using Hale's or Rose's values are not much different despite the considerable difference in measured η values. Morton's fields are both below and above $E/p_0=300$ and since Hale's values of η are lower than Rose's for $E/p_0<300$ and are higher than Rose's for $E/p_0>300$, the errors due to Hale's values of η tend to compensate. If one assumes that the primary ionization is correct as calculated by Rose's values of η and that the discrepancy is due to a γ, it is possible to calculate values of γ from the curves of I_{meas}/I_0 versus pressure as given by Morton. This procedure results in values of γ varying from 0.15 to unity and in a few cases even exceeding unity. These values of γ are much too high (and indeed much too variable) to be accepted as reasonable. The above conclusions are equally valid if one uses Hale's values of η. However, Morton's data for 150 and 180 V when put into the same form as used by Johnson and plotted in Fig. 3 lie considerably higher than Johnson's curve. Thus, there is an experimental discrepancy between Morton's and Johnson's data. This discrepancy is not understood. A further complication is the fact that V^* is not negligible compared to the gap voltages of 100, 150, and 180 V analyzed by Morton. If such an effect were taken into account, the agreement between Morton's measured and calculated current multiplications would be even worse. The basis for doubting the validity of Townsend's ionization function at these low pressures from Morton's data would not seem to be on a firm foundation at the present time.

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Angular Distribution and Thickness Dependence of Transition Radiation from Thin Aluminum Foils*

R. J. Herickhoff,† W. F. Hanson,‡ E. T. Arakawa, and R. D. Birkhoff
Health Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee

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The angular distribution of photons emitted from Al foils 320 Å in thickness bombarded with 80-keV electrons, and the dependence of the peak intensity on foil thickness, have been determined experimentally in the wavelength region from 650 to 1100 Å. The absolute efficiency of the optical spectrometer was determined in the visible-wavelength region from a standard lamp and extrapolated into the vacuum ultraviolet-wavelength region using the measured grating efficiency and quantum efficiency of sodium salicylate. Experimental photon intensities were found to be approximately one half of the theoretical photon intensities. The intensity at the peak wavelength had a maximum at 22° and 157° from the foil normal, with zero intensity at 0°, 180°, and 90°. The angles for which the theoretical curves exhibit maximum intensity are 15 and 168°. The wavelength for which the experimental curves exhibit maximum intensity was found to decrease as the angle of observation increased from 0° to 40° or decreased from 180° to 140°. The plasma wavelength and energy, 835 Å and 14.9 eV, respectively, were obtained by extrapolating to 0°. The intensity at the wavelength of maximum emission was found to decrease rapidly as the foil thickness was increased from 180 to 435 Å and then to remain fairly constant out to 700 Å.

INTRODUCTION

THE study by optical methods of plasma oscillations induced by charged-particle excitation was initiated by Ferrell's[1] prediction in 1958 that these oscillations should decay by the emission of monochromatic photons at the plasma frequency. Numerous observations of electron-irradiated Ag have revealed a peak at 3300 Å with a continuum in the longer wavelength region.[2-6] Ritchie and Eldridge[7] generalized the transition-radiation theory of Frank and Ginsburg[8] for an idealized semi-infinite medium to the case of a finite foil characterized by a dielectric constant $\epsilon(\omega)$. A complete description of the Ag spectrum could not be obtained from the simple plasmon-decay picture but required the transition-radiation theory.

A recent paper reported a sharp peak in the optical-emission spectrum of electron-irradiated Al at 15.2 eV which was identified as plasmon radiation.[9] Similar investigations of Mg, Cd, In, and Zn were reported.[10]

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† ORINS Graduate Fellow, Vanderbilt University, Nashville, Tennessee.

‡ AEC Health Physics Fellow, Vanderbilt University, Nashville, Tennessee.

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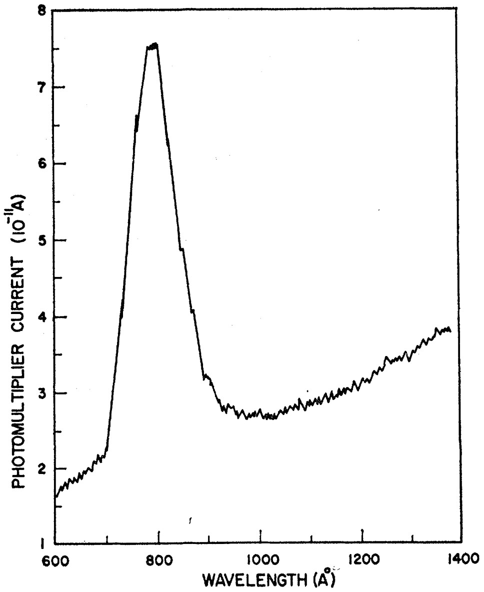

FIG. 1. Recorder trace of photon emission from electron-bombarded Al foil 320 Å thick ($\theta=160°$, $E=80$ keV).

Mg and In gave results in good agreement with the plasma-oscillation theory while the results for Cd and Zn showed considerable discrepancy from the plasma-oscillation theory. With the use of known optical constants the theoretical transition radiation spectra were calculated for Al and In and showed good agreement with the experimental spectra.

This paper presents the results of an investigation on the Al optical-emission peak previously reported and compares these results with the predictions of the transition-radiation theory.

EXPERIMENT

The experimental apparatus consisted of an electron accelerator, a Seya-Namioka vacuum ultraviolet spectrometer, and a sodium salicylate-coated EMI 6256B photomultiplier tube as a detector. The Al foils were irradiated in the center of an angular-distribution chamber which enabled the spectrometer to be rotated around the foil from 0 to 150° with respect to the foil normal without breaking the vacuum. Details of the apparatus can be found in previous papers.[4–6]

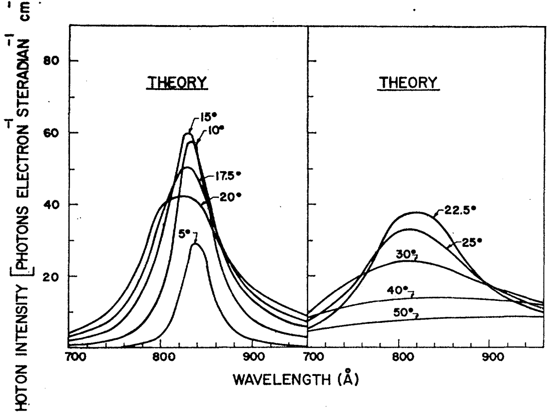

FIG. 2. Theoretical spectra for angles on the front side of an Al foil 320 Å thick ($\theta=0$–90°, $E=80$ keV).

Thin Al foils were prepared at pressures of 4×10^{-6} Torr in a vacuum evaporator by evaporating a wetting agent and Al successively onto a glass slide. The thin foil was then floated off on water and mounted on an aluminum ring which was placed in the target position. Smooth foils of thicknesses down to 180 Å were prepared in this manner. The foil thickness was determined by transmission of 5030 Å wavelength light through the foil. Interferometric thickness measurements were used to calibrate the transmission curve.

The spectral distribution of photons emitted by Al foils bombarded by 80-keV electrons was determined in the wavelength region from 650 to 1100 Å for various angles of observation on the electron-emergence or front side of the foil ($\theta=0°$ to 90°), and on the electron-incidence or back side of the foil ($\theta=90°$ to 180°). A trace of the spectrum recorded at $\theta=160°$ is shown in Fig. 1 to illustrate the signal-to-noise ratio and the breadth of the peak at 800 Å. The spectrometer resolution is about 25 Å. No other peaks were observable from 600 to 1400 Å. Similar spectra were obtained at other angles of observation. These spectra were corrected for the response of the spectrometer and detector system in the following way: The absolute response of the system in the visible-wavelength region was previously determined with a tungsten-strip-filament lamp calibrated by the National Bureau of Standards. The absolute response of the system in the ultraviolet region of interest was determined by making two corrections to this original calibration. The first correction was the

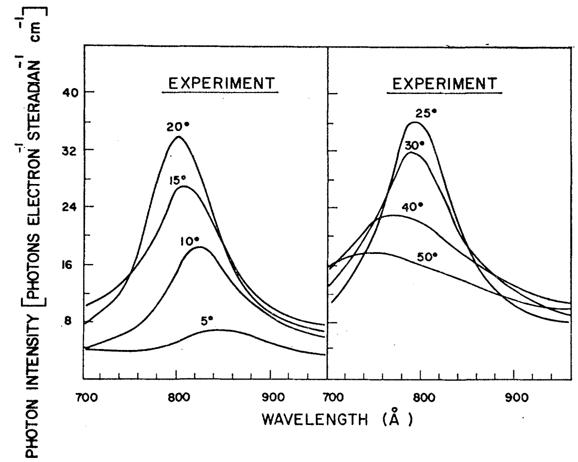

FIG. 3. Experimental spectra for angles on the front side of an Al Foil 320 Å thick ($\theta=0$–90°, $E=80$ keV).

absolute quantum efficiency of sodium salicylate[11] which was taken as 0.2 and constant from 650 to 1100 Å. The second correction was the ratio of the measured grating efficiency[12] at 800 Å to that at 4200 Å, the wavelength of maximum emission from sodium salicylate. The largest uncertainty in this calibration is the value of the absolute quantum efficiency of sodium salicylate since results of different experiments are often inconsistent. The relative quantum efficiency is reported by Johnson *et al.*[13] to be constant between 900 and 2300 Å, by Watanabe *et al.*[14] to be constant between 850 and 3000 Å with one point 15% lower at 584 Å, and by Samson[15] to be constant between 400 and 900 Å at which point it begins to increase. Allison *et al.*[16] report the absolute quantum efficiency to be 0.94 at both 1216 and 1611 Å. The absolute quantum efficiency, for the spectral region of interest in the present experiment, was reported by Brunner *et al.*[11] to be 0.4±0.08 between 400 and 1000 Å with one point 50% higher at 1216 Å. The quantum efficiencies quoted above have been integrated over 4π sr. Since the solid angle subtended by our photomultiplier tube was 2π sr, the value of the quantum efficiency used in our calibration was 0.2, one-half the value reported by Bruner. The other quantities used in the calibration are known to within a few percent and thus the error in the absolute yield determined in this report depends almost entirely on the uncertainty in the value of the absolute quantum efficiency.

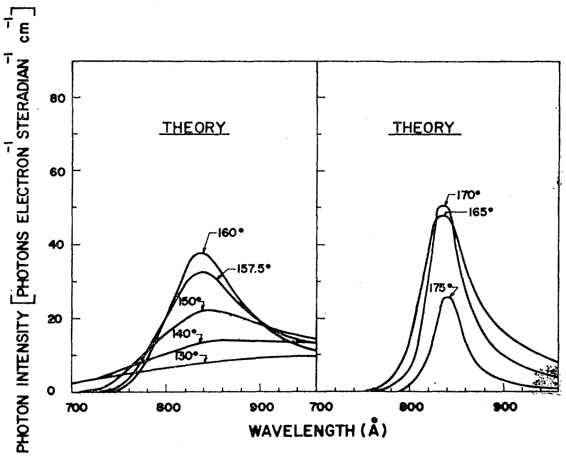

FIG. 4. Theoretical spectra for angles on the back side of an Al foil 320 Å thick ($\theta=90$–$180°$, $E=80$ keV).

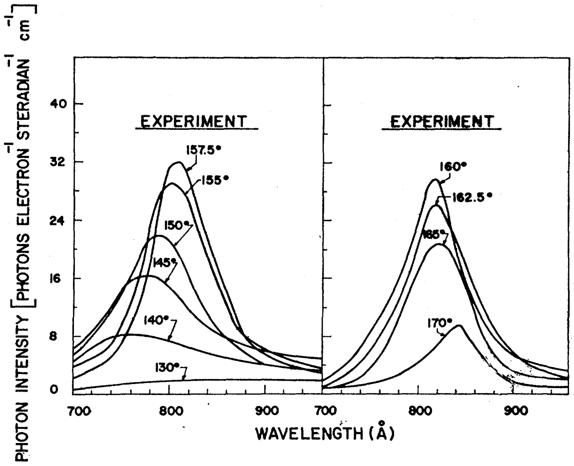

FIG. 5. Experimental spectra for angles on the back side of an Al foil 320 Å thick ($\theta=90$–$180°$, $E=80$ keV).

RESULTS AND DISCUSSION

The comparisons of the theoretical and experimental spectral distributions for angles less than 90° are shown in Figs. 2 and 3 for Al foils 320 Å thick bombarded with 80-keV electrons. Figures 4 and 5 give the corresponding theoretical and experimental curves for angles greater than 90°. These figures show that the experimental absolute photon intensities determined with the present calibration are approximately 40% lower than the theoretical photon intensities. The spectral distributions generally agree, but the experimental curves have a higher background on the front (electron-emergence) side than on the back (electron-incidence) side. This is not explained by the peaking of Bremsstrahlung in the forward direction since calculations show that for the conditions of this experiment, the contribution of bremsstrahlung to the emission is negligible.[17] Perhaps the major factor contributing to this difference in background is that more electrons will be scattered into the spectrometer with the spectrometer on the beam-

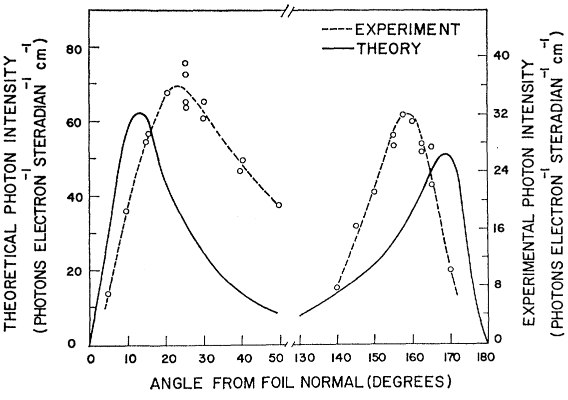

FIG. 6. Theoretical and experimental peak intensities as a function of angle of observation.

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emergence side of the foil than with the spectrometer on the beam-entrance side.

Figure 6, which shows the peak intensity as a function of angle of observation, summarizes the results of Figs. 2–5. The experimental peak intensities for the front side are slightly higher than the experimental intensities for the back side. This relationship also holds for the peak intensities of the theoretical curves. The angles for which the experimental curves have maximum intensity are $\theta=22°$ on the front side and $\theta=157°$ on the back side, while the angles for which the theoretical curves exhibit maximum intensity are $\theta=15°$ and $\theta=168°$. This discrepancy may be due to differences in the optical constants of the foils used in this experiment from those used in the optical constant determination.

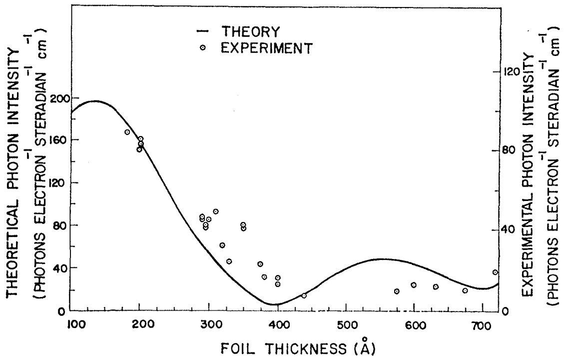

FIG. 8. Photon intensity at peak wavelength as a function of foil thickness.

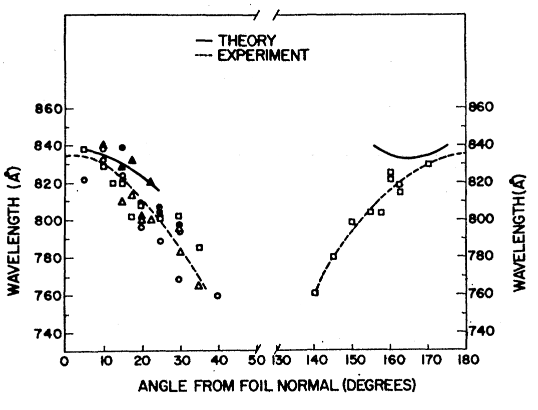

FIG. 7. Variation of peak wavelength as a function of angle of observation.

Figures 3 and 5 show that the energies of the experimental peak intensities shift with angle of observation and also that as the angle between the foil normal and the spectrometer increases beyond the angle of maximum emission, the intensity tends to be reduced. The corresponding theoretical curves of Figs. 2 and 4 agree with these observations. However, in Fig. 7, which shows the values of the wavelength of peak intensity as a function of angle of observation, it can be seen that for increasing angles from the foil normal on the back side, the shift in the experimental wavelength of the peak intensity with angle of observation is opposite to that of the theoretical shift. This discrepancy may be due to the previously mentioned differences in optical constants. A cutoff angle and the shift of peak energy with angle are both predicted by the generalized transition-radiation theory and are qualitatively explained in terms of refraction of light at the surface of the dielectric. In the frequency range $\omega>\omega_p$, where the foil is transparent to photons, photons which originate in the foil interior and strike the surface at an angle θ' with respect to the foil normal will undergo refraction and are bent toward the normal. The exit angle θ is related to θ' by Snell's law $\epsilon^{1/2}\sin\theta'=\sin\theta$. Setting $\theta'=\pi/2$ (the maximum angle of photons in the foil interior) and $\epsilon=1-\omega_p^2/\omega^2$, Snell's law reduces to $\omega=\omega_p/\cos\theta$. Thus for a given frequency $\omega>\omega_p$ there is a value of θ beyond which photons originating inside the foil will not be observed. In the transition-radiation theory the maximum in the photon intensity distribution should occur at $\omega=\omega_p/\cos\theta$. Thus, the curve through the points in Fig. 7 extrapolated to 0° should give the plasma wavelength, $\lambda_p=835$ Å or $E_p=14.9$ eV. This value is in agreement with previous electron energy-loss experiments.[18]

A sine-squared variation of intensity with thickness is predicted by the generalized transition-radiation theory. The first maximum is predicted to occur for a thickness around $\beta\lambda_p/2$ where β is the electron velocity relative to that of light and λ_p is the wavelength of the peak. Secondary maxima are weaker than the first maximum because of increased photon absorption in the foil for increasing foil thickness. Comparison of the experimental results with theory in Fig. 8 confirms the essential correctness of the transition-radiation description.

ACKNOWLEDGMENTS

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