2}\right]^{-1}. \tag{64}$$

In our self-consistent program the masses m_p and m_n are regarded as given external conditions in terms of which the coupling parameters f and g are determined through the mass equations (6′). But the case when m_p is exactly equal to m_n is exceptional, because, instead of the two mass equations (6′), the only equation that one derives for the degenerate baryon mass is Eq. (59). This determines g but says nothing about f. This is the type of degenerate mass solution considered by Nambu and Jona-Lasinio, in which no zero-mass scalar mesons occur. On the other hand, if the degenerate-mass limit (60) of the second of the self-consistent mass equations (6′) is imposed as a condition to fix f, then the isotriplet of massless scalar mesons is present in the theory, even in the case $m_p=m_n$.

If we impose the condition (60) to determine f in the degenerate-mass case, then the limiting process $m_p\to m_n$ can be reversed and all the results of our solutions can be continuously varied from the degenerate-mass, isospin conserving case to the case of nondegenerate baryon masses in which the isospin symmetry $\mathcal{B}$ of our model suffers spontaneous breakdown. Imposing this condition (60) in the degenerate-mass case just selects the correct zero-order solutions, as in degenerate perturbation theory, from which continuous variations to the case of broken isospin symmetry may be made by means of power-series expansions in the perturbation parameter $(m_3/m_1)=(m_p-m_n)/(m_p+m_n)$. We shall make use of this in our consideration of the spin-one mesons, where we shall determine only the gross qualitative properties of these states in the degenerate-mass limit with the condition (60) imposed on the zeroth-order solution, and assume that the fine-structure splittings of these states can be obtained from expansions in powers of (m_3/m_1).[13]

Whereas it was a great convenience that we could anticipate the poles for the spinless mesons at just that value of momentum transfer $q=0$ which simplified our calculations, in the spin-one cases we can anticipate no particular convenience of this sort. We must include the q-proportional gradient coupling terms throughout.

In the case of the transverse vector isovector current we consider the response of our system to a weak perturbation by a charged external probe field[14] $\delta V_\mu{}^{T(-)}$

$$\delta\mathcal{L}_V{}^{(-)}=V^0\bar{\psi}\tau_+ i\gamma_\mu\psi\delta V_\mu{}^{T(-)} +W^0\bar{\psi}\tau_+\sigma_{\mu\nu}\psi\partial_\nu(\delta V_\mu{}^{T(-)}), \tag{65}$$

and the phenomenological matrix element of this vertex that we want to calculate is

$$\langle 2|\delta\mathcal{L}_V{}^{(-)}|1\rangle=\bar{u}_2\tau_+[V^R(q^2)i\gamma_\mu +W^R(q^2)i\sigma_{\mu\nu}q_\nu]u_1\delta V_\mu{}^{T(-)}. \tag{66}$$

The superscript T stands for the transversality of the probe field $\delta V_\mu{}^{T(-)}$. Each of these couplings is modified by the strong interactions of the baryon pairs through the terms $-\frac{1}{2}hV^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ and $-\frac{1}{6}\beta T^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ of the Lagrangian (1′) to form successive closed bubbles of a chain as shown in Fig. 1. The matrix elements which account for the mixing of the two coupling terms in Eq. (66) in adding a single bubble to the chain are

$$c_V(q^2)=\tfrac{1}{3}\delta_{\mu\nu}{}^T c_{\mu\nu}=\tfrac{1}{3}[\delta_{\mu\nu}-(q_\mu q_\nu/q^2)]c_{\mu\nu}=\tfrac{1}{3}(\delta_{\mu\nu}c_{\mu\nu}-a_S), \tag{67}$$

$$c_V(q^2)+\tfrac{1}{3}a_S(q^2)=\frac{-2i(ih)}{3(2\pi)^4}\int d^4p\ \mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\gamma_\mu\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\gamma_\nu\tau_+]\delta_{\mu\nu}, \tag{68}$$

$$b_V(q^2)=\frac{-2i(ih)}{3(2\pi)^4}\int d^4p\ \mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\gamma_\mu\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\sigma_{\nu\alpha}q_\alpha\tau_+]\delta_{\mu\nu}, \tag{69}$$

$$b_V'(q^2)=\frac{-2i(i\beta/3)}{3q^2(2\pi)^4}\int d^4p\ \mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\sigma_{\mu\alpha}q_\alpha\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\gamma_\nu\tau_+]\delta_{\mu\nu}, \tag{70}$$

$$a_V(q^2)=\frac{-2i(i\beta/3)}{3q^2(2\pi)^4}\int d^4p\ \mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\sigma_{\mu\alpha}q_\alpha\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\sigma_{\nu\beta}q_\beta\tau_+]\delta_{\mu\nu}, \tag{71}$$

[13] These arguments are not affected significantly by the cutoff procedure used as long as our philosophy that all the physically meaningful relationships are to be obtained before the application of an *ad hoc* cutoff or subtraction scheme. These arguments should be re-examined if more complicated approximations were to be used for the baryon self-energy and the meson states than our single-bubble approximation.

[14] We continue to consider the charged isovector channel to avoid the problems of the mixing of the isoscalar and isovector modes in the neutral channel, as was considered above in the case of the π^0 and η pseudoscalar states.

where $a_S(q^2)$ in Eqs. (67) and (68) is the longitudinal element calculated in Eqs. (42) and (46), and $\delta_{\mu\nu}{}^T = (\delta_{\mu\nu} - q_\mu q_\nu q^{-2})$ is the transverse-vector projection operator.

As in the previous cases these (divergent) elements of the transverse vector bubble matrix can be reduced [by exploiting the Lorentz invariance of our model, the convention of symmetric integration, and the self-consistency conditions (6′)] to the (divergent) integrals $J(q^2)$ and $I(q^2)$ defined above in Eqs. (18) and (47). We get

$$\begin{aligned} c_V(q^2) &= (-h/3)\{3I(q^2)+J(q^2)-q^{-2}(m_p-m_n)^2[f^{-1}-J(q^2)]\} \\ b_V(q^2) &= (-3q^2h/\beta)b_V{}'(q^2) = h(m_p+m_n)[g^{-1}+I(q^2)] \\ a_V(q^2) &= (-\beta/9)\{3I(q^2)+2J(q^2)-2q^{-2}(m_p-m_n)^2[f^{-1}-J(q^2)]\}. \end{aligned} \tag{72}$$

With these same techniques it is easy also to prove the identity

$$I(q^2)+(m_p+m_n)^2q^{-2}[g^{-1}+I(q^2)] = -J(q^2)+(m_p-m_n)^2q^{-2}[f^{-1}-J(q^2)], \tag{73}$$

with the help of which the matrix elements (72) may be expressed purely in terms of the integral $I(q^2)$:

$$\begin{aligned} c_V(q^2) &= (-h/3)\{2I(q^2)-(m_p+m_n)^2q^{-2}[g^{-1}+I(q^2)]\}, \\ b_V(q^2) &= (-3q^2h/\beta)b_V{}'(q^2) = h(m_p+m_n)[g^{-1}+I(q^2)], \\ a_V(q^2) &= (-\beta/9)\{I(q^2)-2(m_p+m_n)^2q^{-2}[g^{-1}+I(q^2)]\}. \end{aligned} \tag{74}$$

The condition that we have a vector meson of mass μ_V in the bubble approximation, according to Eq. (10), is that

$$0 = \Delta_V(q^2) = \det[1-B_V(q^2)] \tag{75}$$

have a solution at $\mu_V{}^2 = -q^2$. Using the expressions (74) for the matrix elements, this characteristic equation for the vector meson mass is

$$\begin{aligned} 0 &= \det[1-B_V(q^2)], \\ &= 1+(h/3)\{2I(q^2)-(m_p+m_n)^2q^{-2}[g^{-1}+I(q^2)]\}+(\beta/9)\{I(q^2)-2(m_p+m_n)^2q^{-1}[g^{-1}+I(q^2)]\} \\ &\quad +(h\beta/27)\{2I(q^2)-(m_p+m_n)^2q^{-2}[g^{-1}+I(q^2)]\}\{I(q^2)-2(m_p+m_n)^2q^{-2}[g^{-1}+I(q^2)]\} \\ &\quad +(h\beta/9)(m_p+m_n)^2q^{-2}[g^{-1}+I(q^2)]^2. \end{aligned} \tag{76}$$

As it stands, this equation is rather intractable. Recall also that the coupling parameter h is, as yet, an undetermined parameter of our model.

To make a start toward finding what vector meson mass solutions, $q^2 = -\mu_V{}^2$, Eq. (76) has, we will consider the limiting case of $(m_p - m_n)$ very small, and assume also that the vector meson mass is small enough so that

$$I(q^2) \simeq I(0) + q^2[dI(q^2)/dq^2]_{q^2=0} = -g^{-1}+q^2I'. \tag{77}$$

For this case the identity (73) yields in lowest approximation

$$I' \simeq (g^{-1}-f^{-1})/4m_n{}^2, \tag{78}$$

and in this approximation the eigenvalue equation (76) becomes

$$\mu_V{}^2 = -q^2 \simeq \frac{[1+(h/3)(f^{-1}-3g^{-1})][1-(\beta/9)(3g^{-1}-2f^{-1})]}{(g^{-1}-f^{-1})[-h\beta f^{-1}4/9+2h+\beta/3]} 12m_n{}^2. \tag{79}$$

For convenience, let us introduce a relativistic cutoff $\kappa^2 = 100m_n{}^2$ of the type described in Eq. (46) of I in which case $g^{-1} \simeq 9.5m_n{}^2$ and $f^{-1} \simeq 8.8m_n{}^2$, and Eq. (79) then determines the coupling parameter h in terms of $\mu_V{}^2$. We take $\mu_V{}^2 = \frac{1}{4}m_n{}^2$. The value obtained by this procedure is $h \simeq 0.15m_n{}^{-2}$. We can now use this value of h to see what mass is predicted for the axial-vector mesons.

For the axial-vector isovector mesons we study the response of our baryon system to a charged, external, transverse axial-vector probe field $\delta A_\mu{}^T$. The perturbation due to such a probe is

$$\delta\mathcal{L}_A{}^{(-)} = V^0\bar{\psi}\tau_+ i\gamma_\mu\gamma_5\psi\delta A_\mu{}^T + iW^0\bar{\psi}\tau_+\sigma_{\mu\nu}\gamma_5\psi\partial_\nu(\delta A_\mu{}^T), \tag{80}$$

for which we want to calculate the renormalized matrix element

$$\langle 2|\delta\mathcal{L}_A{}^{(-)}|1\rangle = \bar{u}_2\tau_+[V^R(q^2)i\gamma_\mu\gamma_5 + W^R(q^2)\sigma_{\mu\nu}q_\nu\gamma_5 u_1]\delta A_\mu{}^T. \tag{81}$$

The transverse axial-vector coupling mixes with the tensor coupling

$$(-\beta/6)T^2\boldsymbol{\tau}\cdot\boldsymbol{\tau} = (-\beta/6)\tfrac{1}{2}\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi = (-\beta/6)\tfrac{1}{2}\bar{\psi}\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}\psi\bar{\psi}\sigma_{\mu\nu}\gamma_5\boldsymbol{\tau}\psi \tag{82}$$

in the same way as the vector coupling does in the bubble matrix. The elements of the bubble matrix are

$$c_A(q^2)+\tfrac{1}{3}a_p(q^2)=\frac{-2i(h)}{(2\pi)^4}\int d^4p\ \mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)i\gamma_\mu\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\gamma_\nu\gamma_5\tau_+]\delta_{\mu\nu}\,, \tag{83}$$

$$b_A(q^2)=\frac{-2i(h)}{3(2\pi)^4}\int d^4p\ \mathrm{Sr}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)i\gamma_\mu\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\sigma_{\nu\alpha}q_\alpha\gamma_5\tau_+]\delta_{\mu\nu}\,, \tag{84}$$

$$b_A(q^2)=\frac{-2i(-\beta/3)}{3q^2(2\pi)^4}\int d^4p\ \mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\sigma_{\mu\alpha}q_\alpha\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)i\gamma_\nu\gamma_5\tau_+]\delta_{\mu\nu}\,, \tag{85}$$

$$a_A(q^2)=\frac{-2i(-\beta/3)}{3q^2(2\pi)^4}\int d^4p\ \mathrm{Sp}[\tfrac{1}{2}S_F(p+\tfrac{1}{2}q)\sigma_{\mu\alpha}q_\alpha\gamma_5\tau_-\tfrac{1}{2}S_F(p-\tfrac{1}{2}q)\sigma_{\nu\beta}q_\beta\gamma_5\tau_+]\delta_{\mu\nu}\,, \tag{86}$$

where $a_p(q^2)$ in Eq. (83) is the longitudinal element calculated in Eqs. (12) and (17). Commuting the γ_5 through until it annihilates on another γ_5 in each of these elements reduces them to the corresponding element of the transverse vector bubble matrix, Eqs. (68)–(71), with the substitutions of the exchanges

$$\begin{aligned}(m_p+m_n)&\rightleftarrows(m_p-m_n)\,,\\ h&\rightleftarrows h\,,\\ g&\rightleftarrows f;\ \text{or equivalently, } \alpha\rightleftarrows\alpha \text{ and } \beta\rightleftarrows-\beta\,.\end{aligned} \tag{87}$$

The integrals defined in Eqs. (18), (47), and (62) are related according to

$$-I(m_p,m_n,q^2)=J(m_p,-m_n,q^2)\,, \tag{88}$$

so the replacements (87) imply an exchange of $I(q^2)$ with $-J(q^2)$ in going from the transverse vector bubble matrix elements, as given in Eqs. (74), to those for the axial-vector case.[15]

In the limit as $|m_p-m_n|$ becomes arbitrarily small the bubble matrix for the charged axial-vector mesons diagonalizes, $b_A=b_A'=0$, and the characteristic equation for the mass of these mesons becomes

$$0=1-J(q^2)\left(\frac{2}{3}h-\frac{1}{9}\beta\right)-\frac{2}{27}h\beta J^2(q^2)\,. \tag{89}$$

In the approximation for small q^2 according to Eqs. (73) and (77), this gives

$$\mu_A{}^2=-q^2\simeq\frac{3-f^{-1}[2h-\tfrac{1}{3}\beta-(2/9)h\beta f^{-1}]}{(g^{-1}-f^{-1})[2h-\tfrac{1}{3}\beta-(2/9)h\beta f^{-1}]}4m_n{}^2\,. \tag{90}$$

[15] The scalar-longitudinal vector and pseudoscalar longitudinal pseudovector bubble matrices are related by the same exchange (87) that relates the transverse vector and transverse pseudovector cases. The bubble matrix elements for the charged pseudoscalar case, Eqs. (24)–(26), are changed into those of the scalar case, Eqs. (51)–(53), under the substitution of the exchange (87). This exchange can be effected more elegantly by the chiral transformation

$$\psi\to e^{i\frac{1}{4}\pi(\tau_3-1)\gamma_5}\psi\quad \bar\psi\to\bar\psi e^{i\frac{1}{4}\pi(\tau_3-1)\gamma_5}\psi,$$

which, of course, is not contained in the symmetry group of the Lagrangian (1′), but causes it to undergo

$$\mathcal{L}(g,f,h)\to\mathcal{L}(f,g,h).$$

If we now use the relativistic cutoff $\kappa^2=100m_n{}^2$ and assume the presence in our model of vector mesons of mass $\mu_V\simeq\frac{1}{2}m_n$, which we found corresponds to $h=0.15m_n{}^{-2}$, then Eq. (90) gives $\mu_A{}^2\simeq6m_n{}^2$ for the mass of the axial-vector mesons. Thus, the axial-vector meson mass turns out to be so large as to invalidate the power-series expansion approximation by which it was derived. (The same qualitative result holds for any reasonable values of the cutoff $\kappa^2\gtrsim2m_n{}^2$.) Nevertheless, we take this large estimated mass as indicating that the actual mass of the axial-vector meson in our model is too large to contradict the lack of its experimental observation, even if it were calculated in a more exact manner.

We have treated only the charged vector and axial-vector meson states. This is consistent with our hypothesis that the isospin breakdown is small, in which case the neutral mesons in the isovector-like modes can be assumed to be almost like the charged modes with which they are degenerate in the charge-independent limit. And, as we went to some length to point out above, small deviations from the charge-independent limit can be treated by analytic continuation, starting from the symmetric limit. The phenomenological coupling constants for these and all other mesons of our theory can be calculated by the method described in Eqs. (36)–(38) and (61)–(64) for the massless spin-zero mesons, although when the meson pole occurs at nonzero mass more than just one element of the bubble matrix will contribute as in Eq. (37). Thus, we will consider our treatment of the vector and axial-vector isotriplet meson states of our model as essentially complete, in principle, for any type of spontaneous symmetry breakdown.

There remain, however, the neutral vector and axial-vector collective excitations which are identifiable as the isoscalar modes in the limit as the spontaneous symmetry $\mathcal{B}$ of our model is continuously reduced to zero in the solutions that we consider. The most general perturbation to consider in this case is

$$\begin{aligned}\delta\mathcal{L}_V{}^{(0)}=V^0\bar\psi i\gamma_\mu\tau_3\psi V_\mu{}^{T(0)}+X^0\bar\psi\sigma_{\mu\nu}\tau_3\psi\partial_\nu(\delta V_\mu{}^{T(0)})\\ +W^0\bar\psi i\gamma_\mu\psi W_\mu{}^T+Y_0\bar\psi\sigma_{\mu\nu}\psi\partial_\nu(\delta W_\mu{}^T)\end{aligned} \tag{91}$$

by which the fermions of our model are coupled to neutral vector probe fields of isospin $T=1$, $T_3=0$, $\delta V_\mu{}^{T(0)}$, and $T=T_3=0$, $\delta W_\mu{}^T$. With the mixing of isospins $T=1$ and $T=0$ and the mixing of the vector and tensor Dirac couplings there is, in general a 4×4 bubble matrix for this case. However, in the limit $m_p \rightarrow m_n$ the isospin mixing goes to zero, the $T=1$, $T_3=0$ mode becomes just like the charged vector mesons to complete a pure isovector charge triplet, and we are left with only the uncoupled isoscalar mode to consider in a 2×2 submatrix referring to the phenomenological couplings

$$\langle 2, T=0 | \delta \mathfrak{L}_V{}^{(0)} | 1, T=0 \rangle = \bar{u}_2[V^R(q^2) i\gamma_\mu + V^R(q^2) i\sigma_{\mu\nu} q_\nu] u_1 \delta W_\mu{}^T. \quad (92)$$

In the limit $m_p \rightarrow m_n$ the elements of the bubble matrix in the present $T=0$ case are the same as the $m_p \rightarrow m_n$ limits of the elements given in Eqs. (67)–(71) for the charged vector mesons, except that the coupling parameters are those of the terms $-\frac{1}{2}(3h+\alpha)V^2\mathbf{1}^2$ and $(\beta/6)T^2\mathbf{1}^2$ of the Lagrangian (1) instead of $-\frac{1}{2}hV^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$ and $(-\beta/6)T^2\boldsymbol{\tau}\cdot\boldsymbol{\tau}$. Thus we get the mass formula for the isoscalar, vector mode from that of the charged vector mesons, Eq. (79), simply by replacing $\beta \rightarrow -\beta$ and $h \rightarrow -(3h+\alpha)$. Using the relativistic cutoff method and a power-series expansion, as in the charged meson case, we do not find low-mass mesons. With the explicit introduction of a cutoff, we could, in principle, solve the equation $\det[1-B(q^2)]=0$ for arbitrarily large roots $\mu^2=-q^2$. We do not do this because solving for large meson mass would involve details of the dynamics which would depend on the specific nature of the cutoff procedure. This problem is essentially dynamical and would lead us beyond results that are primarily due just to the general symmetry of the model. The considerations outlined above have already shown that any freely propagating collective states in this channel are not of low mass. We shall therefore ignore them.

Similar considerations also show that there are no freely propagating collective states of low mass in any of the remaining channels ($T=1$, $J_P=0^+$), ($T=0$, $J_P=0^-$), and ($T=0$, $J_P=1^+$).

Summing up, then, we say that for solutions of our model in which the spontaneous breakdown of the unitary (isospin) symmetry is small compared to that of the chiral (γ_5) symmetry, the masses of the meson states can be treated by first calculating the masses of the degenerate meson multiplets for the limiting solution in which the unitary symmetry breakdown becomes zero, and then varying the solution continuously to the case of complete symmetry breakdown to get the meson mass splittings. We have assumed that the $T=1$, $J_P=1^-$ isovector, vector meson mass is about $\mu_V \cong \frac{1}{2} m_n$. We have estimated for the solution in which the chiral, but not the unitary, symmetry is broken that there are no freely propagating low-mass states with quantum numbers $T=1$, $J_P=1^+$, isovector, pseudovector, $T=0$, $J^P=1^\pm$, isoscalar, vector, and pseudovector or $T=0$, $J_P=0^\pm$ isoscalar, scalar, and pseudoscalar. We take this to mean that these meson states may be ignored for practical purposes in solutions not grossly different from this one. The principal low-mass mesons in this and nearby solutions are, thus, the massless $T\simeq 1$, $J_P=0^\pm$, isovector mesons. This leaves all the meson states of our model in the bubble approximation accounted for. Mesons of spin higher than one cannot occur because the Lagrangian (1) does not include any gradient couplings and so the baryon-antibaryon pair in the bubbles can be only in states of spin one or zero.