the static values of the total magnetic moments of the particles should be directly related [as in equation (39)] rather than the anomalous moments.

It is perhaps pertinent to conclude by emphasizing the point of view put forward in this work. No redefinition of the spin of a particle is involved. The "spin" transformations[23] defined in Eqs. (1) and (2) and under which invariance is postulated are the intersection of the two "little groups" of the basic $\tilde{U}(4)$ group defined in Eq. (1) corresponding to the momenta p and p'. Thus the momenta specify the allowed set of transformations, but the transformations themselves do not contain the momenta explicitly. In contrast to this, the $P\tilde{U}(4)$ transformations from which the couplings are formed are the generalization of the "spin" transformations when explicit dependence on the momenta is allowed. Notice that some such generalization has to be made, because the generators of the basic invariance group itself do not form a sufficient basis with which to specify the interactions.

Finally, it should be noted that since the allowed "spin" transformations are determined essentially uniquely by two independent four-momenta there is no similar $\tilde{U}(2)$ group defined for four- (or higher) particle vertices. Any restrictions which the theory may place on such interactions are the implicit effects of the three-point functions on these interactions (e.g., through unitarity, or the decomposition of the amplitudes into single particle exchange contributions). The study of these effects and the associated problem of consistently and uniquely specifying the three-meson vertex are clearly matters for further detailed consideration.

[23] See similar attempts for the free particle case: F. Gürsey, Phys. Letters **14**, 330 (1965). Y. Ne'eman, *ibid.* **14**, 327 (1965). K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters **14**, 458 (1965). L. K. Pandit and Riazuddin, *ibid.* **14**, 462 (1965).

ACKNOWLEDGMENTS

The author is indebted to F. L. Gross, F. von Hippel and D. R. Yennie for many stimulating discussions.

PHYSICAL REVIEW VOLUME 139, NUMBER 4B 23 AUGUST 1965

Broken Symmetric Couplings for Decuplet Decays into Baryons and Pseudoscalars*

ELIZABETH JOHNSON†
Imperial College and Battersea College of Technology, London, England

AND

EDWARD R. MCCLIMENT‡
University of Iowa, Iowa City, Iowa

(Received 11 March 1965)

In this approach to $SU(3)$ symmetry breaking we adopt the point of view that perturbation theory may be used to calculate the corrections to the $B^*(10)BP$ coupling constants. The symmetry breaking is introduced via Gell-Mann–Okubo mass splittings of both the internal and external lines of the third-order vertex diagram. In this manner we obtain coupling constants which obey the sum rules of Gupta and Singh. This approach seems consistent with the notion that the Gell-Mann–Okubo formulas are themselves based on first-order symmetry breaking in the masses. Our results are compared with those of other authors.

1. INTRODUCTION

ONE of the greatest successes of the $SU(3)$ symmetry has been the prediction of the Ω based on a decuplet representation for the excited 3,3 baryons.[1] The mass of the Ω, predicted from the well-known mass formulas of Okubo and Gell-Mann, is predicated on a specific transformation property in unitary space for the interaction which breaks the symmetry, viz., that the symmetry-breaking interaction transforms like the $T=0$, $Y=0$ component of an octet.[2,3] Many relations between coupling constants follow from the assumption that the strong interactions are $SU(3)$ symmetric; these relations are often used to estimate experimentally unknown coupling constants. The question arises as to how reliable these estimates of coupling constants are when a symmetry-breaking octet interaction is taken into account. Based on the knowledge of the transformation property of the interaction together with the assumption that first-order perturbation theory suffices, sum rules similar to those for the masses have been obtained for the coupling constants[4] but a dynamical calculation is necessary to discover how individual coupling constants are affected. One such calculation by Wali and Warnock[5] has already been carried out with the result that the coupling constants show large

* Part of this work was performed while both authors were at the University of Wisconsin, Madison, Wisconsin.
† Supported in part by the U. S. Atomic Energy Commission.
‡ Supported in part by the Wisconsin Alumni Research Foundation.

[1] R. E. Behrends, J. Dreitlein, C. Fronsdal, and W. Lee, Rev. Mod. Phys. **34**, 1 (1962).
[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[3] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).
[4] V. Gupta and V. Singh, Phys. Rev. **135**, B1442 (1964).
[5] K. C. Wali and R. L. Warnock, Phys. Rev. **135**, B1358 (1964).

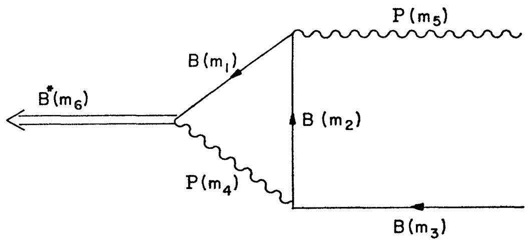

FIG. 1. Third-order diagram on which this calculation is based. The masses (or mass squares) in each of the lines are given by Gell-Mann–Okubo mass formulas.

deviations from their values predicted by unitary symmetry. The present calculation is an effort to determine whether the large deviations obtained by Wali and Warnock were peculiar to their model.

In this work we assume that deviations of coupling constants from their $SU(3)$ values for decays of the decuplet baryon resonances $B^*(10)$ into baryons B and pseudoscalars P may be approximated by introducing the exact Gell-Mann–Okubo (GMO) mass splittings into each line of the third-order vertex diagram and by using Feynman techniques to calculate the changes in the amplitudes to first order in the mass splittings Δm (Δm^2 for bosons). This approach seems to be consistent with the notion that the mass formulas themselves follow from the assumption that the symmetry breaking occurs in first order.

2. DESCRIPTION OF THE MODEL

The general form of the amplitude for the coupling of a spin-$\frac{3}{2}$ particle on its mass shell to a spin-$\frac{1}{2}$ and pseudoscalar particle is given by

$$M_{B^*\to BP}=(M_{B^*}m_B/2\omega_P E_{B^*}E_B)^{1/2} \times i\kappa_\mu \bar{u}_\mu(M_{B^*})u_P(m_B)F_{BP}{}^{B^*}, \quad (1)$$

where $\kappa_\mu=\frac{1}{2}(B_\mu-P_\mu)$, using the convention of denoting the 4-momentum of a particle by its name. $\bar{u}_\mu(M_{B^*})$ is the Rarita-Schwinger spinor in momentum space which describes a particle of spin $\frac{3}{2}$. Its properties are[6,7]

$$\bar{u}_\mu(i\gamma\cdot B^*+M_{B^*})=0\,,\qquad \bar{u}_\mu B_\mu{}^*=0\,,\qquad \bar{u}_\mu\gamma_\mu=0. \quad (2)$$

If we adopt the phenomenological interaction Lagrangians[8]

$$\mathfrak{L}_{B^*BP}=(G_0/M)\epsilon_{cdr}\bar{B}_\mu{}^{*abr}(M_{B^*}) \times B_a{}^c(m_B)\partial_\mu P_b{}^d(\mu_P)+\text{H.c.}, \quad (3)$$

$$\mathfrak{L}_{BBP}=i\sqrt{2}g[(1-2f)\bar{B}_c{}^a\gamma_5 B_a{}^b P_b{}^c+\bar{B}_c{}^a\gamma_5 B_b{}^c P_a{}^b], \quad (4)$$

which are consistent with our assumptions that the couplings are kept symmetric while the symmetry breaking is introduced via the masses, the lowest order amplitude has the obvious form

$$M_{B^*\to BP}{}^{(1)}=(M_{B^*}m_B/2\omega_P E_{B^*}E_B)^{1/2} \times C_{BP}{}^{B^*}(G_0/M)i\kappa_\mu\bar{u}_\mu(M_{B^*})u(m_B)\,, \quad (5)$$

where $C_{BP}{}^{B^*}$ is the appropriate $SU(3)$ symmetric Clebsch-Gordan coefficient for the particular decuplet, baryon, pseudoscalar state considered.[9]

The object of this calculation is to obtain broken symmetric corrections to the coupling constants $C_{BP}{}^{B^*}G_0/M$ which are first order in the deviations of the masses from their symmetric values. The model used is to take the lowest order perturbation-theoretic correction to the above coupling constants which contains the symmetry breaking. This of course occurs in the third order; and by expanding in the deviations of the masses from their degenerate values, one obtains a symmetric term plus terms proportional to the mass differences. The symmetric term, which may contain divergences, contributes to the unrenormalized coupling constants $C_{BP}{}^{B^*}G_0/M$; but if we imagine that the proper counter terms have been added to cancel any divergences, we obtain simply the renormalized symmetric coupling constants to third order. There are no divergences in the terms proportional to the mass differences, so that this method seems to provide a consistent means of calculating changes in the coupling constants due to symmetry-breaking interactions.

Since the calculation is to be carried out in third order it is necessary to evaluate the matrix element for the contribution from the third-order diagram, shown in Fig. 1 to establish notation. Since there are internal masses to be varied as well as those in the external lines, it is more convenient to relabel the masses, as shown in the figure, in the expressions to be expanded in the masses. The m_4 and m_5 in reality are mass squares because the mass formulas for the pseudoscalars are expressed in terms of mass squares rather than masses; but to treat them on the same footing as the baryon masses it is convenient to use a similar symbol. Then in terms of $m_1\cdots m_6$ the third-order matrix element is given by

$$M_{B^*\to BP}{}^{(3)}=\frac{1}{(2\pi)^4}\left(\frac{M_{B^*}m_B}{2\omega_P E_{B^*}E_B}\right)^{1/2} C_{BP}{}^{B^*}\frac{G_0}{M} \times 2g^2 i\kappa_\mu\bar{u}_\mu(M_{B^*})u(m_B)F_{BP}{}^{(3)B^*}(m_i)\,, \quad (6)$$

where $F_{BP}{}^{(3)B^*}(m_i)$ is evaluated by the usual Feynman methods.[10]

We have chosen to derive the effect of mass splittings on the coupling constants by assuming that the particles

[6] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).

[7] P. G. Federbush, M. T. Grisaru, and M. Tausner, Ann. Phys. (N. Y.) **18**, 23 (1962). These authors give the relation Eq. (18) between the resonance width Γ and the square of the coupling constant for spin $\frac{3}{2}^+\to\frac{1}{2}^++0^-$.

[8] The parameter $f=F/(D+F)$. A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

[9] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).

[10] Cf. S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Row Peterson & Company, New York, 1961).

involved are composed of fundamental triplets such that the mass of only the third component of each triplet is perturbed from its $SU(3)$ symmetric value:

$$\Delta m(N)=A\,,\quad \Delta m(\Sigma)=0\,,$$
$$\Delta m(\Xi)=B\,,\quad \Delta m(\Lambda)=\tfrac{2}{3}(A+B)\,. \tag{7}$$

This choice introduces a spurious scalar part into the symmetry-breaking terms and a consequent over-all shift in every coupling constant which is easily subtracted out at the end of the calculation. The contributions from mass splittings in the external lines are directly proportional to the Δm's of these lines. The contributions from the Δm's in the internal lines are proportional to factors given by[11]

$$\delta(B_b{}^a\bar{B}_d{}^c)=\{\delta_d{}^a\delta_b{}^c(\Delta m)_b{}^a-\tfrac{1}{3}\delta_b{}^a\delta_d{}^c[(\Delta m)_b{}^a+(\Delta m)_d{}^c-\tfrac{1}{3}\sum_x(\Delta m)_x{}^x]\}\times\left[\frac{\partial}{\partial m}\left(\frac{1}{i\gamma\cdot p-m}\right)\right]_{m_0}, \tag{8}$$

where, for baryons,

$$(\Delta m)_b{}^a=\delta_3{}^aB+\delta_b{}^3A\,, \tag{9}$$

and $B_b{}^a$ is the (traceless) $SU(3)$ baryon destruction operator. The treatment for internal bosons is analogous. The propagator in Eq. (8) is equivalent to the mass operator

$$\Delta\mathbf{m}=A\bar{B}_3{}^aB_a{}^3+B\bar{B}_a{}^3B_3{}^a\,, \tag{10}$$

which is obviously not traceless and therefore contains some scalar part.

By expanding $F^{(3)}$ of Eq. (6) in the mass differences Δm_i ($m_i=m_i{}^0+\Delta m_i$), we can write

$$F_{BP}{}^{(3)B^*}(m_i)=F_0(m_i{}^0)+\textstyle\sum_i(\partial F_0/\partial m_i)X_i(B^*BP)\Delta m_i\,, \tag{11}$$

where

$$F_0(m_i)=\frac{2}{i}\int_0^1dx\int_0^1dy\int d^4k\,x^2y\frac{\frac{3}{2}k^2+b^2(m_i,x,y)}{[k^2+a^2(m_i,x,y)]^3}\,, \tag{12}$$

with a^2 and b^2 algebraic functions of x, y, and the masses. The $X_i(B^*BP)$ are group-theoretical coefficients which depend on the particular particles in both the external and the intermediate states as well as the mass being varied. While $F_0(m_i{}^0)$ is divergent it is $SU(3)$ symmetric, and if the counter terms are present it combines with the G^0/M in $M^{(1)}$ to give the renormalized coupling constant $G_0{}^R/M$ correctly up to third order. The terms involving the $\partial F_0/\partial m_i$ are all convergent and represent the broken symmetric contributions to the coupling constants. If we make the assumption that $G_0{}^R/M$ does not differ much from the

[11] M. Ikeda, S. Ogawa, and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) 22, 715 (1959).

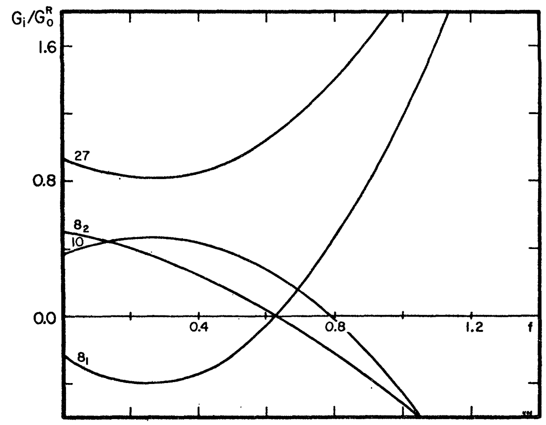

FIG. 2. Contributions to coupling constants in a diagonal-unitary-symmetry representation in units of the renormalized coupling. For pure $SU(3)$ these constants are all zero. The $G_i{}^2$ give the relative probability that (BP) in the $SU(3)$ multiplet (i) will interact with B^* via the $T=0$, $Y=0$ octet interaction.

unrenormalized coupling constant G_0/M, then the correct matrix element up to third order is given by

$$M_{B^*\to BP}\simeq\left(\frac{M_{B^*}m_B}{2\omega_PE_{B^*}E_B}\right)^{1/2}C_{BP}{}^{B^*}\frac{G_0{}^R}{M}i\kappa_\mu\bar{u}_\mu(M_{B^*})u(m_B)\times\left(1+\frac{g^2}{4\pi}\frac{1}{(2\pi^3)}\sum_i\frac{\partial F_0}{\partial m_i}X_i(B^*BP)\Delta m_i\right), \tag{13}$$

where $g^2/4\pi\simeq15$ is the πN coupling constant. The y integrals of

$$\frac{\partial F_0}{\partial m_i}=\pi^2\int_0^1dx\int_0^1dy\,x^2y\times\left(\frac{1}{a^2(m_i{}^0)}\frac{\partial b^2}{\partial m_i}-\frac{3a^2(m_i{}^0)+b^2(m_i{}^0)}{a^4(m_i{}^0)}\frac{\partial a^2}{\partial m_i}\right) \tag{14}$$

can be carried out analytically and the remaining x integration is easily computed by machine. The choice of $(\Delta m)_b{}^a$ in Eq. (9) and its counterpart for bosons implies the degenerate mass choice of $m_B{}^0=m(\Sigma)$ and $\mu_P{}^{0^2}=m^2(\pi)$.[12] Consistent removal of the scalar part however dictates the choice $m_B{}^0=\frac{1}{2}[m(\Sigma)+m(\Lambda)]$ and $\mu_P{}^{0^2}=\frac{1}{2}[m^2(\pi)+m^2(\eta)]$.

In Eq. (13) the terms in parentheses multiplied by $C_{BP}{}^{B^*}G_0{}^R$ are the $B^*\to BP$ coupling constants. Dividing out this factor for convenience we are left with the (normalized) Gupta-Singh[5] coefficients,

$$X(B^*BP)=1+(g^2/4\pi)(1/2\pi^3)\times\sum_i\frac{\partial F_0}{\partial m_i}X_i(B^*BP)\Delta m_i\,, \tag{15}$$

[12] The integrals are rather insensitive to the choice of degenerate masses. For instance, taking $m_B{}^0=m(\Sigma)$ and $(\mu_P{}^0)^2=m^2(\pi)$ gives results which are quite similar to those quoted herein. $M_B{}^{*0}$ is taken to be $M(Y_1{}^*)$.

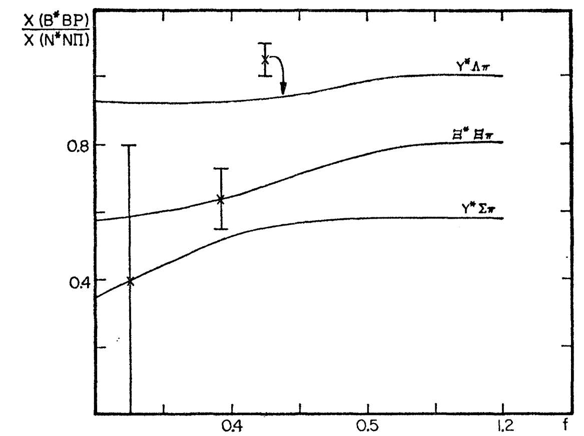

FIG. 3. Experimentally observable couplings normalized to $X(N^*N\pi)$ as a function of f with $a=1.00$. The experimental values along with their uncertainties are shown by the vertical error bars.

which by definition all equal one for unbroken symmetry. Gupta and Singh[5] (G.S.) have shown that, when $SU(3)$ is broken to first order by an interaction transforming like the $T=0$, $Y=0$ component of an octet, the coefficients $X(B^*BP)$ may be written in terms of five independent parameters a, p, q, r, s, where $a=1$, and $p=q=r=s=0$ correspond to unbroken symmetry. Altogether there are 12 B^*BP coupling constants so that among them there exist seven relations in the form of sum rules. Coupling constants calculated to first order in the mass splittings of any of the particle multiplets involved automatically obey these sum rules provided the masses in the multiplet obey the appropriate GMO mass formula.

3. RESULTS AND DISCUSSION

The principal result of this calculation is an enumeration of the group-theoretically arbitrary G.S. parameters p, q, r, s (setting the scalar part $a=1.00$). These values obviously depend upon the parameter f occurring in the $B\rightarrow BP$ Yukawa vertices. We obtain

$$\begin{aligned} a &\equiv 1.00\,,\\ p &= 0.56-0.61f+1.21f^2\,,\\ q &= -0.046-0.28f+0.57f^2\,,\\ r &= 0.22-0.16f-0.30f^2\,,\\ s &= 0.13+0.28f-0.57f^2\,. \end{aligned} \tag{16}$$

It is perhaps more illuminating to consider instead the parameters $G_i/G_0{}^R$,

$$G_{27}/G_0{}^R=(5/3)p\,,\quad G_{8_2}/G_0{}^R=5^{1/2}r\,,$$
$$G_{8_1}/G_0{}^R=5q\,,\qquad G_{10}/G_0{}^R=2\sqrt{2}s\,, \tag{17}$$

where G_i^2 gives the relative probability that (BP) in the $SU(3)$ multiplet (i) will interact with B^* via the usual symmetry-breaking interaction. Referring to Fig. 2 in which the G_i are plotted against f, it is evident that the symmetry-breaking effects are considerable.

Figure 3 contains a plot of experimentally observable coupling constants, normalized to that of $N^*N\pi$, against the parameter f. Also indicated are the current experimental values along with their uncertainties, obtained from the data of Ref. 13 and the relation[7]

$$G^2(B^*BP)=M_{B^*}{}^2\Gamma(B^*BP)/ Q^3[(M_{B^*}+m_B)^2-\mu_P{}^2]\,, \tag{18}$$

where Q is the c.m. momentum of either of the decay particles.

The range of f near 0.4 is of particular interest. Martin and Wali[8] have shown on the basis of an N/D calculation that a decuplet baryon resonance is expected to be more strongly bound than any other in this region of f. This is also the range favored by the Cabibbo theory of weak interactions.[14] In addition $f\simeq\frac{2}{5}$ is a unique prediction of $SU(6)$[15] and of $\tilde{U}(12)$.[16] It is encouraging that our results agree more closely with experiment for this region of f than for any other.

The results of Wali and Warnock,[5] with the assumption $f=0.35$, suggest that the dominant symmetry-breaking contribution to the coupling constants comes from the coupling of the B^* to the 27-fold (BP) states (referred to as "27-dominance"). One sees from Fig. 2 that our calculations show a similar effect, insofar as the probability of 27-type coupling-constant splitting is predicted to be approximately four times larger than that of any other type for this range of f.[17]

In Table I our results are compared with those of other authors. The coupling constants (normalized to the $N^*N\pi$ coupling constant) rather than p, q, r, s, are used since those of Wali and Warnock contain nonlinear contributions and therefore do not satisfy the sum rules. Referring to the experimentally observable couplings in the first four columns it is evident that our method gives comparable results and is in principle simpler to evaluate. Wali and Warnock's calculation of broken couplings contains a subtraction constant and the coupling constant $g^2/4\pi$ which they have chosen so as to give best agreement with experiment for their simultaneous calculation of the B^* masses. We also

[13] *Proceedings of the 1964 International Conference on High-Energy Physics at Dubna* (Moscow, 1965).

[14] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[15] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964); B. Sakita, *ibid.* **13**, 643 (1964).

[16] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Royal Soc. **284**, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 267 (1965); B. Sakita and K. C. Wali, *ibid.*, p. 404.

[17] All group-theoretical effects, which slightly favor 27-type coupling for most values of f, enter this calculation linearly; hence strong 27-dominance arises only if forced by the choice of f. Based on our calculation alone there is no *a priori* reason to choose a different f than the one which gives best agreement with experiment. On the other hand, Wali and Warnock obtained strong 27-dominance because their 27-fold channels were closer to binding than any of the other nonresonant channels for their natural choice of f, that which gives the binding of the decuplet resonances.

TABLE I. Comparison of our coupling constants with those of other authors.

X_{DBP}	$N^*N\pi$	$Y^*\Lambda\pi$	$Y^*\Sigma\pi$	$\Xi^*\Xi\pi$	$Y^*\Sigma\eta$	$\Xi^*\Xi\eta$	$N^*\Sigma K$	$Y^*\Xi K$	$Y^*N\bar{K}$	$\Xi^*\Lambda\bar{K}$	$\Xi^*\Sigma\bar{K}$	$\Omega\Xi K$
Pure $SU(3)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Wali and Warnock	1.00	0.78	0.66	0.64	0.58	0.58	0.63	0.56	0.87	0.75	0.63	0.64
Freund and Nambu	1.00	0.88	0.58	0.66	0.58	0.66	0.59	0.67	0.98	0.86	0.57	0.64
$f=0.4$, $a=1.0$	1.00	0.93	0.53	0.65	0.43	0.55	0.14	0.26	1.24	1.16	0.76	0.89
$f=0.4$, $a=0.8$	1.00	0.90	0.45	0.59	0.34	0.48	0.00	0.23	1.27	1.19	0.73	0.87

obtain agreement similar to Freund and Nambu[18] who have made a two-parameter fit to the four observable coupling constants. These authors introduce another parameter when comparing their results with Wali and Warnock for the non-measurable couplings.

It is to be emphasized that the difference between the bare and renormalized coupling constant has been ignored in the calculation of p, q, r, s. Had this not been done the effect would have been to reintroduce the scalar part $a\neq 1$ which would then have to be taken as another parameter leading to a two-parameter fit. Alternatively, one could assume a reasonable value of f and again make a one-parameter fit to the experimental data. As an example, we choose $f=\frac{2}{5}$ and consider the effect of a variation in the parameter a. The results are plotted in Fig. 4 where it is evident that we obtain agreement with experiment similar to that with $a=1.00$ and $f\sim 0.4$ implying that our assumption $G_0{}^R\simeq G_0$ is reasonable.

Neither of our comparisons, nor those of the other authors, agree with the experimental ratio $X(Y^*\Lambda\pi)/X(N^*N\pi)$ which casts doubt on all of the procedures. A possible two-parameter fit does not improve this situation, for in order to obtain $X(Y^*\Lambda\pi)/X(N^*N\pi)\sim 1.05$ it is necessary to have $a\sim -3$, in which case it is impossible to fit any of the remaining experimental values.

Up to this point deviations of the BBP coupling constants from their $SU(3)$ values have been ignored. In order to account for them within the framework of this approach it would be necessary to first properly renormalize the perturbation theory explicitly, which is too ambitious since the viewpoint has been adopted that perturbation theory is only capable of treating deviations from unitary symmetry. The problem then in dealing with internal variation of coupling constants is the treatment of the symmetric part which this method is incapable of handling. However, the work of Freund and Nambu[18] suggests that the BBP couplings remain within 15% of their symmetric values with the implication that is it not a bad approximation to ignore the deviation and account for the change in B^*BP coupling constants by the mass splittings alone.

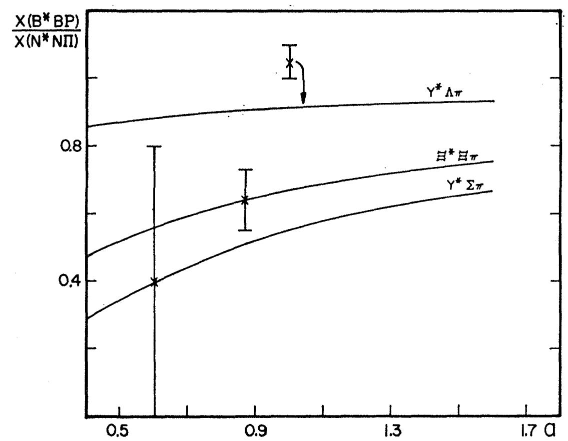

FIG. 4. Experimentally observable couplings normalized to $X(N^*N\pi)$ as a function of a with $f=0.4$. The experimental values are indicated as in Fig. 3.

ACKNOWLEDGMENTS

The authors would like to acknowledge many interesting discussions and encouragement from Dr. C. J. Goebel and Dr. R. F. Sawyer. They also wish to thank the Wisconsin Alumni Research Foundation for support of computer time on the University of Wisconsin CDC 1604 computer. Part of this work was also carried out on the University of Iowa 7040 computer. One of us (E. J.) would like to thank Miss M. Carter for help in checking the calculations.

[18] P. G. O. Freund and Y. Nambu, Phys. Rev. Letters 13, 221 (1964).