P- and D-State Contributions to the Magnetic Moment Form Factors of H^3 and He^3 †

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The effects of reasonable admixtures of P- and D-state ($J^P=\frac{1}{2}^+$, $T=\frac{1}{2}$) components to the ground-state wave function of H^3 and He^3 on their magnetic moment form factors are calculated. It is found that these form factors cannot be accounted for in this way. Inclusion of the S' state and a typical $T=\frac{3}{2}$ state still leaves the magnetic form factors unexplained, although such an admixture is shown to account for the difference between the observed charge form factors. The empirical isoscalar and isovector exchange magnetic moment form factors that are needed to fit the experimental data are calculated.

I. INTRODUCTION

ELASTIC scattering of high-energy electrons from H^3 and He^3 provides an important source of information about the three-nucleon ground state. The experimental data have been analyzed in terms of the electric charge and magnetic moment form factors by means of the Rosenbluth equation for spin-$\frac{1}{2}$ systems. The basic formulas, which take into account the charge, mass, and anomalous moments of the nuclei,[1] have been used to express $F_{\text{ch}}(H^3)$, $F_{\text{ch}}(He^3)$, $F_{\text{mag}}(H^3)$, and $F_{\text{mag}}(He^3)$ as functions of the four-momentum transfer q^2.

A summary of the previous attempts to understand these four experimental form factors on the basis of various assumptions concerning the three-nucleon system may be found in Ref. 2. In this paper the nonrelativistic analysis of Schiff[3] is extended to include the contributions to the magnetic moment form factors that arise from the P and D states with $T=\frac{1}{2}$. (The error incurred in such an impulse-approximation, nonrelativistic treatment is on the order of $q^2/6M^2$, where M is the nucleon mass.[4]) In addition a typical $T=\frac{3}{2}$ state, which may appear in the He^3 wave function,[5,6] is included in a re-analysis of the charge-form-factor data combining the results of Refs. 2 and 3.

The dominant component of the ground-state wave function is the fully space-symmetric ${}^2S_{\frac{1}{2}}$ state with $T=\frac{1}{2}$. Of the nine additional even-parity, $J=T=\frac{1}{2}$ states, three are neglected as having such small amplitudes as to be unimportant: the fully space-antisymmetric ${}^2S_{\frac{1}{2}}$ and ${}^2P_{\frac{1}{2}}$ states and the ${}^4P_{\frac{1}{2}}$ state. The remaining ${}^2S_{\frac{1}{2}}$ and ${}^2P_{\frac{1}{2}}$ states contribute significantly only through interference with the dominant S state. Only the three ${}^4D_{\frac{1}{2}}$ states are thought to be present with sufficient probability so as to be important in other than interference terms. The $T=\frac{3}{2}$, $J=\frac{1}{2}$ states have not been classified. However, there is no such fully space-symmetric state, and since the amplitude of a $T=\frac{3}{2}$ state is small, a mixed-symmetry type-S state is assumed to be the main component. This state should have the largest interference with the dominant S state.

For the convenience of the reader, the symmetry properties and formalism of the state functions are reviewed in the next two sections. The magnetic-moment form-factor expressions are obtained in the following three sections. The last section presents some numerical results including fits to the charge form factors and estimates of the magnetic-exchange-moment form factors.

II. SYMMETRY PROPERTIES

Three classifications of symmetry with respect to interchanges of the three nucleons are encountered repeatedly in calculations of this type. All quantities introduced below in the wave functions which carry subscript s are completely symmetric under such interchanges, while those labeled with subscript a are fully antisymmetric. Except for the Pauli spin matrices, quantities in the wave functions which carry subscripts 1 and 2 have mixed symmetry with respect to such interchanges and transform according to the permutation table given in Eq. (3) of Ref. 3 or Eq. (1) of Ref. 2:

$$\begin{aligned} &P_{23}\phi_1=\phi_1\,,\quad P_{12}\phi_1=\tfrac{1}{2}(3^{1/2}\phi_2-\phi_1)\,,\quad P_{13}\phi_1=-\tfrac{1}{2}(3^{1/2}\phi_2+\phi_1)\\ &P_{23}\phi_2=-\phi_2\,,\quad P_{12}\phi_2=\tfrac{1}{2}(\phi_2+3^{1/2}\phi_1)\,,\quad P_{13}\phi_2=\tfrac{1}{2}(\phi_2-3^{1/2}\phi_1)\,. \end{aligned}\tag{1}$$

The combinations of mixed symmetry quantities

$$\begin{aligned} \phi_s&=\chi_2\eta_2+\chi_1\eta_1\\ \phi_a&=\chi_2\eta_1-\chi_1\eta_2\\ \phi_1&=\chi_2\eta_2-\chi_1\eta_1\\ \phi_2&=\chi_2\eta_1+\chi_1\eta_2 \end{aligned}\tag{2}$$

transform as indicated above.

Two vectors which satisfy Eq. (1) are

$$\mathbf{R}_1=(\tfrac{4}{3})^{1/2}\boldsymbol{\rho}\,,\quad \mathbf{R}_2=-\mathbf{r}\,;\tag{3}$$

where $\boldsymbol{\rho}$ and $\mathbf{r}$ are the internal space coordinates of the 3-nucleon system defined as in Appendix A of Ref. 3:

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[1] H. Collard and R. Hofstadter, Phys. Rev. **131**, 416 (1963).
[2] B. F. Gibson and L. I. Schiff, Phys. Rev. **138**, B26 (1965).
[3] L. I. Schiff, Phys. Rev. **133**, B802 (1964).
[4] G. B. West, Phys. Rev. **139**, B1246 (1965).
[5] T. A. Griffy, Phys. Letters **11**, 155 (1964).
[6] K. Okamoto, Phys. Letters **11**, 150 (1964).

$\mathbf{r}$ is the vector from nucleon 2 to nucleon 3 and $\boldsymbol{\rho}$ is the vector from the midpoint of $\mathbf{r}$ to nucleon 1. It is easily seen from Eqs. (2) that

$$S_s=R_2^2+R_1^2,\quad S_1=R_2^2-R_1^2,\quad S_2=2\mathbf{R}_2\cdot\mathbf{R}_1 \tag{4}$$

have the appropriate symmetry properties, and that the corresponding S_a is identically zero. Space functions with mixed symmetry may also be defined as in Eq. (6) of Ref. 2. If $g(12,3)$ is a scalar function of the internal coordinates $\mathbf{R}_1,\mathbf{R}_2$ that is symmetric under interchange of nucleons 1 and 2 but neither symmetric nor antisymmetric under interchange of 3 with 1 or 2, then

$$\begin{aligned}V_1&=6^{-1/2}[g(12,3)+g(13,2)-2g(23,1)] \\ V_2&=2^{-1/2}[g(12,3)-g(13,2)]\end{aligned} \tag{5}$$

transform according to Eq. (1).

The doublet spin states are

$$\begin{aligned}\chi_1&=6^{-1/2}[(++-)+(+-+)-2(-++)] \\ \chi_2&=2^{-1/2}[(++-)-(+-+)],\end{aligned} \tag{6}$$

where a $+$ (or$-$) means that the nucleon corresponding to that position in the parenthesis has spin up (or down). Both χ_1 and χ_2 have a total spin component of $+1/2$ in the "up" direction; the conjugate functions, which interchange $+$ and $-$ in the arguments, have total spin components of $-1/2$. The doublet isospin functions η_1 and η_2 have the same form as Eq. (6), where a $+$ (or $-$) means that the nucleon is a proton (or neutron). The η's describe He^3, while their conjugate functions describe H^3.

III. WAVE FUNCTIONS

A concise derivation of the P- and D-state functions to be considered may be found in Ref. 2. Using

$$\boldsymbol{\sigma}_1,\ \boldsymbol{\sigma}_2,\ \boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_{23}, \tag{7}$$

where 1, 2, and 3 refer to the nucleons, $\boldsymbol{\sigma}_{23}=\boldsymbol{\sigma}_2-\boldsymbol{\sigma}_3$, and the components of $\boldsymbol{\sigma}_i$ are the three Pauli spin matrices (with unit elements) which act on the ith nucleon, the P-state spin functions $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ may be expressed in terms of χ_2:

$$\boldsymbol{\pi}_1=(12)^{-1/2}[\boldsymbol{\sigma}_{23}+i\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_{23}]\chi_2,\quad \boldsymbol{\pi}_2=\boldsymbol{\sigma}_1\chi_2. \tag{8}$$

The D-state functions may also be generated from χ_2:

$$\begin{aligned}D_s&=[(\boldsymbol{\sigma}_1\cdot\mathbf{R}_2)(\boldsymbol{\sigma}_{23}\cdot\mathbf{R}_2)+(\boldsymbol{\sigma}_1\cdot\mathbf{R}_1)(\boldsymbol{\sigma}_{23}\cdot\mathbf{R}_1) \\ &\qquad-\tfrac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_{23})(R_2^2+R_1^2)]\chi_2, \\ D_1&=[(\boldsymbol{\sigma}_1\cdot\mathbf{R}_2)(\boldsymbol{\sigma}_{23}\cdot\mathbf{R}_2)-(\boldsymbol{\sigma}_1\cdot\mathbf{R}_1)(\boldsymbol{\sigma}_{23}\cdot\mathbf{R}_1) \\ &\qquad-\tfrac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_{23})(R_2^2-R_1^2)]\chi_2, \\ D_2&=[(\boldsymbol{\sigma}_1\cdot\mathbf{R}_2)(\boldsymbol{\sigma}_{23}\cdot\mathbf{R}_1)+(\boldsymbol{\sigma}_1\cdot\mathbf{R}_1)(\boldsymbol{\sigma}_{23}\cdot\mathbf{R}_2) \\ &\qquad-\tfrac{2}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_{23})(\mathbf{R}_2\cdot\mathbf{R}_1)]\chi_2.\end{aligned} \tag{9}$$

The P states to be considered are then

$$\begin{aligned}\psi_3&=(\boldsymbol{\pi}_2\eta_2+\boldsymbol{\pi}_1\eta_1)\cdot(\mathbf{R}_1\times\mathbf{R}_2)f_3, \\ \psi_4&=[(\boldsymbol{\pi}_2S_1+\boldsymbol{\pi}_1S_2)\eta_2+(\boldsymbol{\pi}_2S_2-\boldsymbol{\pi}_1S_1)\eta_1] \\ &\qquad\cdot(\mathbf{R}_1\times\mathbf{R}_2)f_4,\end{aligned} \tag{10}$$

and the D states are

$$\begin{aligned}\psi_6&=[(5D_sS_2-2D_2S_s)\eta_1-(5D_sS_1-2D_1S_s)\eta_2]f_6, \\ \psi_7&=[D_2S_s\eta_1-D_1S_s\eta_2]f_7, \\ \psi_8&=[(D_2S_1+D_1S_2)\eta_1-(D_2S_2-D_1S_1)\eta_2]f_8,\end{aligned} \tag{11}$$

where the argument of the real functions f_i is $S_s=R_1^2+R_2^2=R^2$, and the state numbering of Sachs,[7] as modified by Gibson and Schiff,[2] is used. The normalization integrals for the D states are

$$\begin{aligned}\int\psi_6^*\psi_6d^3r_i&=\left(3^{1/2}\frac{35\pi^3}{16}\right)\int_0^\infty f_6^2R^{13}dR, \\ \int\psi_7^*\psi_7d^3r_i&=\left(3^{1/2}\frac{5\pi^3}{8}\right)\int_0^\infty f_7^2R^{13}dR, \\ \int\psi_8^*\psi_8d^3r_i&=\left(3^{1/2}\frac{7\pi^3}{24}\right)\int_0^\infty f_8^2R^{13}dR.\end{aligned} \tag{12}$$

The dominant S state has the form

$$\psi_1=(\chi_2\eta_1-\chi_1\eta_2)f_1 \tag{13}$$

with the normalization

$$\int\psi_1^*\psi_1d^3r_i=\left(\frac{3^{3/2}\pi^3}{4}\right)\int_0^\infty f_1^2R^5dR. \tag{14}$$

The total wave function may be written as in Eq. (22) of Ref. 2:

$$\psi=\sum\psi_i=u_2\eta_1-u_1\eta_2, \tag{15}$$

where the summation is over states 1,3,4,6,7,8 and the u's are space-spin functions which transform according to Eq. (1).

The functions $f_i(S_s)$ considered are of the Irving-Gunn type $e^{-\alpha R/2}/R^{n/2}$. The corresponding $g(12,3)$ functions have the exponential argument shown in Eq. (25) of Ref. 3.

IV. GENERAL STRUCTURE OF THE MAGNETIC FORM FACTOR

The moment-density operator in the impulse approximation is taken to be

$$\begin{aligned}\rho_M(\mathbf{r},\mathbf{r}_i)=\sum_{i=1}^{3}&[\tfrac{1}{2}(1+\tau_{iz})\sigma_{iz}\mu_pf_{\text{mag}}{}^p(\mathbf{r}-\mathbf{r}_i) \\ &+\tfrac{1}{2}(1-\tau_{iz})\sigma_{iz}\mu_nf_{\text{mag}}{}^n(\mathbf{r}-\mathbf{r}_i)],\end{aligned} \tag{16}$$

where it is assumed that the nucleons contribute without interference or distortion. The τ's are isospin matrices and the μ's are the static moments of the

[7] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1953), pp. 180–187; a different classification of states has been given by L. Cohen and J. B. Willis, Nucl. Phys. **32**, 114 (1962).

proton and neutron. The quantities $f_{\rm mag}^n$ and $f_{\rm mag}^n$ are the nucleon spatial distribution functions of the moment densities about the centers of the nucleons.

In the impulse approximation there is no orbital-angular-momentum term in the moment-density operator. The interacting nucleons are treated as free particles during the scattering process. As free particles they make no orbital-angular-momentum contribution to ρ_m; it contains only spin terms corresponding to the free nucleons.[4] However, the presence of an orbital-angular-momentum term in the $q^2=0$ limit, where internal binding is not ignored, has been noted by several authors (for example, see Ref. 14). The contribution of such a term to the static magnetic moments of H^3 and He^3 is less than 2% for a D-state probability of 6% (D^2 terms are the only ones considered in this paper which would give a nonzero expectation value). This is about one-third the D^2 contribution of the spin terms contained in Eq. (16), because of the size of the anomalous moments of the proton and neutron. Since the D^2 contributions to the magnetic form factors for this ρ_m are found to be of the order of the experimental errors involved, this impulse approximation to the moment-density operator is considered a valid one in the region where electron-scattering data are available, $q^2\geq 1 f^{-2}$.

The expectation value of ρ_m defined in Eq. (16) is to be taken with respect to initial and final states which have the same spin because the z axis is chosen as the spin-quantization axis. If the x axis were chosen as the quantization axis, a "spin flip" between initial and final states would occur. With the choice (16) for ρ_m, the momentum transfer $\mathbf{q}$ is restricted to lie in the plane normal to the spin direction determined by σ_{iz}, i.e., in the x-y plane. If ρ_m were defined in terms of σ_{ix} (which would correspond to a spin flip between initial and final states in the scattering process), then $\mathbf{q}$ would be restricted to the y-z plane.

The Fourier transform of the expectation value of the moment-density operator is then

$$\int\int \exp(i\mathbf{q}\cdot\mathbf{r})\psi^*\rho_M(\mathbf{r},\mathbf{r}_i)\psi d^3r d^3r_i. \tag{17}$$

The integration over $\mathbf{r}$ is performed by changing variables from $\mathbf{r}$ to $\mathbf{r}-\mathbf{r}_i$, which causes the nucleon form factors $F_{\rm mag}^p$ and $F_{\rm mag}^n$ to appear as multiplying factors. The isospin sums may be carried out and the resulting expression reduced by means of the permutation table in Eq. (1) to the form

$$\begin{aligned}\mu_p F_{\rm mag}^p \int & \exp(i\mathbf{q}\cdot\mathbf{r}_1) \\ &\times[\langle u_2|\sigma_{1z}|u_2\rangle+3\langle u_1|\sigma_{1z}|u_1\rangle]d^3r_i \\ &+\mu_n F_{\rm mag}^n\int \exp(i\mathbf{q}\cdot\mathbf{r}_1)[2\langle u_2|\sigma_{1z}|u_2\rangle]d^3r_i \\ &=\mu_p F_{\rm mag}^p(3F_{am}+F_{bm})+2\mu_n F_{\rm mag}^n F_{bm},\end{aligned} \tag{18}$$

where

$$\begin{aligned}F_{am}&=\int \exp(i\mathbf{q}\cdot\mathbf{r}_1)\langle u_1|\sigma_{1z}|u_1\rangle d^3r_i, \\ F_{bm}&=\int \exp(i\mathbf{q}\cdot\mathbf{r}_1)\langle u_2|\sigma_{1z}|u_2\rangle d^3r_i,\end{aligned} \tag{19}$$

and $\langle u_k|\sigma_{1z}|u_k\rangle$ indicates that a spin sum must still be performed. For the purpose of this paper it is more useful to express the form factor as

$$\mu_n F_{\rm mag}^n F_{1m}+(\mu_p F_{\rm mag}^p+\mu_n F_{\rm mag}^n)F_{2m}, \tag{20}$$

where F_{1m} and F_{2m} are linear combinations of F_{am} and F_{bm}

$$F_{1m}=F_{bm}-3F_{am},\quad F_{2m}=F_{bm}+3F_{am}. \tag{21}$$

These are the interesting combinations since their effects are easily compared to those of the S^2 term and the SS' cross term of Ref. 3.

Since $F_{\rm mag}^p$ and $F_{\rm mag}^n$ are normalized to unity, Eq. (18) does not reduce to the correct static moment $\mu(He^3)$. This is to be expected since, as noted above, the impulse approximation ignores binding and is not valid for small q^2. The difference can be ascribed to empirically determined isovector and isoscalar exchange moments. Similar isovector and isoscalar terms are required in order to fit the form-factor data for $q^2>0$. When the exchange terms are included, the complete expression for the magnetic form factor of He^3 is

$$\begin{aligned}\mu(He^3)F_{\rm mag}(He^3)=\mu_n F_{\rm mag}^n F_{1m} \\ +(\mu_p F_{\rm mag}^p+\mu_n F_{\rm mag}^n)F_{2m}+F_{xs}-F_{xv},\end{aligned} \tag{22}$$

where F_{xs} and F_{xv} are unnormalized isoscalar and isovector magnetic-exchange form factors. The corresponding expression for H^3 is obtained by replacing He^3 by H^3, interchanging p and n, and changing the sign of F_{xv}

$$\begin{aligned}\mu(H^3)F_{\rm mag}(H^3)=\mu_p F_{\rm mag}^p F_{1m} \\ +(\mu_n F_{\rm mag}^n+\mu_p F_{\rm mag}^p)F_{2m}+F_{xs}+F_{xv}.\end{aligned} \tag{23}$$

The magnetic form factors may also be written in terms of the body form factors for the odd and like nucleons

$$\begin{aligned}\mu(He^3)F_{\rm mag}(He^3)&=\mu_n F_{\rm mag}^n F_{Om} \\ &\quad+2\mu_p F_{\rm mag}^p F_{Lm}+F_{xs}-F_{xv}, \\ \mu(H^3)F_{\rm mag}(H^3)&=\mu_p F_{\rm mag}^p F_{Om} \\ &\quad+2\mu_n F_{\rm mag}^n F_{Lm}+F_{xs}+F_{xv},\end{aligned} \tag{24}$$

where the odd-nucleon magnetic-body form factor is

$$F_{Om}=F_{1m}+F_{2m}=2F_{bm} \tag{25}$$

and the like nucleon body form factor is

$$F_{Lm}=\tfrac{1}{2}F_{2m}=\tfrac{1}{2}(3F_{am}+F_{bm}). \tag{26}$$

V. P-STATE CONTRIBUTION TO THE MAGNETIC FORM FACTORS

As indicated above, only the interference of the $^2P_{\frac{1}{2}}$ states ψ_3 and ψ_4 with ψ_1 should contribute significantly to the form factor. An analysis of F_a and F_b quite similar to that in Sec. VI of Ref. 2 shows that: the contribution of ψ_3 to the integrands vanishes identically; the contributions of ψ_4 to each term is proportional to

$$(\mathbf{R}_1\times\mathbf{R}_2)_Z S_2 f_1 f_4, \qquad (27)$$

so that the integration over the direction of $\mathbf{R}_2$ causes both F_a and F_b to vanish. Thus the ${}^2P_{\frac{1}{2}}$ states ψ_3 and ψ_4 do not contribute to the magnetic form factors through interference with the dominant S state. The same result holds if S_1 and S_2 are replaced by the more general V_1 and V_2.

VI. D-STATE CONTRIBUTIONS TO THE MAGNETIC FORM FACTORS

The largest D-state contribution to the magnetic form factors should be through interference with ψ_1. Since the spin product

$$\langle\chi_2|\sigma_{1z}\sigma_{1i}\sigma_{23j}|\chi_2\rangle\equiv 0;\quad i,j=x,y,z \qquad (28)$$

only F_a is nonzero. This implies that only the odd nucleon contributes through this type of term. The momentum transfer $\mathbf{q}$ is taken to define the x direction, and the integrations are then carried out as in Ref. 2. The resulting expression is

$$F_a(S,D)=\frac{32}{9}\times 6\pi^3\int_0^\infty \frac{R^9dR}{z^2}\times[2f_6f_1J_6(z)-f_7f_1J_4(z)+\tfrac{4}{5}f_8f_1J_6(z)], \qquad (29)$$

where $z=3^{-\frac{1}{2}}qR$.

The D^2 term is expected to give the next largest contribution to the magnetic form factors. The integrations and reductions involved are similar to those of the SD term and the D^2 term of the charge form factor. The results are

$$F_a=-\frac{\pi^3\sqrt{3}}{24}\int_0^\infty \frac{R^{13}dR}{z^2}[f_6^2(70J_2-65\tfrac{3}{5}J_4+108(9/35)J_6-223(17/21)J_8+208\tfrac{1}{3}J_{10})+f_7^2(20J_2-22\tfrac{2}{5}J_4+21\tfrac{3}{5}J_6)+f_8^2(9\tfrac{1}{3}J_2-11(11/15)J_4+45(33/35)J_6-5(13/21)J_8+21\tfrac{2}{3}J_{10})+f_6f_7(-11\tfrac{1}{5}J_4+152(8/35)J_6-28(4/7)J_8)+f_6f_8(-40\tfrac{2}{5}J_4+68(6/7)J_6-19(3/7)J_8+17\tfrac{1}{3}J_{10})+f_7f_8(-6(2/15)J_4+12(12/35)J_6-27(11/21)J_8)], \qquad (30)$$

$$F_b=-\frac{\pi^3\sqrt{3}}{24}\int_0^\infty \frac{R^{13}dR}{z^2}[f_6^2(70J_2-65\tfrac{3}{5}J_4-67(26/35)J_6+23(16/21)J_8-41\tfrac{2}{3}J_{10})+f_7^2(20J_2-22\tfrac{2}{5}J_4-42\tfrac{2}{5}J_6)+f_8^2(9\tfrac{1}{3}J_2-11(11/15)J_4+8(16/35)J_6-4(5/9)J_8-26\tfrac{1}{3}J_{10})+f_6f_7(-11\tfrac{1}{5}J_4+5(17/35)J_6+5(5/7)J_8)+f_6f_8(22J_4-21(11/35)J_6+15\tfrac{1}{3}J_8+6\tfrac{2}{3}J_{10})+f_7f_8(26(2/15)J_4-12(12/35)J_6+16(10/21)J_8)]. \qquad (31)$$

From these expressions, one can verify that the inclusion of D states in the three-nucleon ground-state wave function does decrease the static expectation value of the moment-density operator, but only because of the reduction in the allowed probability of the fully space-symmetric S state.

VII. NUMERICAL RESULTS

As stated in Sec. III, functions $f_i(S_i)$ of the form $e^{-\alpha R/2}/R^{n/2}$ were considered. The qualitative features of the calculations were insensitive to the choice of $n=0,1,2$. For this reason the numerical results[8] presented here are restricted to the case $n=0$ (Irving function). The integrals were done analytically, but the resulting expressions were not very enlightening, so that only the numerical evaluations are quoted.

First a re-analysis of the most recent form factor data[9] for He^3 and H^3 was carried out. A reasonable fit is obtained if the S' or mixed symmetry S state is included in the analysis along with a typical $T=3/2$ state, which may occur in the He^3 wave function. It is assumed that the percentages of states are: $P_S=92\%$, $P_{S'}=2\%$, $P_D=6\%$, $P_{T=3/2}=0.25\%$. The 2% S'-state probability is based on the variational calculation of Blatt and Delves,[10] the electron-scattering calculation of Griffy and Oakes,[11] and the slow neutron-deuteron capture calculation of Meister, Radha, and Schiff.[12] The 6% D-state probability was chosen as the lower limit of the variational results for $P_{S'}=2\%$. The 0.25% probability

[8] Computer time was supported by National Science Foundation Grant No. NSF-GP948.

[9] H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearean, R. B. Day, and R. T. Wagner, Phys. Rev. **138**, B57 (1965).

[10] J. M. Blatt and L. M. Delves, Phys. Rev. Letters **12**, 544 (1964); estimates of S' and D states by B. S. Bhakar and A. N. Mitra [Phys. Rev. Letters **14**, 143 (1965)] are somewhat smaller.

[11] T. A. Griffy and R. J. Oakes, Phys. Rev. **135**, B1161 (1964).

[12] N. T. Meister, T. K. Radha, and L. I. Schiff, Phys. Rev. Letters **12**, 509 (1964); see also Ref. 19.

for the $T=3/2$ state is an upper-limit estimate by Okamoto.[13]

The estimate of an exchange moment[14] of 0.27 nm[14] is based on the assumption that there is only an isovector contribution, in which case the exchange moments of H^3 and He^3 are equal and opposite. For such a case the sum of the moments gives an estimate of $P_D \approx 3.8\%$ if only S and D states are assumed. At the time this calculation was made, the energy variational calculation[15] on H^3 had estimated $P_D \approx 4\%$, which would appear to substantiate the result. However, at present, variational calculations[10] which give $P_{S'}$ of the order of 2% (actually 1.6–2.7%) also give a P_D of 7.5 down to 5.6%. Hence if $P_{S'}$ and P_D are taken to be 2 and 6%, respectively, one can still produce the correct $\mu(He^3)$ and $\mu(H^3)$ by assuming both an isovector and an isoscalar contribution to the moments.

With the above combination of states, it is possible to fit the charge data without the inclusion of additional unknown charge-exchange terms. The charge-form-factor formulas may be summarized as follows:

$$2F_{ch}(He^3) = (2F_{ch}{}^p + F_{ch}{}^n)F_{1c} + (F_{ch}{}^n - F_{ch}{}^p)(F_{2c} + F_{3c}), \quad (32)$$

$$F_{ch}(H^3) = (F_{ch}{}^p + 2F_{ch}{}^n)F_{1c} + (F_{ch}{}^p - F_{ch}{}^n)F_{2c},$$

where

$$F_{1c} = F(S,S) + F_{1c}(D,D),$$
$$F_{2c} = F(S,S') + F_{2c}(D,D), \quad (33)$$
$$F_{3c} = (P_{T=3/2}/P_{S'})^{1/2} F(S,S').$$

Note that $F(S,S')$ is $\frac{2}{3}$ of F_2 as defined in Ref. 3 and that F_{3c} is the contribution of the $T=\frac{3}{2}$ state.

From Ref. 3, $F(S,S)$ has the form

$$(1+2q^2/9\alpha^2)^{-7/2},$$

so that under the assumption that $F_{ch}{}^n$ is zero for $q^2 \geq 1$ F^{-2},[16] a graph of $F_1(q)^{-2/7}$ versus q^2 (Fig. 1) gives $\alpha_S = 1.34$ F^{-1} where the plot has been corrected for the 92% probability of the dominant S state. This is a shift in the right direction over α(C.E.) $= 1.27$ F^{-1} determined by the bare-nucleon Coulomb-energy expression, since finite-size effects tend to reduce the bare-nucleon value of the Coulomb energy.[6]

Using this value of α_S, the curve $[F_{ch}(H^3) - F_{ch}(He^3)] \times [F_{ch}{}^p]^{-1}$ which eliminates the contribution of $F(S,S)$ and is therefore more sensitive to the D states, can be fitted to determine α_D. Under the assumption that ψ_6 ψ_7, ψ_8 have equal probabilities, α_D is found to be approximately $\sqrt{2}\alpha_S$. The largest D-state contribution comes

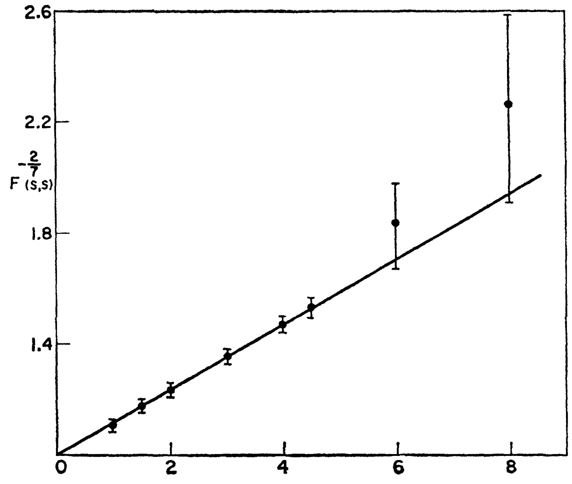

FIG. 1. Straight-line plot of $F(S,S)$ versus $q^2(f^{-2})$. The data have been corrected for $P_S=0.92$ and D-state effects. The size parameter α_S is 1.34 F^{-1}.

from the interference of ψ_7 and ψ_8, and the above parameter α_D is essentially unchanged if ψ_6 is absent from the ground state.

The magnetic-form-factor expressions, complete with S' and $T=\frac{3}{2}$ terms, are[2,3,5]

$$\mu(He^3)F_{mag}(He^3) = \mu_n F_{mag}{}^n F_{1m} + (\mu_p F_{mag}{}^p + \mu_n F_{mag}{}^n)F_{2m} + (\mu_p F_{mag}{}^p - \mu_n F_{mag}{}^n)F_{3m} + F_{xs} - F_{xv}, \quad (34)$$

$$\mu(H^3)F_{mag}(H^3) = \mu_p F_{mag}{}^p F_{1m} + (\mu_p F_{mag}{}^p + \mu_n F_{mag}{}^n)F_{2m} + F_{xs} + F_{xv},$$

where

$$F_{1m} = F(S,S) + F_1(S,D) + F_{1m}(D,D),$$
$$F_{2m} = F(S,S') + F_2(S,D) + F_{2m}(D,D),$$
$$F_{3m} = F_{3c}.$$

The D^2 contributions are almost negligible, being of the same order of magnitude as the experimental errors, or smaller. The SD interference term is smaller than expected because of the sign differences in the three terms. If ψ_6 were absent, which would affect the charge results only to a slight degree, the SD interference effects could be doubled; however, this would still not reduce the calculated F_{xs} to zero. The largest contribution after the S' term is from the $T=\frac{3}{2}$ state, since its amplitude and not the probability enters the calculation and since it contributes like the sum of the absolute values of the moments and not their difference.

The isovector and isoscalar form factors which are required to fit the experimental data are given in Table I. They are similar in shape to those found by Levinger and Srivastava,[17] as they should be, since the

[13] K. Okamoto, Phys. Letters (to be published); C. Werntz and H. S. Valk [Phys. Rev. Letters 14, 910 (1965)] give a smaller value but seem to overestimate the energy of the $T=\frac{3}{2}$ level.

[14] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1953), pp. 245–252.

[15] E. Gerjuoy and J. Schwinger, Phys. Rev. 61, 138 (1942).

[16] R. Hofstadter, in *Science and Humanity* (Yearbook of the "Znanie" Publishing House, Moscow, to be published).

[17] J. S. Levinger and B. K. Srivastava, Phys. Rev. 137, B426 (1965).

TABLE I. Form factors.

q^2 (F^{-2})	F_{xv}	F_{xs}	Error[a]
1.0	0.30	0.016	0.097
1.5	0.17	0.021	0.061
2.0	0.09	0.019	0.043
2.5	0.04	0.039	0.028
3.0	−0.010	0.046	0.020
3.5	−0.009	0.042	0.015
4.0	−0.008	0.038	0.013
4.5	0.010	0.013	0.008
5.0	0.008	0.004	0.013
6.0	−0.012	0.008	0.009
8.0	−0.005	−0.003	0.004

[a] The statistical error given includes only errors in measurements of the magnetic moment form factors of H^3 and He^3.

major S and S' terms are included in that work. However they are smaller and fall off more rapidly. The isovector form factor is essentially zero for $q^2 > 2.5F^{-2}$. The isoscalar form factor is zero within the experimental error except the region $2.5 \leq q^2 \leq 4.5$. It was hoped that the inclusion of D and $T=\frac{3}{2}$ state effects would reduce F_{xs} to zero. But the D states do not aid in this; F_{xs} is closer to zero if P_D is reduced and P_S increased.

Thus it is possible to fit the charge-form-factor data with a wave function composed of a completely symmetric S state and a small admixture of S', D, and $T=\frac{3}{2}$ states, without including additional parameters in the form of isoscalar and isovector charge-exchange form factors.[18] The choice of $P_{S'}$, P_D, and $P_{T=3/2}$ for a good fit to the data depends, of course, on the assumption that $F_{ch}^n(q^2)=0$ for $q^2 \geq 1F^{-2}$. For this F_{ch}^n, a significant reduction in $P_{S'}$ must be accompanied by an increase in $P_{T=3/2}$, or the fit to the charge data in Fig. 2 is destroyed. If $P_{S'}^{1/2}+\frac{1}{3}P_{T=3/2}^{1/2}$ is kept constant, the charge results are unchanged.

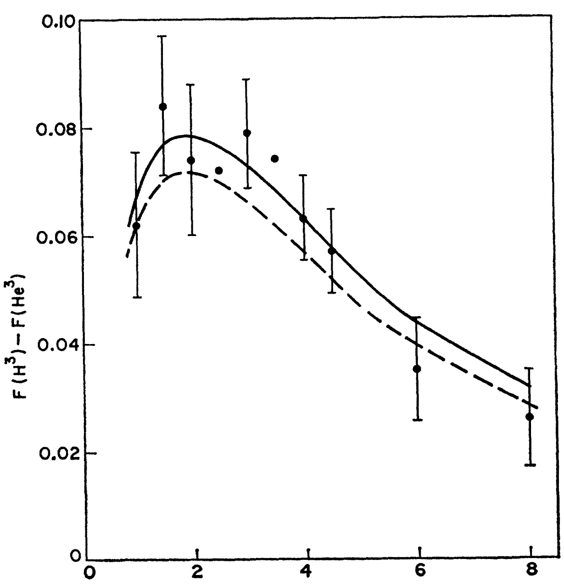

FIG. 2. Plot of $[F_{ch}(H^3)-F_{ch}(He^3)][F_{ch}^p]^{-1}$ versus $q^2(F^{-2})$. The solid curve includes S', D, and $T=\frac{3}{2}$ state effects. The dashed curve excludes the $T=\frac{3}{2}$ state contribution.

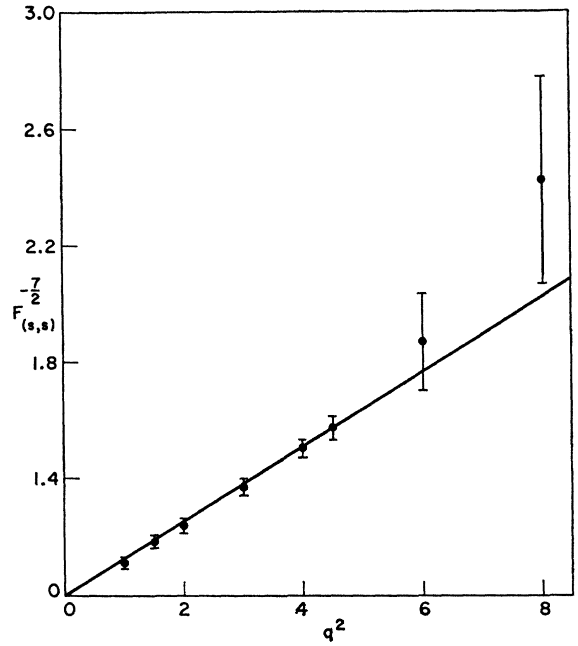

FIG. 3. Straight-line plot of $F(S,S)$ versus $q^2(F^{-2})$ for $F_{ch}^n>0$.

In order to fit the magnetic data with this same wave function, empirical exchange terms are required. This is not surprising in view of the long-known values of the static magnetic moments. If $P_{S'}$ and $P_{T=3/2}$ are varied as indicated above, F_{xv} and F_{xs} become smaller if $P_{T=3/2}$ is increased.

The 2% choice for $P_{S'}$ is considered an upper limit. The slow neutron capture reaction[19] $H^2(n,\gamma)H^3$ and its inverse[20] $H^3(\gamma,n)H^2$, as well as the inelastic electron scattering,[11] are more compatable with $P_{S'} \leq 1\%$. The

TABLE II. Form factors.

q^2 (F^{-2})	F_{ch}^n	F_{xv}^a	F_{xs}^a
1.0	0.02	0.29	0.042
1.5	0.03	0.17	0.054
2.0	0.04	0.09	0.050
2.5	0.05	0.07	0.068
3.0	0.06	−0.005	0.073
3.5	0.07	0.007	0.066
4.0	0.08	0.004	0.060
4.5	0.09	0.020	0.036
5.0	0.10	0.016	0.022
6.0	0.11	−0.004	0.023
8.0	0.11	−0.001	0.007

[a] The errors are the same as quoted in Table I.

[18] A. Q. Sarker, Phys. Rev. Letters 13, 375 (1964); Nuovo Cimento 36, 392 (1965).

[19] T. K. Radha and N. T. Meister, Phys. Rev. 136, B388 (1964); Phys. Rev. 138, AB7 (E) (1965).

[20] R. Bösch *et al.*, Phys. Letters 15, 243 (1965).

β decay process[21] is in serious disagreement with any admixture of states except $T=\frac{3}{2}$. But $P_{T=3/2}=0.25\%$ is also an upper limit. Reducing both $P_{S'}$ and $P_{T=3/2}$ can be accomplished without adding charge-exchange terms if $F_{ch}{}^n$ is assumed to be positive. Then $P_{S'}$ and $P_{T=3/2}$ can be made to take most any combination of values (consistent with the above upper limits) by the proper choice of $F_{ch}{}^n$. Consider $F_{ch}{}^n$ to have the form given in Table II (see Ref. 17); this is in some disagreement with inelastic electron scattering from H^2. A plot of $F_1(q^2)^{-2/7}$ versus q^2 (Fig. 3) then gives $\alpha_S=1.32$ F^{-1}. In this case the straight line does not pass within the error brackets at $q^2=8$ F^{-2}; this perhaps is because of the smaller than expected value of $F_{ch}(H^3)$ at this point. Using this α_S, the curve

$$X(q^2)=\frac{(B-A)+\Delta(\frac{1}{2}B-2A)}{1-\Delta^2},$$
$$A=\frac{F_{ch}(He^3)}{F_{ch}{}^p},\quad B=\frac{F_{ch}(H^3)}{F_{ch}{}^p},\quad \Delta=\frac{F_{ch}{}^n}{F_{ch}{}^p}, \tag{36}$$

which eliminates $F(S,S)$, is fitted (Fig. 4) for $\alpha_D=\sqrt{2}\alpha_S$ and a choice of $P_{S'}=0.6\%$, $P_{T=3/2}=0.1\%$; P_D has been kept at 6%. The isoscalar and isovector exchange form factors required to fit the magnetic data with this wave function are given in Table II. As remarked above they are larger than in the case in which $F_{ch}{}^n$ is assumed to be zero. If $P_{T=3/2}$ is taken to be zero, the charge analysis is essentially the same when $P_{S'}=0.65\%$. Both F_{xs} and F_{xv} are increased slightly (<10%) over their values in Table II. Again a comparison with the exchange terms of Levinger and Srivastava can be made only with regard to the general shape because of the different method used to account for the D-state effects, which changes the definitions of F_{xv} and F_{xs}.

In general, the effect of the D states on the form factors is seen to be small. For the present experimental errors involved, they can essentially be neglected in the magnetic-form-factor calculations. In the charge-form factors, the D states are important for low q^2 as can be seen in Fig. 4. This is as expected since the extended spatial distribution of the D states should be reflected in the form factor at low q^2.

[21] R. Blin-Stoyle, Phys. Rev. Letters 13, 55 (1964).

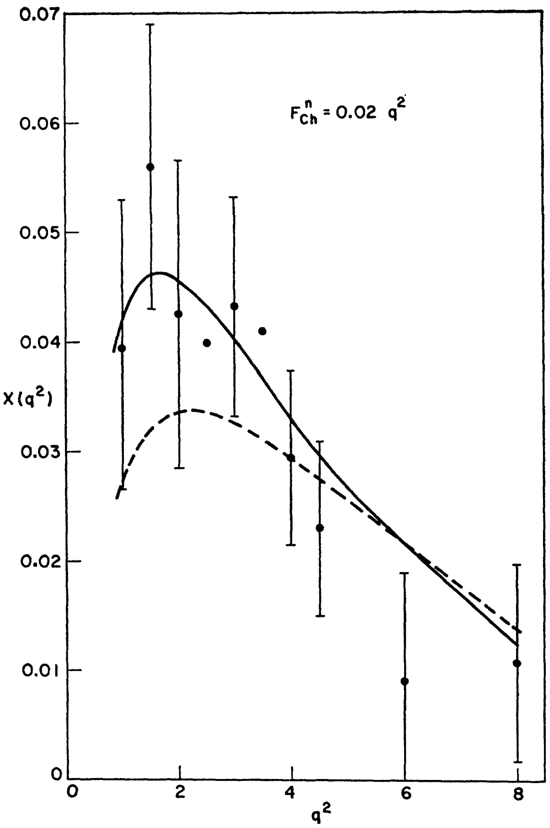

FIG. 4. Plot of $X(q^2)$ versus $q^2(F^{-2})$. The solid curve includes S',D, and $T=\frac{3}{2}$ state effects. The dashed curve includes only S' and $T=\frac{3}{2}$ states. Points were computed using $F_{ch}{}^n(q^2)$ given in Table II.

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