Relativistic Merging of Spin and Isotopic Spin*

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A calculation is presented in which spin and isotopic spin are combined within one larger symmetry. By treating the spin symmetry in a novel way the theory is made completely relativistic. Excellent agreement with experiment is achieved and, in particular, the experimentally well verified form equality between the electric and magnetic form factors of the proton emerges as a natural consequence of the theory.

I. INTRODUCTION

A NEW technique has recently been proposed[1] which provides a relativistic method of treating the spin independence of strong interactions. This allows for the construction of theories in which the spin and internal symmetries are combined within a larger group of symmetries in a completely relativistic manner. The new method is easily seen to yield the conventional results in the static limit, and thus the crucial test of its validity lies in the comparison of its predictions with experiment in a relativistic situation. One such test is presented in this paper: a calculation in which spin and isotopic spin are combined within one larger relativistic symmetry, thus allowing predictions to be made of relations between the nucleon electromagnetic form factors.

The proposed treatment of the spin independence is given in some detail in Sec. II. Section III treats the problem of combining the spin and isotopic-spin symmetries and the results are discussed in the final section.

II. THE TREATMENT OF SPIN

The new approach to spin independence is based on the idea that any such symmetry should not be regarded as a basic property of the underlying space-time but rather that it must be dictated by the physics of the specific process under consideration, namely the momenta of the particles involved and their interactions. It is therefore proposed that the dependence on the momenta of the particles involved be *explicitly* included in the specification of the symmetry. In the absence of internal symmetries, the most general set of transformations which may be performed on a spinor $U(p)$ may be defined in infinitesimal form by

$$U(p)\rightarrow SU(p)=[1+i\alpha_\mu\Gamma^\mu]U(p)$$
$$=[1+i(a_I1+a_5\gamma_5+a_\mu\gamma^\mu+ia_{\mu5}\gamma^\mu\gamma_5+\tfrac{1}{2}a_{\mu\nu}\sigma^{\mu\nu})]U(p), \quad (1)$$

where the scalar product $\bar{U}(p)U(p)$ is invariant if the parameters α_μ are real. If now $U(p)$ and $\bar{U}(p')$ are to represent incoming and outgoing free fermions at some vertex, then the generators in S must commute with $(\not p-m)$ and $(\not p'-m)$ in order that the transformed spinors $SU(p)$ and $\bar{U}(p')S^{-1}$ shall remain solutions of the Dirac equation with the appropriate mass and momentum. This immediately restricts the transformations defined in (1) to the four-parameter subset specified by

$$a_5=a_\mu=p^\nu a_{\nu5}=p'^\nu a_{\nu5}=p^\nu a_{\nu\mu}=p'^\nu a_{\mu\nu}=0. \quad (2)$$

The remaining four operators form a closed set with a $\tilde{U}(2)$ structure which may be interpreted as a representation of a spin group, and this will be taken as defining the set of "spin transformations" under which the strong interactions of free fermions with virtual bosons is assumed invariant.

It is probably of interest to provide a more concrete construction of the generators defined above which allows for a simple and clear inspection of their properties. This is most easily achieved in terms of a basis of four mutually orthogonal unit four-vectors. Firstly, the four-vectors $P=p'+p$ and $q=p'-p$ are orthogonal (for free spinors which represent particles of equal mass) and therefore provide the definitions of a unit timelike vector ϵ_0 and a unit spacelike vector ϵ_3 which are orthogonal. The existence of two more spacelike orthogonal unit vectors ϵ_1 and ϵ_2 which are orthogonal also to ϵ_0 and ϵ_3 may be ensured by a consideration of the Breit frame of the two fermions where $p=(E,0,0,k)$ and $p'=(E,0,0,-k)$ and it is clear that the unit vectors along the x and y directions in this frame may be Lorentz-transformed to provide a covariant definition of ϵ_1 and ϵ_2 in any general frame. Thus it is possible to represent the four operators of interest as

$$1,\quad S_1=i\epsilon_1{}^\mu\gamma_\mu\gamma_5,\quad S_2=i\epsilon_2{}^\mu\gamma_\mu\gamma_5,\quad S_3=\epsilon_1{}^\mu\epsilon_2{}^\nu\sigma_{\mu\nu} \quad (3)$$

and it is easy to verify directly that

$$S_iS_j=\delta_{ij}+i\epsilon_{ijk}S_k \quad (4)$$

and that S_i commutes with $(\not p-m)$ and $(\not p'-m)$.

This treatment generalizes in an obvious way for the higher dimensional representations if the free spinors of higher rank are assumed to obey the Bargmann-Wigner[2] equations

$$\begin{aligned}(\not p)_\alpha{}^\sigma\Psi_{\sigma\beta\gamma}\cdots&=m\Psi_{\alpha\beta\gamma}\cdots\\(\not p)_\beta{}^\sigma\Psi_{\alpha\sigma\gamma}\cdots&=m\Psi_{\alpha\beta\gamma}\cdots \text{ etc.}\end{aligned} \quad (5)$$

for these are effectively the Dirac equation applied separately to each index. In particular, if the second-

* Work supported in part by the U. S. Office of Naval Research.
[1] K. J. Barnes, Phys. Rev. Letters 14, 798 (1965).

[2] V. Bargmann and E. Wigner, Proc. Nat. Acad. Sci. U. S., 34, 211 (1948).

rank mixed spinor $\Phi_\alpha{}^\beta$ which may be written in the form

$$\Phi_\alpha{}^\beta=[\theta+\phi\gamma_5+i\gamma^\mu\gamma_5F_{\mu5}+\gamma^\mu\phi_\mu+\tfrac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}]_\alpha{}^\beta \quad (6)$$

is assumed[3,4] to transform in the same way as $\Psi_\alpha(q)\bar{\Psi}^\beta(q)$, then it provides[5] a possible description[6] of pseudoscalar and vector particles described, respectively, by the sets of functions $(\phi,F_{\mu5})$ and $(\phi_\mu,F_{\mu\nu})$. This may be seen by the direct application of the Bargmann-Wigner equations with mass μ and momentum q which gives the equations

$$\begin{gathered}\theta=0\\ i\mu F_{\mu5}=q_\mu\phi,\qquad \mu\phi=iq^\mu F_{\mu5}\\ i\mu F_{\mu\nu}=q_\mu\phi_\nu-q_\nu\phi_\mu,\quad i\mu\phi_\mu=q^\nu F_{\mu\nu}.\end{gathered} \quad (7)$$

Similarly the third-rank spinor $\Psi_{\alpha\beta\gamma}$ provides a possible description of both spin $\frac{3}{2}$ particles and spin $\frac{1}{2}$ particles each with even parity relative to the basic spinor Ψ_α. This made possible by an index symmetry decomposition of the spinor. Since the transformations defined by Eq. (3) contain only the Dirac matrices 1, $i\gamma^\mu\gamma_5$, and $\sigma^{\mu\nu}$, it is possible to define a matrix B (within the Dirac algebra) by the requirement that Ψ^TB^{-1} transform in the same way as $\bar{\Psi}$, and where B has the properties

$$B^{-1}\gamma^\mu B=(\gamma^\mu)^T, \quad (8)$$

$$B^T=-B, \quad (9)$$

$$B^\dagger=\gamma_0B^{-1}\gamma_0. \quad (10)$$

As is easily verified[7] the matrices (1B), $(\gamma^\mu B)$, and (γ_5B) are antisymmetric while $(\sigma^{\mu\nu}B)$ and $(i\gamma^\mu\gamma_5B)$ are symmetric, so that concrete representations of higher rank spinors of specified symmetry may easily be made. Thus the completely symmetric part of $\Psi_{\alpha\beta\gamma}$ may be written in the form

$$D_{\alpha\beta\gamma}=D_\alpha{}^\mu(i\gamma_\mu\gamma_5B)_{\beta\gamma}+\tfrac{1}{2}\Psi_\alpha{}^{\mu\nu}(\sigma_{\mu\nu}B)_{\beta\gamma}, \quad (11)$$

where the full symmetry is ensured by the conditions

$$\gamma_\mu D^\mu=0=\sigma_{\mu\nu}\Psi^{\mu\nu}, \quad (12)$$

$$D^\mu+\gamma_5\gamma_\sigma\Psi^{\mu\sigma}=0. \quad (13)$$

If the Bargmann-Wigner equations are applied with mass m and momentum p, this finally takes the form

$$mD_{\alpha\beta\gamma}(p)=[(p\!\!\!/+m)i\gamma_\mu\gamma_5B]_{\alpha\beta}D_\gamma{}^\mu(p), \quad (14)$$

where $(p\!\!\!/-m)D^\mu=0$, $\gamma_\mu D^\mu=0$, and $p_\mu D^\mu=0$. Hence $D_{\alpha\beta\gamma}$ provides a possible description of spin $\frac{3}{2}$ particles.[8] Again the mixed symmetry part of $\Psi_{\alpha\beta\gamma}$, specified by

$$M_{\alpha\beta\gamma}+M_{\beta\alpha\gamma}=0 \quad (15)$$

$$M_{\alpha\beta\gamma}+M_{\beta\gamma\alpha}+M_{\gamma\alpha\beta}=0, \quad (16)$$

may be written in the form

$$M_{\alpha\beta\gamma}=B_{\alpha\beta}N_\gamma+(\gamma_5B)_{\alpha\beta}\Psi_\gamma+(\gamma_\mu B)_{\alpha\beta}\Psi_\gamma{}^\mu \quad (17)$$

with the constraint

$$N+\gamma_5\Psi+\gamma_\mu\Psi^\mu=0. \quad (18)$$

When the Bargmann-Wigner equations are applied this takes the form

$$mM_{\alpha\beta\gamma}(p)=[(p\!\!\!/+m)B]_{\alpha\beta}N_\gamma(p) \quad (19)$$

where $(p\!\!\!/-m)N=0$. Hence $M_{\alpha\beta\gamma}$ provides another possible description of spin-$\frac{1}{2}$ particles, and this will in fact be the choice made in the present work for a description of the physical nucleons.

The specification of the basic symmetry transformations and the treatment of the free particle states is now essentially complete but the interactions between the particles are as yet undetermined. It is proposed that the interaction of free fermions with virtual bosons be described by the interaction vertex

$$\bar{\Psi}^\alpha(p')\Lambda_\alpha{}^\beta(q)\Psi_\beta(p), \quad (20)$$

where the second-rank mixed spinor $\Lambda_\alpha{}^\beta$ is to be formed from $\Phi_\alpha{}^\beta$, p, p' and scalar functions of the invariant four-momentum transfer squared. This interaction is not only a Lorentz-invariant but is also invariant under the spin transformations, since the generators of these spin transformations commute with both $p\!\!\!/$ and $p\!\!\!/'$. There is still a great deal of arbitrariness in this interaction, and in order to remove this it is speculated that $\Lambda_\alpha{}^\beta$ must be formed from generators of the $P\tilde{U}(4)$ set of transformations. These are the full set of transformations

$$U(p)\to S_p(p,p')U(p), \quad (21)$$

where S_p has the properties

(A) $(p\!\!\!/-m)S_pU(p)=0=\bar{U}(p')S_p(p\!\!\!/'-m)$,

(B) $\bar{U}(p)U(p)$ and $\bar{U}(p')U(p)$ are invariants,

(C) the product of any two generators of these transformations shall again be a generator.

Thus this closed set of generators defines the most general set of transformations compatible with the free particle Dirac equations and preserving scalar products, while possessing the property that the generators of the corresponding transformations

$$\bar{U}(p')\to\bar{U}(p')S_{p'}{}^{-1}(p,p') \quad (22)$$

are identical to those in S_p. This last requirement is, of course, essential if the generators are to be identified as couplings in the manner suggested above.

A straightforward but somewhat tedious application of the above conditions utilizing the properties of the Dirac matrices and equation shows that a suitable

[3] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965).

[4] B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965).

[5] F. J. Belinfante, Physica **6**, 870 (1939).

[6] N. Kemmer, Proc. Roy. Soc. (London) **A173**, 91 (1939).

[7] See, for example, J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955).

[8] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

form for S_p is

$$S_p(p,p')=1+if(q^2)\alpha_\mu(p+m)[(p'+m)\Gamma^\mu(p+m)+(4m^2-q^2)\Gamma^\mu +(4m^2-q^2)^{1/2}[\Gamma^\mu(p+m)+(p'+m)\Gamma^\mu]](p'+m)\,, \quad (23)$$

where $f(q^2)$ is an arbitrary function of q^2. The normalization requirement that the generator with $\Gamma^\mu=1$ shall be the identity operator determines $f(q^2)$ to be given by

$$f^{-1}(q^2)=2(4m^2-q^2)[(4m^2-q^2)+2m(4m^2-q^2)^{1/2}]. \quad (24)$$

It therefore follows that the interaction defined in Eq. (20) takes the form (which is now nonlocal),

$$\tfrac{1}{2}g[(4m^2-q^2)+2m(4m^2-q^2)^{1/2}]^{-1} \times\bar\Psi(p')[(4m^2-q^2)\Phi(q)+(p+m)\Phi(q)(p'+m)+(4m^2-q^2)^{1/2}[(p+m)\Phi(q)+\Phi(q)(p'+m)]]\Psi(p)\,, \quad (25)$$

where g is an arbitrary coupling constant which may depend on q^2. If now it is decided to maintain the mass-shell relations

$$\theta=0,\quad i\mu F_{\mu5}=q_\mu\phi,\quad \text{and}\quad i\mu F_{\mu\nu}=q_\mu\phi_\nu-q_\nu\phi_\mu \quad (26)$$

given in Eq. (7) also in the interaction region, then the interaction takes the particularly simple form

$$g\bar\Psi(p')[(4m^2-q^2)^{-1/2}\phi_\mu[P^\mu+r^\mu/\mu]+\phi\gamma_5((4m^2-q^2)^{1/2}/\mu)]\Psi(p) \quad (27)$$

where $r^\mu=\epsilon^{\mu\nu\rho\lambda}q_\nu P_\rho\gamma_\lambda\gamma_5$. This then completes the formal apparatus needed to treat spin symmetry within the context of a relativistic S-matrix theory dealing with free-particle states and three-particle vertices.

III. THE COMBINED SYMMETRY

The treatment of spin independence discussed in the previous section provides a natural framework for the relativistic merging of such spin independence with internal symmetries. Since the purpose of this paper is to provide a relativistic test of the present treatment of spin independence rather than to investigate the internal symmetries, the simplest possible choice of internal symmetry will be made, namely, invariance will be assumed under the transformations which have as their generators all possible products of the generators of the spin transformations and those of the isotopic spin group; specifically, for the lowest-dimensional representation,

$$U(p)\to[1+i\alpha_{ij}S^i\tau^j]U(p)\,, \quad (28)$$

where the τ^j are the usual Pauli matrices and the S^i were defined in Eq. (3). The particle spinor thus has two sets of indices, the usual spin indices on which the spin transformations act, and the isotopic-spin indices on which the isotopic-spin transformations act. This gives then a relativistic version of Wigner's[9] theory of supermultiplets, but in view of the recent successes[10] of $SU(6)$ the approach of Leung and Barut[11] will be followed and the physical baryons will be assigned to the fully symmetric multispinor which transforms as the product of three basic spinors (or quarks). Under the direct product of the spin transformations and those of isotopic spin, this symmetric third-rank spinor decomposes in the following manner:

$$\Psi_{ABC}=\Psi_{\alpha i,\beta j,\gamma k}=D_{\alpha\beta\gamma,ijk}+(1/2\sqrt{6})[\epsilon_{ij}M_{\alpha\beta\gamma,k}+\epsilon_{jk}M_{\beta\gamma,\alpha,i}+\epsilon_{ki}M_{\gamma\alpha\beta,j}]\,, \quad (29)$$

where the Greek indices represent spin variables and lie in the range 0–3, while the Latin indices represent isotopic-spin variables and take the values 1 and 2. Have ϵ_{ij} is the two-dimensional Levi-Civita symbol, $D_{\alpha\beta\gamma,ijk}$ is completely symmetric in both Greek and Latin indices (and will be taken to represent N^* states), and $M_{\alpha\beta\gamma,i}$ which has mixed symmetry (of the type described in the last section) in the Greek indices describes the physical nucleons. If the mesons are now described by the second-rank traceless mixed spinor

$$\Phi_A{}^B=\Phi_{\alpha i}{}^{\beta j}=[\phi_i{}^j(\gamma_5+(\gamma^\mu\gamma_5/\mu)q_\mu)+(\phi_\mu)_i{}^j(\gamma^\mu+i\sigma^{\mu\nu}q_\nu/\mu)]_\alpha{}^\beta \quad (30)$$

as suggested by the previous section, then the baryon-meson interaction vertex takes the form

$$g\bar\Psi(p')^{\alpha i,C,D}[(4m^2-q^2)^{-1/2}\phi_\mu(q)_i{}^j(P^\mu+r^\mu/\mu)_\alpha{}^\beta+\phi(q)_i{}^j(\gamma_5)_\alpha{}^\beta((4m^2-q^2)^{1/2}/\mu)]\Psi(p)_{\beta j,C,D}. \quad (31)$$

Using the results given earlier in this paper, the part of this interaction which represents the interaction of vector

[9] E. P. Wigner, Phys. Rev. **51**, 105 (1937).
[10] See for example: M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965) and Refs. (1) and (2) contained therein. K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, *ibid.* **13**, 698 (1964). F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964). M. A. B. Bég, B. W. Lee, and A. Pais, *ibid.* **13**, 514 (1964). B. Sakita, *ibid.* **13**, 643 (1964). K. Johnson and S. B. Treiman, *ibid.* **14**, 189 (1965).
[11] Y. C. Leung and A. O. Barut (unpublished).

mesons and nucleons may be extracted in the form

$$(g/12)(4m^2-q^2)^{-1/2}\phi_\mu(q)_i{}^j(P^\mu+r^\mu/\mu)_\alpha{}^\beta[\bar{M}^{\alpha\gamma\delta,r}M_{\beta\gamma\delta,s}(\delta_j{}^i\delta_r{}^s+4\delta_r{}^i\delta_j{}^s)+\bar{M}^{\alpha\gamma\delta,r}M_{\beta\delta\gamma,s}(\delta_j{}^i\delta_r{}^s-5\delta_r{}^i\delta_j{}^s)], \quad (32)$$

where

$$mM(p)_{\alpha\beta\gamma,i}=[(p\!\!\!/+m)B]_{\alpha\beta}N_{\gamma,i}(p). \quad (33)$$

Assuming that the coupling of the photon to the nucleons is dominated by vector meson coupling, an immediate physical test of the theory is available. For the conventional nucleon electromagnetic current[12,13]

$$J^\nu=e(1-q^2/4m^2)^{-1}\bar{N}(p')[(P^\nu/2m)(G_E{}^S+\tau_3G_E{}^V)+(r^\nu/4m^2)(G_M{}^S+\tau_3G_M{}^V)]N(p) \quad (34)$$

may be seen from the above to have the form

$$J^\nu=eF(q^2)(1-q^2/4m^2)^{1/2}\bar{N}(p')[(P^\nu/2m)\tfrac{1}{2}(1+\tau_3)+(r^\nu/4m^2)(2m/\mu)\tfrac{1}{6}(1+5\tau_3)]N(p), \quad (35)$$

where $F(q^2)=\mu^2f(q^2)[\mu^2-q^2]^{-1}$ contains the dependence on the couplings and the effect of the meson propagators, and $f(q^2)=1$ if the dominance by the vector mesons describes the process completely.

IV. DISCUSSION OF THE RESULTS

The specific form of the current obtained in (35) strongly supports the point of view put forward by Gourdin[14] that the physical nucleon form factors be taken not as G_E and G_M, but as

$$G_0=G_E\left(1-\frac{q^2}{4m^2}\right)^{-1/2}, \qquad G_1=G_M\left(1-\frac{q^2}{4m^2}\right)^{-1/2}, \quad (36)$$

a form first defined by Yennie[15] *et al.*, and leaves the hope that G_0 and G_1 may yet be analytic. According to Gourdin, the threshold arguments of Bergia and Brown[16] apply equally to these form factors as to G_E, G_M, implying $G_0=G_1$ at $q^2=4m^2$ in the time-like region. The present work is compatible with this, suggesting as it does that the form factors both vanish at this point, and seems to require that $f(q^2)$ tends strongly to zero for large values of the momentum transfer, if the form factors G_E and G_M, are to have the usually assumed asymptotic behavior.[17]

However, in order that an easy comparison with the experimental results may be made, the results of the last section are best expressed as relations between the more usual electromagnetic form factors of the proton and neutron, defined by

$$G_{E,M}{}^p=G_{E,M}{}^S+G_{E,M}{}^V \qquad G_{E,M}{}^n=G_{E,M}{}^S-G_{E,M}{}^V. \quad (37)$$

It is easily seen that this results in the equations

$$G_E{}^n(q^2)=0, \quad (38)$$

$$G_M{}^n(q^2)=-\tfrac{2}{3}G_M{}^p(q^2), \quad (39)$$

$$G_M{}^p(q^2)=\frac{2m}{\mu}G_E{}^p(q^2). \quad (40)$$

Equation (38) and the form equalities is (39) and (40) have all been speculated[12] in the past from the experimental data and have recently been most impressively confirmed[18] by the electron scattering data from Stanford (in the range $0<-q^2<30\,F^{-2}$), although that (38) is not exact is clearly indicated by the accurate measurements of low-energy neutron scattering.[19] Of these relations, the first two have been obtained and discussed before,[20] but the third is unique to the present treatment of spin independence. The numerical value of $2m/\mu$ obtained for the static magnetic moment of the proton is difficult to evaluate reliably. Since the experimental value is 2.79 in units of nuclear magneton it seems sensible to insist on the physical nucleon mass for m, and to conclude that the mean vector-meson mass is 674 MeV if this vector-meson contribution gives the full physical effect. This value lies between the physical masses of the vector mesons and the mean meson mass proposed by Bég and Singh[21] on the basis of $SU(6)$ (of which the isotopic-spin group is a subgroup) and seems not entirely unreasonable. Perhaps the most satisfactory feature of this prediction $(2m/\mu)$, however, is that it derives entirely from the tensor $\sigma^{\mu\nu}$ coupling and no trace remains of the Dirac γ^μ magnetic-moment contribution. This emphasizes that the nucleon is in no sense a Dirac particle minimally coupled to the electromagnetic field, and clarifies[22] why

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[16] S. Bergia and L. Brown, Proceedings of the Stanford Conference on Nucleon Structure, 1963 (unpublished).

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[18] E. B. Hughes *et al.*, Bull. Am. Phys. Soc. **10**, 95 (1965).

[19] See Ref. (13) and the references contained therein.

[20] K. J. Barnes, P. Carruthers, and Frank von Hippel, Phys. Rev. Letters **14**, 82 (1965). See also Refs. (3) and (4).

[21] M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418, 681 (E) (1964).

[22] The curious result that the $SU(6)$ prediction $\mu_n=-\frac{2}{3}\mu_p$ related the total moments while numerically the induced (anomalous) moments (which one might intuitively expect to be related) did not dominate these values was pointed out to the author by D. R. Yennie.

the static values of the total magnetic moments of the particles should be directly related [as in equation (39)] rather than the anomalous moments.

It is perhaps pertinent to conclude by emphasizing the point of view put forward in this work. No redefinition of the spin of a particle is involved. The "spin" transformations[23] defined in Eqs. (1) and (2) and under which invariance is postulated are the intersection of the two "little groups" of the basic $\tilde{U}(4)$ group defined in Eq. (1) corresponding to the momenta p and p'. Thus the momenta specify the allowed set of transformations, but the transformations themselves do not contain the momenta explicitly. In contrast to this, the $P\tilde{U}(4)$ transformations from which the couplings are formed are the generalization of the "spin" transformations when explicit dependence on the momenta is allowed. Notice that some such generalization has to be made, because the generators of the basic invariance group itself do not form a sufficient basis with which to specify the interactions.

Finally, it should be noted that since the allowed "spin" transformations are determined essentially uniquely by two independent four-momenta there is no similar $\tilde{U}(2)$ group defined for four- (or higher) particle vertices. Any restrictions which the theory may place on such interactions are the implicit effects of the three-point functions on these interactions (e.g., through unitarity, or the decomposition of the amplitudes into single particle exchange contributions). The study of these effects and the associated problem of consistently and uniquely specifying the three-meson vertex are clearly matters for further detailed consideration.

[23] See similar attempts for the free particle case: F. Gürsey, Phys. Letters **14**, 330 (1965). Y. Ne'eman, *ibid.* **14**, 327 (1965). K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters **14**, 458 (1965). L. K. Pandit and Riazuddin, *ibid.* **14**, 462 (1965).

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Broken Symmetric Couplings for Decuplet Decays into Baryons and Pseudoscalars*

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In this approach to $SU(3)$ symmetry breaking we adopt the point of view that perturbation theory may be used to calculate the corrections to the $B^*(10)BP$ coupling constants. The symmetry breaking is introduced via Gell-Mann–Okubo mass splittings of both the internal and external lines of the third-order vertex diagram. In this manner we obtain coupling constants which obey the sum rules of Gupta and Singh. This approach seems consistent with the notion that the Gell-Mann–Okubo formulas are themselves based on first-order symmetry breaking in the masses. Our results are compared with those of other authors.

1. INTRODUCTION

ONE of the greatest successes of the $SU(3)$ symmetry has been the prediction of the Ω based on a decuplet representation for the excited 3,3 baryons.[1] The mass of the Ω, predicted from the well-known mass formulas of Okubo and Gell-Mann, is predicated on a specific transformation property in unitary space for the interaction which breaks the symmetry, viz., that the symmetry-breaking interaction transforms like the $T=0$, $Y=0$ component of an octet.[2,3] Many relations between coupling constants follow from the assumption that the strong interactions are $SU(3)$ symmetric; these relations are often used to estimate experimentally unknown coupling constants. The question arises as to how reliable these estimates of coupling constants are when a symmetry-breaking octet interaction is taken into account. Based on the knowledge of the transformation property of the interaction together with the assumption that first-order perturbation theory suffices, sum rules similar to those for the masses have been obtained for the coupling constants[4] but a dynamical calculation is necessary to discover how individual coupling constants are affected. One such calculation by Wali and Warnock[5] has already been carried out with the result that the coupling constants show large

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