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Critical Currents in Superconducting Films of Indium

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Critical currents in the region near the transition temperature T_c have been measured for thin indium films, of thickness 585 to 3600 Å, deposited on sapphire cylinders. The coherence length obtained from the normal resistivities is compared with previous values and the increase of T_c with decreasing film thickness is determined. Measurements are reported on a 585-Å-thick film which combine the use of a compensated geometry avoiding the difficulties associated with specimen edges, and current pulses with a rise time of 7 nsec in which the transition is not obscured by specimen heating. Two regions are observed; for $(T_c-T) \leq 0.11$°K the critical currents vary with the 1.5 power of the temperature difference, in accord with the Ginzburg-Landau theory, while for 0.11°K $\leq (T_c-T) \leq 0.30$°K the temperature dependence becomes nearly linear. The approach to this behavior depends on (a) shortening the measuring-pulse rise time, and, particularly, (b) reducing the fraction of normal resistance restored. Paradoxically, because of (b), even dc currents lead to similar results.

I. INTRODUCTION

THERE have been a number of attempts to determine critical currents from various theories of superconductivity. In a bulk specimen, the magnetic energy of the excluded field produced by the electric current determines the critical currents (Silsbee's[1] hypothesis). In thin specimens, the kinetic energy of the electrons dominates the free-energy expression and determines the critical currents. Ginzburg[2] calculated the critical current for a thin cylindrical film; his derivation was based on the Ginzburg–Landau (G.L.)[3] theory. Rogers[4] calculated the critical current on the basis of the BCS[5] theory by taking into account the change in distribution of quasiparticles and energy gap with current. His values are found to be 25% lower than the ones calculated on the basis of the semiphenomenological theories. Bardeen[6] used a variational method and chose the energy gap as a free parameter. By minimizing the free-energy expression which had its contributions from pairing interaction and kinetic energy of the supercurrents, he obtained an expression for the critical current. The derivations, whether based on the G. L. theory or the microscopic theory, assume particularly simple form when worked out in the London limit, which is easier to satisfy when working with thin films and near the transition temperature.

In most of the experimental results previously reported, the critical currents have been determined for films deposited on flat substrates. The results obtained in such experiments are affected by the nonuniformity of currents at the specimen edges. The use of direct current or current pulses having a slow rise time introduces further complications due to the occurrence of Joule heating. Bardeen[6] has emphasized the need for experiments in which precautions are taken to eliminate both these difficulties. The use of films deposited on cylindrical rods with critical-current measurements employing fast-rising pulses, described in the present paper, has enabled the study of the superconductive-normal transition with the obscurities introduced by the specimen edges and heating minimized.

II. SAMPLE PREPARATION AND MOUNTING

The films were prepared by vacuum evaporation of Cominco Brand 99.999% pure indium metal at a deposition rate of slightly over 20 Å per second. Two sizes of sapphire rods, 1.5 and 0.4 mm in diameter and 4 cm long, were used as substrates. The substrate was rotated at 3 revolutions per second while in thermal contact with liquid nitrogen. Pressure in the vacuum space rose from an initial value of approximately 10^{-7} to 10^{-5} mm during deposition. Prior to cooling the substrate, the charge boat was heated to drive off adsorbed gases, with a shield inserted between it and the substrate.

The film thickness was determined by weighing the substrate before and after deposition. It was noted that the ratio of weight of the deposit to that of the charge evaporated to completion was constant to within 12%.

Earlier films deposited on glass rods were found to lack conductivity if they were thinner than 2000 Å. This was due to the formation of agglomerates apparently caused by a rise of surface temperature. The use of sapphire rods, however, whose thermal conductivity at nitrogen temperatures is approximately 300 times that of glass, maintained continuity in the films for the thinnest ones used (585 Å). Electron micrographs of the films deposited on glass and sapphire substrates show the formation of bigger sized grains on the former.

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[1] F. B. Silsbee, J. Wash. Acad. Sci. **6**, 597 (1916).
[2] V. L. Ginzburg, Dokl. Akad. Nauk SSSR **118**, 464 (1958) [English transl.: Soviet Phys.—Doklady **3**, 102 (1958)].
[3] V. L. Ginzburg and L. D. Landau, Zh. Eksperim. i Teor. Fiz. **36**, 1918 (1959) [English transl.: Soviet Phys.—JETP **9**, 1364 (1959)].
[4] K. T. Rogers, Ph.D. thesis, Illinois University, 1960 (unpublished).
[5] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[6] J. Bardeen, Rev. Mod. Phys. **34**, 667 (1962).

The specimen was mounted co-axially in the detachable end of a 50-Ω line into the cryostat, thereby protecting the specimen from the magnetic fields of currents in the return lead. For the pulse measurements, a coaxial terminating resistor was connected directly above the specimen. An inner bakelite mounting block carried the specimen on a strain-free support of No. 30-wire copper hooks, which served as current and potential leads. A smaller adjustable bakelite block facilitated bringing the potential lead glued to it very close and parallel to the specimen. The bottom of this lead was brought out through the end of the line through an Amphenol co-axial "subminax" connector to a separate returning 50-Ω potential line. Extreme care was necessary in mounting the potential lead so that the area between it and the specimen, and hence the extraneous inductive pulse, were minimized.

The contact between the current leads and the film was made by very carefully melting indium. For this purpose the film was made thick in the ends by depositing a second coat of indium. The potential contact was made by using silver print. In view of the stringent requirement of keeping the potential lead very close to the film, only one potential lead was used.

III. APPARATUS AND MEASUREMENTS

The temperature regulation was secured by using a Cartesian-diver type of pressure stabilizer and a heater at the bottom of the cryostat to stir the helium bath. The stability of the temperature was monitored by a carbon-resistance thermometer but the temperature was measured by using the 1958 helium vapor pressure scale, with the hydrostatic correction applied.

The dc film resistivity was determined by passing currents of 0.5 to 2.0 mA through the specimen. The earth's field was first compensated by adjusting Helmholtz pairs so as to minimize the resistance with the specimen approximately at the transition temperature, defined as that for which the resistance has half its value in the normal state. The transition temperature was found to be the same whether direct current or a series of pulses of increasing magnitude was used.

A Rutherford Model B7B pulse generator was found convenient for providing pulses of up to 50 V into the 50-Ω matched current line, with a rise time of nominally 15 nsec. For heavier currents, two other sources were used. The first was a thyratron producing single-shot pulses of up to 15 A with a rise time of 1.2 μsec. The other produced smoothed fast pulses of rise time 7 nsec and duration 80 μsec by discharging a line through a Potter and Brumfield JMI mercury-wetted relay. With the latter, the voltage pulses were displayed on a Tektronix 661 sampling oscilloscope having a rise time of 0.35 nsec.

TABLE I. Resistivity and mean free path for indium films.

T (°K)	ρ^b (Ω cm)	l^b (Å)	$\rho^b l^b$ (Ω cm²)
273	10.07×10^{-6}	119	$1.2\pm0.42\times10^{-11}$
4.2	0.130	7940	1.03 ± 0.23

TABLE II. Resistivity mean-free-path product and coherence lengths for indium.

Author	$\rho^b l^b$ (Ω cm²)	ξ_0 (Å)
Roberts[a]	0.89×10^{-11}	2900
Dheer[b]	0.57	4400
Toxen[c]	0.98	2600 ± 400
This work	1.03	2500 ± 550

[a] See Ref. 11.
[b] See Ref. 12.
[c] See Ref. 10.

IV. RESULTS AND DISCUSSION

1. Film Resistivity

The ratio of the electrical resistivity of a thin film to that of the bulk metal, ρ/ρ^b, can be expressed in terms of the bulk mean free path l^b and the thickness of the film d. Assuming diffuse surface reflection, Fuchs[7] treatment leads to

$$\rho/\rho^b = 1+\tfrac{3}{8}(l^b/d)\,, \quad \text{for } l^b \ll d, \tag{1}$$

$$\frac{\rho}{\rho^b} = \frac{4l^b}{3d[\ln(l^b/d)+0.4228]}\,, \quad \text{for } l^b \gg d. \tag{2}$$

Our measured resistivities for indium films of different thicknesses were plotted in terms of Eq. (1) for room temperature as ρ versus d^{-1} and in terms of Eq. (2) as $(\rho d)^{-1}$ versus $\ln d$ for helium temperature. From the slope and the intercept of straight lines through the data, values for the resistivity and mean free path at high and low temperatures were obtained, as shown in Table I. Thus (ρl) is found to be approximately constant, independent of temperature.[8]

Following Toxen[9,10] (ρl) can be used to calculate the coherence length ξ_0. The various published estimates are compared below in Table II.

The data of Roberts[11] and Dheer[12] are based on measurements of high-frequency surface impedance. Toxen's estimate is based principally on critical magnetic-field measurements.

[7] K. Fuchs, Proc. Cambridge Phil. Soc. **34**, 100 (1938).
[8] It has been pointed out to us that the rather small value of l^b at 4.2°K may be related to the fine grain size of the films deposited on cold sapphire substrates, and that the resistivity of small polycrystalline specimens of indium is discussed in detail by R. T. Bate, B. Martin, and P. F. Hille, Phys. Rev. **131**, 1482 (1963).
[9] A. M. Toxen, Phys. Rev. **123**, 442 (1961).
[10] A. M. Toxen, Phys. Rev. **127**, 382 (1962).
[11] T. E. Faber, Proc. Roy. Soc. (London) **A241**, 531 (1957).
[12] P. N. Dheer, Proc. Roy. Soc. (London) **A260**, 333 (1961).

2. Transition Temperature and the Transition Width

As first pointed out by Lock,[13] the transition temperature for thin films departs from its value in the bulk material. Toxen[9] found that for the indium films deposited on vitreous silica substrate, the relationship between T_c and d is given by

$$\delta T_c = (52/d) - (750/d^2)\,, \tag{3}$$

where δT_c is the difference in T_c for the film and bulk material and d is the film thickness in angstrom units. In Fig. 1 is shown a plot of T_c versus d from the present work. The data are found to fit the equation

$$\delta T_c = (73.4/d) - (21\,300/d^2)\,. \tag{4}$$

The fact that T_c for the films is higher than T_c for the bulk material indicates that the films are under tensile stress which, of course, is to be expected because the coefficient of expansion for indium is greater than that for sapphire.

The transition width, defined to be the temperature interval between 10% and 90% of the resistance in the normal state, lies between 3 and 8 millidegrees for the indium films. There is no systematic variation of the transition width with film thickness.

3. Critical Currents

The indium films investigated lie in the range from 580 to 3540 Å in thickness. The dependence of critical currents on temperature is governed by the ratio $\psi_{0k}d/\lambda_0$ as given by Ginzburg.[2] The condition $(\psi_{0k}d/\lambda_0)<1$ is satisfied by all the films except the one 3540 Å thick, for the region $(T_c-T)=\Delta T\leq 0.150$°K. If we assume the critical current I_c to be given by $I_c \propto (\Delta T)^n$, then for the case $(\psi_{0k}d/\lambda_0)\ll 1$ Ginzburg gives $n=1.5$; for $(\psi_{0k}d/\lambda_0)\gg 1$, $n=1.12$.

The measurement of critical currents presents difficulty because, while the superconducting to normal transition takes place at extremely high speed, the very low thermal capacity of the film leads to a rise in film temperature in a submicrosecond interval. The results obtained from measurements using both pulses and steady direct current will be discussed in turn below. Except where otherwise stated, the critical current has been taken as that required to restore half the resistance in the normal state.

Most of the critical current data were obtained using pulses of rise time 1.2 μsec. These results, normalized to $\Delta T=0.3$°K, are plotted in Fig. 2. Two regions are evident and for each of these a curve was fitted by the method of least squares using computer iteration. The form of the curve was taken as $I_c=B(\Delta T)^n$ for the first region and $I_c=A'+B'(\Delta T)^m$ for the second region. The exponent of ΔT was found to be $n=1.27$ for the first region up to 0.08°K, and subsequently $m=0.85$ for $\Delta T\approx 0.08$°K to $\Delta T\approx 0.30$°K.

These values may be compared with those of Alekseevskii and Mikheeva[14] for discs of tin on glass. They had reported $n\approx 0.6$ for rounded pulses of 0.1-sec duration, but had found for pulses of rise time 250 μsec that films deposited at nitrogen temperature gave $n\approx 1$, while films deposited at room temperature showed a break in slope of the I_c versus ΔT curve similar to that reported here, with n appearing from their graph to be about 1.4 in the region near the transition.

In the present investigation it was found that films of tin deposited at nitrogen temperatures could be made conducting to smaller thicknesses than those deposited at room temperature, but unlike the latter, such films showed a plateau in the transition curve.

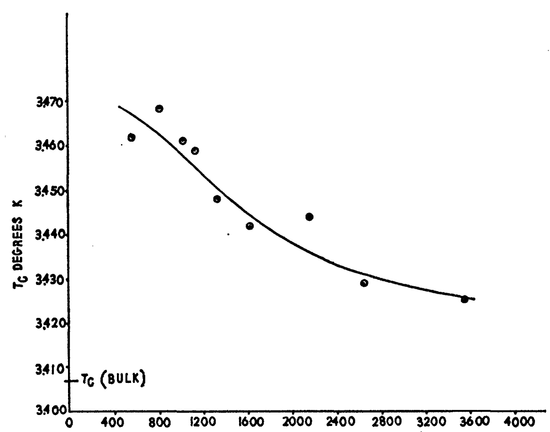


FIG. 1. Dependence of transition temperature of In films on thickness. Fitted curve corresponds to
$\delta T_c = (73.4/d) - (21{,}300/d^2)$.

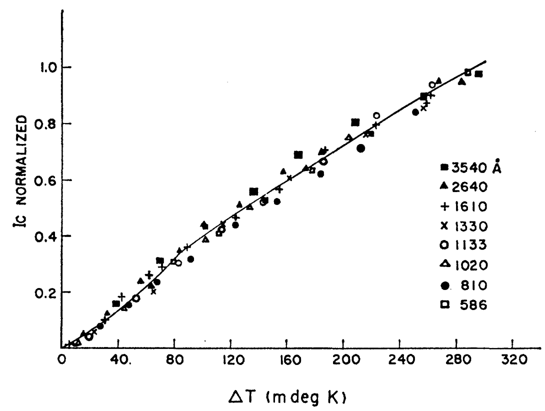
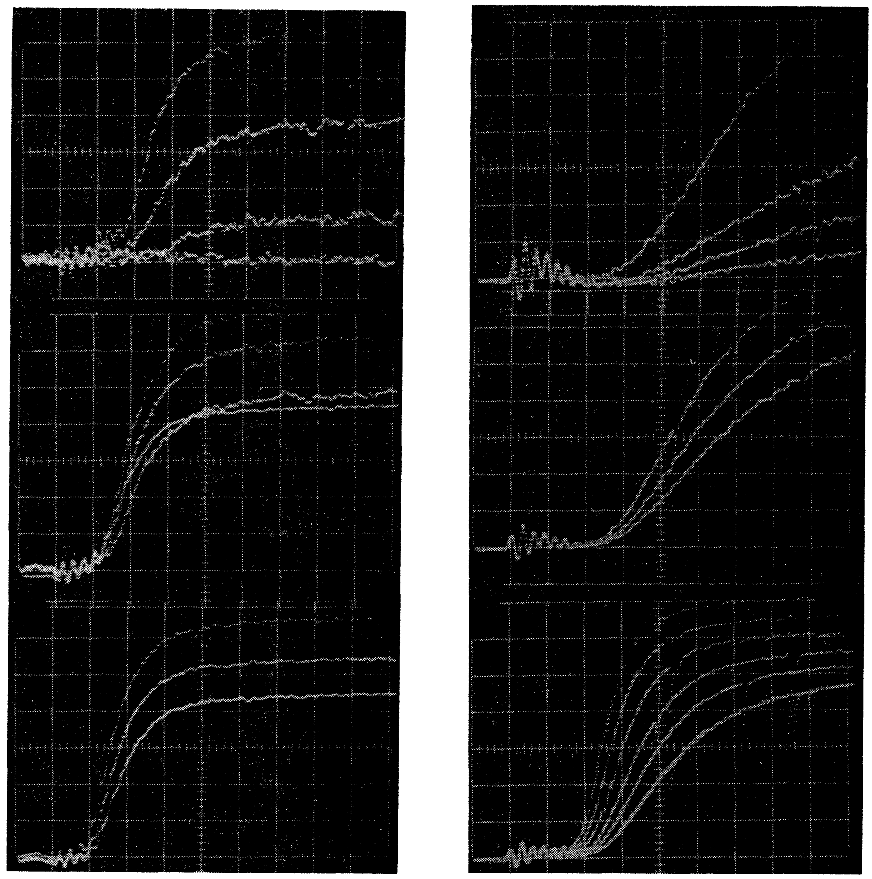


FIG. 2. Critical currents for In films using pulses of 1.2-μsec rise time, normalized to $\Delta T=0.3$°K. The regions above and below $\Delta T=80$ mdeg K have been fitted by separate curves, as explained in the text.

[13] J. M. Lock, Proc. Roy. Soc. (London) **A208**, 391 (1951).

[14] N. E. Alekseevskii and M. N. Mikheeva, Zh. Eksperim. i Teor. Fiz. 38, 292 (1960) [English transl.: Soviet Phys.—JETP **11**, 211 (1960)].



(a) (b)

FIG. 3. Oscillograms of current-induced transitions for pulses of 7-nsec rise time in 585-Å-thick In film. $T_c=3.460$°K. Horizontal sensitivity: 5 nsec/cm. (a) $T=3.413$°K. Top: 5 mV/cm; middle: 10 mV/cm; bottom: 20 mV/cm. Traces, from top: 25, 20, 15, 10, 40, 35, 30, 30, 70, 60, 50-mA pulse current. (b) $T=1.488$°K. Top: 50 mV/cm; middle: 100 mV/cm; bottom: 200 mV/cm. Traces, from top: 480, 460, 440, 420, 540, 520, 500, 800, 750, 700, 650, 620, 580-mA pulse current.

Electron micrographs showed a pattern characteristic of twinning in the former. Since such difficulties were not encountered with any of the indium films, only the results for indium are presented at the present time.

It was felt that the departure from the theoretical value of $n=1.5$ could be due to Joule heating after a certain resistance was restored in the film. In fact with pulses of increasing magnitude it was seen that the voltage pulse consisted of a rapid initial rise followed by a further increase with change of slope as Joule heating carried the film over the rest of the transition.

The fast pulses used to investigate the current transition with Joule heating minimized had a rise time of 7 nsec. The specimen was a film of indium 585 Å thick coated on a sapphire rod of 0.4 mm diameter. The ends where current leads were attached had a heavy coat of indium, as described. This thick indium coating had one disadvantage; the transition temperature for the end region is slightly lower than that for the thin part. Lead and tin were tried on the ends but the contacts did not prove reliable. However, if the indium film is made quite thick, as was finally done, the residual resistance of the thick part will be very small and the heat developed negligible. Such appeared to be the case for transitions in the temperature region close to T_c. Voltage pulses for $\Delta T=0.049$°K are shown in Fig. 3(a). The shape does not change with pulse magnitude; the transition thus appears to be free from heating. A small temperature fall due to the latent heat could occur within the rise time of the pulse, as mentioned later. The voltages read for calculation of the appearance of resistance are the ones 20 nsec after the start of the current pulse, which is then at practically full value. At lower temperatures than useful for the present measurements the nature of the transition changes, and as the current is increased to the critical values there, heating of the film becomes evident, as shown in Fig. 3(b).

The critical currents (corresponding to $0.5R$) determined by using the fast pulses are plotted as a function of ΔT in Fig. 4. As before, the curve has two regions; in the first region for $0°\text{K}\leq\Delta T\leq 0.11°\text{K}$, $n=1.41$, and for the second region for $0.11°\text{K}\leq\Delta T\leq 0.300°\text{K}$, $n=1.06$.

To examine further whether the transitions observed were free from heating, currents corresponding to 12%, 25%, 50%, 75%, and 90% of the resistance R in the normal state were determined graphically. In case some heating had occurred, the value of n should be less for the currents corresponding to the appearance of a larger fraction of the resistance. This is found to be so;

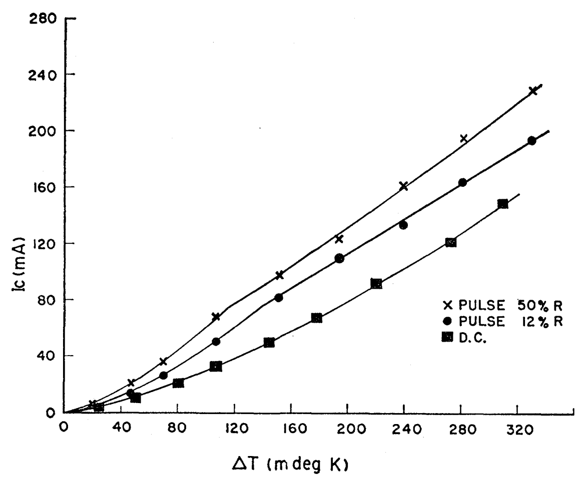


FIG. 4. Critical currents for 585-Å-thick In film for steady dc and for pulses of 7-nsec rise time. The latter results have been fitted by separate curves for points below and above $\Delta T=0.11$°K, as explained in the text.

the values of n for the appearance of 12%, 25%, 50%, 75% and 90% of the normal resistance are 1.50±0.02, 1.45±0.04, 1.41±0.01, 1.40±0.09 and 1.22±0.06 respectively in the region $0°\text{K}\leq\Delta T\leq 0.11°\text{K}$. A plot of critical current for the restoration of 12% resistance is also shown in Fig. 4. It thus appears that the critical currents observed for restoring 50% resistance were influenced by heating.

The second region, where $n=1.06$, commences at $\Delta T\approx 0.11$°K. The penetration depth for a film of 585-Å thickness is 2960 Å[15] at $\Delta T=0.3$°K. Hence $(\psi_{0\kappa}d/\lambda_0)<1$ and one would expect the region for $n=1.5$ might extend to $\Delta T=0.3$°K. Earlier commencement of the second region could result from Joule heating at the larger values of ΔT, but it is to be noted that the boundary value between the two regions appears to change only from $\Delta T\approx 0.11$°K to $\Delta T\approx 0.08$°K when the pulse rise time is lengthened from 7 nsec to 1.2 μsec. (Figure 2.)

Recently, Hagedorn[16] published his Silsbee-limit critical-current results for a 1700-Å-thick film of tin on a cylindrical substrate. By using current pulses of rise time 2×10^{-10} sec, he found the transition to occur in times of the order of 10^{-10} sec or less. His critical currents are found to be given by Silsbee's hypothesis provided that a correction is applied for the latent heat of the cooling during transition. Joule heating commences soon after transition.

By considering the entropy difference between the completely normal film and the film with no current in it, it can be seen that even for a second-order transition in a thin film a latent heat is absorbed, but not discontinuously. For our thin film with $\Delta T\leq 0.3$°K, however, calculations based on the data of Bryant and Keesom[17] show that the effect would be simply to multiply the scale of ΔT in this region by a factor of 1.16, for restoration of 100% of the normal state. For the smaller fractions of normal resistance restored here, not necessarily corresponding to a simple fraction of the cylindrical sample restored to the normal state, no correction has been applied.

In our work, the transition times seem to be smaller than the limit of the pulse rise time, in agreement with Hagedorn and other workers.[18–22] Near the transition temperature the critical current is found to be $\propto(\Delta T)^{1.5}$ in agreement with Bardeen and Ginzburg. However, the magnitude of critical current to restore the normal resistance (to take an example for $\Delta T=0.07$°K) in our work is 0.048 A as compared to 0.15 A based on Ginzburg's expression for an ideal thin film.

4. Thermal Effects

As mentioned earlier, the transitions at lower temperatures are marked by generation of Joule heat. For traces such as those in the upper section of Fig. 3(b), where an almost linear potential rise much slower than that of the current pulse is evident, it is possible to attempt an approximate estimate of the temperature increase between two instants, one say 20 nsec after the initiation of the pulse and the other 40 nsec after, provided the following assumptions are made:

(i) At the first instant, corresponding to the center line in Fig. 3, the appearance of the resistance is a consequence of the transition, practically unaffected by Joule heating. This is not rigorously true if we imagine the transition to be very fast.

(ii) During the interval of 20 nsec between the first instant and the second, the entire Joule heat warms up the film alone. This seems to be a fair assumption, despite the high thermal conductivity of sapphire, as the heat transfer to the sapphire rod is governed by the Kapitza boundary resistance between the film and the substrate. If one makes use of the data by Little,[23] it can be shown that the heat lost to the sapphire rod during the 20 nsec interval is negligible; furthermore, the heat capacity of the rod is only 2.5% that of the film. The heat transfer to the helium bath is known to be a relatively slow process.

The temperature rise of the film can now be estimated by first constructing curves of resistance versus temperature with I as parameter. For this purpose the data used is that for 20 nsec, originally plotted as

[15] Determined from critical-magnetic-field measurements on these films; R. D. Chaudhari, Ph.D. thesis, University of British Columbia, 1964 (unpublished).

[16] F. B. Hagedorn, Phys. Rev. Letters **12**, 322 (1964).

[17] C. A. Bryant and P. H. Keesom, Phys. Rev. **123**, 491 (1961).

[18] D. Abraham, Solid State Electron. **1**, 340 (1960).

[19] D. L. Feucht and J. B. Woodford, Jr., J. Appl. Phys. **32**, 1882 (1961).

[20] A. H. Nethercot, Jr., and R. J. von Gutfeld, Phys. Rev. **131**, 576 (1963).

[21] K. Rose and M. D. Sherril, Bull. Am. Phys. Soc. **8**, 233 (1963).

[22] E. M. Hartlin, R. M. Wertheimer, and G. M. Graham, Can. J. Phys. **42**, 1282 (1964).

[23] W. A. Little, Can. J. Phys. **37**, 334 (1959).

resistance versus current characteristics, taken at constant temperature. From the observed δR, the change in resistance at 40 nsec, 1 cm from the end of the traces in Fig. 3, the change in temperature is then obtained from the R-T curve for the particular current. This δT may be called the observed temperature rise.

The temperature rise may also be estimated by calculating Q, the energy dissipated, using the applied current and observed potential trace; from the known thermal capacity of the film the temperature rise is then determined. This may be called the calculated temperature rise. In this calculation it is assumed that the entire thin film (not counting the thicker part at the ends) is heated uniformly.

Comparison of δT observed and calculated reveals some interesting points (see Fig. 5). For low values of the currents, δT observed is found to be greater than δT calculated, and in general the ratio goes on diminishing from a value 10 for $Q\approx 4\times 10^{-12}$ J to a value of unity for $Q\approx 10^{-10}$ J. For increasing currents the decrease continues and for $Q\approx 10^{-9}$ J the ratio=0.2. The higher value of the ratio could be interpreted by assuming that, for smaller power input, the temperature rise occurs only at the resistive parts of the film and if the velocity of propagation of the thermal waves is small, the effective thermal capacity of the film is smaller than the calculated value. For larger energy dissipation, the ratio decreases for two reasons: firstly, the resistance appears at more points; and secondly, because of the increased velocities of thermal propagation, not only the entire thin film but also the thicker film at the ends act as a heat sink. This explanation is consistent with the existence of thermal wave fronts, moving with great speeds.

It must be noted, however, that the above analysis is qualitative only; no attempt has been made to include a latent heat in the calculation. The first effect of including a latent heat would be to lower the ratio of observed to calculated temperature by a constant fraction for small ΔT; for large ΔT, such as for the last point, which corresponds to the upper trace in Fig. 3(b), the maximum latent heat could be 10% of the Joule heat Q.

From the above description it might appear that current pulses rising with great speed would enable the study of the transition without the accompanying thermal effects. A difficulty arises, however, because the surface impedance at high frequencies introduces additional power losses and will tend to set a limit to the useable speed for the rise of the current pulse.

The surface resistance becomes perceptible when the photon energies of the high frequency component of the pulse become comparable with or exceed the energy gap at that temperature. In view of the dependence of the energy gap on temperature, for the temperature region close to T_c, these frequencies are $\approx 10^{10}$ cps. The highest frequencies used in this work are $\approx 10^{8}$ cps.

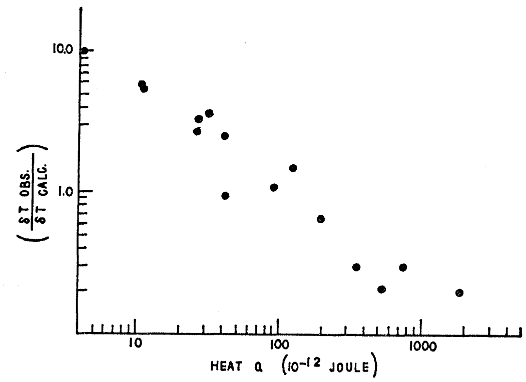


FIG. 5. Ratio of temperature rise in film observed from resistance change to that calculated from Joule heat Q released in film, assuming uniform heating. Film thickness 585 Å. Temperature rise taken is that for 40 nsec after beginning of the pulse.

Kolchin *et al.*[24] find that, with faster rising pulses, the critical current decreases, this being interpreted by them as heating due to eddy currents and amounting to an argument for not using fast rising pulses for observing isothermal transitions. In the present work, no evidence for eddy-current heating was obtained.

5. dc Transitions

Ginzburg and Shalnikov[25] measured the critical currents for their tin films using steady direct currents and found that for the temperature range $\Delta T\leq 0.4$°K, $I_c\propto(\Delta T)^{1.5}$. The critical currents, using dc, measured in our work for the 585-Å-thick indium film, are shown in Fig. 4. With dc, taking 50% R restored as the criterion, the transitions are sharp, but show hysteresis for increasing and decreasing currents.[26] A comparison of the critical currents obtained by using pulses and direct currents (Fig. 4) shows that the dc values are lower, indicating that these transitions belong to the thinner parts of the film. For the steady dc $n=1.42\pm 0.02$ in the temperature range $\Delta T\leq 0.300$°K. The dc curve does not have a discontinuity in slope such as that evident for the pulse results, but if the range is divided in two, then, for $0\leq\Delta T\leq 0.11$°K, it is found that $n=1.57\pm 0.11$.

6. Dependence of Critical Currents on Film Thickness

According to Ginzburg,[2] for a given value of T near T_c, $I_c\propto d$. The critical currents for indium films have been plotted as a function of film thickness in Fig. 6 for

[24] A. M. Kolchin, Y. G. Mikhailov, N. M. Reinov, A. V. Rumyantseva, A. P. Smirnov, and V. N. Totubalin, Zh. Eksperim. i Teor. Fiz. **40**, 1543 (1961) [English transl.: Soviet Phys.—JETP **13**, 1083 (1961)].

[25] N. I. Ginzburg and A. I. Shalnikov, Zh. Eksperim. i Teor. Fiz. **37**, 399 (1959) [English transl.: Soviet Phys.—JETP **10**, 285 (1960)].

[26] J. W. Bremer and V. L. Newhouse, Phys. Rev. **116**, 309 (1959).

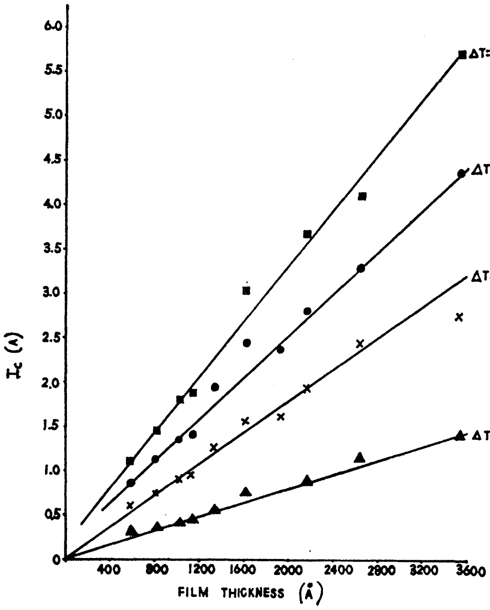


FIG. 6. Dependence of critical currents in In films on thickness for various values of $\Delta T = T_c - T$.

various values of ΔT. The dependence is seen to be linear, in agreement with the theory.

V. SUMMARY AND CONCLUSIONS

The measurement of resistivity at room temperature and helium temperature enabled the calculation of ρl and ξ_0, and these results have been listed in Table II along with the results of other workers. Our estimates are in fair agreement with other values.

The small-current transition width for indium films, as measured by resistance measurements, was found to lie between 3 and 8 millidegrees. The transition temperature was found to increase with decreasing film thickness.

For a 585-Å-thick indium film, the critical currents were measured by using pulses of a rise time of 7 nsec, the transitions being observed with the aid of a sampling oscilloscope having a rise time of 0.35 nsec. When the fraction of normal resistance restored was reduced to 12%, these measurements gave $n=1.50$ for $0 \le \Delta T \le 0.11$°K and $n=1.02$ for $0.11°\text{K} \le \Delta T \le 0.300$°K. The Ginzburg–Landau theory predicts $n=1.5$ for the region under observation, i.e., $0 \le \Delta T \le 0.300$°K for a film of this thickness.

For the indium films generally, but with restoration of 50% R, $n=1.27$ for the region $0 \le \Delta T \le 0.08$°K, and $n=0.87$ for $0 \le \Delta T \le 0.300$°K, when using pulses of rise time 1.2 μsec. The dc measurements on the 580-Å-thick film give a value $n=1.42$ in the range $0 \le \Delta T \le 0.300$°K, with values near 1.5 as the range is further restricted. The magnitude of the critical current for the pulses of 7 nsec rise time is about 0.3 that predicted by Ginzburg; for the dc measurements it is about 0.12.

It thus appears that the variation of critical current with temperature for thin films approaches that predicted by the Ginzburg–Landau theory as (a) the rise time of the measuring pulses is decreased; and particularly (b) as the fraction of resistance restored is made smaller. Thus, paradoxically, even dc measurements show this variation, although for a still smaller fraction of the film; even if measurements are not restricted to a deliberately small resistance restored,[27] heating effects beginning in the weakest regions will have time to spread, as they would not with pulses of intermediate duration. Presumably, taking smaller fractions of normal resistance restored as the transition criterion ensures isothermal conditions momentarily for correspondingly smaller fractions of film embedded in surrounding thicker regions.

The transition near the critical temperature with negligible Joule heating is found to be faster than 10^{-9} sec, in agreement with other workers, and is independent of current amplitude. The dependence of transition time on the current amplitude, as reported by Schmidlin *et al.*,[28] is clearly an aspect of transitions highly influenced by Joule heating.

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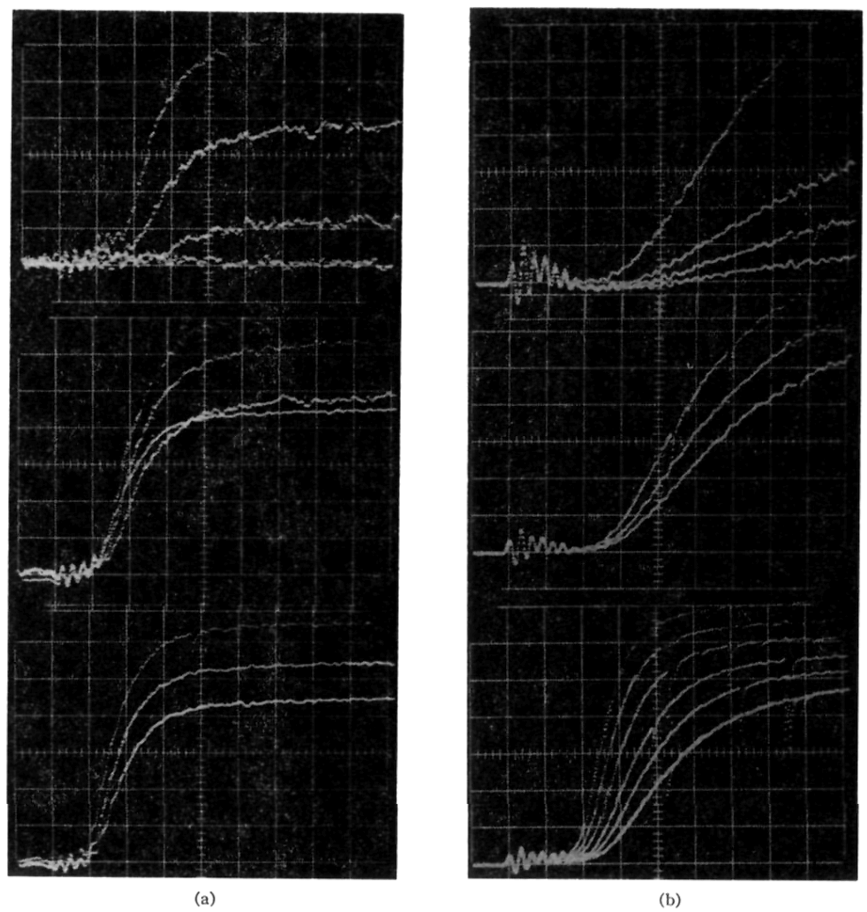

FIG. 3. Oscillograms of current-induced transitions for pulses of 7-nsec rise time in 585-Å-thick In film. $T_c = 3.460°K$. Horizontal sensitivity: 5 nsec/cm. (a) $T = 3.413°K$. Top: 5 mV/cm; middle: 10 mV/cm; bottom: 20 mV/cm. Traces, from top: 25, 20, 15, 10, 40, 35, 30, 30, 70, 60, 50-mA pulse current. (b) $T = 1.488°K$. Top: 50 mV/cm; middle: 100 mV/cm; bottom: 200 mV/cm. Traces, from top: 480, 460, 440, 420, 540, 520, 500, 800, 750, 700, 650, 620, 580-mA pulse current.