Some Simple Consequences of $SU(4)$ Symmetry

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The electromagnetic properties of the nucleons and N^*, as well as the meson-baryon coupling constants, are examined within the framework of an $SU(4)$ symmetry combining spin and isospin. The motivation is to examine the consequences of such a symmetry at low energies without having to worry about the large mass differences within the $SU(3)$ multiplets in the corresponding $SU(6)$ theory. The coupling-constant ratios are in qualitative agreement with the experimental values. The arbitrariness in the proton-neutron magnetic-moment ratio is discussed. In the model where the electromagnetic interaction of the baryons is mediated by the vector mesons in the adjoint representation of $SU(4)$, the magnetic-moment ratios agree with the $SU(6)$ results.

ONE of the difficulties facing the $SU(6)$ theory[1] is the following: Several efforts to generalize $SU(6)$ to the relativistic domain seem to indicate that the symmetry is not exact[2] and that the realm of applicability for an approximate spin-independent theory lies mostly in the low-energy region. On the other hand, $SU(3)$ is in principle expected to work better in the high-energy limit where the mass differences can be neglected. Since $SU(6)$ combines both, it is not clear in what region it can be expected to be strictly applicable. The unusually good agreement between some of the theoretical and experimental numbers, such as the proton-neutron magnetic-moment ratio,[3] is therefore slightly surprising. It is perhaps not completely useless to return from time to time to the smaller symmetry group combining spin and isospin,[4] and to examine what consequences already follow from it. This might be more meaningful for considerations of some low-energy phenomena, since isospin invariance is not badly broken at low energies. We discuss in this note two simple aspects by a straightforward application to an $SU(4)$ theory of calculations already performed in the "nonrelativistic" SU (6) theroy.

1. ELECTROMAGNETIC INTERACTION

Let us first consider the charge part of the electromagnetic vertex of nucleons in $SU(2)$, the isotopic spin group. One can construct from $N=\begin{pmatrix}p\\n\end{pmatrix}$ bilinear expressions for the effective currents J_1 and J_3, which transform like tensors of 1 and 3 dimensions in isotopic spin space, and which upon coupling with the photon field lead to low-frequency "charge vertices" $e\varphi(\bar{N}N)$ and $e\varphi(\bar{N}\tau_3N)$, respectively. (Here φ is the electrostatic potential.) Since the vertex at zero frequency determines the proton and neutron charges, the effective electromagnetic current must consist of the linear combination (J_1+J_3). Turning to $SU(4)$, and assigning the nucleons and N^* to the totally symmetric **20** representation, one can similarly construct effective currents J_1 and J_{15}, which upon coupling with the photon field lead to low-frequency "charge vertices":

$$J_1 \rightarrow e\varphi[\bar{\psi}^{\alpha i,\beta j,\gamma k}\delta_i{}^{i'}\delta_\alpha{}^{\alpha'}\psi_{\alpha' i',\beta j,\gamma k}] \tag{1a}$$

$$J_{15} \rightarrow e\varphi[\bar{\psi}^{\alpha i,\beta j,\gamma k}\delta_i{}^{i'}(\tau_3)_\alpha{}^{\alpha'}\psi_{\alpha' i',\beta j,\gamma k}] \tag{1b}$$

$$i,\ j,\ k=1,\ 2;\quad \alpha,\beta,\gamma=1,\ 2.$$

Assuming the absence of terms that transform according to higher representations in the effective electromagnetic current, the nucleon and N^*-charge assignments require the current to consist of the combination $(\frac{1}{3}J_1+J_{15})$.

We now make an assumption analogous in form but somewhat more arbitrary than the one made in the $SU(6)$ calculation.[5] We assume that the effective low-energy vertex including the magnetic term is of the form:

$$3\bar{\psi}^{\alpha i,\beta j,\gamma k}[e\varphi\delta_i{}^{i'}+\mu(p)i(\boldsymbol{\sigma}\cdot\mathbf{q}\times\boldsymbol{\varepsilon})_i{}^{i'}]Q_\alpha{}^{\alpha'}\psi_{\alpha' i',\beta j,\gamma k}. \tag{2}$$

That is, the same operator Q (which in this case is $\frac{1}{3}+\tau_3$) multiplies both the charge and the magnetic-moment terms in Eq. (2). The following relations immediately follow:

$$\mu(p)/\mu(n)=-\tfrac{3}{2}, \tag{3a}$$

$$\mu(N^*)=\text{charge}\times\mu(p), \tag{3b}$$

$$\langle p|\mu|N_+{}^*\rangle=\tfrac{2}{3}\sqrt{2}\mu(p). \tag{3c}$$

These agree with the $SU(6)$ results.

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1 F. Gursey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); F. Gursey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964); B. Sakita, Phys. Rev. (to be published).

2 One category of the theories has the symmetry "intrinsically" broken [K. Bardakci, J. Cornwall, P. Freund, and B. W. Lee, Phys. Rev. Letters **13**, 698 (1964); M. A. B. Bég and A. Pais, Phys. Rev. **138**, B692 (1965) and other similar works]. A second category has the free Lagrangian invariant, but has a nonlocal interaction which breaks the symmetry for general values of the particle momenta [K. T. Mahanthappa and E. C. G. Sudarshan (unpublished); and other similar works]. For both of these categories, the symmetry is approximated best at low momenta. A third category enforcing strict $SU(6)$ symmetry faces some serious difficulties [S. Coleman (unpublished)].

3 M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters **13**, 514 (1964); B. Sakita, Phys. Rev. Letters **13**, 643 (1964).

4 E. P. Wigner, Phys. Rev. **51**, 105 (1937).

5 The arbitrariness in this assumption will be discussed further in Sec. 3.

2. MESON-BARYON COUPLINGS

As suggested in Ref. 1, π, ρ and the physical ω are assigned to the **15** representation. The meson wave function at low frequency is of the form:

$$M_{\alpha i}{}^{\beta j}=P_\alpha{}^\beta\delta_i{}^j+V_\alpha{}^\beta(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})_i{}^j, \tag{4}$$

where

$$\begin{aligned} &V_1{}^1=(\rho^0/\sqrt{2})+(\omega/\sqrt{2}), && V_2{}^2=-(\rho^0/\sqrt{2})+(\omega/\sqrt{2}),\\ &V_1{}^2=\rho^-, && V_2{}^1=\rho^+,\\ &P_1{}^1=(\pi^0/\sqrt{2}), && P_2{}^2=-(\pi^0/\sqrt{2}),\\ &P_1{}^2=\pi^-, && P_2{}^1=\pi^+. \end{aligned} \tag{5}$$

Although $SU(4)$ does not predict the D/F ratios for meson-baryon coupling, it does predict the relative coupling strengths between $(\rho\bar{N}N)$ and $(\omega\bar{N}N)$. Since $\mathbf{20}\times\mathbf{20}^*$ contains **15** only once, the "minimal" meson-baryon vertex[1] is unique when one takes into account the parities of the mesons.[6] One finds

$$f_{\rho^0\bar{p}p}:f_{\omega\bar{p}p}=1:3. \tag{6}$$

This is in agreement with the $SU(6)$ prediction, and also with previous studies of low-energy nuclear forces,[7] of the high-energy pp cross section,[8] and of the nucleon isoscalar charge form factor.[9] All seem to indicate that $f_{\omega\bar{N}N}{}^2$ is considerably larger than $f_{\rho\bar{N}N}{}^2$. In addition, the ratio of pseudovector coupling to vector coupling, $g_A=(5/3)g_V$, is also reproduced in $SU(4)$.

3. REMARKS

It may be thought that, since $SU(4)$ is a subgroup of $SU(6)$, it should be trivially true that all $SU(6)$ predictions concerning particles which lie in the $SU(4)$ "sector" are reproduced in $SU(4)$. That this is not obviously the case can be illustrated by the following consideration:

One might write out the baryon current with free indices:

$$(J_1)_{\alpha i}{}^{\alpha' i'}=\tfrac{1}{4}\delta_\alpha{}^{\alpha'}\delta_i{}^{i'}\bar{\psi}^{\beta j,\gamma k,\delta l}\psi_{\beta j,\gamma k,\delta l},$$
$$(J_{15})_{\alpha i}{}^{\alpha' i'}=\bar{\psi}^{\alpha' i',\beta j,\gamma k}\psi_{\alpha i,\beta j,\gamma k}-(J_1)_{\alpha i}{}^{\alpha' i'},$$

and assume that the low-frequency electromagnetic vertex has the form

$$(\tfrac{1}{3}J_1+J_{15})_{\alpha i}{}^{\alpha' i'}[\delta_{i'}{}^ie\varphi+\mu(p)i(\boldsymbol{\sigma}\cdot\mathbf{P}\times\boldsymbol{\varepsilon})_{i'}{}^i]Q_{\alpha'}{}^\alpha. \tag{7}$$

This amounts to an assumption distinct from the assumption of Eq. (2).[10] The proton-neutron magnetic-moment ratio that follows from this assumption would be -4.

To remove the ambiguity it is necessary to specify the relation between the isospin property at the charge vertex and that at the magnetic-moment vertex. One way is to take a model in which the photon-baryon interaction is mediated by the vector mesons in the adjoint representation. Just as the low-frequency effective meson-baryon "minimal" coupling is unique, so is the low-frequency effective meson-baryon "magnetic" coupling. The answer one gets is in agreement with Eq. (2) and not with Eq. (7). In view of the higher mass of the physical φ, as well as the weakness of the nucleon-φ coupling as indicated in the study of the nucleon isoscalar form factor,[9] the assumption of the dominance of vector mesons in the adjoint representation does not seem unreasonable. The agreement between the $SU(6)$ and $SU(4)$ results is also clear physically, for the physical φ, which is not in the **15** representation of $SU(4)$, is also not coupled to the nonstrange baryons in $SU(6)$.

Needless to say, if one follows the treatment of the baryon electromagnetic interaction by Delbourgo *et al.*[11] in $\tilde{U}(12)$, and does a corresponding calculation in $\tilde{U}(8)$ using mesons in the **63** representation, one also obtains agreement with Eq. (2).

Note added in proof. After this note was submitted for publication, we received an unpublished report by L. C. Biedenharn, J. Nuyts, and N. Straumann (CERN) in which the magnetic moments in $SU(4)$ were also discussed, and in which an earlier report by Y. C. Leung and A. O. Barut (Trieste, unpublished report No. IC/65/16) was criticized.

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[6] M. A. B. Bég and A. Pais, Ref. 2.

[7] R. S. McKean, Phys. Rev. **125**, 1399 (1962); D. Amati, E. Leader, and B. Vitale, Phys. Rev. **130**, 750 (1963).

[8] Riazuddin and Fayyazuddin, Phys. Rev. **132**, 873 (1963).

[9] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).

[10] Of course the two corresponding assumptions in $SU(6)$ become indistinguishable since Q transforms like **8** in $SU(3)$.

[11] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **284**, 146 (1965).