Interaction of Alfvén Waves and Spin Waves in a Ferromagnetic Metal

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The interaction between spin waves and Alfvén waves is investigated in bulk ferromagnetic metals. It is shown that the Alfvén wave will interact strongly with the spin-wave system. In the presence of the interaction, the modes can no longer be identified as pure Alfvén- or spin-wave modes. The wave number at which resonance occurs can be varied by changing the strength of the external dc field. The results are applied to iron, in which it is expected that this interaction can occur.

I. INTRODUCTION

RECENTLY, the occurrence of magnetoplasma oscillations was predicted in high conductivity solids.[1] These oscillations have been found in certain semiconductors[2] and in metals[3] at very low temperatures. If only one type of charge carrier is present, or if several types are present and the density of electrons is unequal to the density of holes, the magnetoplasma mode that occurs is the helicon mode. When the numbers of electrons and holes are equal, Alfvén waves[4,5] can propagate. Alfvén waves have been observed experimentally in bismuth.[5]

Recent magnetoresistance measurements indicate that in iron, the numbers of electrons and holes are equal.[6] Therefore, it is possible that Alfvén waves can propagate in ferromagnetic iron. Helicon-spin wave interactions have been predicted in ferromagnetic metals by Stern and Callen.[7] They treated a one-carrier system, but applied their results to iron. In this paper, we extend their treatment so as to investigate the interaction of the Alfvén wave with spin waves in a compensated, ferromagnetic metal like iron.

II. THEORY

Electromagnetic radiation can propagate in bulk metals in the presence of a dc magnetic field $\mathbf{B}_0$. The conditions for this to occur are

$$\omega_{ci}\tau_i \gg 1 \quad \text{and} \quad \omega < \omega_{ci}. \tag{1}$$

Here, $\omega_{ci}=eB_0/m_ic$ is the cyclotron frequency of a carrier with effective mass m_i, where e is the absolute value of electronic charge and $\mathbf{B}_0=\mathbf{H}_{dc}+4\pi\mathbf{M}_s$ is the dc magnetic induction inside the ferromagnetic material; ω is the frequency of the electromagnetic wave; and τ_i is the relaxation time of the carriers.

When one type of carrier is dominant and the magnetoplasma wave which propagates is the helicon mode, the above conditions are sufficient. However, if the numbers of electrons and holes are equal and Alfvén wave propagation occurs, the additional condition

$$\omega\tau_i \gg 1 \tag{2}$$

is also required. Also, if for a compensated material, the condition $\omega_{ci}\tau_i \gg 1$ is satisfied for one type of carrier but not for the other, helicon propagation occurs instead of Alfvén wave propagation.

The model used here is that of a two-component plasma interacting with a spin continuum. The magnetization of the spin system is described in terms of the density of the spin magnetic moment. We assume a transverse circularly polarized wave propagating along the magnetic field which we take as the z axis of our coordinate system.

The equations of motion for the magnetization $\mathbf{M}$ of a coupled spin system have been given by Herring and Kittel[8]

$$d\mathbf{M}/dt=-(ge/2mc)\mathbf{M} \times[C\nabla^2\mathbf{M}+\mathbf{H}_0+\mathbf{H}_A-\gamma\mathbf{M}_s+\mathbf{H}], \tag{3}$$

where $\mathbf{H}_0$ is the external dc field, $\mathbf{H}_A$ is the anisotropy field, γ is the demagnetization factor, $\mathbf{M}_s$ is the saturation magnetization,

$$\mathbf{H}=\mathbf{H}_1 e^{i(kz-\omega t)}, \tag{4}$$

g is the gyromagnetic ratio, and m is the free electron mass.

According to Herring and Kittel,[8] $C=2A/M^2$, where A is the Bloch wall coefficient which is of the order of $(2JS^2/a)$, J is the exchange energy, S is the spin per atom, and a is the lattice spacing. Equation (3), which has a validity beyond the particular atomic model chosen for the ferromagnet, can be derived phenomenologically.[9] The first term in (3) is valid only in the long-wavelength limit, i.e., $ka\ll 1$.

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We take the case where the wave is propagating along the direction of the saturation magnetization $\mathbf{M}_s$ which is also parallel to the dc field $\mathbf{H}_{dc}$ where $\mathbf{H}_{dc}$ is the total magnetic field inside the material

$$\mathbf{H}_{dc}=\mathbf{H}_0+\mathbf{H}_A-\gamma\mathbf{M}_s. \quad (5)$$

The magnetization can be written as

$$\mathbf{M}=\mathbf{M}_s+\mathbf{M}_1e^{i(kz-\omega t)}. \quad (6)$$

Upon substituting (5) and (6) into (3) and linearizing, we obtain the following

$$M_\pm=\pm(\omega_s/4\pi)/[\omega\pm(\eta k^2+\omega_c')]H_\pm. \quad (7)$$

Here,

$$M_\pm=M_{1x}\pm iM_{1y},\quad \omega_c'=(g/2)(e/mc)H_{dc},$$

$$\omega_s=(g/2)(e/mc)4\pi M_s,\quad \text{and}\quad \eta=g(e/mc)A/M_s.$$

The upper and lower signs in (7) refer to the directions of polarization, the upper sign being associated with a left-circularly polarized (LCP) wave while the lower sign is associated with a right-circularly polarized wave (RCP).

For a circularly polarized wave propagating in the z direction, Maxwell's equations yield the following relations for the electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$:

$$ikH_\pm=\mp[(4\pi i/c)j_\pm+(\omega/c)E_\pm], \quad (8a)$$

$$ikE_\pm=\pm(\omega/c)(H_\pm+4\pi M_\pm). \quad (8b)$$

The current density $j_\pm$ is related to the electric field $E_\pm$ by the relation

$$j_\pm=\sigma_\pm E_\pm, \quad (9)$$

where $\sigma_\pm$ is the sum of the conductivities for the various carriers. For a two-component plasma consisting of electrons and holes, we have, neglecting collisions,

$$4\pi\sigma_\pm=i\omega_{pe}^2/(\omega\pm\omega_{ce})+i\omega_{ph}^2/(\omega\mp\omega_{ch}) \quad (10)$$

where, ω_{pe}, ω_{ph} and ω_{ce}, ω_{ch} are the electron and hole plasma frequencies and cyclotron frequencies, respectively. Equation (10) is valid as long as we can neglect nonlocal effects,[10] i.e.,

$$k<|(\omega\pm\omega_{ce})/v_{Fe}|,\quad |(\omega\mp\omega_{ch})/v_{Fh}|,$$

where v_{Fe} and v_{Fh} are the Fermi velocities for the electrons and holes.

When the conditions $\omega\ll\omega_{ce}$, ω_{ch} hold, the conductivity is given by

$$4\pi\sigma_\pm=i[\pm(4\pi ec/B_0)(n_e-n_h)-\omega(c/v_A)^2], \quad (11)$$

where v_A is the Alfvén velocity and is given by

$$v_A=B_0/[4\pi(n_em_e+n_hm_h)]^{1/2}. \quad (12)$$

When the number of electrons equals the number of holes, the first term in (11) is zero and we have Alfvén wave propagation.

The dispersion relation can be found by substituting (7) and (11) into (8) and solving for either $H_\pm$ or $E_\pm$. This yields

$$[\omega^2-\omega_A^2][\omega\pm\omega_M]=\mp\omega_s\omega^2, \quad (13)$$

where $\omega_A=kv_A$ is the dispersion for the pure Alfvén wave and $\omega_M=\eta k^2+\omega_c'$ is the dispersion for the pure spin wave. Also, it can be seen from (7), (8), and (11) that the dielectric constant and permeability are given by

$$\epsilon_\pm=1+(c/v_A)^2 \quad (14a)$$

and

$$\mu_\pm=1\pm[\omega_s/(\omega\pm\omega_M)], \quad (14b)$$

respectively, when $\omega\ll\omega_{ce}$, ω_{ch}.

Solving the dispersion relation (13) for k as a function of ω, we obtain

$$k_u^2=\tfrac{1}{2}\left\{\left(\frac{\omega}{v_A}\right)^2-\frac{(\omega_c'\pm\omega)}{\eta}+\left[\left(\frac{\omega^2}{v_A^2}+\frac{\omega_c'\pm\omega}{\eta}\right)^2+4\frac{\omega_s}{\eta}\left(\frac{\omega}{v_A}\right)^2\right]^{1/2}\right\} \quad (15a)$$

and

$$k_l^2=\tfrac{1}{2}\left\{\left(\frac{\omega}{v_A}\right)^2-\frac{(\omega_c'\pm\omega)}{\eta}-\left[\left(\frac{\omega^2}{v_A^2}+\frac{\omega_c'\pm\omega}{\eta}\right)^2+4\frac{\omega_s}{\eta}\left(\frac{\omega}{v_A}\right)^2\right]^{1/2}\right\}. \quad (15b)$$

If ω_s were equal to zero, (15) would yield the unperturbed Alfvén and spin-wave dispersion relations. The saturation magnetization term, therefore, couples the Alfvén wave to the spin waves. The interaction has its maximum effect when the first term in the square root vanishes. This occurs when $(\omega/v_A)^2=(\omega-\omega_c')/\eta$, i.e., when the wave numbers of the unperturbed Alfvén and spin waves are equal, with the lower sign of polarization. However, with the upper sign, this term never vanishes and the maximum interaction does not occur.

At long wavelengths, the interaction is so strong that for a RCP wave, [lower sign in (15)] the modes can no longer be identified as pure Alfvén waves or pure spin waves. In Fig. 1, the dispersion relation (15) with the lower sign is shown for a single crystal of iron. The following values of the constants are used: $A=2\times10^{-6}$ erg cm^{-1}, $M_s=1700$ G, $g=2.14$, $v_A=5\times10^7$ cm sec^{-1}, and $H_{dc}=1000$ G. For these values, the maximum interaction occurs at $k\simeq300$ cm^{-1}.

III. DISCUSSION

In the above we have shown a strong Alfvén wave-spin wave interaction. However, we have neglected the effects of dissipation. Now we will consider these effects. The effect of collisions on the spin-wave–Alfvén-wave interaction can be taken into account by replacing ω by $\omega+i/\tau_e$ and $\omega+i/\tau_h$, respectively, in the first and second terms of (10). The damping introduced by the collisions can be neglected if $|\omega_{ci}-\omega|\tau_i\gg1$ and $\omega\tau_i\gg1$

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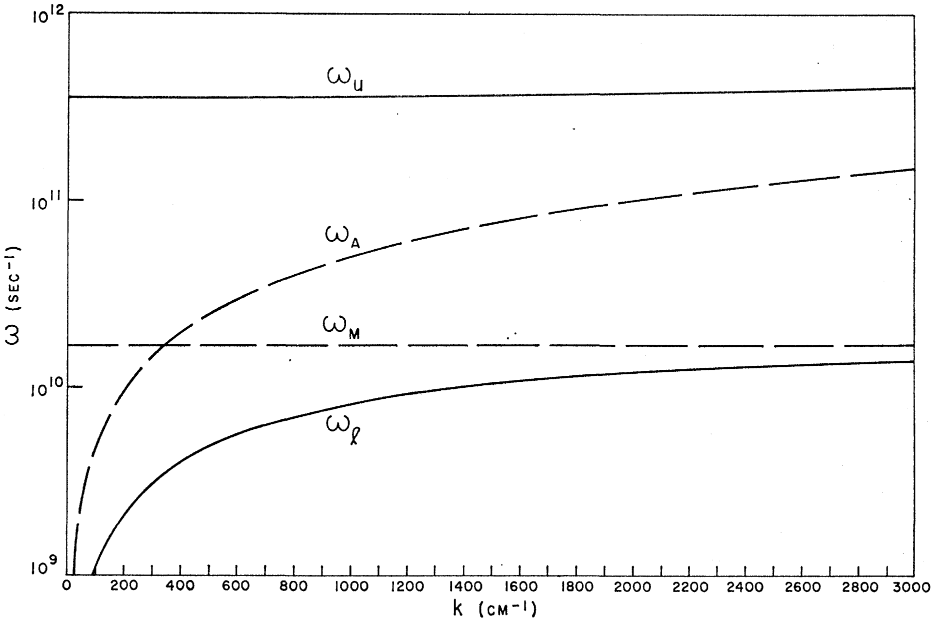

FIG. 1. The dispersion relations are shown on a semilog scale for the coupled and uncoupled Alfvén-magnon modes. The modes in the absence of the interaction are indicated by the dashed lines. The frequencies ω_u and ω_l are associated with the k_u and k_l of Eq. (15).

in a compensated material. If the temperature is sufficiently low and the material sufficiently pure so that the conditions $\omega_{ci}\tau_i \gg 1$ and $\omega\tau_i \gg 1$ are satisfied for both types of carriers, the losses due to scattering of the electrons are negligible. However, because the carriers have a velocity on the order of their Fermi velocity v_{Fi}, Doppler-shifted cyclotron resonance can occur at wave numbers above a certain minimum.[10] Equivalently, $\epsilon_\pm$ will have an imaginary part for such values of k and the modes will be damped. The condition that Doppler-shifted cyclotron resonance occur is that

$$k > \omega_{ci}/v_{Fi} \simeq 3500\ \text{cm}^{-1}. \tag{16}$$

Here we have used $v_{Fh} = v_{Fe} \simeq 10^8$ cm sec^{-1} and the *free*-electron mass in both ω_{ch} and ω_{ce}. At wave numbers at which the condition for Doppler-shifted cyclotron resonance to occur is satisfied, the local theory we used for the conductivity tensor is no longer valid. A nonlocal theory must then be used which yields in place of (10) the following

$$4\pi\sigma_\pm = -(3i/2)\{(\omega_{pe}^2/kv_{Fe})x_e f(x_e) + (\omega_{ph}^2/kv_{Fh})x_h f(x_h) + (i\pi/2k)[(\omega_{pe}^2/v_{Fe})(1-x_e^2) + (\omega_{ph}^2/v_{Fh})(1-x_h^2)]\}, \tag{17}$$

where $x_e = (\omega \pm \omega_{ce})/kv_{Fe}$, $x_h = (\omega \mp \omega_{ch})/kv_{Fh}$, and

$$f(x) = 1 + [(1-x^2)/2x] \ln|(1+x)/(1-x)|.$$

In this region of k, the waves are very heavily damped.

In considering damping in the spin-wave system, one can assume a loss term of the Bloch type[11] or use the approach of Landau and Lifschitz.[9] However, the damping mechanism for spin waves does not seem to be completely understood.[12] The Alfvén-wave–spin-wave interaction could be used as a tool to obtain a better understanding of the loss mechanism for the spin waves. It is for this reason that the RCP modes are of much greater interest than the LCP modes. These modes are mixtures of Alfvén waves and spin waves in the allowed region ($k \lesssim 3500$ cm^{-1}). Therefore, energy can be pumped into these modes and the effects of the damping due to spin waves can be observed. For the LCP case [upper sign in (15)], the modes are distorted by the interaction from their unperturbed values, but they still retain their identity as pure Alfvén waves or pure spin waves. When the spin coupling arises from the electrons, as is assumed in (3), the LCP mode is completely out of phase with the spin waves.

Without considering damping, there are certain frequency regions where the incoming wave will be reflected. For the RCP modes this occurs when

$$\epsilon_-\mu_- < 0.$$

Examination of (14) shows that $\epsilon_- > 0$ for all ω less than ω_{ce} while $\mu_- > 0$ for ω less than ω_M and greater than $\omega_M + \omega_s$ and $\mu_- < 0$ for $\omega_M < \omega < \omega_M + \omega_s$. Therefore, transmission takes place in the regions $0 < \omega < \omega_M$ and $\omega_M + \omega_s < \omega < \omega_{ce}$ while reflection occurs when $\omega_M < \omega < \omega_M + \omega_s$. Strictly speaking, the reflection band should not occur as the condition $\mu_- < 0$ will never be satisfied for the lower mode since ω_M increases with k. However, the values of k where this occurs are well into the Doppler-shifted region where the attenuation is extremely large. There is then an effective gap between

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the two modes and a certain amount of reflection should take place.

The criteria for the existence of Alfvén waves imposes certain conditions on the effective masses of the charge carriers. These conditions require that $\omega \ll \omega_{ce}$, ω_{ch}. For the upper mode the frequency is $\omega_u \simeq \omega_c' + \omega_s$ in the region of propagation. Therefore, the condition for this mode to satisfy the criteria for Alfvén wave propagation becomes

$$(g/2)(m_i/m) \ll 1 . \tag{18a}$$

For the lower mode, the conditions become much less severe. Since the maximum value of ω_l is ω_c' in the region of propagation, we require that

$$(g/2)(m_i/m)(H_{\mathrm{dc}}/B_0) \ll 1 . \tag{18b}$$

As B_0 is always greater than H_{dc}, (18b) is always a less severe condition than (18a). For $H_{\mathrm{dc}} = 10^3$ G and $B_0 = 2\times 10^4$ G, the ratio m_i/m can be as large as 10 without the condition for Alfvén waves being violated.

When the above criteria are violated, the dispersion relations shown in Fig. 1 are no longer valid. This is because the electromagnetic wave which can propagate in the region where the above criteria fails is no longer an Alfvén wave. However, qualitative statements can still be made about the transmission characteristics by using the full expression for ϵ_-,

$$\epsilon_- = 1 - \omega_{pe}^2/\omega_{ce}(\omega - \omega_{ce}) + \omega_{ph}^2/\omega_{ch}(\omega + \omega_{ch}) . \tag{19}$$

The above expression for ϵ_- is valid within the limits of the local theory and with collisions neglected. In the case when $\omega_u > \omega_{ce}$, transmission will occur in the regions $0 < \omega < \omega_M$ and $\omega_{ce} < \omega < \omega_M + \omega_s$ while reflection occurs in the bands, $\omega_M < \omega < \omega_{ce}$ and $\omega_M + \omega_s < \omega$. Whether transmission in the region $\omega_{ce} < \omega < \omega_M + \omega_s$ actually takes place depends on whether the dispersion is such that these frequencies do not fall in the Doppler-shifted cyclotron resonance region. The dispersion relations for the general case are given by

$$k_u^2 = \tfrac{1}{2}\left\{ \left(\frac{\omega}{c}\right)^2 \epsilon_- + \frac{\omega - \omega_c'}{\eta} + \left[\left(\frac{\omega^2}{c^2}\epsilon_- + \frac{\omega_c' - \omega}{\eta} \right)^2 + 4\left(\frac{\omega}{c}\right)^2 \epsilon_- \frac{\omega_s}{\eta} \right]^{1/2} \right\},$$
$$k_l^2 = \tfrac{1}{2}\left\{ \left(\frac{\omega}{c}\right)^2 \epsilon_- + \frac{\omega - \omega_c'}{\eta} - \left[\left(\frac{\omega^2}{c^2}\epsilon_- + \frac{\omega_c' - \omega}{\eta} \right)^2 + 4\left(\frac{\omega}{c}\right)^2 \epsilon_- \frac{\omega_s}{\eta} \right]^{1/2} \right\}. \tag{20}$$

When $\omega \approx \omega_{ce}$, $\epsilon_- \gg 1$ and the dispersion of the lower mode is unaffected. Thus, for the general case, only the upper mode has its dispersion affected when the criteria for Alfvén wave propagation are violated.

The observation of the interaction discussed in this paper would require iron of high purity so that collisional damping would not be important. In iron, there is some uncertainty about the value of the effective mass. The value of m determined from de Haas–van Alphen experiments is less than the free-electron mass.[13] However, there seem to be arguments that lead to an expectation of a mass larger than the free-electron mass. If, indeed, this is the case, we would expect the dispersion for the upper mode to be altered since condition (18a) would no longer be satisfied. We would then have to go to the more general dispersion relation (20). In fact, the presence of a carrier with a mass larger than the free-electron mass could be determined by doing a transmission experiment in the region $\omega_{ce} < \omega < \omega_M + \omega_s$. The magnetoresistance measurements of Reed and Fawcett[14] seem to indicate that at fields much below 100 kOe and in thin whiskers below 40 kOe, the conditions $\omega_{ci}\tau_i \gg 1$ are satisfied for all the carriers in the iron they used. Thus, iron of the required purity to observe the interaction is available. Frequencies in the microwave range would have to be used to satisfy the condition $\omega\tau_i \gg 1$. At fields such that $\omega_{ce}\tau_e \gg 1$ but $\omega_{ch}\tau_h \ll 1$, helicon waves would propagate instead of Alfvén waves and the theory of Stern and Callen[7] would be applicable.

In the above discussion, we have not mentioned the effects of demagnetization that would occur in a finite sample. As can be seen from (5), the demagnetization factor can reduce the field inside the material to a value considerably below the external field. We will not discuss this matter except to point out that the demagnetization factor vanishes for the field along the axis of a long circular cylinder and has its maximum value of 4π for the field perpendicular to a thin disc or slab. Therefore, in doing experiments on the interaction between Alfvén waves and spin waves, account has to be taken of the appropriate demagnetization factors.

In summary, we have shown that under certain conditions, there is a strong interaction between Alfvén waves and spin waves in bulk ferromagnetic metals. This presents the possibility of directly exciting spin waves of finite wavelength. The wave number at which resonance takes place can be varied by changing the external dc magnetic field.

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