or

$$E=\frac{N}{6\pi^3}\int_0^{2\pi}dy\int_0^{4\pi-y}dx\int_0^{\pi}dz \times\left\{\frac{\epsilon^+(x,y,z)}{\exp(\epsilon^+/kT)-1}+\frac{\epsilon^-(x,y,z)}{\exp(\epsilon^-/kT)-1}\right\}. \quad \text{(A7)}$$

We obtained E by numerically integrating Eq. (A6), with N set equal to one half of Avogadro's number, as is appropriate for one mole of $CrBr_3$. The magnetic heat capacity was then taken as the temperature derivative of E.

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Longitudinal-Dielectric-Permittivity Criterion for Current Instability in a Polar Crystal*

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Electrodynanic phenomena in a spatially dispersive medium are governed by the dielectric permittivity tensor. It appears that this tensor is useful in describing small departures from a steady state as well as small departures from equilibrium. The existence of a simple relationship between the longitudinal parts of the permittivity tensor for certain systems in equilibrium and the same systems in a steady state is noted and then applied in a calculation of the longitudinal dielectric permittivity $\epsilon^L(\mathbf{q},\omega;\mathbf{v}_D)$ of a polar crystal in which there is a stream of degenerate carriers with drift velocity $\mathbf{v}_D$. Vanishing of $\epsilon^L(\mathbf{q},\omega;\mathbf{v}_D)$ is associated with longitudinal electric waves in the system. Damping of these waves is governed by Im $\epsilon^L(\mathbf{q},\omega;\mathbf{v}_D)$. It is argued that in some circumstances Im $\epsilon^L(\mathbf{q},\omega;\mathbf{v}_D)$ can be made to vanish by adjusting $\mathbf{v}_D$. This general argument is applied to the case of carriers drifting in GaAs, and it is found that the longitudinal electric waves (plasmons coupled to longitudinal optical phonons) become unstable for values of v_D not much larger than the value reported by Gunn as the threshold for the onset of current oscillations in GaAs. (It is assumed that $\omega\tau_{el}\gg 1$, where τ_{el} is the relaxation time for the distribution of electrons.) The significance of this result is discussed.

I. INTRODUCTION

ONE of the principal goals of the theory of solids is to elucidate as completely as possible the connections between the macroscopic electrodynamic phenomena observable in a medium and its microscopic constitution. A very useful tool in this work is the dielectric permittivity tensor, $\boldsymbol{\varepsilon}(\mathbf{r}, \mathbf{r}', t-t')$. In the next section this tensor is defined in the course of a review of the equations of the electromagnetic field and the description of plane electromagnetic waves in an infinite homogeneous medium. We allow the medium to be spatially as well as temporally dispersive and pay special attention to the longitudinal electric waves (plasma oscillation waves) which can exist when the longitudinal part of the permittivity tensor vanishes.

Most of this analysis applies not only to a medium in equilibrium but also to one in which constant currents are maintained. The problem of establishing relations between the permittivity of a current-carrying medium and that of the same medium in equilibrium is of considerable interest. A simple relation of this kind for the case of a translationally invariant medium is derived here (Sec. V). This relation is used in a brief investigation of longitudinal electric waves in current carrying media. The investigation begins with a calculation of the longitudinal permittivity for a plasma of two components, each of which may have a drift velocity. This result and some results of Born and Huang[1] lead to an expression for the longitudinal permittivity of a polarizable medium carrying a current. Using the relation between the permittivities of the equilibrium and current-carrying medium, and exploiting a general symmetry property of the permittivity, we obtain information about the transfer of energy between the medium and a monochromatic longitudinal electric wave (for the system considered this wave is a collective mode of the lattice, the streaming electrons, and the electromagnetic field).

To illustrate the utility of these ideas we apply them to the case of a crystal of GaAs carrying a current. Gunn[2] has observed interesting current instabilities in this system. We calculate the longitudinal dielectric permittivity for one of Gunn's samples of GaAs and find that the smallest drift velocity for which the calculated permittivity vanishes approximates the drift velocity value reported by Gunn as the threshold for the appearance of instability. This result is interpreted and its significance is discussed.

* This work was supported by the National Science Foundation.

[1] M. Born and K. Huang, *Dynamical Theory of Crystal Lattices* (Oxford University Press, New York, 1954), Secs. 7, 8, and 10.
[2] J. B. Gunn, Solid State Commun. **1**, 88 (1963); IBM J. Res. Develop. **8**, 141 (1964).

II. REVIEW OF THE ELECTRODYNAMICS OF A SPATIALLY DISPERSIVE MEDIUM

The equations of the electromagnetic field in any medium take the form (Gaussian units)

$$\text{(a) } \operatorname{div}\mathbf{D}=4\pi\rho_0, \quad \text{(b) } \operatorname{curl}\mathbf{E}=-c^{-1}\partial\mathbf{B}/\partial t,$$
$$\text{(c) } \operatorname{div}\mathbf{B}=0, \quad \text{(d) } \operatorname{curl}\mathbf{B}=c^{-1}\partial\mathbf{D}/\partial t+(4\pi/c)\mathbf{j}_0. \quad (1)$$

Here ρ_0 and $\mathbf{j}_0$ are the charge density and current density of the external sources of the field. The fields $\mathbf{E}$ and $\mathbf{B}$ are those which enter the expression for the force $\mathbf{F}$ acting on a point test charge e moving in the medium with velocity $\mathbf{v}$:

$$\mathbf{F}=e(\mathbf{E}+c^{-1}\mathbf{v}\times\mathbf{B}). \quad (2)$$

The effect of the external field sources is to induce in the medium a charge density $\rho(\mathbf{r},t)$ and a current density $\mathbf{j}(\mathbf{r},t)$ which satisfy the continuity equation

$$\partial\rho/\partial t+\operatorname{div}\mathbf{j}=0. \quad (3)$$

D is defined by

$$\mathbf{D}(\mathbf{r},t)=\mathbf{E}(\mathbf{r},t)+4\pi\int_{-\infty}^{t} dt'\mathbf{j}(\mathbf{r},t'), \quad (4)$$

and the dielectric permittivity $\boldsymbol{\varepsilon}(\mathbf{r},\mathbf{r}',t)$ is introduced via

$$\mathbf{D}(\mathbf{r},t)=\int_{-\infty}^{t} dt'\int d\mathbf{r}'\boldsymbol{\varepsilon}(\mathbf{r},\mathbf{r}', t-t')\cdot\mathbf{E}(\mathbf{r}',t'), \quad (5)$$

where it is assumed that the properties of the medium are independent of time. The current density $\mathbf{j}(\mathbf{r},t)$ includes a contribution c curl $\mathbf{M}$ where $\mathbf{M}(\mathbf{r},t)$ is the magnetization of the medium.

If the medium is unbounded in space and homogeneous, the dependence of $\boldsymbol{\varepsilon}$ in Eq. (5) on r and r' involves only the combination $\mathbf{r}-\mathbf{r}'$, and then if all fields are expanded as Fourier integrals of plane monochromatic waves, Eq. (5) transforms into

$$\mathbf{D}(\mathbf{q},\omega)=\boldsymbol{\varepsilon}(\mathbf{q},\omega)\cdot\mathbf{E}(\mathbf{q},\omega), \quad (6)$$

where

$$\boldsymbol{\varepsilon}(\mathbf{q},\omega)=\int_{0}^{\infty} dt e^{i\omega t}\int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\boldsymbol{\varepsilon}(\mathbf{r},t). \quad (7)$$

We refer to $\boldsymbol{\varepsilon}(\mathbf{q},\omega)$ as the complex (or transformed) dielectric permittivity of the medium.

The dielectric permittivity determines the number and kind of plane monochromatic electromagnetic waves which can exist in the medium. For a wave proportional to $e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$ the field equations (1b) and (1d) reduce to

$$\mathbf{q}\times\mathbf{E}=(\omega/c)\mathbf{B}, \quad (8)$$
$$\mathbf{q}\times\mathbf{B}=-(\omega/c)\mathbf{D}.$$

Inserting $\mathbf{B}$ from the first of these equations in the second, expanding, and using Eq. (6) one finds that the amplitude $\mathbf{E}(\mathbf{q},\omega)$ of the electric field intensity in the wave must satisfy

$$[(\omega^2/c^2)\boldsymbol{\varepsilon}(\mathbf{q},\omega)+\mathbf{q}\mathbf{q}-q^2\mathbf{1}]\cdot\mathbf{E}(\mathbf{q},\omega)=0, \quad (9)$$

where $\mathbf{1}$ is the unit tensor. Essentially all of the preceding results are discussed in a review paper by Rukhadze and Silin[3]; they have been quoted to establish the notation and the theoretical framework underlying the present work. The connection between the dielectric permittivity and the conductivity is given in RS(2.28).

Equations (1)–(9) apply equally well to a medium in which there is a steady (time-independent) current.

III. THE LONGITUDINAL DIELECTRIC PERMITTIVITY

In what follows we assume that $\boldsymbol{\varepsilon}(\mathbf{q},\omega)$ is uniaxial and that $\mathbf{q}$ is directed along its axis. Then the vector $\mathbf{E}$ can be resolved into components in the direction of $\mathbf{q}$ ($\hat{e}_L$ is the corresponding unit vector) and transverse to that direction (unit vector $\hat{e}_T$):

$$\mathbf{E}(\mathbf{q},\omega)=E_L(\mathbf{q},\omega)\hat{e}_L+E_T(\mathbf{q},\omega)\hat{e}_T \quad (10)$$

and

$$\boldsymbol{\varepsilon}(\mathbf{q},\omega)=\epsilon^L(\mathbf{q},\omega)\hat{e}_L\hat{e}_L+\epsilon^T(\mathbf{q},\omega)(\mathbf{1}-\hat{e}_L\hat{e}_L) \quad (11)$$

so that

$$\boldsymbol{\varepsilon}\cdot\mathbf{E}=\epsilon^L E_L\hat{e}_L+\epsilon^T E_T\hat{e}_T. \quad (12)$$

In these circumstances Eq. (9) breaks up into

$$\epsilon^L(q,\omega)E_L(q,\omega)=0, \quad (13a)$$
$$[\omega^2\epsilon^T(\mathbf{q},\omega)-c^2q^2]E_T(\mathbf{q},\omega)=0. \quad (13b)$$

From Eq. (13a) we see that

$$\epsilon^L(\mathbf{q},\omega)=0 \quad (14)$$

is the dispersion relation for longitudinal electromagnetic waves in the medium. In media containing itinerant electrons or holes these waves are often called plasma oscillation waves. Note that $\epsilon^L(\mathbf{q},\omega)$ is generally complex, which means that Eqs. (13) associate complex values of $\mathbf{q}$ with real values of ω (and vice versa), i.e., solutions of (14) are of the form

$$\mathbf{q}(\omega)=\mathbf{q}_1(\omega)+i\mathbf{q}_2(\omega). \quad (15)$$

Thus longitudinal electromagnetic waves generally change in amplitude in the direction of propagation. Usually $q_2(\omega)>0$, so that the change is a decay; the possibility that $q_2(\omega)$ might vanish or change sign is one of the main concerns of this paper and is discussed further in Sec. V.

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[3] A. A. Rukhadze and V. P. Silin, Usp. Fiz. Nauk. **74**, 223 (1961) [English transl.: Soviet Phys.—Usp. **4**, 459 (1961)]. In what follows, this paper is referred to as RS and equations in it are referred to by the numbers they have there prefixed by RS, e.g., RS(2.7).

IV. CALCULATION OF $\epsilon^L(q,\omega)$ FOR TWO SYSTEMS

Having seen the importance of the function $\epsilon^L(\mathbf{q},\omega)$ in determining the properties of longitudinal electromagnetic waves, we turn now to the problem of calculating $\epsilon^L(\mathbf{q},\omega)$ for simple models of two important systems. Let us assume that in the system under investigation there is established for all times, by some unspecified means, a longitudinal electric field $\mathbf{E}$ of the form $E_0\hat{e}_L \exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)+\eta t]$, where η is a small positive quantity at our disposal. Then Eqs. (6) and (12) imply

$$\mathbf{q}\cdot\mathbf{D}(\mathbf{q},\omega)=q\epsilon^L(\mathbf{q},\omega)E_0\,, \tag{16}$$

and

$$\mathbf{q}\cdot\mathbf{D}(\mathbf{r},t)=qE_0\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)+\eta t]+4\pi\int_{-\infty}^{t}dt'\mathbf{q}\cdot\mathbf{j}(\mathbf{r},t) \tag{17}$$

is a consequence of Eq. (4).

We first consider the system discussed by Harrison,[4] a two-component degenerate plasma in which the electrons and holes may have drift velocities $\mathbf{v}_{D-}$ and $\mathbf{v}_{D+}$. From Harrison's Eqs. (3) and (8) and subject to his approximations, one obtains for the departures from equilibrium of the densities of electrons and holes, respectively (to first order in E):

$$n_-(\mathbf{r},t)=2iem_-^2(2\pi\hbar)^{-3}\int d\mathbf{v}\nabla_v f_0(\mathbf{v}-\mathbf{v}_{D-};N_-)\times\mathbf{E}/(\omega-\mathbf{v}\cdot\mathbf{q}+i\eta)$$

$$n_+(\mathbf{r},t)=-2iem_+^2(2\pi\hbar)^{-3}\int d\mathbf{v}\nabla_v f_0(\mathbf{v}-\mathbf{v}_{D+};N_+)\times\mathbf{E}/(\omega-\mathbf{v}\cdot\mathbf{q}+i\eta)\,, \tag{18}$$

where $f_0=f_0(v;N)$ is the Fermi-Dirac distribution function over velocity for a degenerate gas containing N particles per unit volume. These expressions are of the form

$$n_-(\mathbf{r},t)=n_-(\mathbf{q},\omega)\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)+\eta t]\,.$$

From Eqs. (3), (18) we find for the current, to first order in E,

$$\mathbf{q}\cdot\mathbf{j}(\mathbf{r},t)=\mathbf{q}\cdot\mathbf{j}(\mathbf{q},\omega)\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)+\eta t] \tag{19}$$

with

$$\mathbf{q}\cdot\mathbf{j}(\mathbf{q},\omega)=-e\omega[n_+(q,\omega)-n_-(q,\omega)]\,. \tag{20}$$

Substitution of Eq. (20) into Eq. (17) and evaluation of the time integral leads to

$$\mathbf{q}\cdot\mathbf{D}(\mathbf{r},t)=\mathbf{q}\cdot\mathbf{D}(\mathbf{q},\omega)\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)+\eta t] \tag{21}$$

with

$$\begin{aligned}\mathbf{q}\cdot\mathbf{D}(\mathbf{q},\omega)&=qE_0+4\pi\mathbf{q}\cdot\mathbf{j}(\mathbf{q},\omega)/(-i\omega+\eta)\\&=qE_0-4\pi ie[n_+(q,\omega)-n_-(q,\omega)]\end{aligned} \tag{22}$$

in the limit as $\eta\to 0$.

Comparing Eqs. (16) and (22) we find (introducing $\omega_{P\pm}^2\equiv 4\pi N_\pm e^2/m_\pm$)

$$\begin{aligned}\epsilon^L(\mathbf{q},\omega)=1&+2(m_-/2\pi\hbar)^3(\omega_{P-}^2/N_-q^2)\\&\times\int d\mathbf{v}[\mathbf{q}\cdot\nabla_v f_0(\mathbf{v}-\mathbf{v}_{D-};N_-)/(\omega-\mathbf{v}\cdot\mathbf{q}+i\eta)]\\&+2(m_+/2\pi\hbar)^3(\omega_{P+}^2/N_+q^2)\\&\times\int d\mathbf{v}[\mathbf{q}\cdot\nabla_v f_0(\mathbf{v}-\mathbf{v}_{D+};N_+)/(\omega-\mathbf{v}\cdot\mathbf{q}+i\eta)]\,,\end{aligned} \tag{23}$$

which is H(9), if in Harrison's expression for $\omega_{P\pm}^2$ the factor ϵ_0 is replaced by unity. Equating $\epsilon^L(\mathbf{q},\omega)$ from Eq. (23) to zero gives the dispersion relation for the longitudinal collective excitations discussed by Harrison in the remainder of his article.

The other system to be considered here is a one-component plasma in a polar crystal.[5–7] Varga[8] has shown that for a wide range of circumstances in typical compound semiconductors the lattice and electronic contributions to the induced current are additive. Then for long wavelengths (small q)

$$\epsilon^L(\mathbf{q},\omega)=[\epsilon_{\text{el}}{}^L(q,\omega)-1]+\epsilon_{\text{lat}}{}^L(\omega) \tag{24}$$

with

$$\epsilon_{\text{lat}}{}^L(\omega)=\epsilon_\infty+(\epsilon_0-\epsilon_\infty)/[1-(\omega/\omega_t)^2-i(\omega_t\tau_l)^{-1}(\omega/\omega_t)]\,. \tag{25}$$

Here $\epsilon_{\text{el}}{}^L(q,\omega)$ refers to the electrons alone, and $\epsilon_{\text{lat}}{}^L(\omega)$ is the permittivity according to the Born-Huang theory of long-wavelength optical lattice vibrations.[1] We readily obtain an approximation to $\epsilon_{\text{el}}{}^L(q,\omega)$ for an electron plasma by letting N_+ vanish in Eq. (23):

$$\epsilon_{\text{el}}{}^L(\mathbf{q},\omega)=1+(4\pi e^2/q^2)\zeta(\mathbf{q},\omega) \tag{26}$$

with

$$\zeta(\mathbf{q},\omega)=[2m_-^2/(2\pi\hbar)^3]\times\int d\mathbf{v}\mathbf{q}\cdot\nabla_v f_0(\mathbf{v}-\mathbf{v}_{D-};N_-)/(\omega-\mathbf{v}\cdot\mathbf{q}+i\eta)\,. \tag{27}$$

In what follows, we usually write $\mathbf{v}_D$ for $\mathbf{v}_{D-}$.

[4] M. J. Harrison, J. Phys. Chem. Solids **23**, 1079 (1962). In what follows this paper is referred to as H, and equations in it are referred to by the numbers they have there prefixed by H, e. g., H(7).

[5] V. L. Gurevich, Fiz. Tverd. Tela **4**, 1380 (1962) [English transl.: Soviet Phys.—Solid State **4**, 1015 (1962)].

[6] T. O. Woodruff, Bull. Am. Phys. Soc. **8**, 254 (1963); Phys. Rev. **132**, 679 (1963).

[7] J. B. Gunn, Phys. Letters **4**, 194 (1963).

[8] B. B. Varga, Phys. Rev. **137**, A1896 (1965).

V. THE LONGITUDINAL PERMITTIVITY AND THE DECAY OR GROWTH OF LONGITUDINAL WAVES

By relating the longitudinal permittivity for a system in which the carriers are drifting to the longitudinal permittivity for the same system without drift, it is possible to secure useful insight into the decay (or growth) of longitudinal collective modes. Our purpose is to illustrate this point using the results obtained thus far. Consider $\epsilon_{\text{el}}{}^L(\mathbf{q},\omega)$ as given in Eqs. (26) and (27); since this is the longitudinal permittivity of electrons drifting with velocity $\mathbf{v}_D$, it is more completely described by $\epsilon_{\text{el}}{}^L(\mathbf{q},\omega;\mathbf{v}_D)$. Changing the variable of integration in Eq. (27) to $\mathbf{v}'\equiv\mathbf{v}-\mathbf{v}_D$, we see that

$$\epsilon_{\text{el}}{}^L(\mathbf{q},\omega;\mathbf{v}_D)=\epsilon_{\text{el}}{}^L(\mathbf{q},\omega-\mathbf{v}_D\cdot\mathbf{q};0). \tag{28}$$

It is clear that this relation is valid for carriers drifting in any medium which is invariant under translation, providing the carrier distribution can be represented adequately by displacing the equilibrium distribution in velocity space. The meaning of Eq. (28) is that if electrons with no net drift velocity see a wave of frequency ω and wavevector $\mathbf{q}$, then electrons drifting with velocity $\mathbf{v}_D$ see (in their rest frame) a wave of the same wavevector but of Doppler-shifted frequency $\omega-\mathbf{v}_D\cdot\mathbf{q}$.

Another useful relation easily shown to be valid for the longitudinal permittivity of an isotropic and nongyrotropic medium is

$$\text{Im}\epsilon^L(\mathbf{q},\omega;0)=-\text{Im}\epsilon^L(\mathbf{q},-\omega;0). \tag{29}$$

The important consequence of Eqs. (28) and (29) is that the sign of $\text{Im}\epsilon^L(\mathbf{q},\omega;\mathbf{v}_D)$ changes if the sign of $\omega-\mathbf{v}_D\cdot\mathbf{q}$ changes. Hence, it is possible by adjusting $\mathbf{v}_D$ to make the sign of $\text{Im}\epsilon_{\text{el}}{}^L(\mathbf{q}_0,\omega_0;\mathbf{v}_D)$ opposite to that of $\text{Im}\epsilon_{\text{lat}}{}^L(\omega_0)$ in Eq. (24), and to make their magnitudes equal (at least when $\text{Im}\epsilon_{\text{lat}}{}^L(\omega_0)$ is sufficiently small) so that $\text{Im}\epsilon^L(q_0,\omega_0)=0$, where $\mathbf{q}_0$ and ω_0 are solutions of $\text{Re}\epsilon^L(\mathbf{q}_0,\omega_0)=0$. Then the longitudinal electromagnetic wave $(\mathbf{q}_0,\omega_0)$ neither decays nor grows. If now $\mathbf{v}_D$ is increased further, making $\text{Im}\epsilon^L(\mathbf{q}_0,\omega_0)<0$, RS(4.12) implies that the current-carrying medium will transfer energy to the wave $(\mathbf{q}_0,\omega_0)$.

The preceding general arguments will now be applied to the case of a gallium arsenide crystal carrying a current. The longitudinal dielectric permittivity of this system is given by Eqs. (24)–(27). For the limiting case of zero temperature we have from H(11)–H(18):

$$[\epsilon_{\text{el}}{}^L(q,\omega)-1]=\frac{q_-{}^2}{q^2}\left\{1+\frac{\omega-\mathbf{v}_D\cdot\mathbf{q}}{2qv_F}\times\left[\ln\left|\frac{1-(\omega-\mathbf{v}_D\cdot\mathbf{q})/qv_F}{1+(\omega-\mathbf{v}_D\cdot\mathbf{q})/qv_F}\right|+i\pi\right]\right\}, \tag{30}$$

provided $|\omega-\mathbf{v}_D\cdot\mathbf{q}|<qv_F$, where v_F is the velocity of electrons on the Fermi surface, i.e., $v_F=(\hbar/m_-)(3\pi^2N)^{1/3}$, and $q_-{}^2=3(\omega_P/v_F)^2$. We take $\mathbf{v}_D$ and $\mathbf{q}$ to be in the same direction and assign to the parameters the following values characteristic of the GaAs sample No. RC 28303 of length $L=2.5\times10^{-3}$ cm used by Gunn in his study of current instabilities in III–V semiconductors[2]: $\epsilon_0=12.5$ (Ref. 9), $\epsilon_\infty=10.9$ (Ref. 9), $\omega_t=5.16\times10^{13}$ sec^{-1} (a value suggested by Ref. 9), $m_-=0.07$ $N_-=1.25\times10^{16}$ cm^{-3} m_0 (Refs. 10, 11). These parameter values imply $v_F=1.19\times10^7$ cm/sec and $q_-{}^2=1.21\times10^{13}$ cm^{-2} (to within a few percent—no greater accuracy is claimed or is appropriate for any of the calculations reported here). I have been unable to find values for $\beta\equiv\omega_t\tau_l$ in the literature. The reflectivity curves of Ref. 12 suggest that $15<\beta<1000$ at room temperature.[12a]

For each of several values of β we now find a value of v_D which makes $\text{Im}\epsilon^L(q_l,\omega_l)=0$, where q_l and ω_l are values of q and ω which make $\text{Re}\epsilon^L(q,\omega)=0$. One way to do this is to determine ω_l from the condition $\text{Re}\epsilon_{\text{lat}}{}^L(\omega_l)=0$, and then to find that value of $x\equiv(\omega-v_Dq)/qv_F$ which makes $1+(x/2)\ \ln|(1-x)/(1+x)|=0$. It is easy to check that $x=-0.833$ is the required value; let it be symbolized by x_l. Clearly, $\text{Re}\epsilon^L(q_l,\omega_l)=0$ if q_l and v_D are adjusted to make $x=x_l$. The magnitude of q_l can be fixed by making $\text{Im}\epsilon_{\text{el}}{}^L(q_l,\omega_l)$, which is given by $\pi x_l q_-{}^2/2q_l{}^2$, equal

$$-\text{Im}\epsilon_{\text{lat}}{}^L(\omega_l)=-(\omega_l/\beta\omega_t)(\epsilon_0-\epsilon_\infty)/\{[1-(\omega_l/\omega_t)^2]^2+(\omega_l/\beta\omega_t)^2\},$$

so that $\text{Im}\epsilon^L(q_l,\omega_l)=0$. It then remains only to adjust v_D so that $(\omega_l-v_Dq_l)/q_lv_F=x_l$, i.e., $v_D=(\omega_l/q_l)-x_lv_F$. Proceeding thus, we find:

β	ω_l	q_l (cm^{-1})	v_D (cm/sec)
15	5.41×10^{13}	1.45×10^6	4.71×10^7
100	5.53×10^{13}	4.57×10^6	2.20×10^7
1000	5.53×10^{13}	1.44×10^7	1.37×10^7

It should be noted that these values of v_D have been arrived at by a procedure which in addition to making $\epsilon^L(q_l,\omega_l)=0$ makes $\text{Re}\epsilon_{\text{lat}}{}^L(\omega_l)$ and $\text{Re}\epsilon_{\text{el}}{}^L(q_l,\omega_l)$ each vanish separately; all that is required for a growing wave is for their sum, $\text{Re}\epsilon^L(q_l,\omega_l)$, to vanish. Thus there appear to be other values of v_D (possibly smaller) which would make $\epsilon^L(q_l,\omega_l)$ vanish for differently chosen q_l,ω_l. This possibility is being investigated. Note that the first value of ω_l tabulated above is less than $(\epsilon_0/\epsilon_\infty)^{1/2}\omega_t$ because of the damping term, $-i\beta^{-1}(\omega/\omega_t)$, which appears in the denominator of $\epsilon_{\text{lat}}{}^L(\omega)$, [Eq. (25)].

[9] K. G. Hambleton, C. Hilsum, and B. R. Holeman, Proc. Phys. Soc. (London) **77**, 1147 (1961).

[10] T. S. Moss and A. K. Walton, Proc. Phys. Soc. (London) **74**, 131 (1959).

[11] C. Hilsum and A. C. Rose-Innes, *Semiconducting III-V Compounds* (Pergamon Press, Inc., New York, 1961).

[12] G. Picus, E. Burstein, B. W. Henvis and M. Hass, J. Phys. Chem. Solids **8**, 282 (1959).

[12a] *Note added in proof.* After this article was submitted for publication, C. A. Baumgardner directed my attention to the work of M. Hass and B. W. Henvis [J. Phys. Chem. Solids **23**, 1099 (1962)], who obtained the value $(0.007)^{-1}\approx140$ for β at liquid-helium temperature.

Because of the way ω_l has been determined, it differs from the "plasma shifted" longitudinal optical phonon frequency discussed by Varga.[8]

The example just considered suggests that the threshold value of v_D reported by Gunn[2] for the sample discussed here might be closely related to the smallest value of v_D for which a longitudinal electromagnetic wave (q_l,ω_l) begins to receive energy from the current-carrying crystal rather than losing energy to it. It might be that the oscillations in current which Gunn observes are resolvable into trains of waves describable by values of q and ω close to the values of q_l and ω_l determined here. These waves would first grow and then decay in a period of the order of a nanosecond to give the observed current pulses. The preceding analysis shows that under some circumstances the (q,ω) waves grow as they propagate. They would decay if they were to encounter trains of optical phonons reflected from the end of the crystal from which the electron current issues.

One question arises immediately concerning the relevance of our application of the vanishing permittivity criterion to the current instability in GaAs: to what extent is it affected by the fact that the carriers in Gunn's experimental specimen are nondegenerate when at equilibrium? Our expression for $\epsilon_{el}(q,\omega)$ was obtained on the assumption that the distribution of streaming carriers is given by displacing through $\mathbf{v}_D$ the distribution function for a Fermi gas at the absolute zero of temperature. The actual or "true" distribution of streaming carriers in GaAs for currents just less than the threshold for the Gunn instability is unknown. Elsewhere[13] we have argued that the small fraction of electrons with velocities such that they can couple to the polarization waves to yield the unstable coupled wave are well described by a displaced Fermi distribution, even at high temperatures, because of the effects of forward-favoring asymmetries in the scattering processes and the speed with which fast electrons radiate optical phonons.

The discrepancy between the threshold value of v_D determined experimentally[2] by Gunn (1.44×10^7 cm/sec) and that given by our criterion (some value between 1.4 and 4.7×10^7 cm/sec, if we assume that the correct value of $\omega_t\tau_l$ lies between 15 and 1000, which seems reasonable) is also related to the fact that our criterion actually yields the threshold v_D for a fictitious "approximating" Fermi distribution, and some further analysis is needed to relate the true and the approximating distributions. The explanation[13] of the observed[14] inverse dependence of the threshold v_D on temperature is believed to arise out of the fact that the relation between the true and approximating distributions is temperature-dependent.

An important limitation on the validity of the preceding analysis should be noted. The derivation of our expression $\epsilon_{el}(\mathbf{q},\omega)$ is not valid unless $\omega\tau_{el}\gg1$, where τ_{el} is the relaxation time for the distribution of electrons. This condition is very well satisfied for the application considered here ($\omega_l\sim5\times10^{13}$ sec^{-1}, $\tau_{el}\sim2\times10^{-10}$ sec). Several earlier theoretical discussions of the interaction of waves of current and polarization[5–7] can be valid only in the opposite limit, $\omega\tau_{el}\ll1$.

It is to be hoped that the present work will encourage further investigation of at least two of the problems not adequately treated here:

(1) How is the electrodynamics of a medium in a steady but nonequilibrium state different from that of a medium in equilibrium?

(2) How are the conditions for growth and decay of electromagnetic wave packets propagating through a medium different from the conditions discussed by Rukhadze and Silin,[3] which apply only to waves unlimited in either spatial or temporal extent?

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